The Final HiggsTools Meeting, IPPP, Durham

13 September 2017

Higgs Boson Pair Production: Monte-Carlo Generator Interface & Parton Shower

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Borowka, Greiner, Heinrich, Kerner, Luisoni, Schlenk, Schubert, Vryonidou, Zirke

MAX-PLANCK-GESELLSCHAFT

JHEP 08 (2017) 088 [1703.09252]
JHEP 10 (2016) 107 [1608.04798]
PRL 117 (2016) 012001, Erratum 079901 [1604.06447]

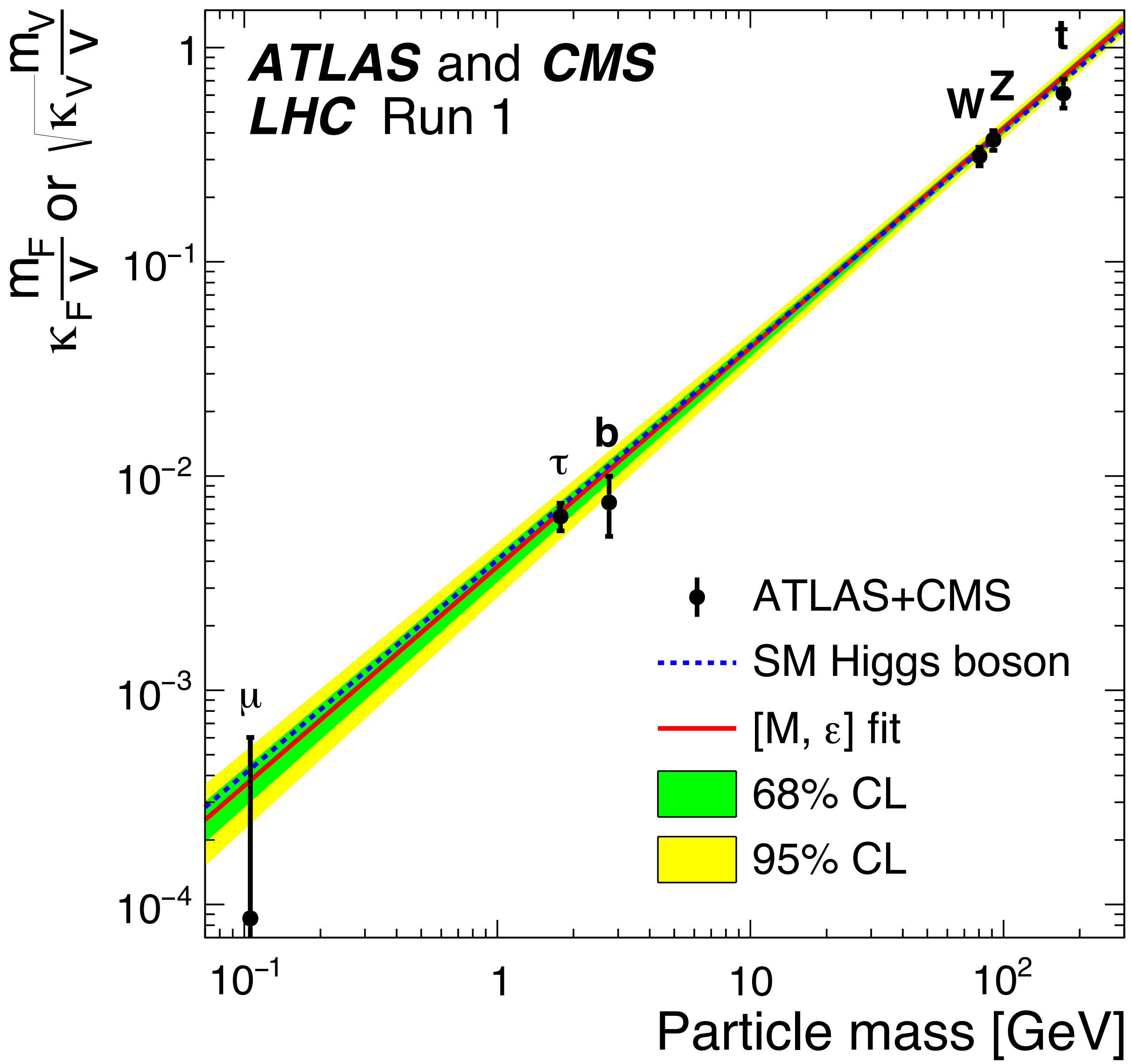
higgstools

Motivation

Higgs mass is known to good precision:

$$m_H = 125.09 \pm 0.21(\text{stat.}) \pm 0.11(\text{syst.})\ \text{GeV}$$

ATLAS, CMS 15



So far, measured couplings agree with the Standard Model

But: Higgs self coupling not yet well measured

Motivation (II)

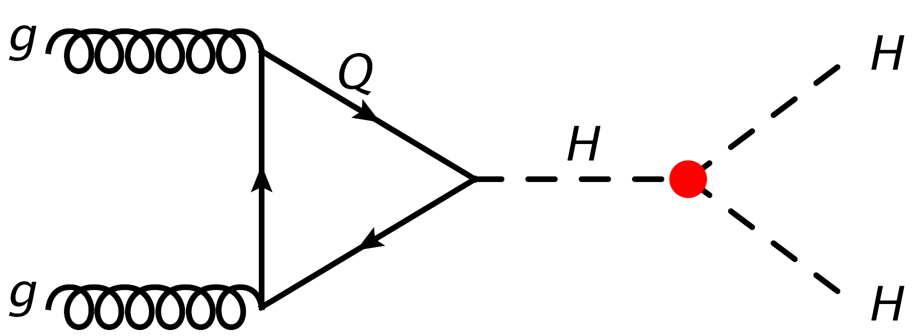
SM Higgs Lagrangian:

$$\mathcal{L} \supset -V(\Phi), \quad V(\Phi) = \frac{1}{2}\mu^2\Phi^2 + \frac{1}{4}\lambda\Phi^4$$

EW symmetry breaking

$$\frac{m_H^2}{2}H^2 + \boxed{\frac{m_H^2}{2v}H^3} + \frac{m_H^2}{8v^2}H^4$$

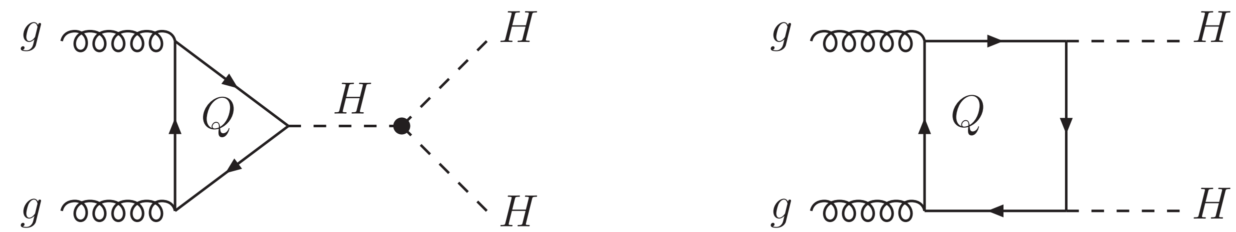
Higgs pair production probes triple-Higgs coupling



Production Channels

$\sigma(pp \to HH + X)$ @ 14 TeV

Gluon Fusion

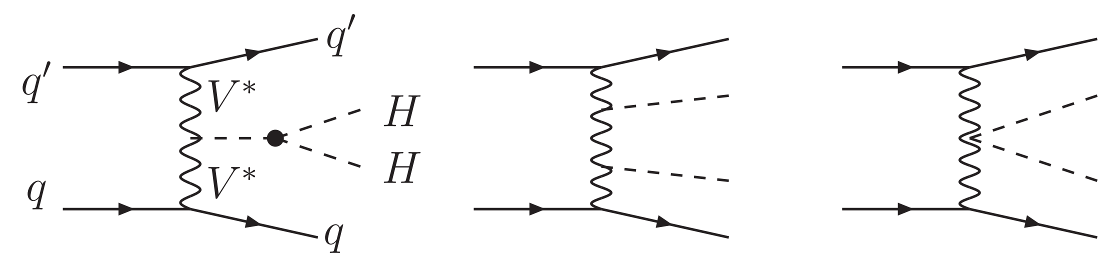


Vector Boson Fusion (VBF)

NLO [1,2] NNLO [3]

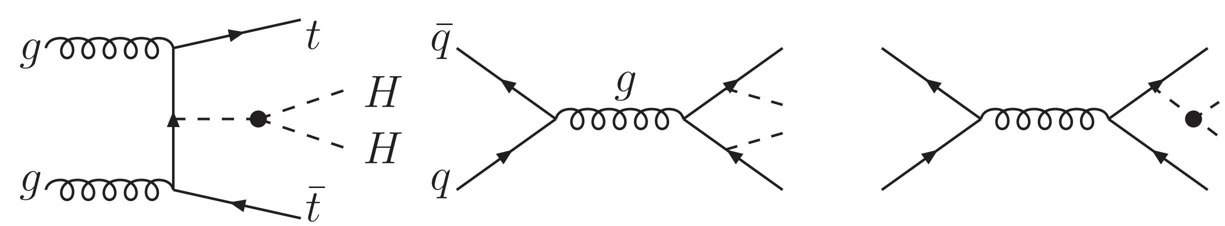
+ non-negligible contribution

from $gg \to HHjj$ LO [5]



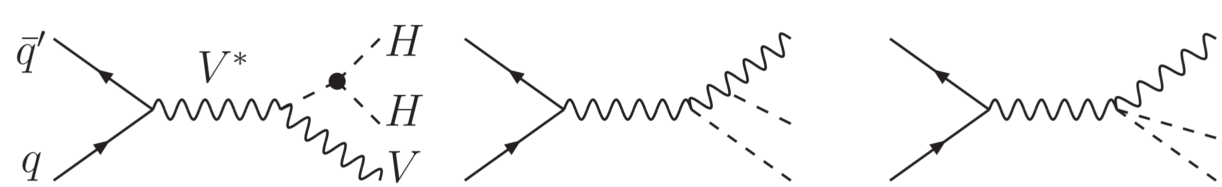
Top-Quark Associated

NLO [2]



Higgs-strahlung

NLO [1,2] NNLO [1,4]



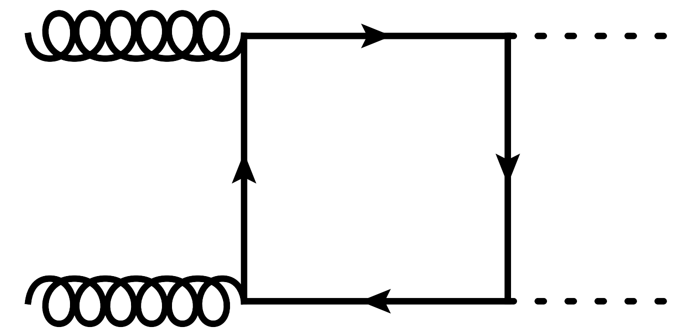
[1] Baglio, Djouadi, Gröber, Mühlleitner, Quevillon, Spira 12;

[2] Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Torrielli, Vryonidou, Zaro 14;

[3] Ling, Zhang, Ma, Guo, Li, Li 14 [4] Li, Wang 16

[5] Dolan, Englert, Greiner, Nordstrom, Spannowsky 15;

HH Production (Gluon Fusion)



1. LO (1-loop), Dominated by Top (Bottom <1%)
   Glover, van der Bij 88

HEFT: $m_T \to \infty$

Effective tree-level gluons/Higgs couplings

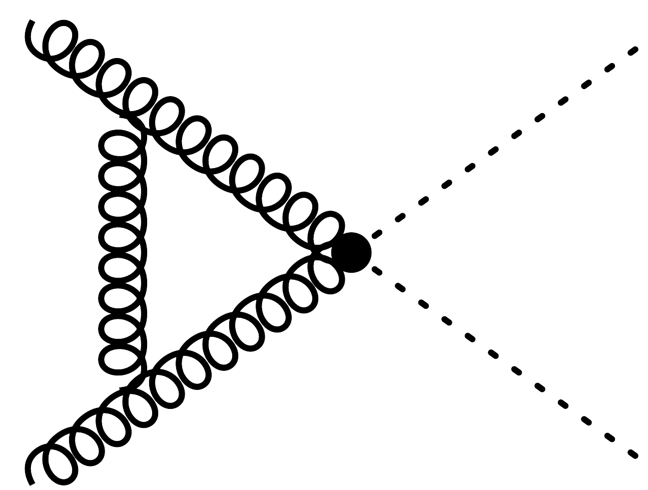
Lowers number of loops by 1

HEFT valid for

$$\sqrt{\hat{s}} \ll 2m_T$$

HH production for

$$2m_H < \sqrt{\hat{s}}$$



2. Born Improved NLO H(iggs)EFT $m_T \to \infty$ **K≈ 2**
   Dawson, Dittmaier, Spira 98

A. Including m_T in Real Radiation "FTapprox" **-10%**
   Maltoni, Vryonidou, Zaro 14

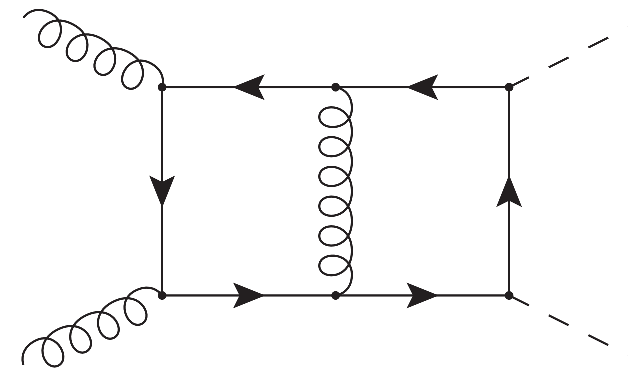
B. Including $\mathcal{O}(1/m_T^{12})$ Terms in Virtual MEs **±10%**
   Grigo, Hoff, Melnikov, Steinhauser 13; Grigo, Hoff 14; Grigo, Hoff, Steinhauser 15

HH Production (Gluon Fusion) (II)

3. NLO QCD (2-loop) with Full Top Mass
   Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Schubert, Zirke 16;
   Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Zirke 16

   (Transverse momentum) NLL + NLO
   Ferrera, Pires 16

   Parton Shower (POWHEG/MG5_aMC@NLO)
   Heinrich, SPJ, Kerner, Luisoni, Vryonidou 17

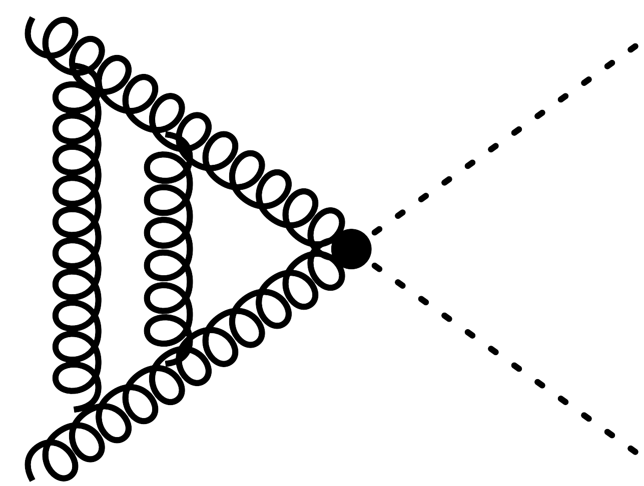


4. Born Improved NNLO HEFT **+20%**
   De Florian, Mazzitelli 13

   Including Matching Coefficients
   Grigo, Melnikov, Steinhauser 14

   Including Terms $\mathcal{O}(1/m_T^4)$ in Virtual MEs
   Grigo, Hoff, Steinhauser 15

   (Threshold) NNLL + NNLO Matching **+9%**
   (SCET) Shao, Li, Li, Wang 13; de Florian, Mazzitelli 15



5. NNLO HEFT (Differential)
   de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16

Shopping List

Virtual MEs (HH):

Channels:

$$gg \to HH \qquad \cancel{q\bar{q} \to HH}$$

Contributes at NNLO

	Diagrams
Tree	0
1-loop	8
2-loop	122

Real Radiation (HH + j…):

1j Channels:

$$gg \to HH + g$$
$$gq \to HH + q \qquad g\bar{q} \to HH + \bar{q}$$
$$q\bar{q} \to HH + g$$

Huge simplification!

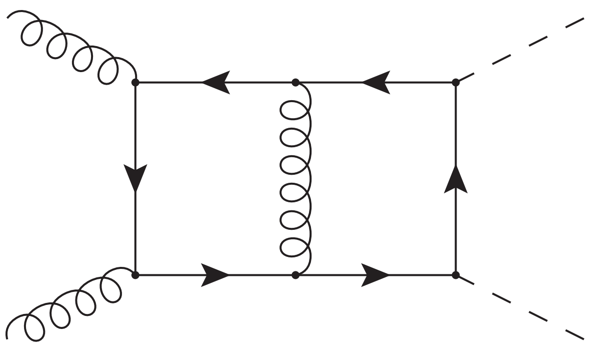
	Diagrams
Tree ⊗ Double	0
1-loop ⊗ Single	54+8+8+8

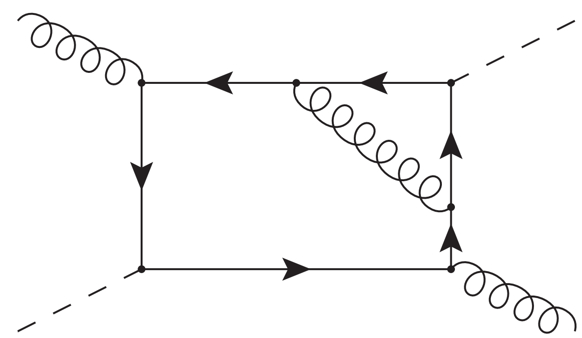
`GoSam` for MEs + Catani-Seymour Dipole Subtraction

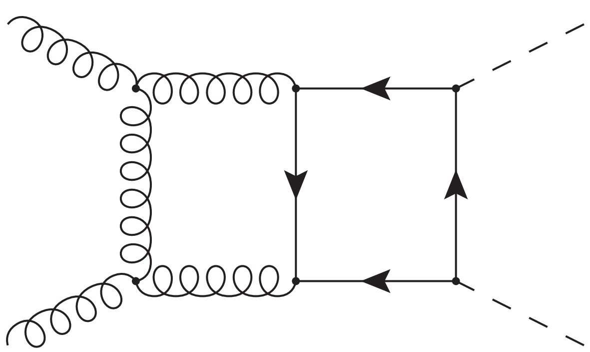
Catani, Seymour 96

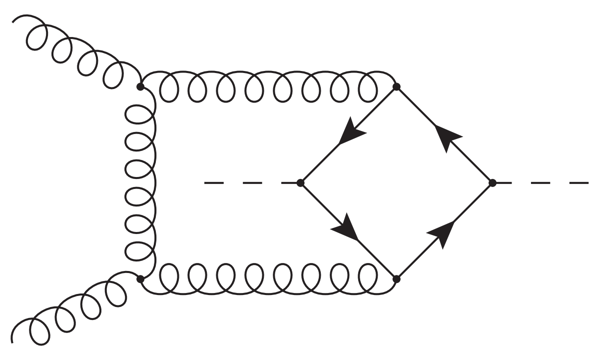
Virtual Contribution

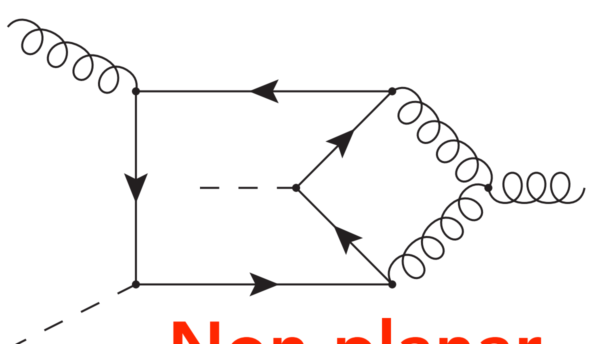
Yukawa only (≤ 4-point)



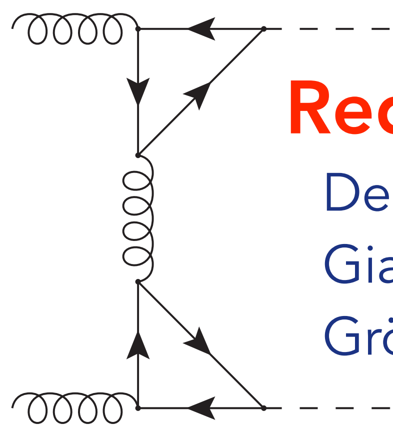








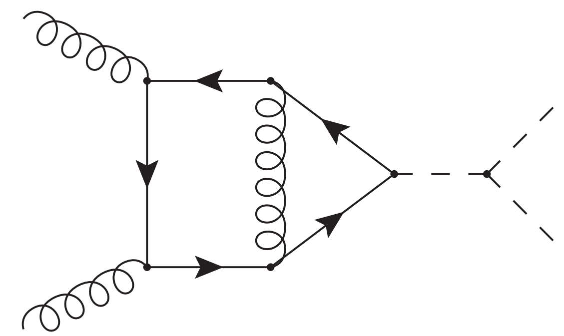
Non-planar

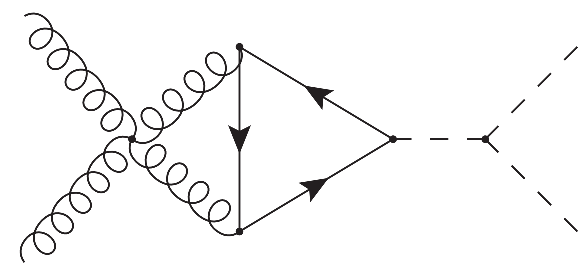


Reducible

Degrassi, Giardino, Gröber 16

Self-coupling (≤3-point)





Integrals Known

$gg \to H$

Spira, Djouadi et al. 93, 95; Bonciani, P. Mastrolia 03,04; Anastasiou, Beerli et al. 06;

Many integrals not known analytically, except:

$H \to Z\gamma$ Bonciani, Del Duca, Frellesvig et al. 15; Gehrmann, Guns, Kara 15;

Evaluating the Amplitude

Planar Integrals: Reduce to finite basis with **REDUZE 2**

von Manteuffel, Studerus 12; Panzer 14; von Manteuffel, Panzer, Schabinger 15

Non-Planar Integrals: Evaluate integrals directly

All Integrals evaluated numerically with **SecDec** (implements sector decomposition)

Borowka, Heinrich, Jahn, SPJ, Kerner, Schlenk, Zirke

Hepp 66; Denner, Roth 96; Binoth, Heinrich 00

Entire 2-loop amplitude evaluated with a single code

$$F = \sum_i \underbrace{\left(\sum_j C_{i,j} \epsilon^j \right)}_{\text{coeff.}} \underbrace{\left(\sum_k I_{i,k} \epsilon^k \right)}_{\text{integral}} = \epsilon^{-2} \left[C^{(L)}_{1,-2} I^{(L)}_{1,0} + \ldots \right] + \epsilon^{-1} \left[C^{(L)}_{1,-1} I^{(L)}_{1,0} + \ldots \right] + \ldots$$

compute once

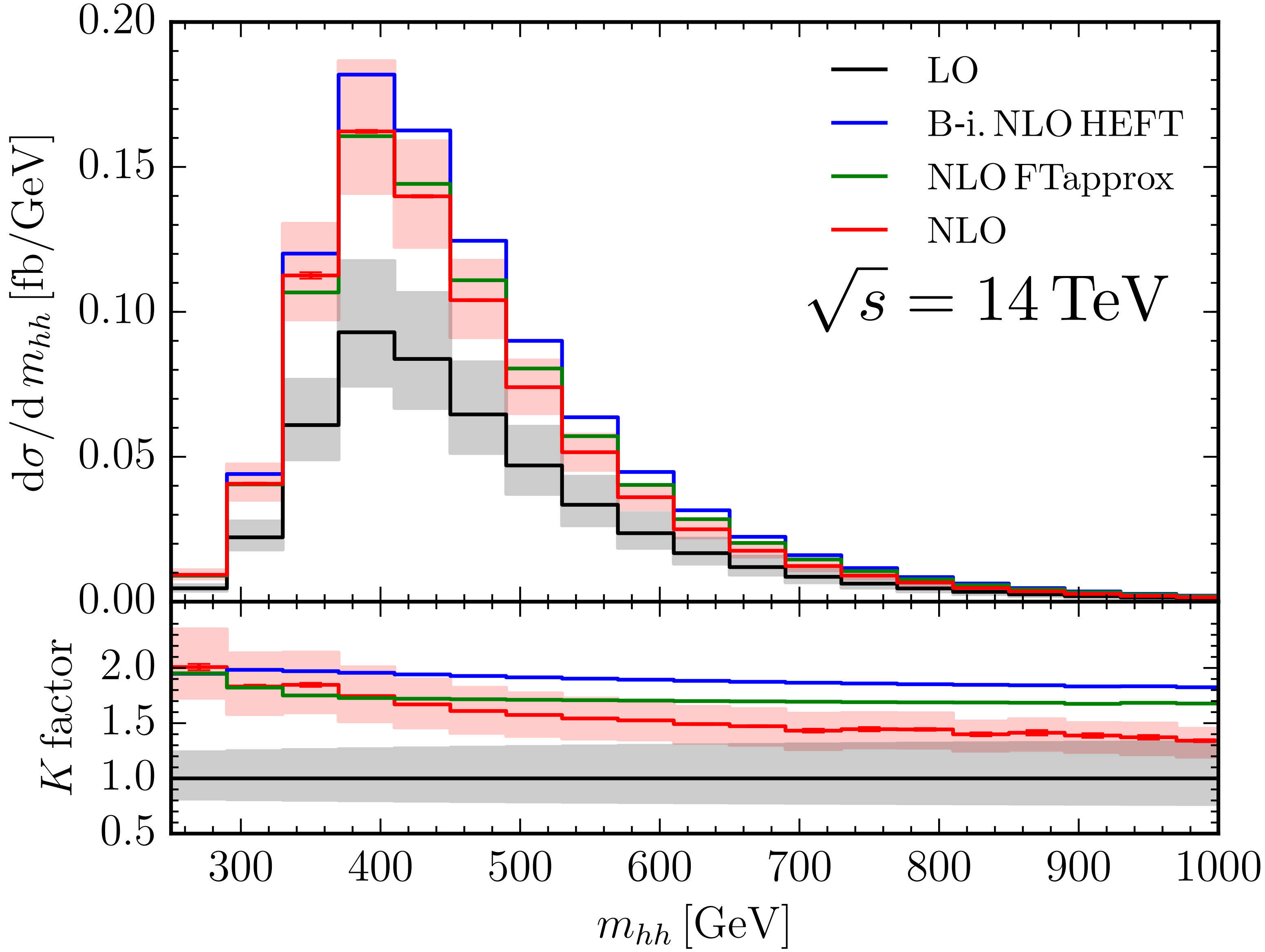
Dynamically set target precision for each sector, minimising time

Use Quasi-Monte-Carlo (QMC) integration $\mathcal{O}(n^{-1})$ error scaling

Li, Wang, Yan, Zhao 15; (Review: Dick, Kuo, Sloan 13)

Implemented in OpenCL, evaluated on GPUs

Results (I): Invariant Mass



PDF4LHC15_nlo_30_pdfas

$m_H = 125$ GeV

$m_T = 173$ GeV

Uncertainty:

$\mu_0 = \frac{m_{HH}}{2}$

$\mu_{R,F} \in \left[\frac{\mu_0}{2}, 2\mu_0\right] \quad (7-\text{point})$

HEFT: Outside scale var.

$$m_{hh} > 420\,\text{GeV}$$

FTapp: Outside scale var.

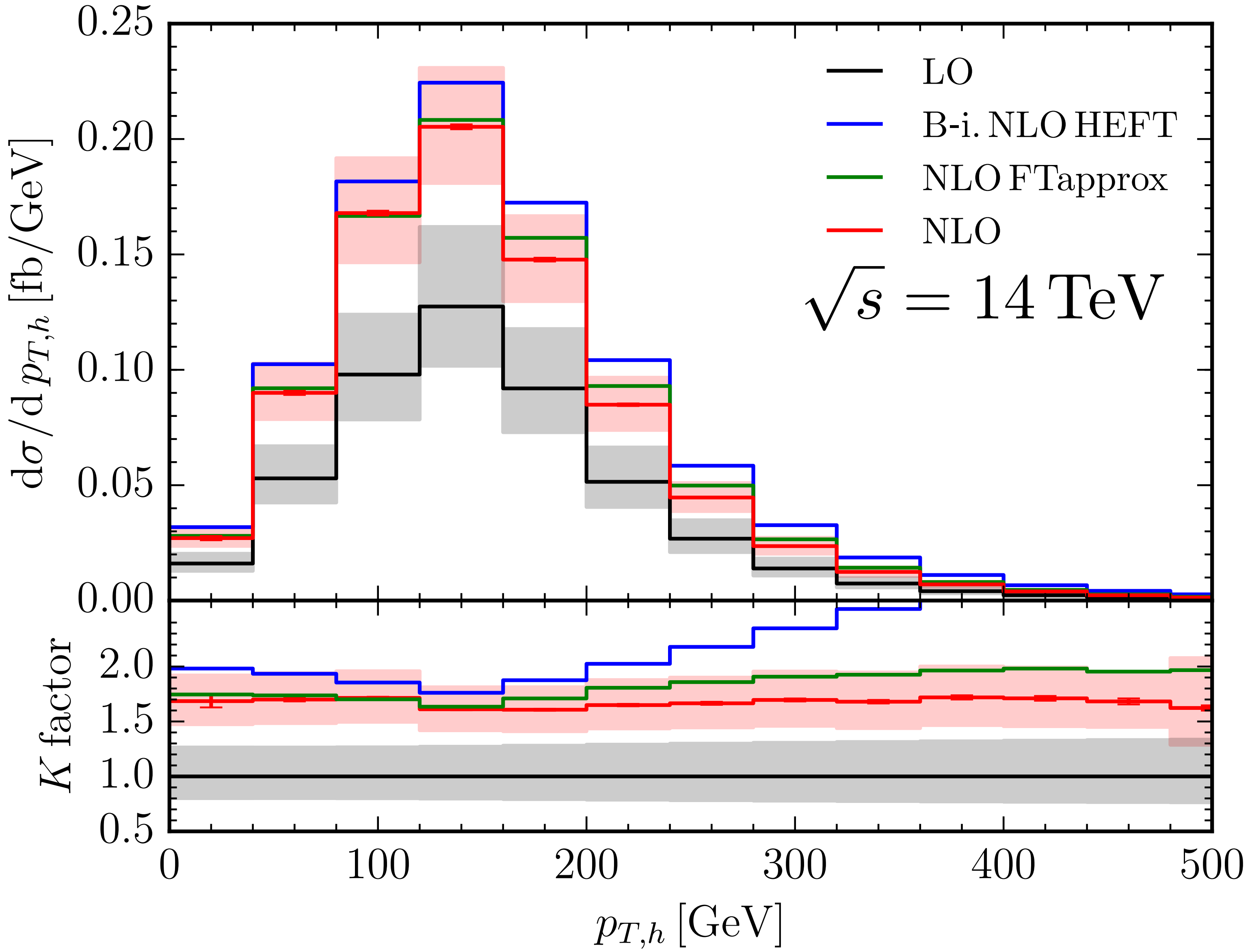
$$m_{hh} > 620\,\text{GeV}$$

	σ_{LO} (fb)	σ_{NLO} (fb)
B.I. HEFT	$19.85^{+27.6\%}_{-20.5\%}$	$38.32^{+18.1\%}_{-14.9\%}$
FTapprox	$19.85^{+27.6\%}_{-20.5\%}$	$34.26^{+14.7\%}_{-13.2\%}$
Full Theory	$19.85^{+27.6\%}_{-20.5\%}$	$\mathbf{32.91^{+13.6\%}_{-12.6\%}} \pm \mathbf{0.3\%}(\text{stat.}) \pm \mathbf{0.1\%}(\text{int.})$

HEFT overestimates by 16%

FTap. overestimates by 4%

Results (II): pT Either Higgs



HEFT: Can be poor approx. for larger $p_{T,h}$

Note: Ambiguous how to rescale HEFT real radiation by full LO born differentially

FTapp: Significantly better but still overestimating

Real radiation plays larger role for large $p_{T,h}$
Including m_T in real radiation does improve over HEFT in tails

Monte Carlo Interface

Amplitude depends on 2 form factors:

$$\mathcal{M}^{\mu\nu} = F_1(\hat{s}, \hat{t}, m_h^2, m_t^2, D) T_1^{\mu\nu} + F_2(\hat{s}, \hat{t}, m_h^2, m_t^2, D) T_2^{\mu\nu}$$

Amplitude is slow to evaluate:

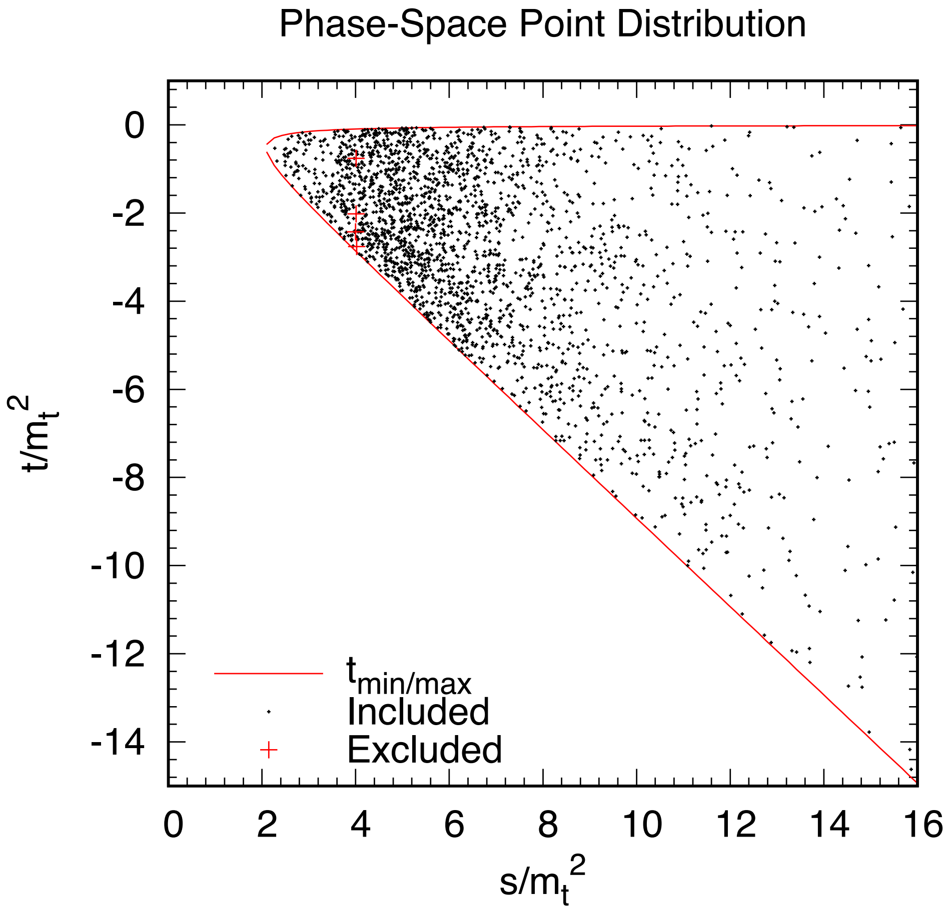
Accuracy goal: 3% for F_1, 5-20% for F_2 (depending on F_2/F_1)

GPU Time/PS point: 80 min - 2 days (median 2 hours)

Can not put directly into a Monte Carlo

But: Virtual matrix element depends only on $\hat{s}, \hat{t}$ (fixed m_T, m_H)

Can build 2D grid of our phase-space points and interpolate between 3741 pre-calculated points



Monte Carlo Interface (II)

Parametrisation:

$$x = f(\beta(\hat{s})), \quad c_\theta = |\cos\theta| = \left|\frac{\hat{s} + 2\hat{t} - 2m_H^2}{\hat{s}\beta(\hat{s})}\right|, \quad \beta = \left(1 - \frac{4m_H^2}{\hat{s}}\right)^{\frac{1}{2}}$$

Choose $f(\beta)$ according to cumulative distribution function of phase space points used in the original calculation

Obtain nearly uniform distribution in (x, c_θ) unit square

Two-step interpolation procedure:

1. Choose equidistant grid points, estimate result at each grid point with linear interpolation of amplitude results in vicinity
2. Clough-Tocher interpolation (as implemented in `SciPy`) to estimate amplitude at arbitrary sampling points

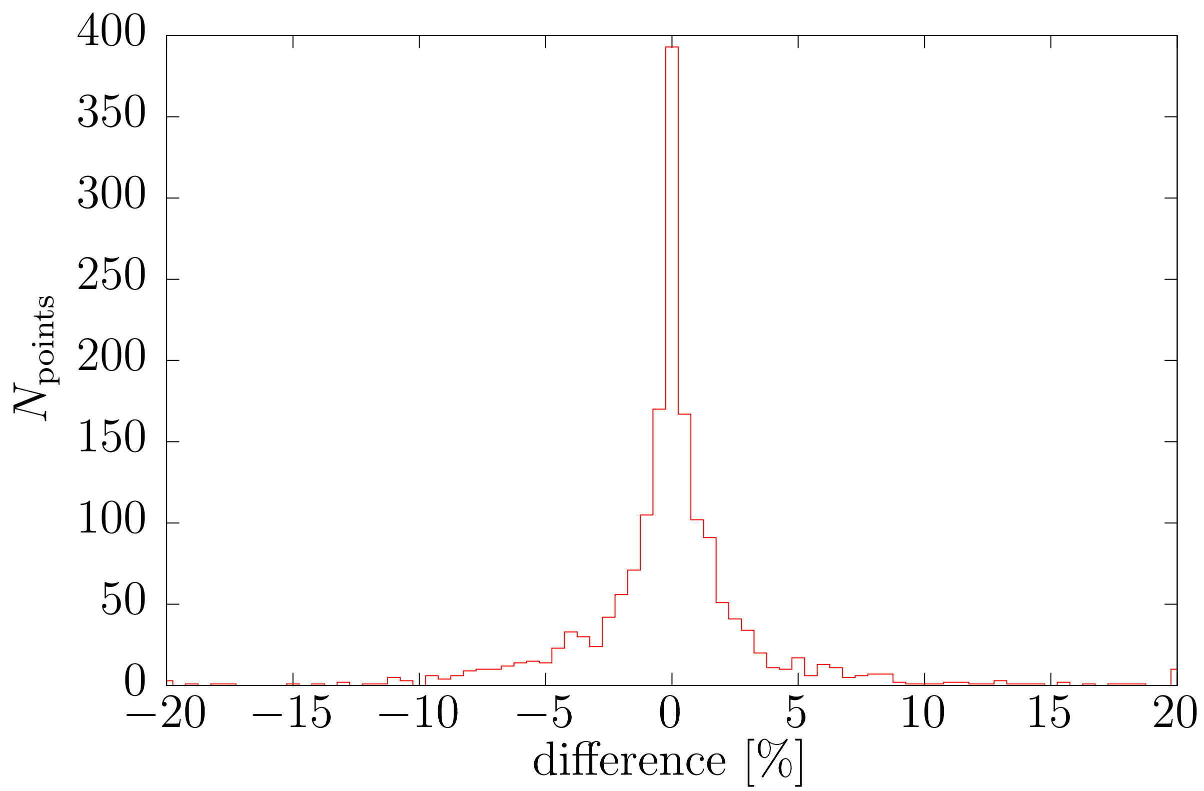
Procedure reduces sensitivity to uncertainties of input data points

Monte Carlo Interface (III)

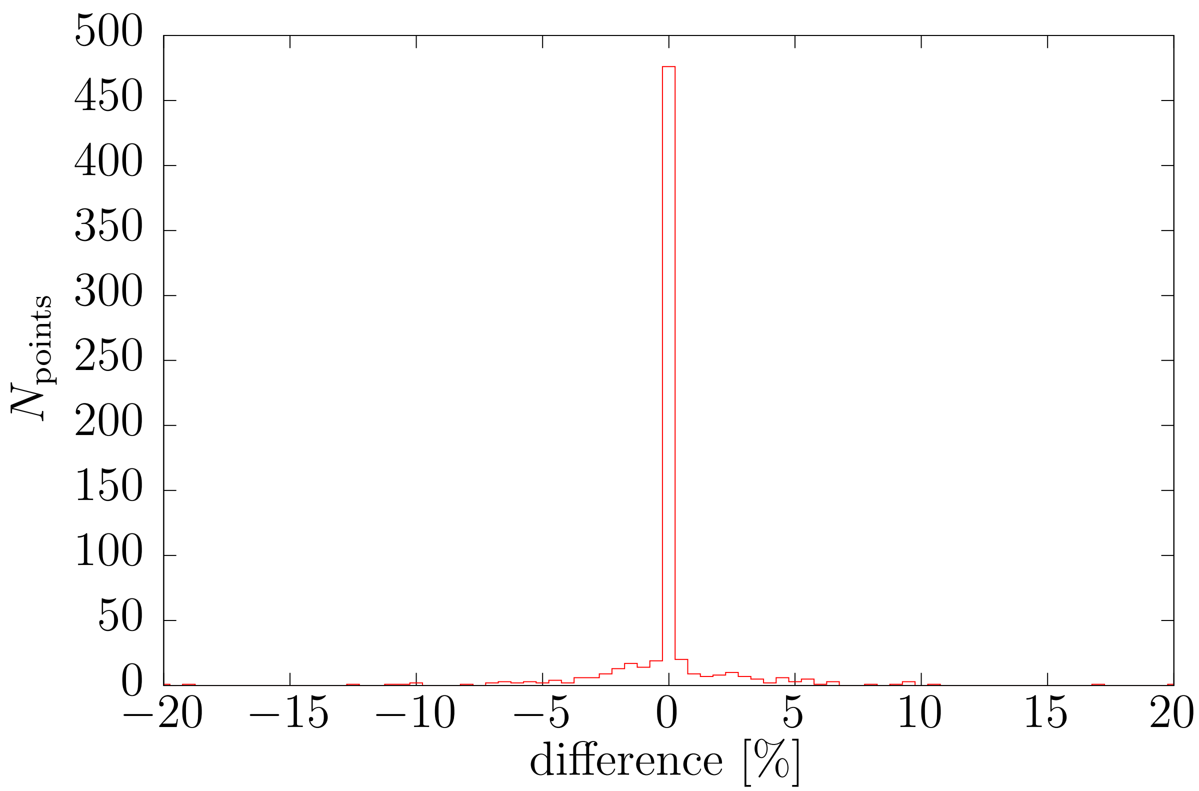
Grid of $\mathcal{V}_{\text{fin}}$ (1-loop x 2-loop interference) implemented in `Python`
Interfaced to `FORTRAN`, `C`, `C++` via `Python/C API` (examples in repo)

https://github.com/mppmu/hhgrid

Grid Stability:



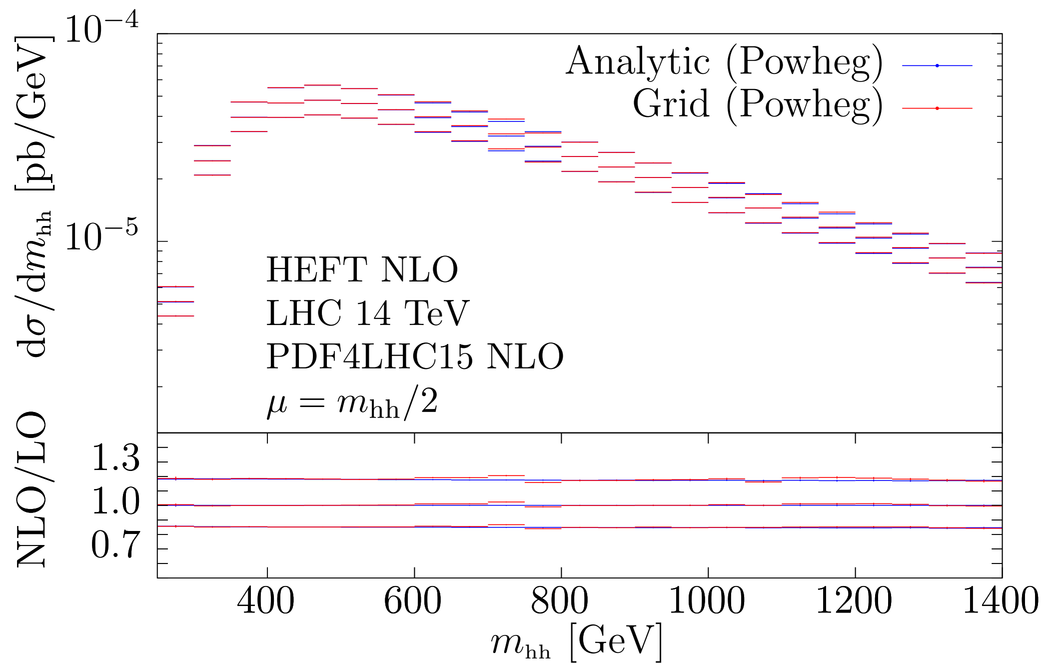
Grid vs grid with 50% of points

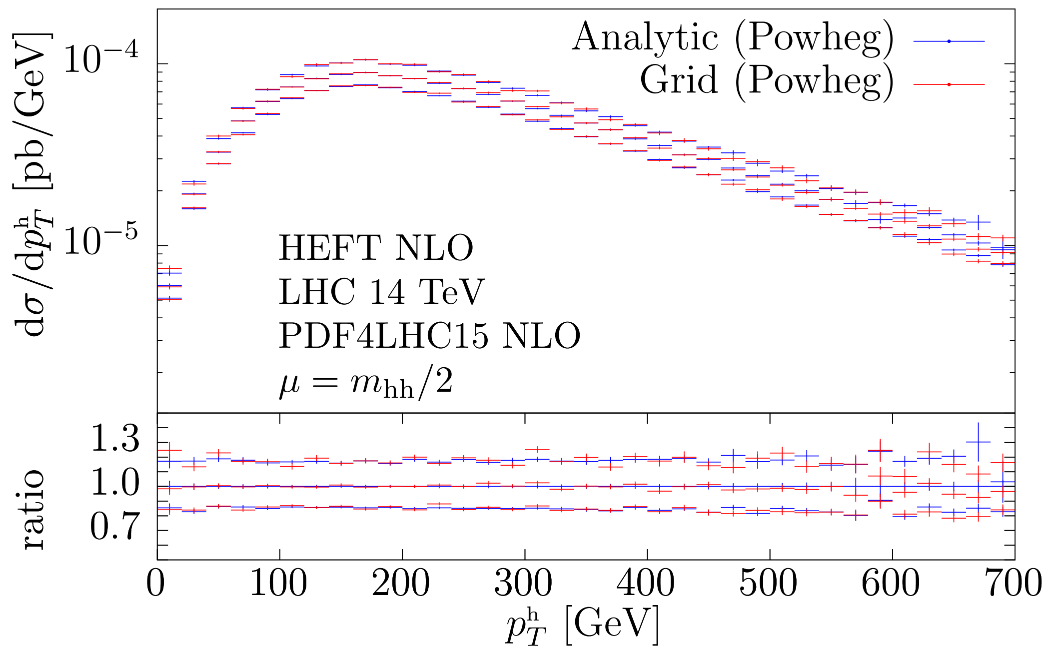


Grid vs grid with 80% of points

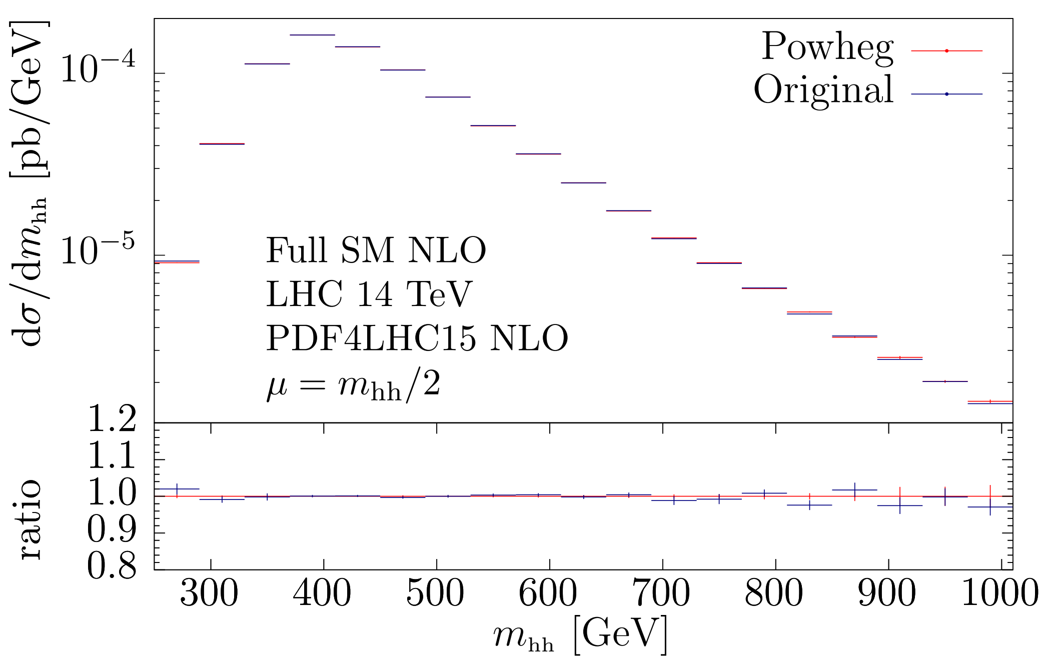
Grid Validation

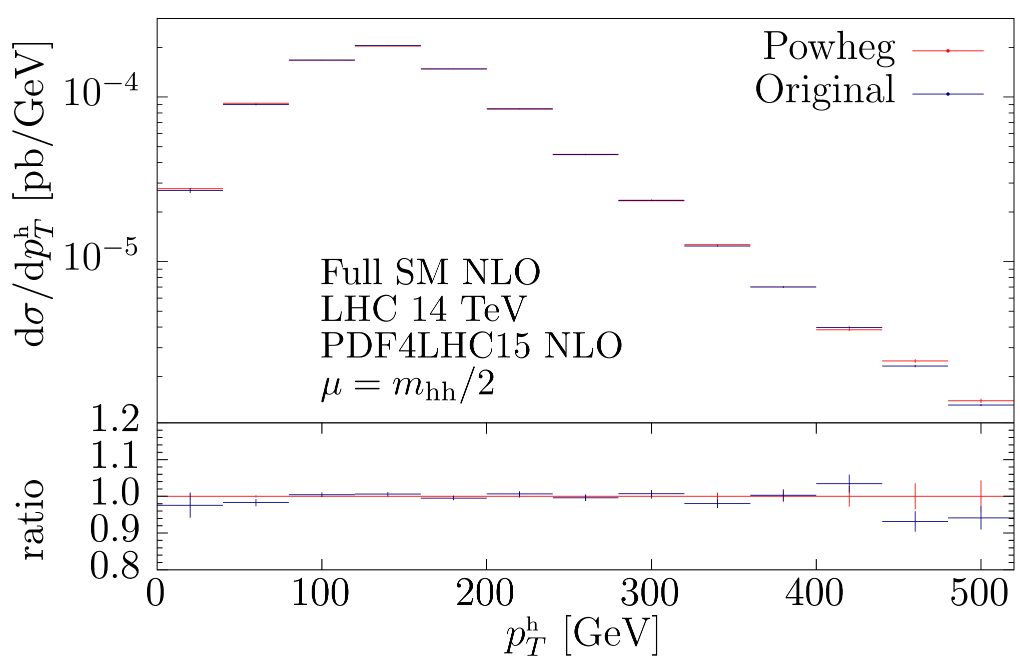
Use HEFT to study validity of grid





Full SM compare **`POWHEG`** (grid) with our original results

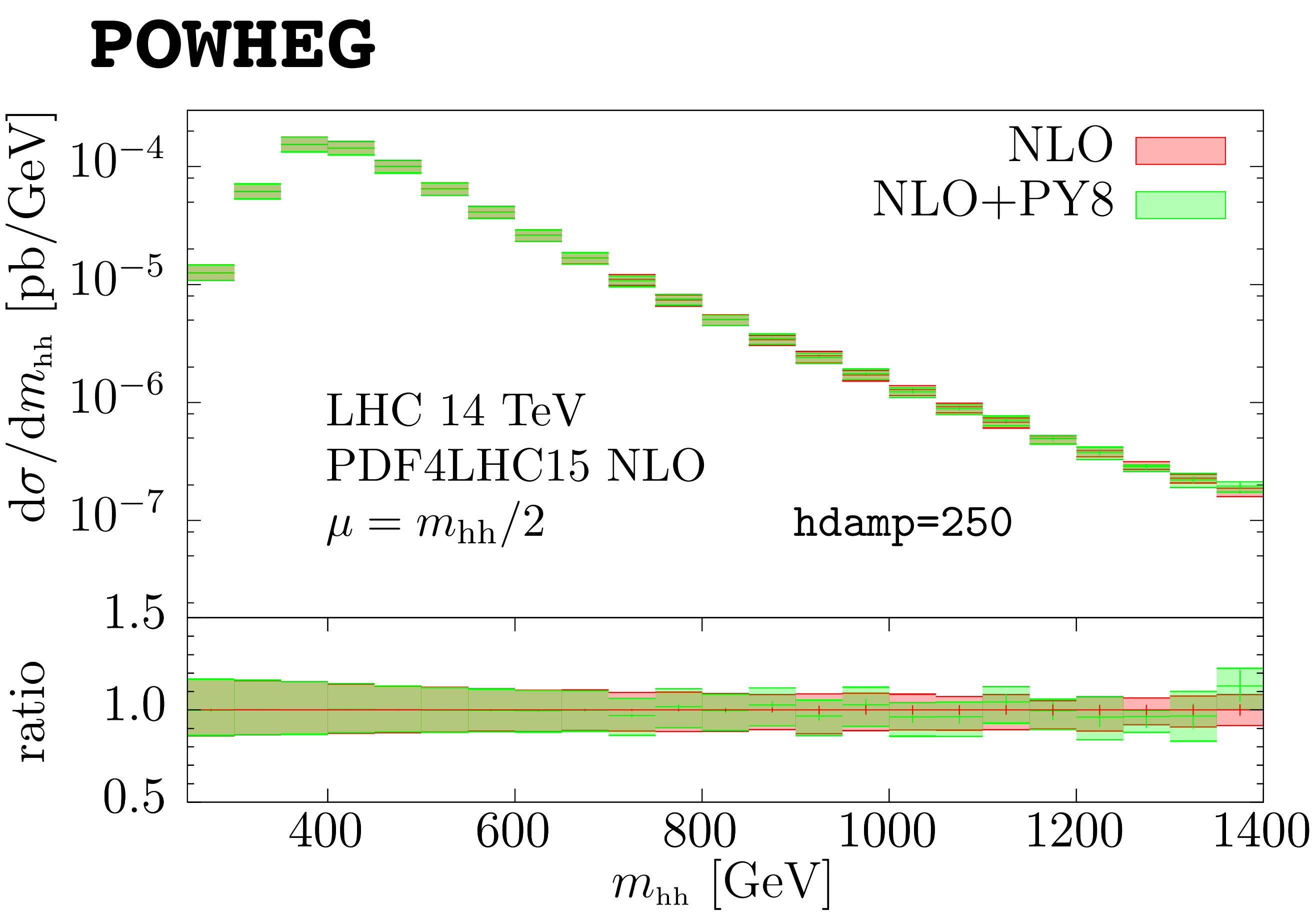


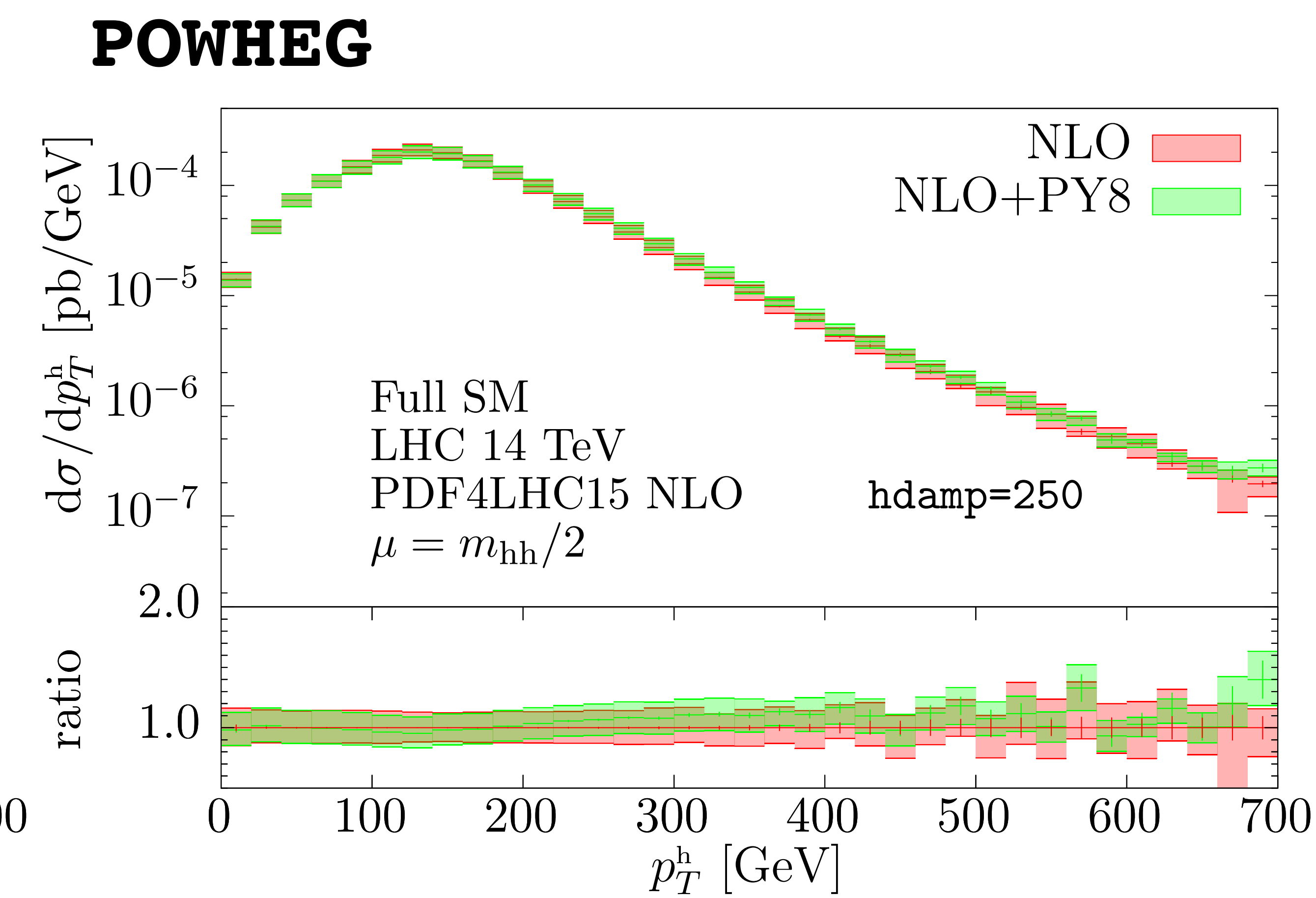


Showered Results

No Higgs decay or hadronization included

Assume $\Gamma_h = 0$ (decay can be attached e.g. in narrow width approx.)



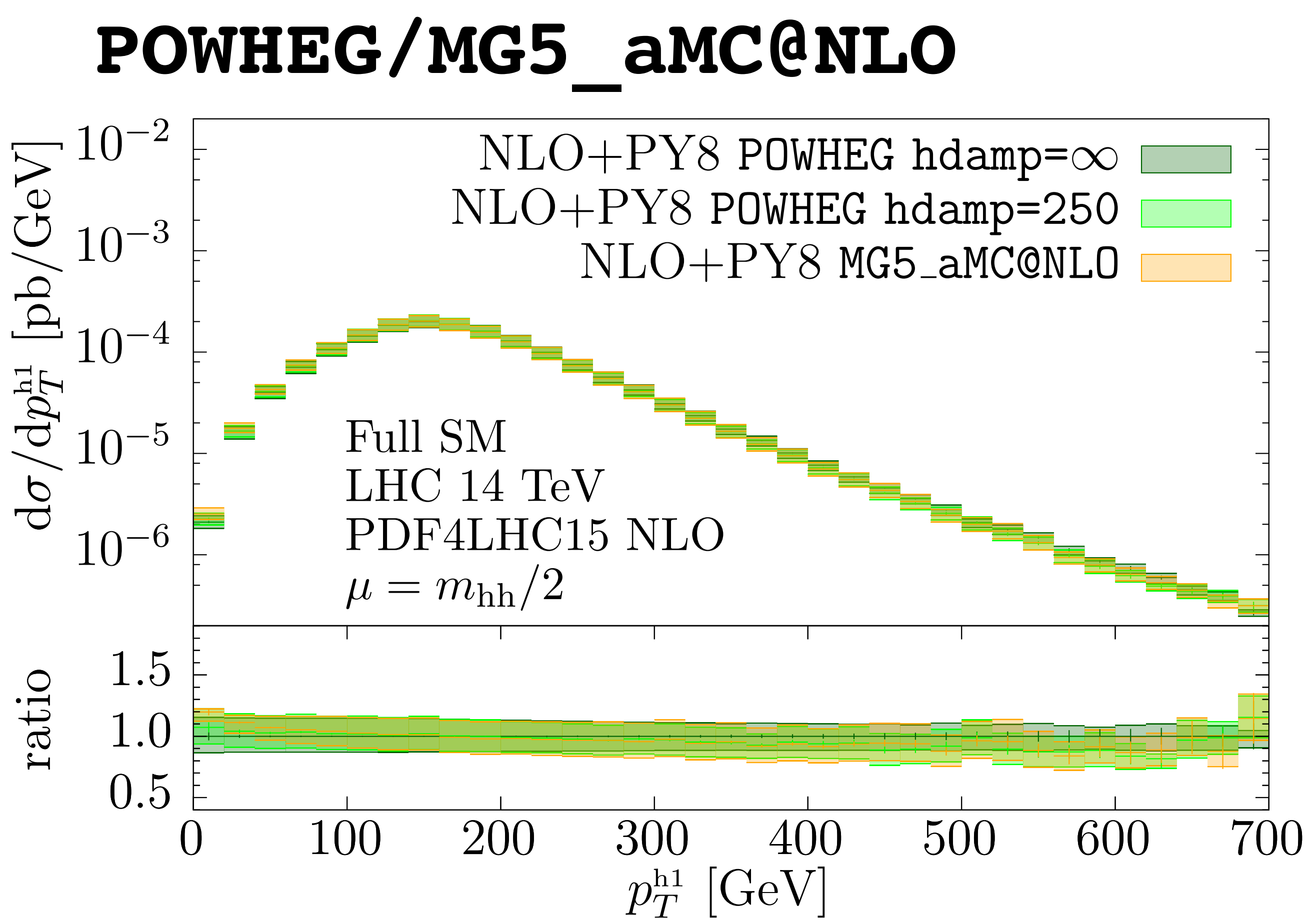


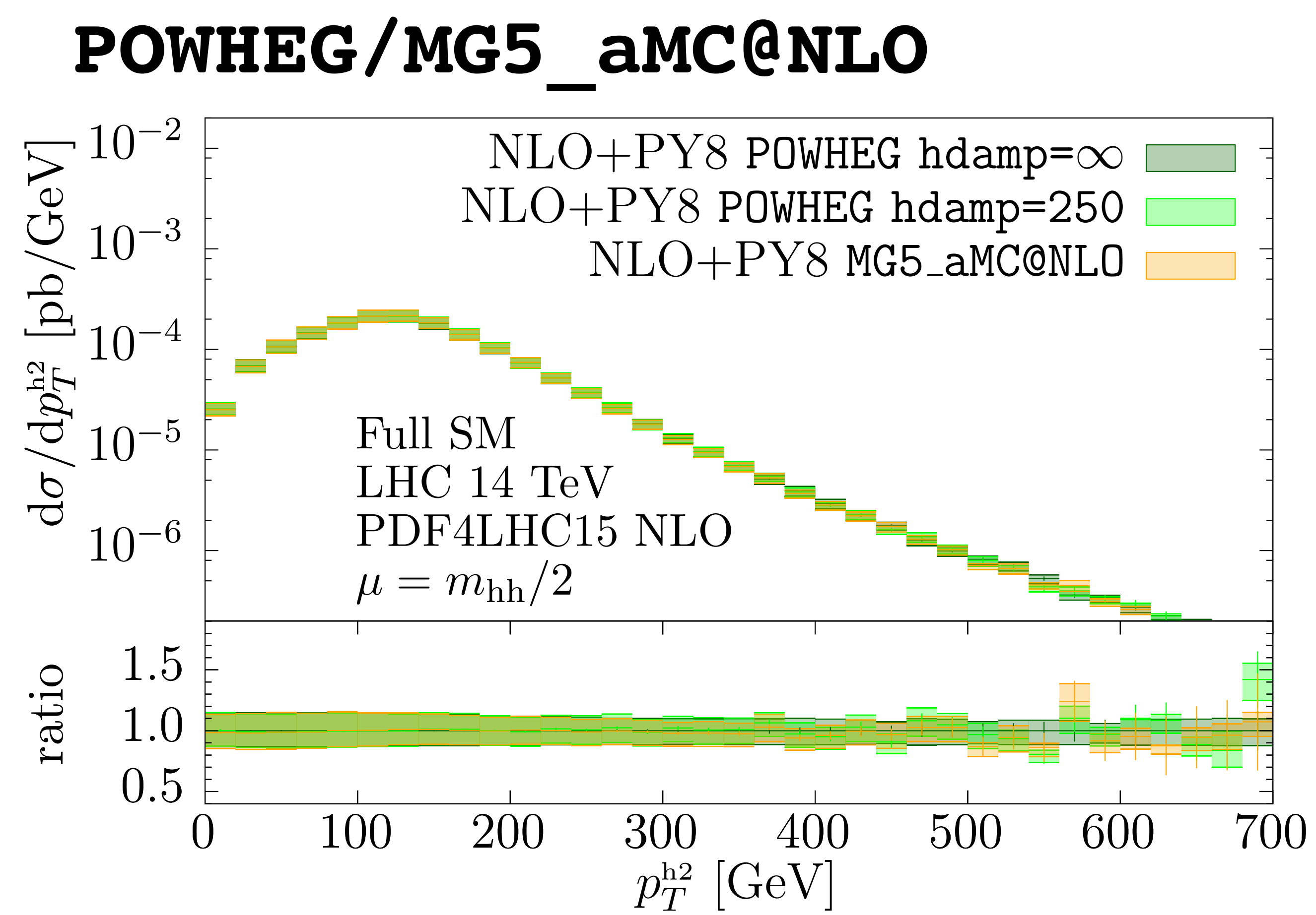
More inclusive variables not sensitive to shower

Shower has moderate impact on NLO accurate observables

Showered Results (II)

We use same `Pythia8.2` shower for both `POWHEG/MG5_aMC@NLO`





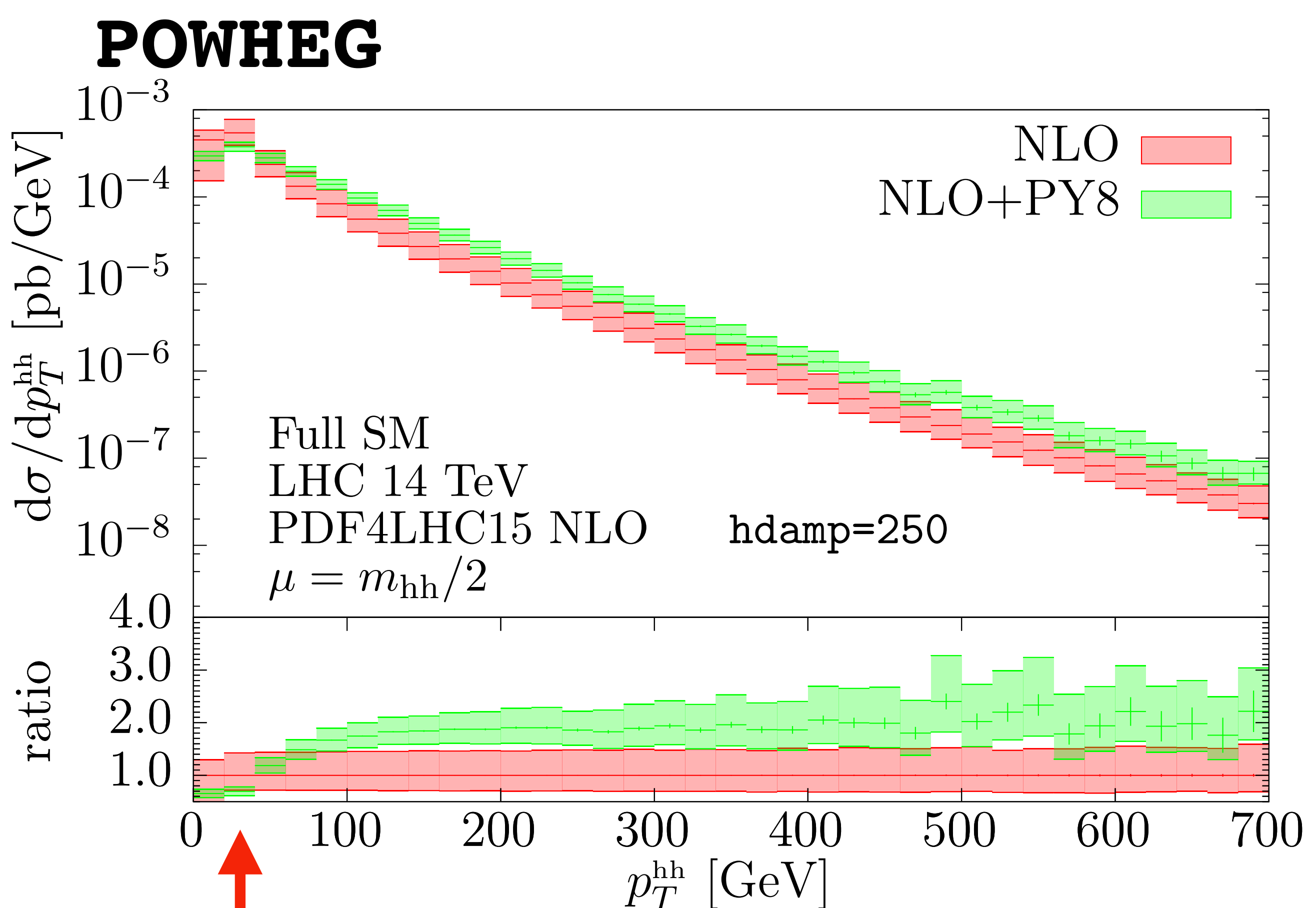
Different matching schemes have very moderate impact on NLO accurate observables

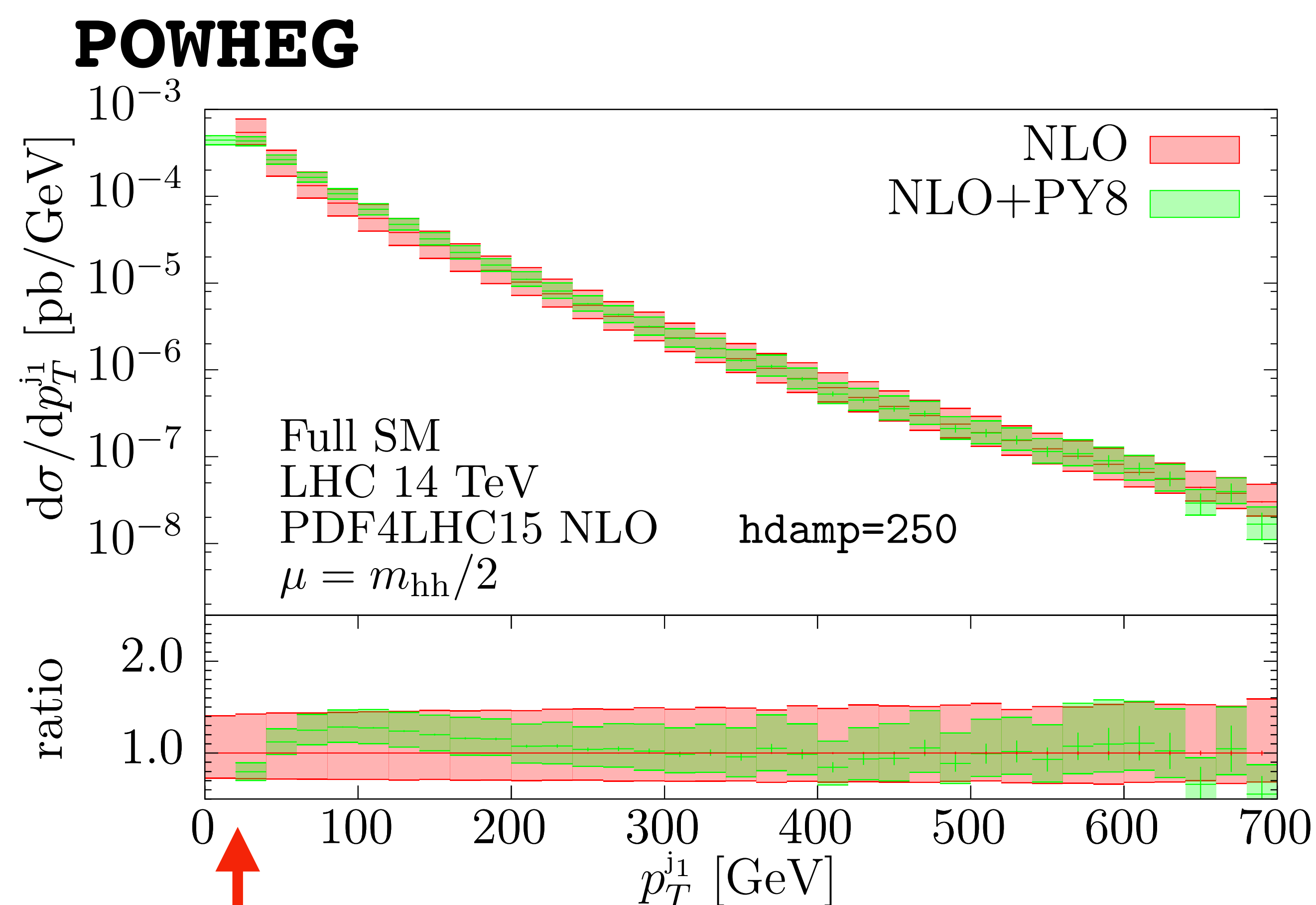
Showered Results (III)

Shower has larger impact on LO accurate observables

Fastjet anti$-k_T$ $R = 0.4$

$p^{j}_{T,\mathrm{min}} = 20$ GeV

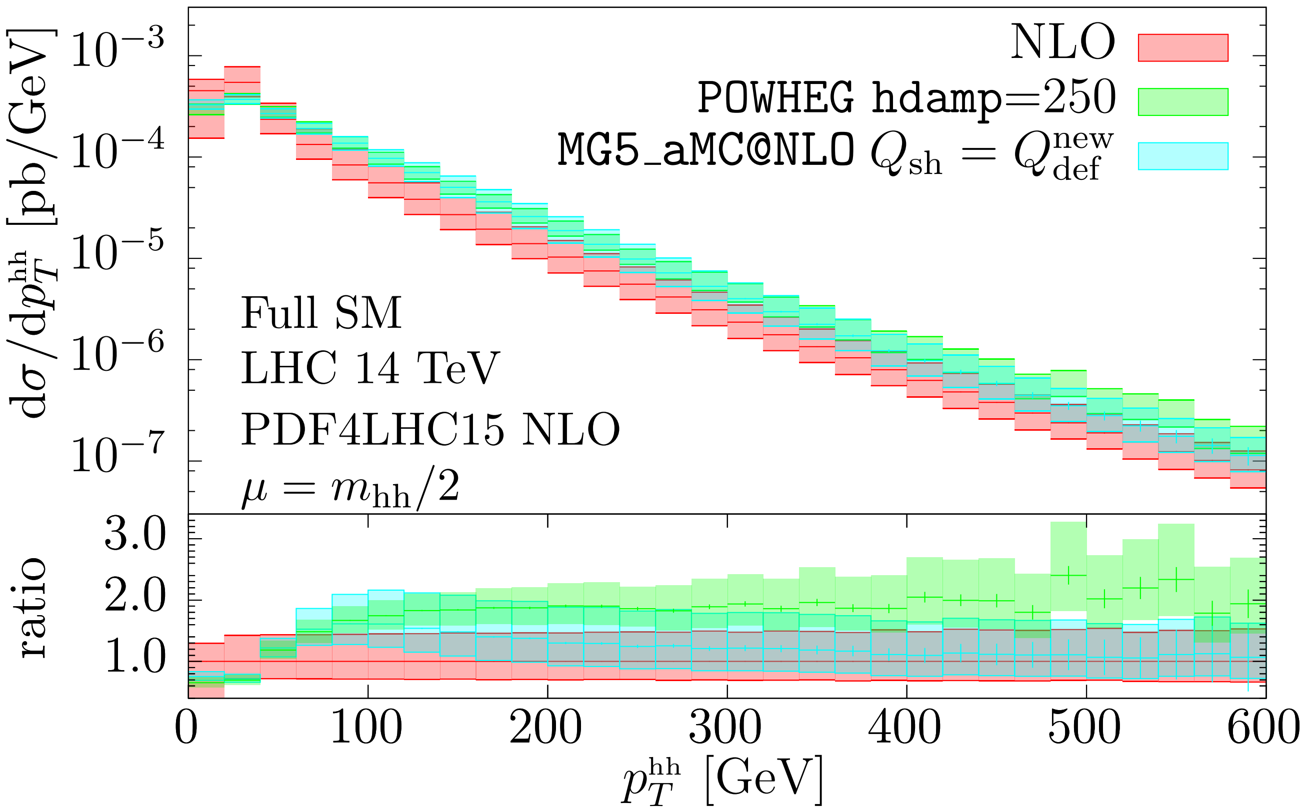




Parton shower needed to provide reliable predictions at low p_T^{hh}, $p_T^{j_1}$

Showered Results (IV)

POWHEG/MG5_aMC@NLO



p_T^{hh} is sensitive to matching

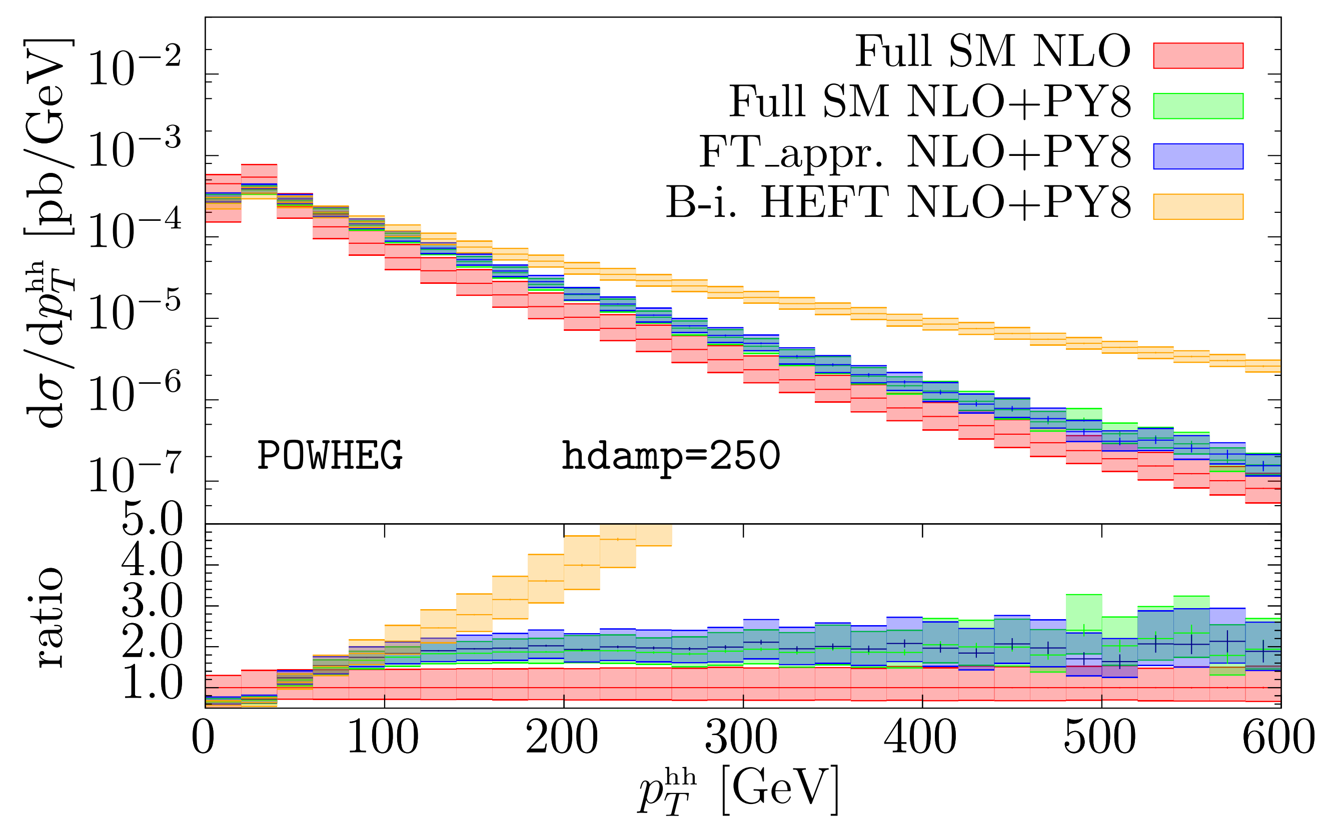
MC@NLO (2.5.3) shower scale:

$\texttt{shower\_scale\_factor} \times [0.1 H_T/2, H_T/2]$

Reducing shower scale gives softer distributions

B.I HEFT: much harder than exact computation

FTapp: gives good description at high p_T^{hh} (contains exact real ME)



POWHEG: hdamp & matching

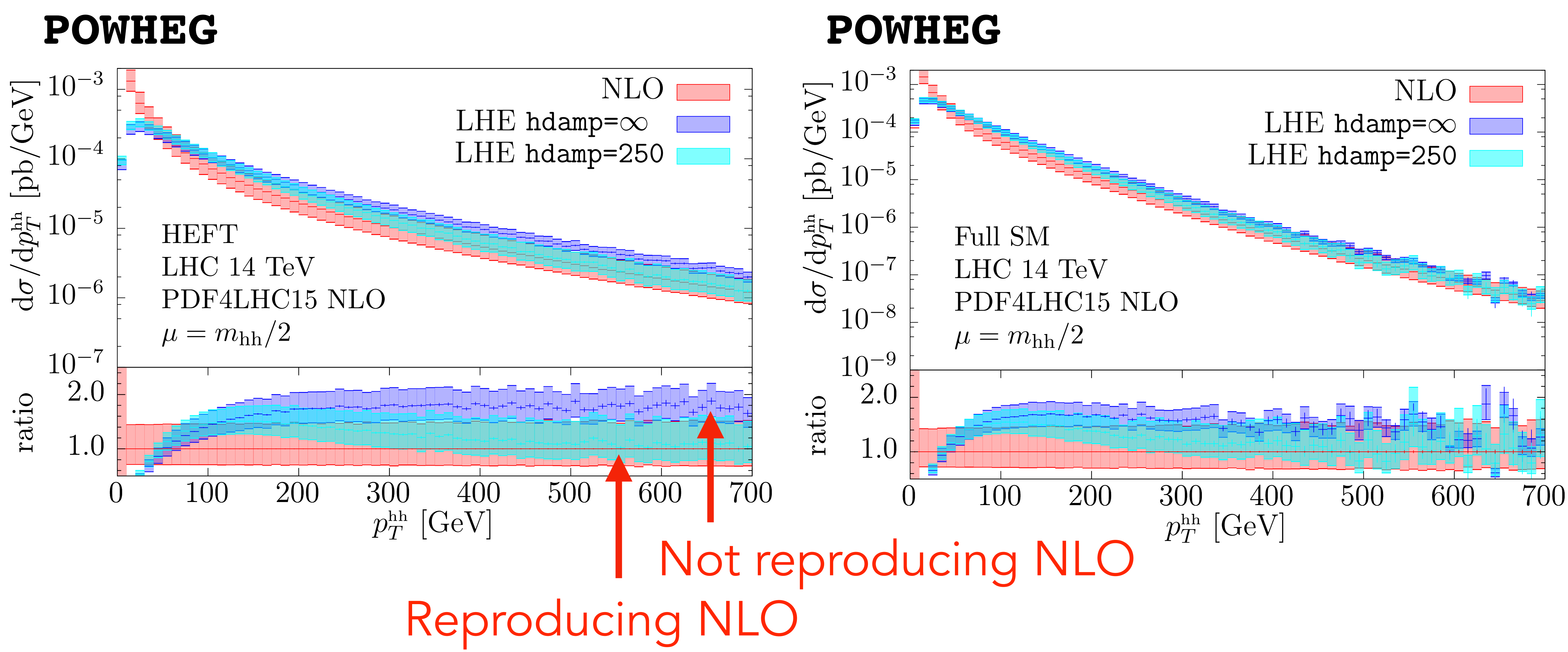
LHE events (before shower): at large p_T^{hh} we do not reproduce NLO

Can introduce h_{damp} to limit amount of hard radiation exponentiated:

Exponentiated → $R_{\mathrm{sing}} = R \times F$

$$R_{\mathrm{reg}} = R \times (1 - F)$$

$$F = \frac{h_{\mathrm{damp}}^2}{(p_T^{hh})^2 + h_{\mathrm{damp}}^2}$$

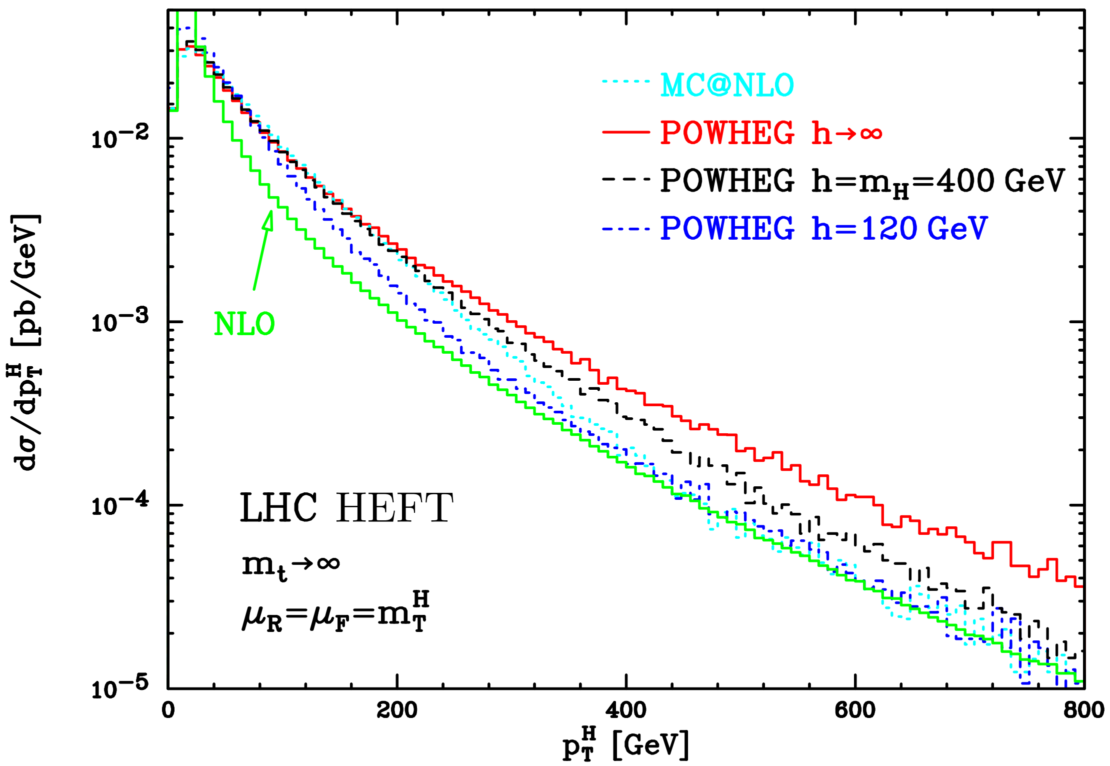


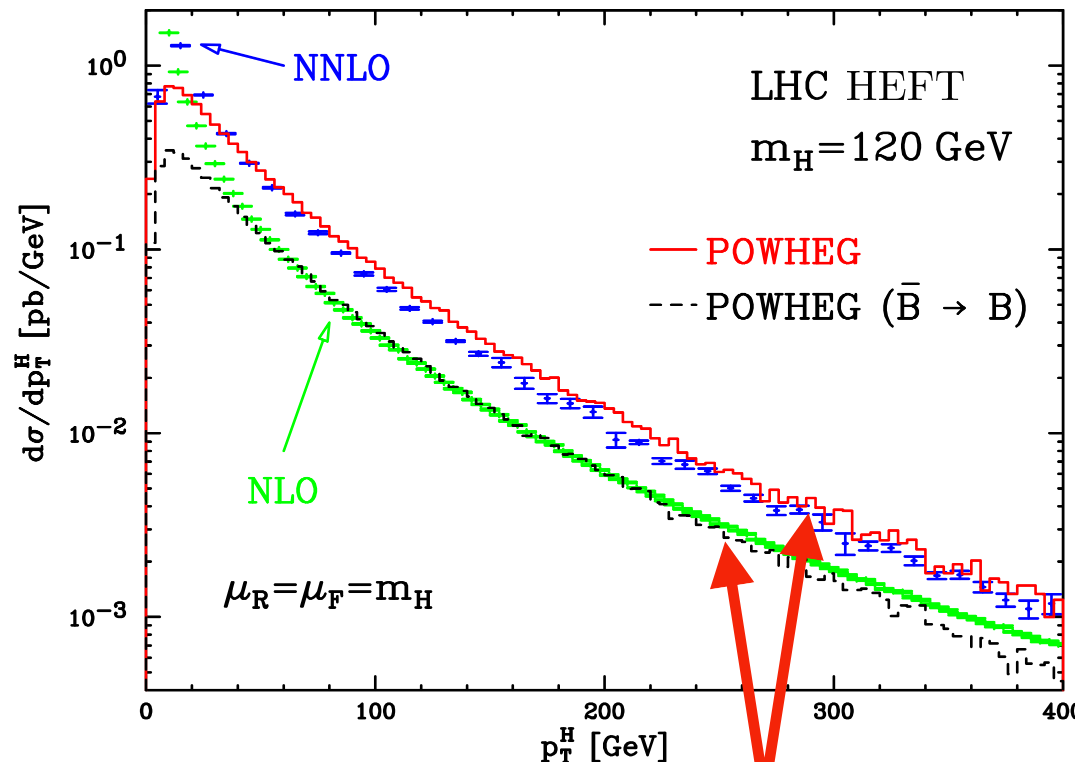
POWHEG: hdamp & matching (II)

Similar situation for Higgs p_T

Without $h_{\rm damp}$ **POWHEG** events look similar to NNLO

Difference between NLO/**POWHEG** due to terms of $\mathcal{O}(\alpha_s)$





Alioli, Nason, Oleari, Re 09

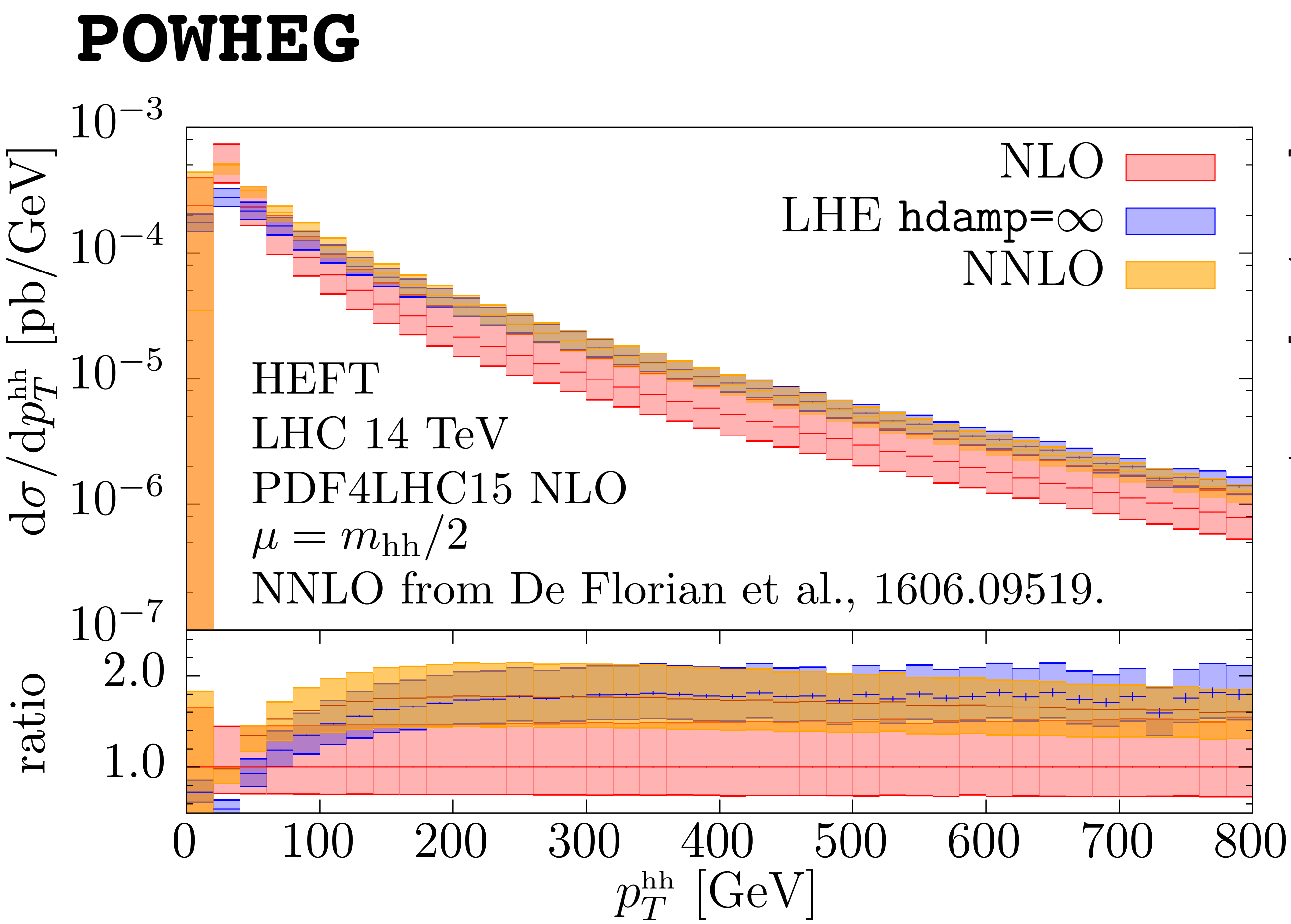
Difference $\mathcal{O}(\alpha_s)$

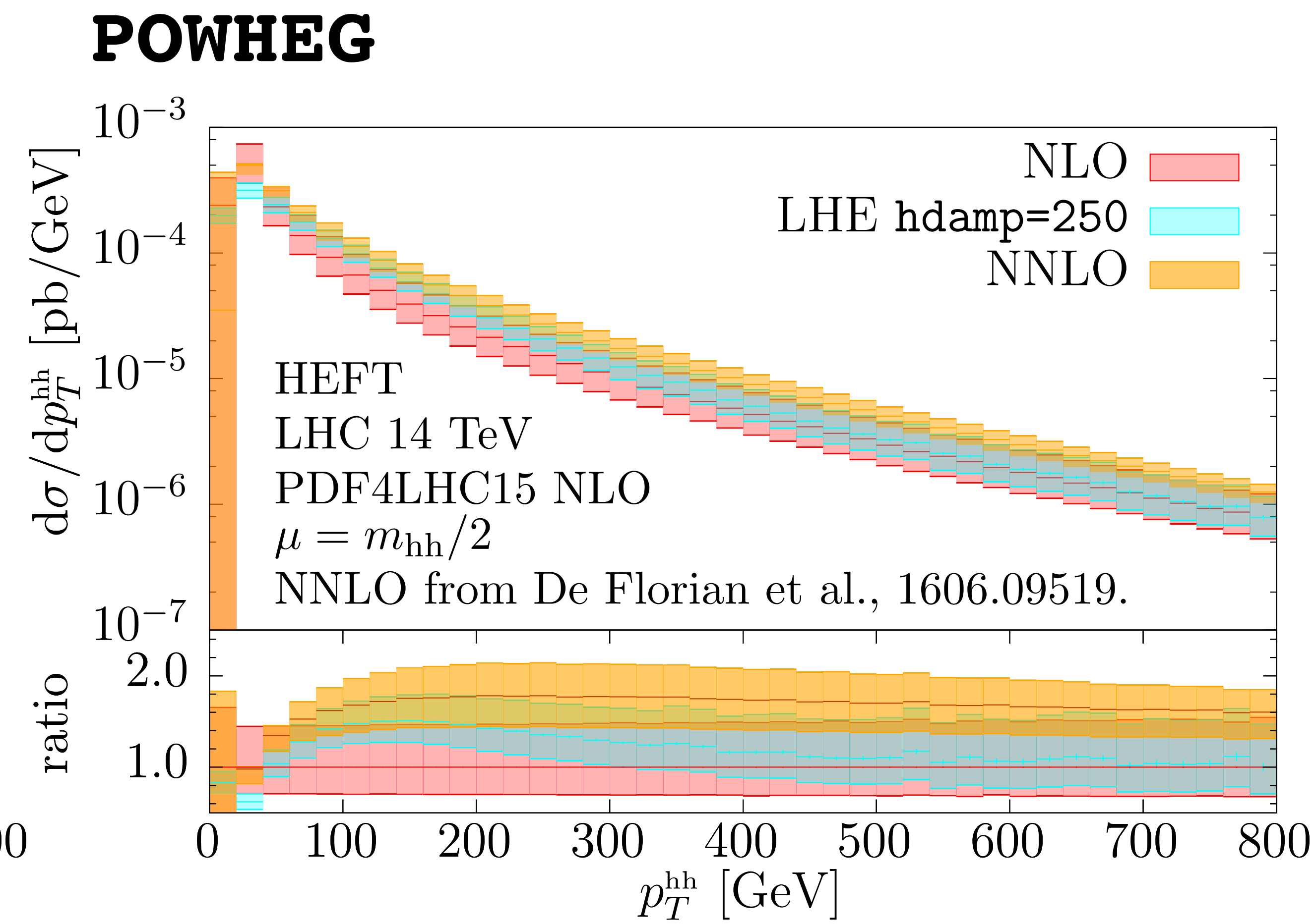
Only real part of the NLO contributes to p_T tail (variable is LO accurate)

NLO corrections are large (K≈2), can expect sensitivity to matching

POWHEG: hdamp & matching (III)

In the HEFT we know the NNLO result, can compare to LHE at NLO





$h_{\mathrm{damp}} = \infty$ leads to NLO+PS that agrees more with NNLO

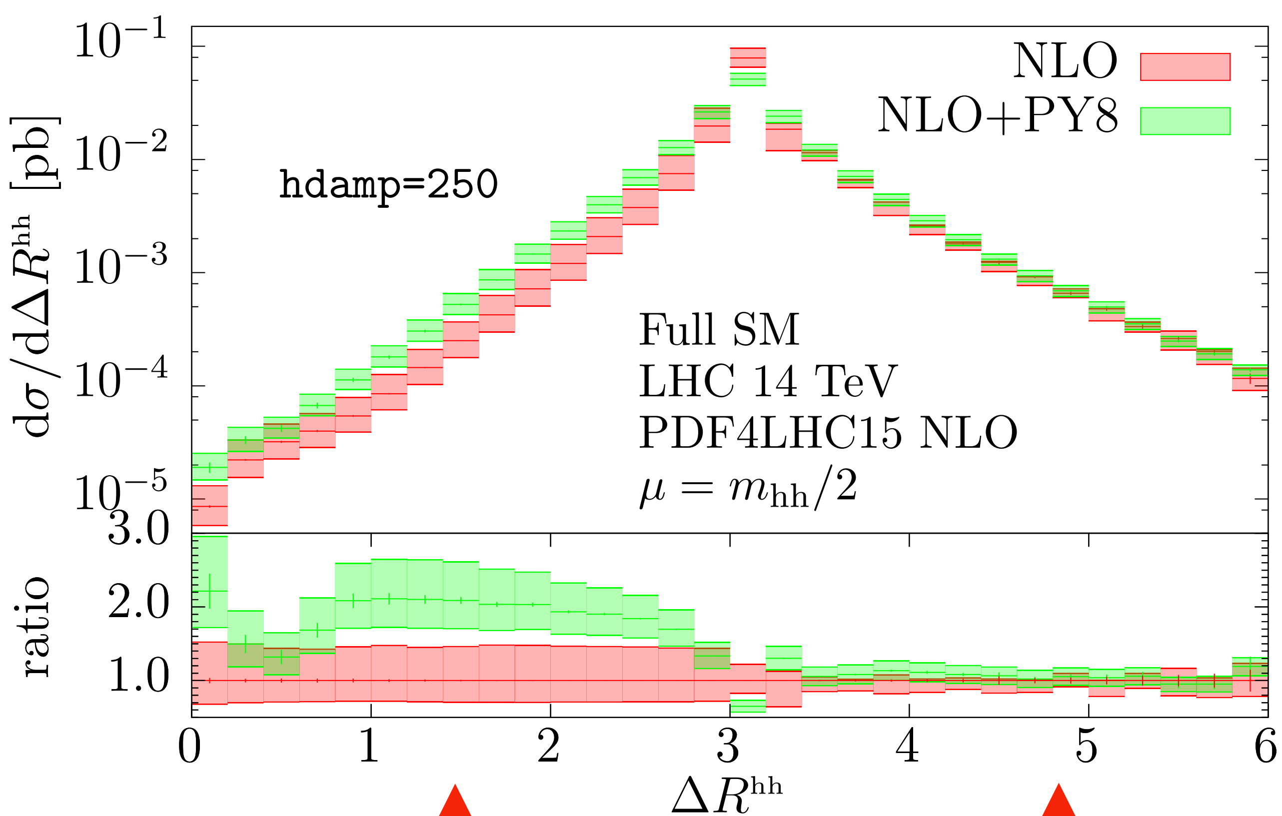
$h_{\mathrm{damp}} = 250\ \mathrm{GeV}$ gives tail closer to NLO (also in full theory)

NNLO not known in the full theory

Radial Separation

Radial separation: $\Delta R^{\mathrm{hh}} = \sqrt{(\eta_1 - \eta_2)^2 + (\Phi_1 - \Phi_2)^2}$

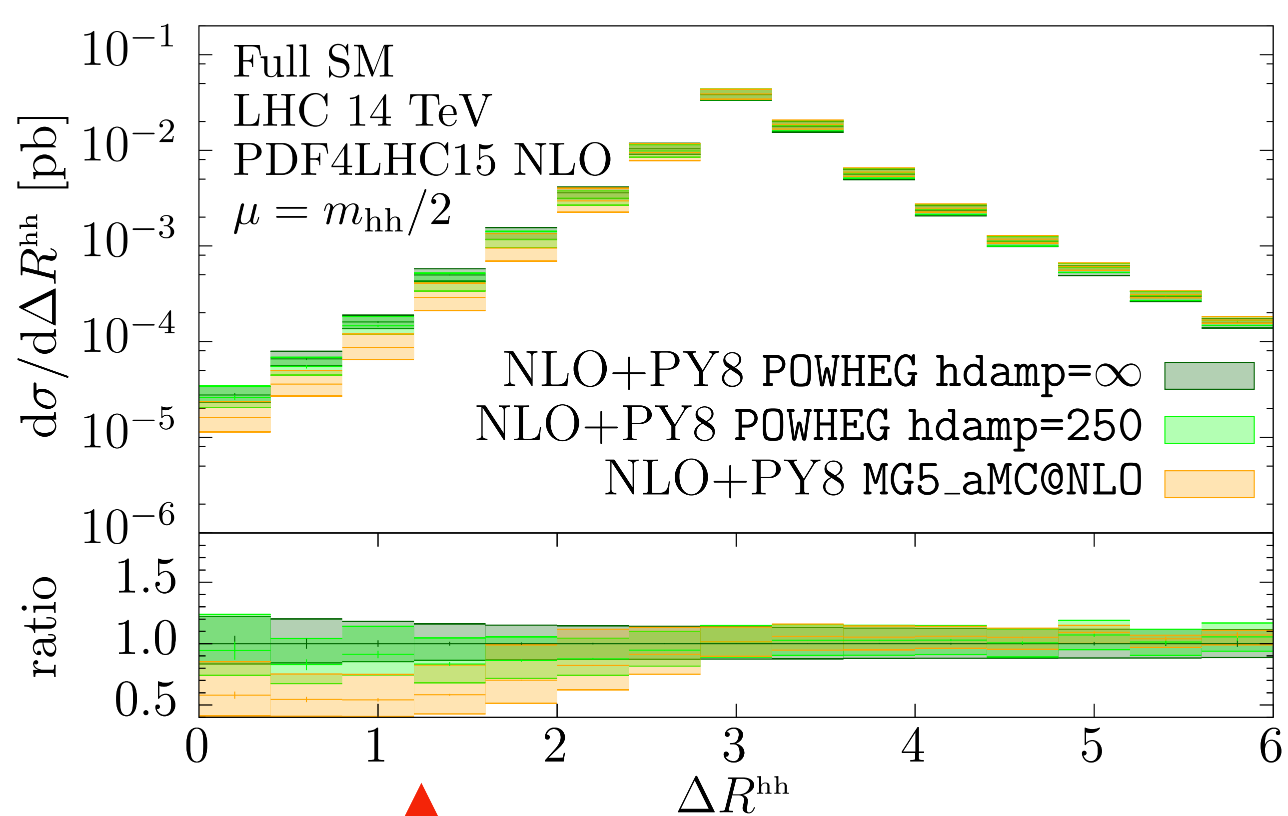
POWHEG



LO accurate

NLO accurate

POWHEG/MG5_aMC@NLO



Accordingly, matching scheme uncertainties larger for small radial separation

Conclusion

Higgs Boson Pair Production via Gluon Fusion

- Important measurement for probing the self coupling (HL-LHC era)
- Large ($K \approx 2$) NLO correction, deviates from Born Improved HEFT
  - -14% @ 14 TeV, -24% @ 100 TeV
- Distributions altered significantly
- NLO result interfaced to `POWHEG/MG5_aMC@NLO` for parton shower

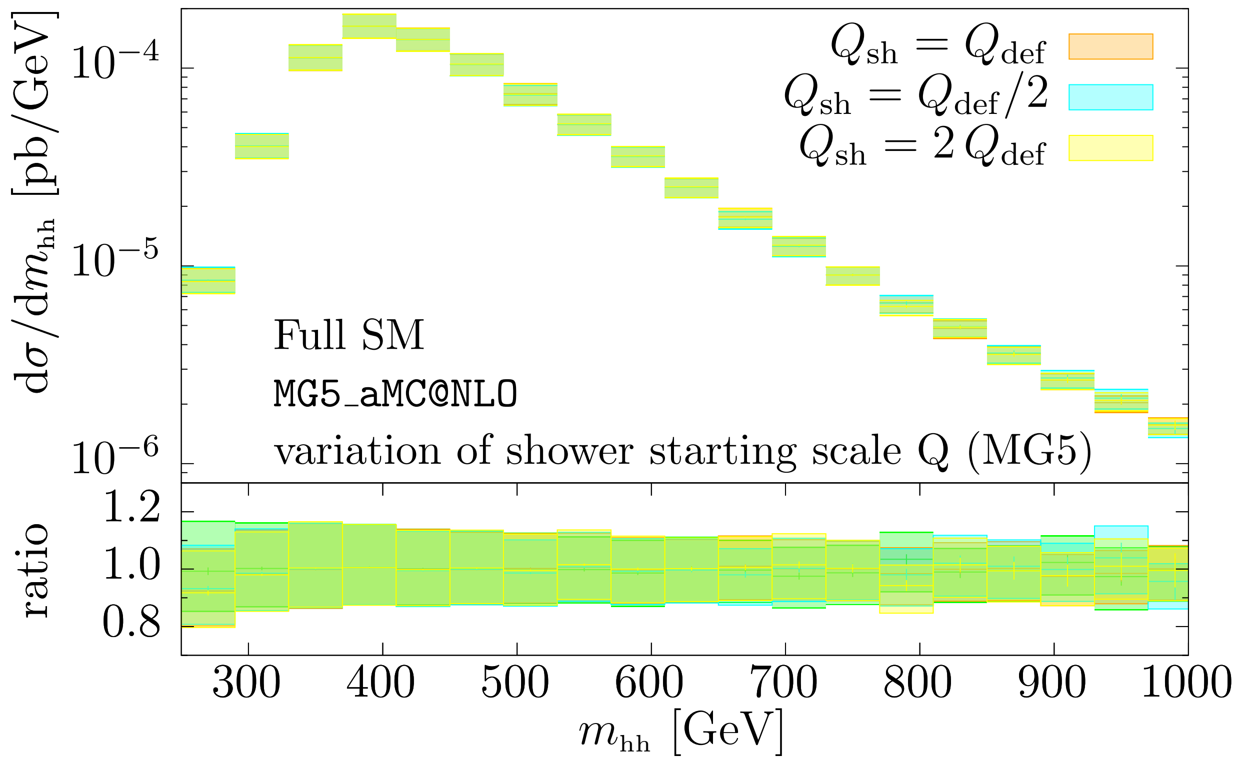
Ongoing Work…

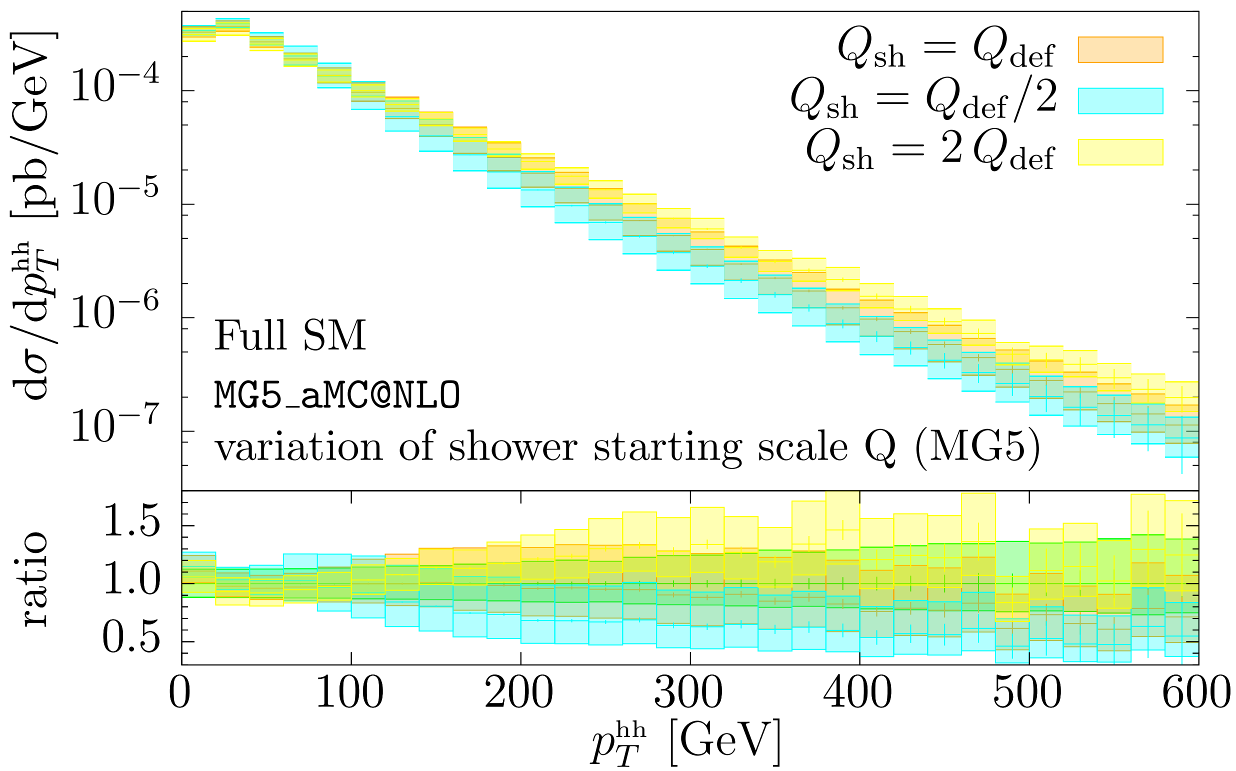
- Make result available within `SHERPA, HERWIG`
- Make fully differential/improved combination with NNLO HEFT
- Apply methods/framework to calculate other processes

Thank you for listening!

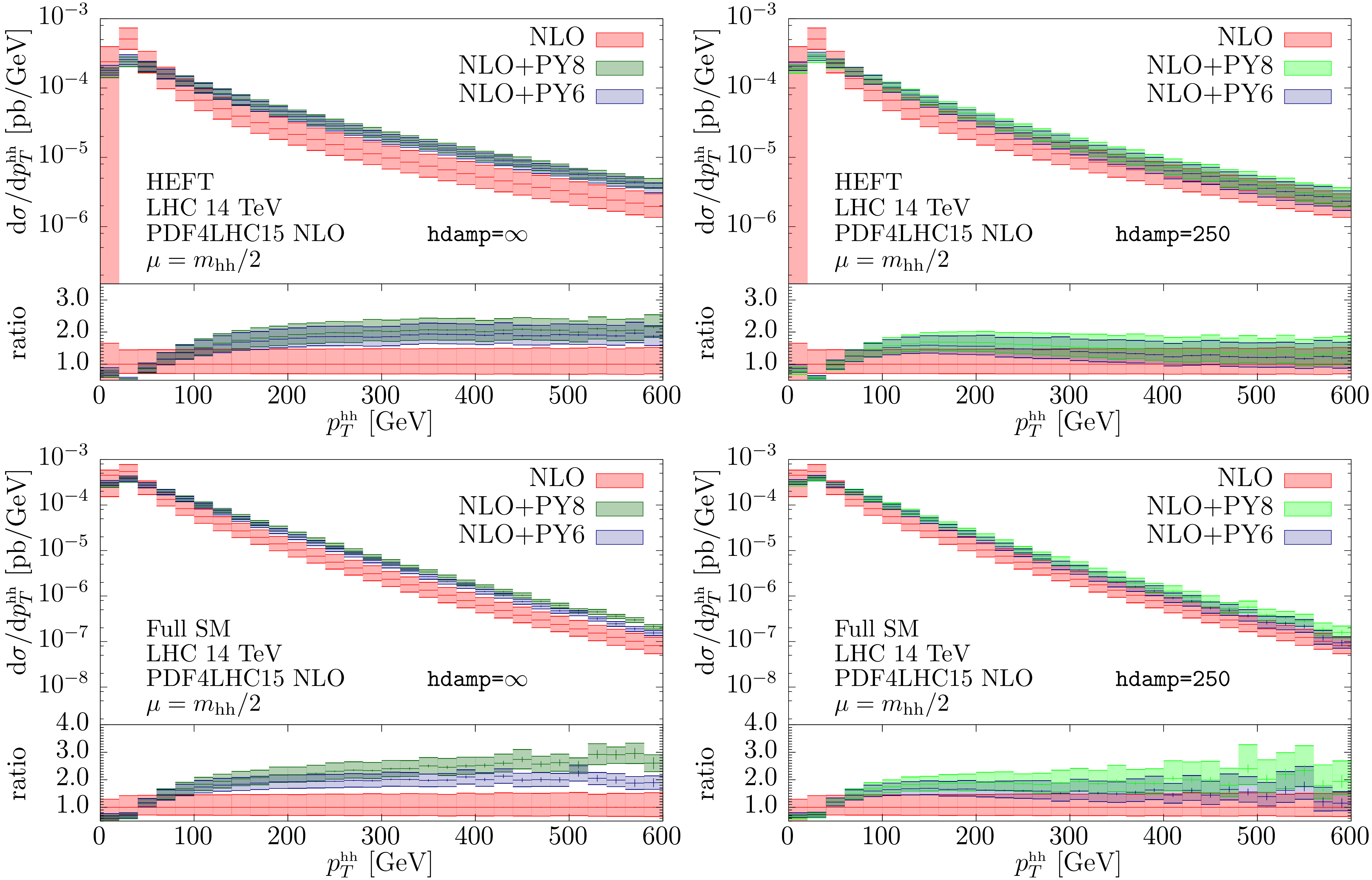
Backup

SM: MG5_aMC@NLO Shower Scale

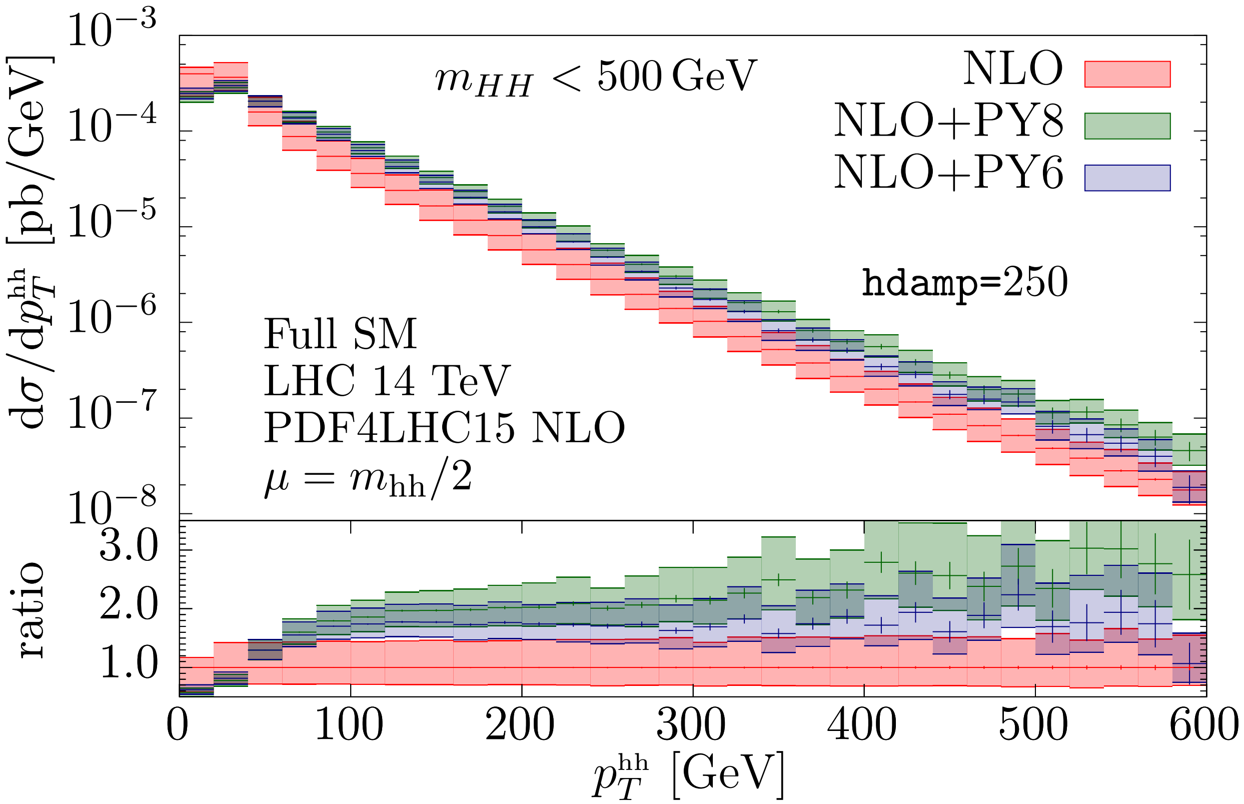


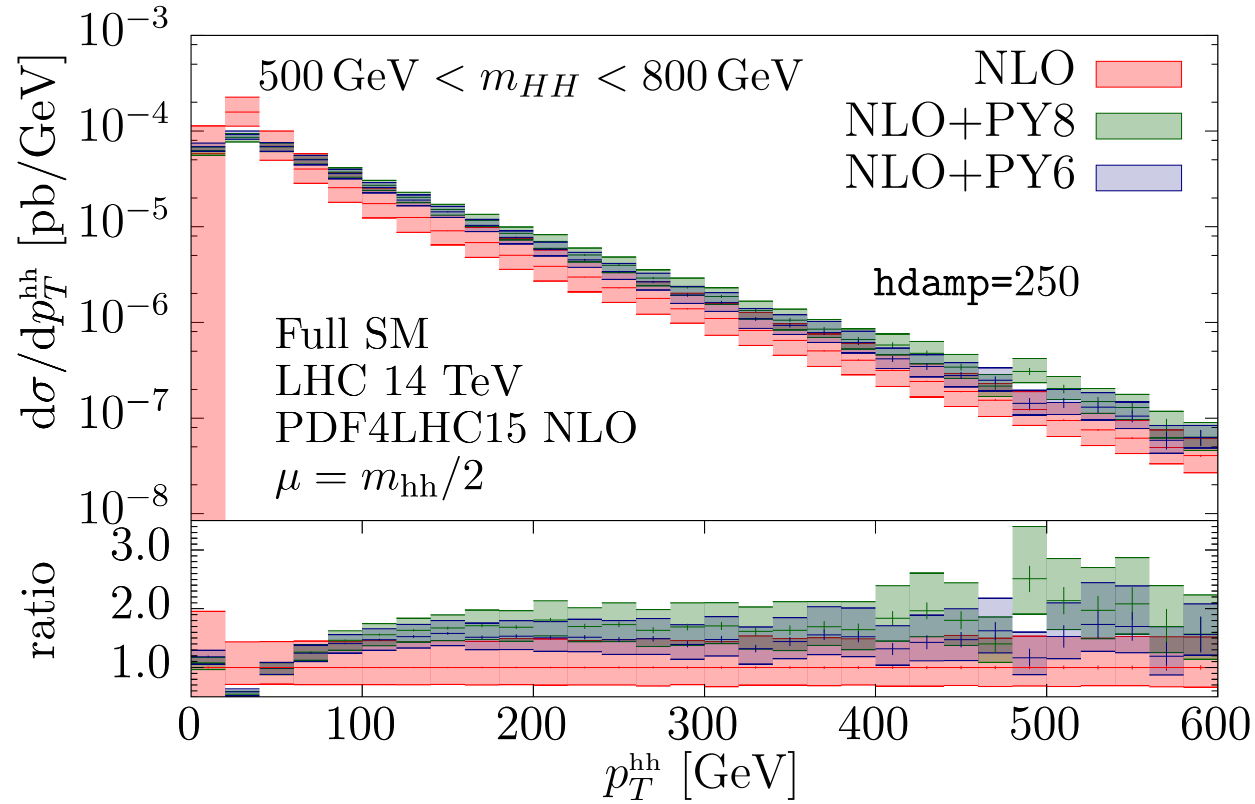


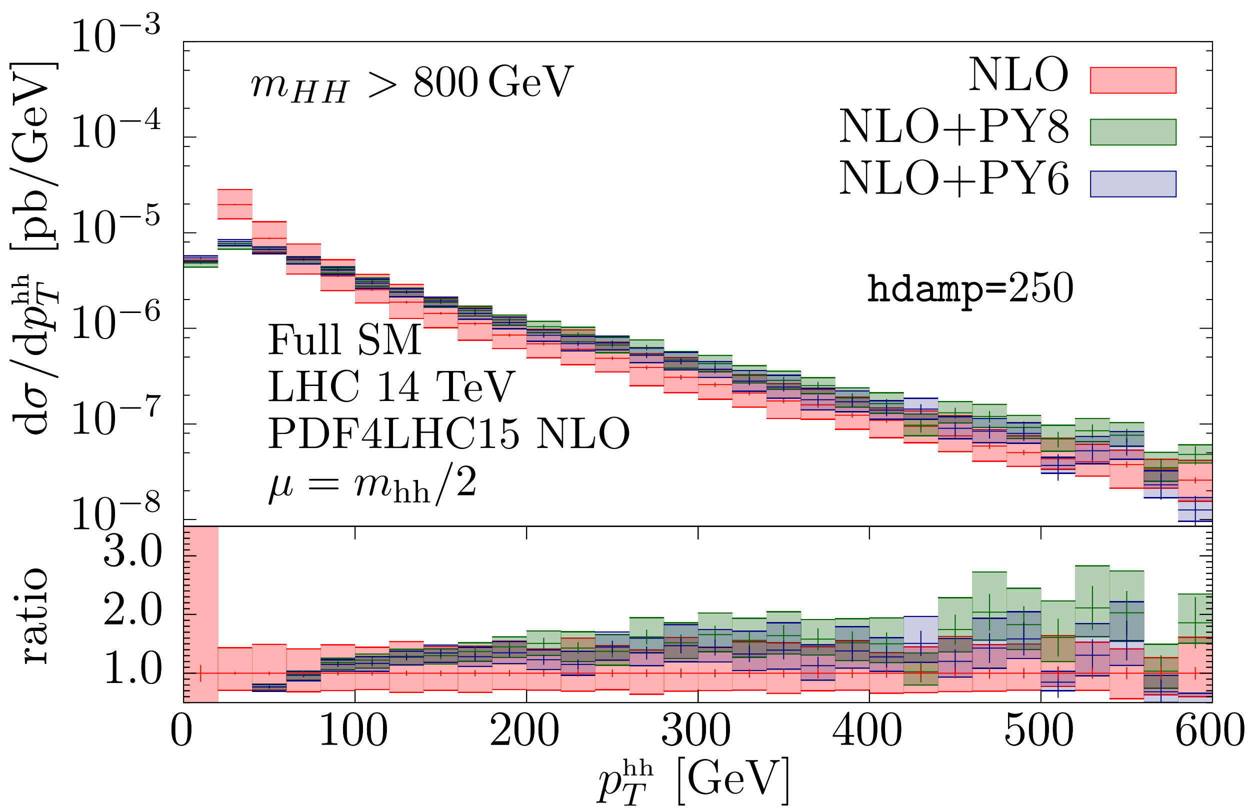
Pythia6 vs Pythia8



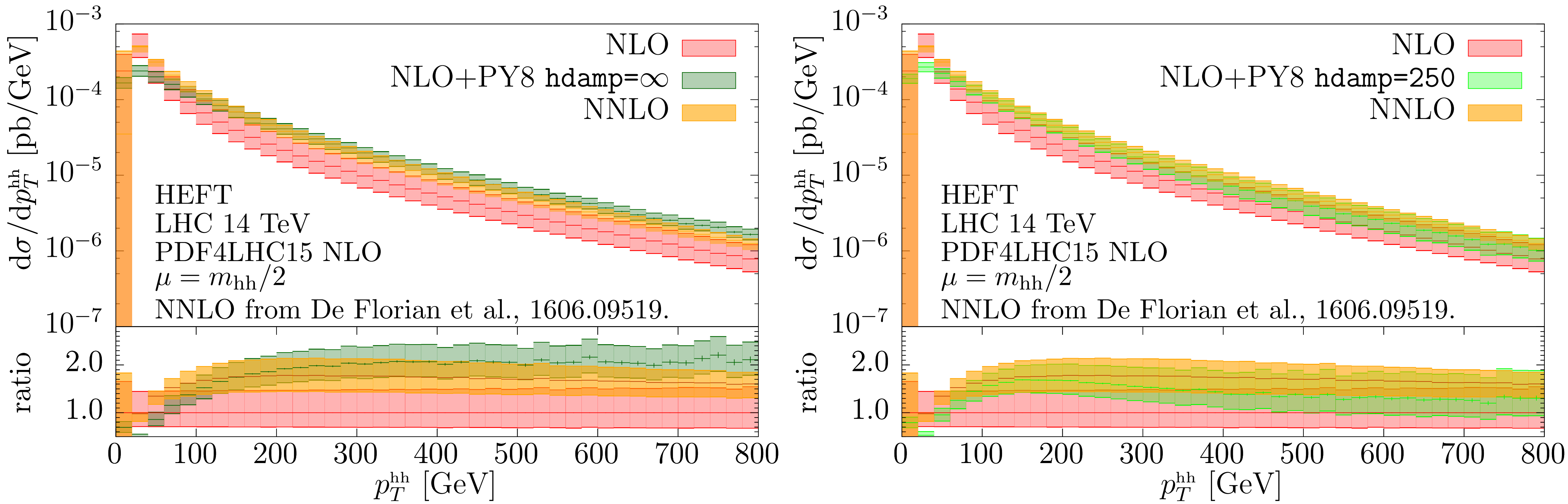
p_T^{hh} in bins of m_{hh}



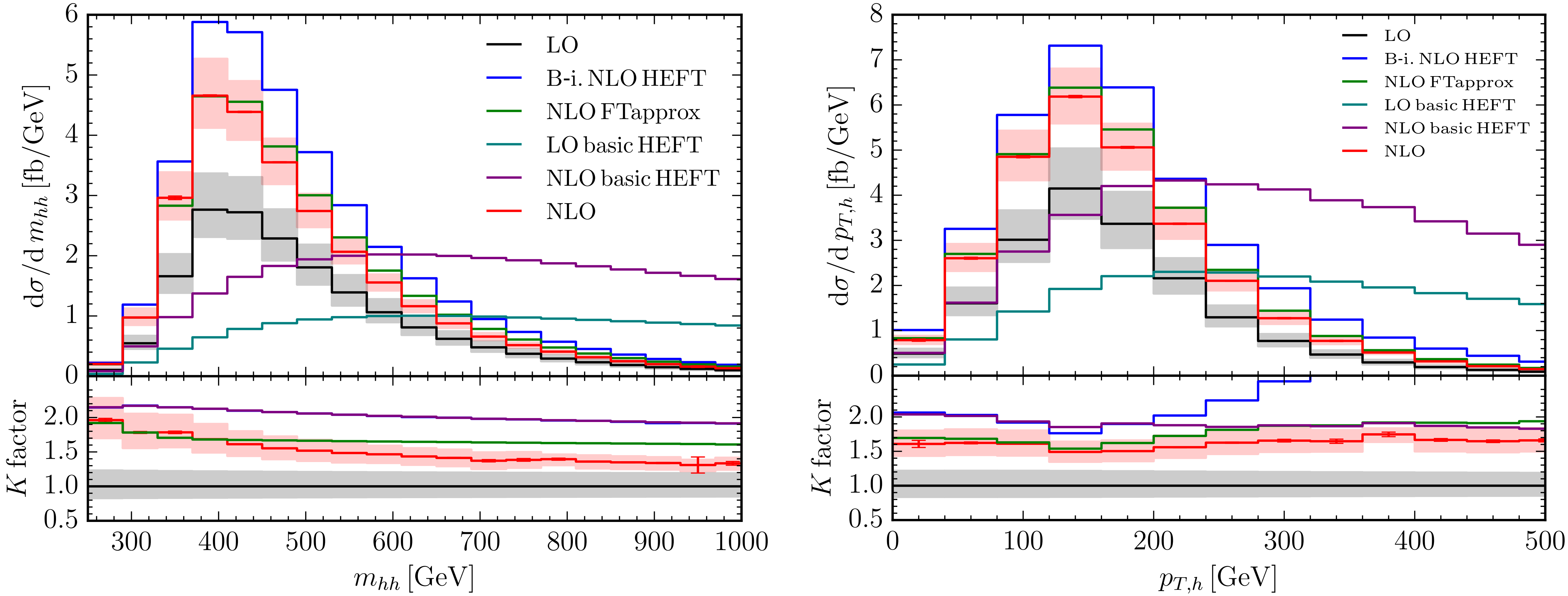




HEFT: Showered Results



Results: 100TeV



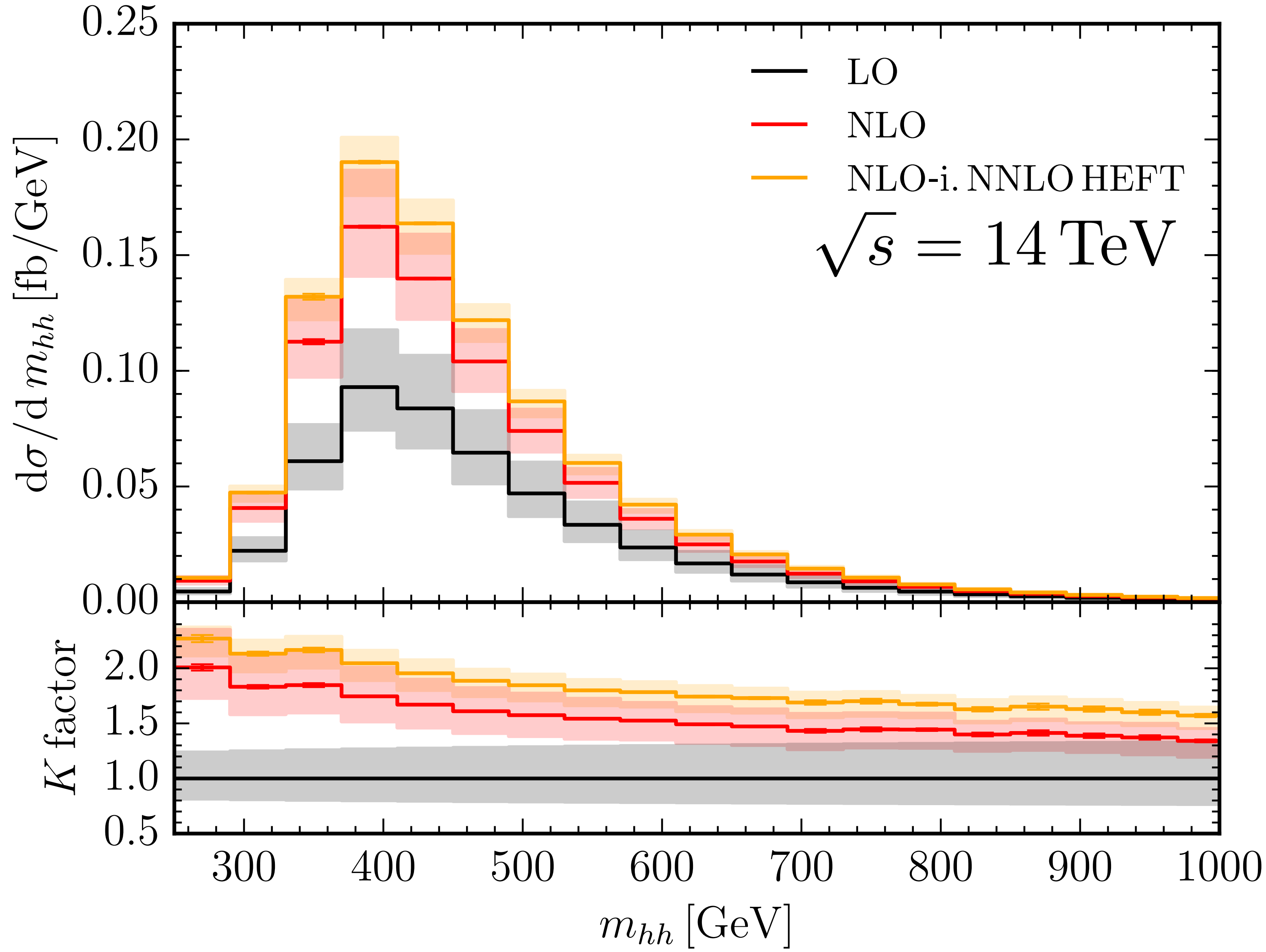
	σ_{LO} (fb)	σ_{NLO} (fb)
B.I. HEFT	–	$1511^{+16.0\%}_{-13.0\%}$
FTapprox	–	$1220^{+11.9\%}_{-10.7\%}$
Full Theory	$731.3^{+20.9\%}_{-15.9\%}$	$1149^{+10.8\%}_{-10.0\%}$

HEFT overestimates by 32%

FTap. overestimates by 6%

Difference between full theory and HEFT more pronounced

NLO Improved NNLO HEFT



First attempt to combine full NLO

Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Zirke 16

+

NNLO HEFT (Differential)

de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16

$$\frac{\mathrm{d}\sigma^{\mathrm{approx.}}}{\mathrm{d}m_{hh}} \equiv \frac{\mathrm{d}\sigma_{\mathrm{NLO}}}{\mathrm{d}m_{hh}} \times \frac{\mathrm{d}\sigma^{\mathrm{HEFT}}_{\mathrm{NNLO}}/\mathrm{d}m_{hh}}{\mathrm{d}\sigma^{\mathrm{HEFT}}_{\mathrm{NLO}}/\mathrm{d}m_{hh}}$$

Bin-by-bin rescaling of NLO by NNLO HEFT K-factor

$$\sigma^{approx.} = 38.67^{+5.2\%}_{-7.6\%}$$

Checks

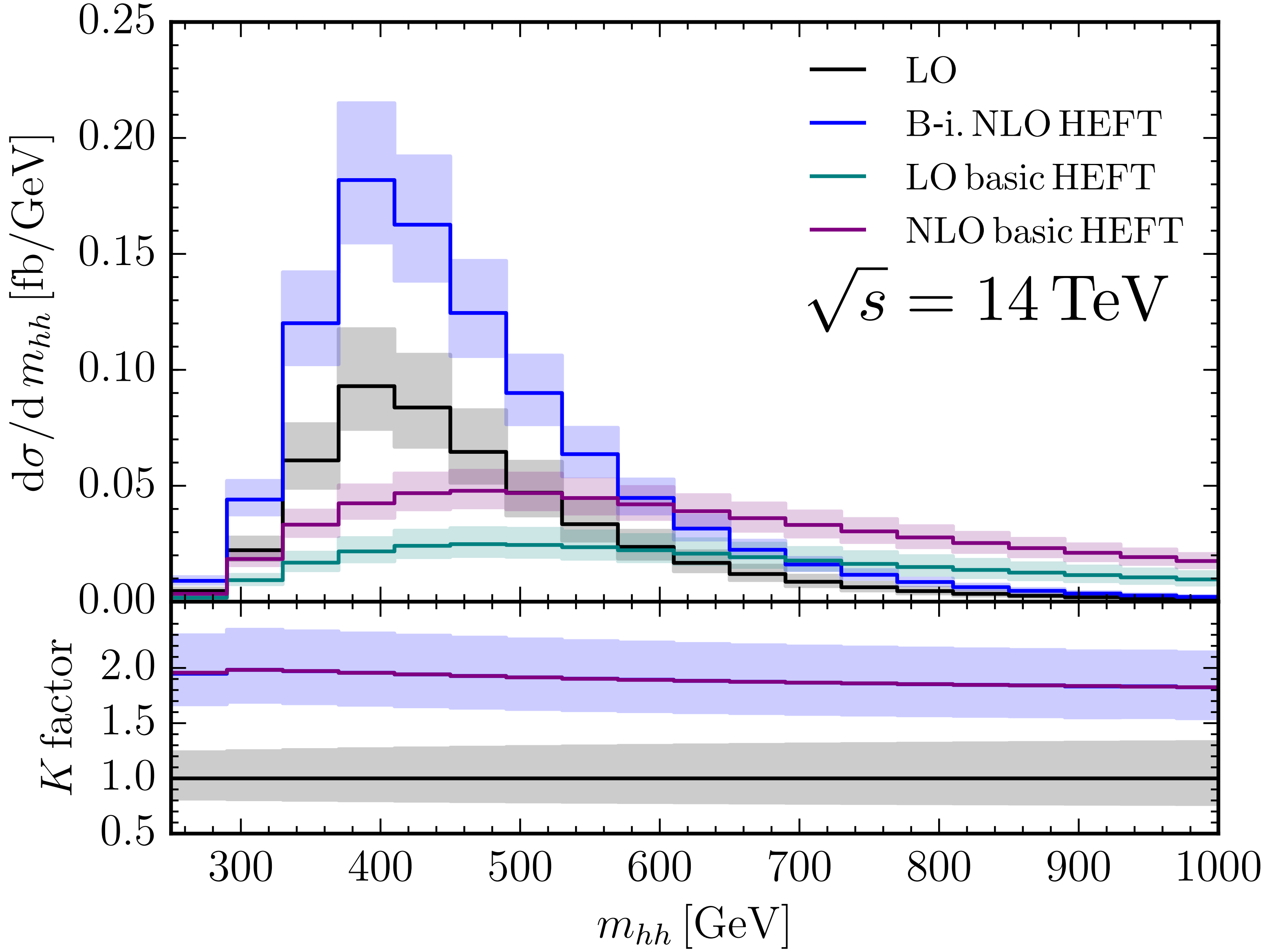
Real Emission / Subtraction Terms

- Independence of dipole-cut $\alpha_{\rm cut}$ parameter Nagy 03
- Agreement with literature Maltoni, Vryonidou, Zaro 14
- Agreement with FKS (`POWHEG/MG5_amc@NLO`)
  Frixione, Kunszt, Signer 96; Nason 04; Frixione et al 07; Alioli et al. 10; J. Alwall et al. 14

Virtual Corrections

- Two calculations of amplitude up to reduction
- Amplitude result invariant under $t \leftrightarrow u$
- Pole cancellation
- Mass renormalization using two methods:
  counter-term insertion vs. calculating $\mathrm{d}\mathcal{M}^{\rm LO}/\mathrm{d}m_T^2$ numerically
- Agreement of contributions $gg \to H \to HH$ with `SusHi` Harlander, Liebler, Mantler 13,16
- Convergence of $1/m_T$ expansion to full result where agreement is expected

LO & Born Improved NLO HEFT



PDF4LHC15_nlo_30_pdfas
$m_H = 125$ GeV
$m_T = 173$ GeV
Uncertainty:
$\mu_0 = \frac{m_{HH}}{2}$
$\mu_{R,F} \in \left[\frac{\mu_0}{2}, 2\mu_0\right] \quad (7-\text{point})$

LO: HEFT describes distributions poorly, underestimates XS @ LO by 14%

	σ_{LO} (fb)	σ_{NLO} (fb)
HEFT	$17.07^{+30.9\%}_{-22.2\%}$	$31.93^{+17.6\%}_{-15.2\%}$
B.I. HEFT	$19.85^{+27.6\%}_{-20.5\%}$	$38.32^{+18.1\%}_{-14.9\%}$
Full Theory	$19.85^{+27.6\%}_{-20.5\%}$	$\ldots$

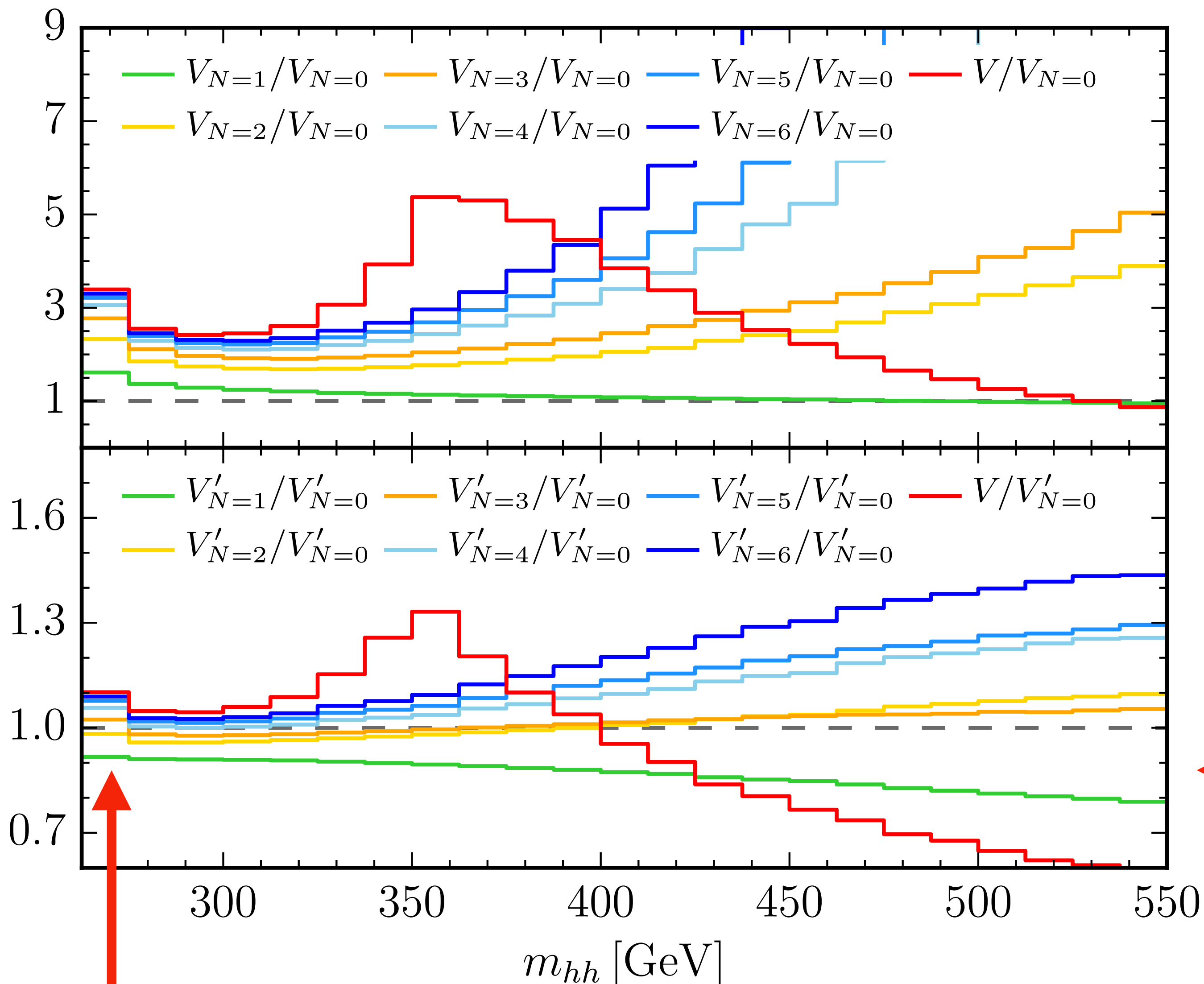
NLO: HEFT indicates

$$K \approx 2$$

Comparison to Expansion

Can compare just virtual ME to expansion:

$$d\hat{\sigma}_N = \sum_{\rho=0}^{N} d\hat{\sigma}^{(\rho)} \left(\frac{\Lambda}{m_t}\right)^{2\rho} \qquad \Lambda \in \left\{\sqrt{\hat{s}}, \sqrt{\hat{t}}, \sqrt{\hat{u}}, m_h\right\}$$



$$V_N = \left(d\hat{\sigma}_N^V + d\hat{\sigma}_N^{LO} \otimes \mathbf{I}\right)$$

$$V'_N = V_N \frac{d\hat{\sigma}^{LO}}{d\hat{\sigma}_N^{LO}}$$

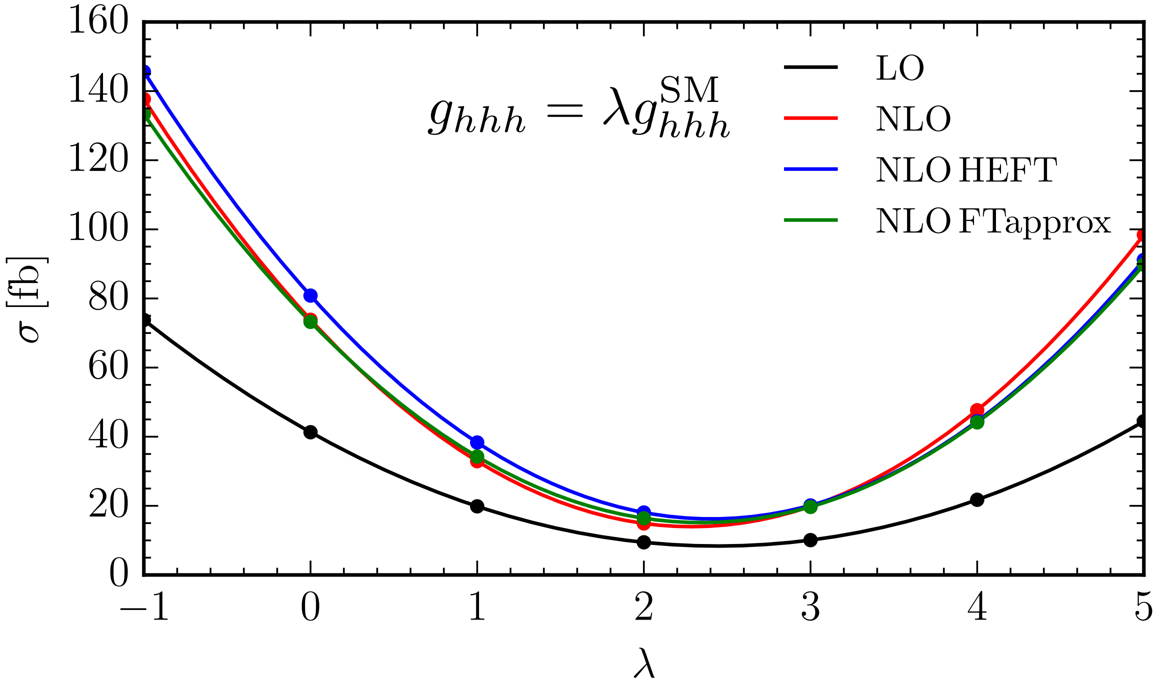
Rescaled better but does not describe full above threshold

Expansion converges on full $\sqrt{\hat{s}} < 2m_T$

$V_{N\geq4}$ thanks to J. Hoff

Grigo, Hoff, Steinhauser 15

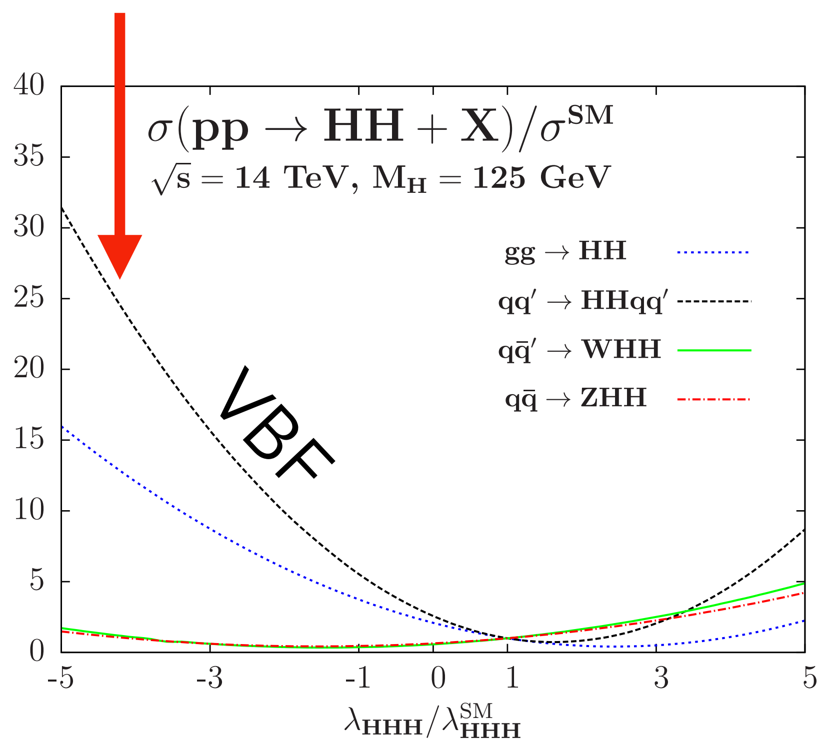
Triple-Higgs Coupling Sensitivity



SM: Destructive interference between g_{hhh} and y_T^2 contrib.

Distributions: Can help to distinguish between λ values

VBF: More sensitive (but small XS)



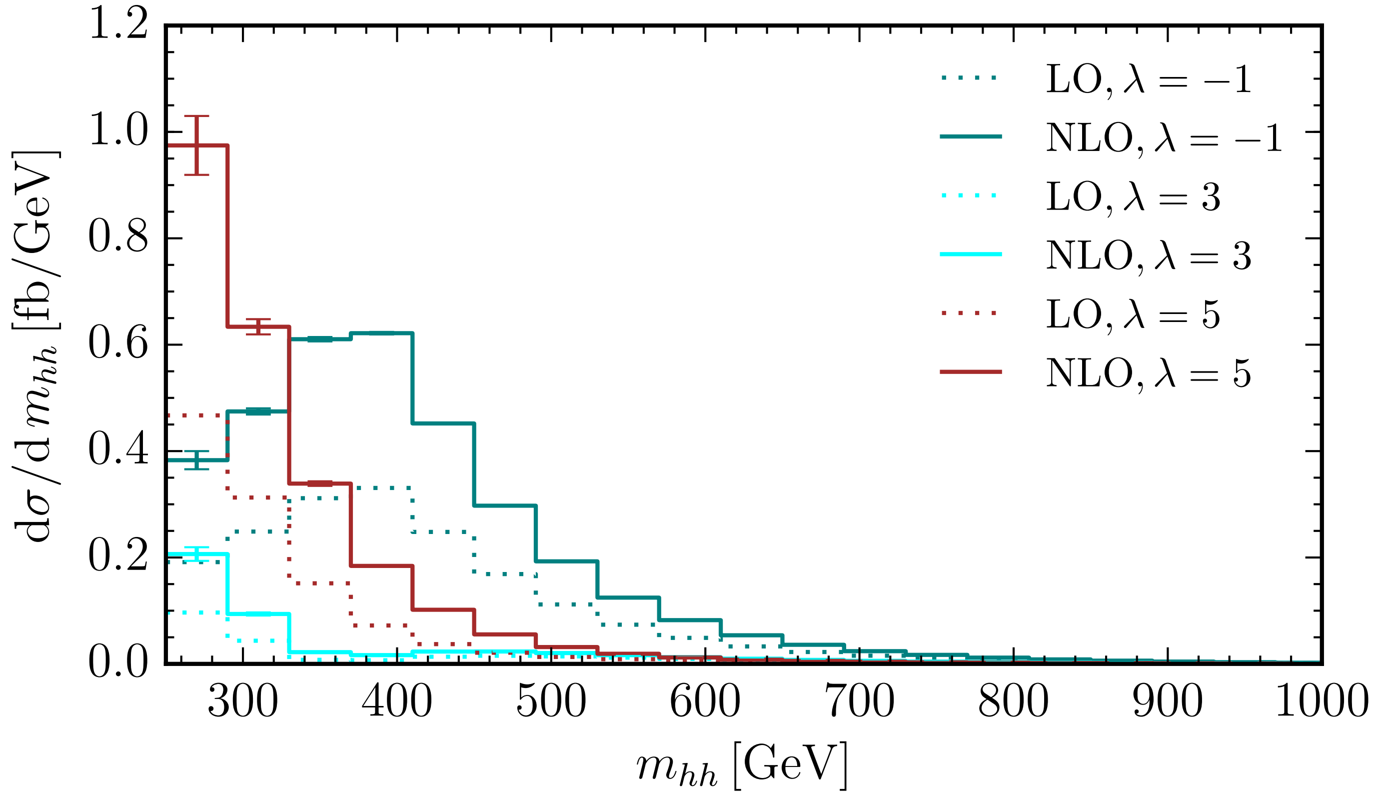
Baglio, Djouadi, Gröber, Mühlleitner, Quevillon, Spira 12

Can increase sensitivity to HH:

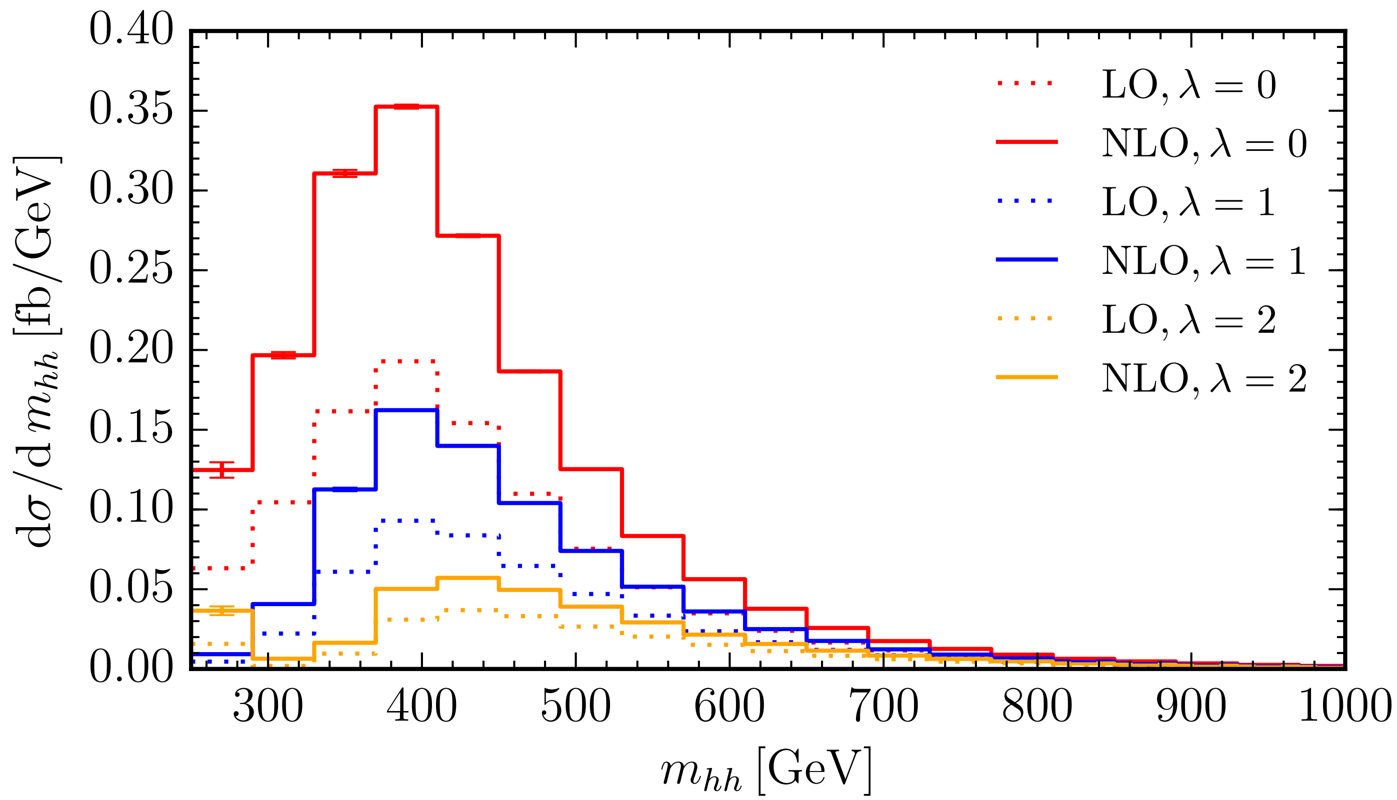
- $p_{T,jet}^{min}$ cut
- $\sigma(gg \to HH)/\sigma(gg \to H)$
- Multivariate $b\bar{b}b\bar{b}$

Barr, Dolan, Englert, Ferreira de Lima, Spannowsky 15; Mangano et al. 16; Goertz, Papaefstathiou, Yang, Zurita 13; Behr, Bortoletto, Frost, Hartland, Issever, Rojo 15

Lambda Variation

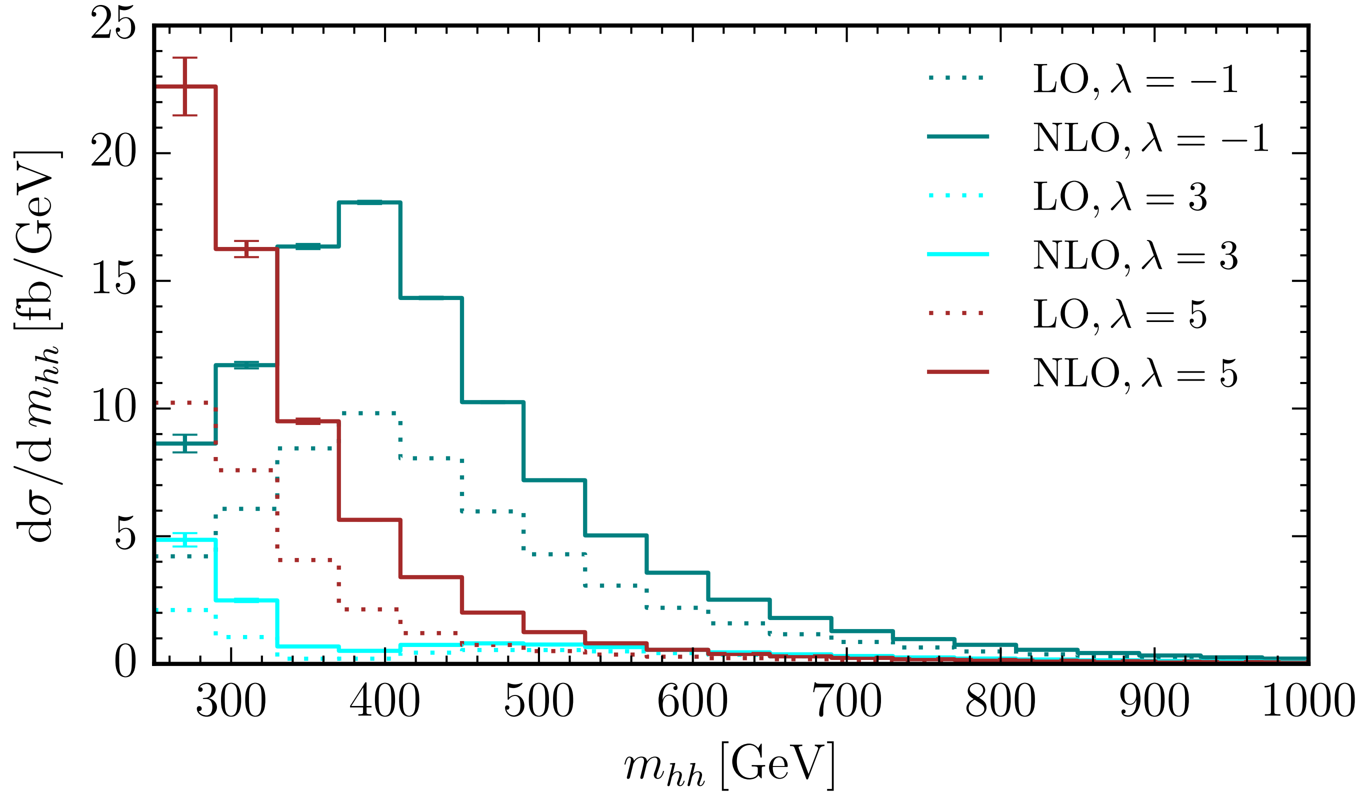


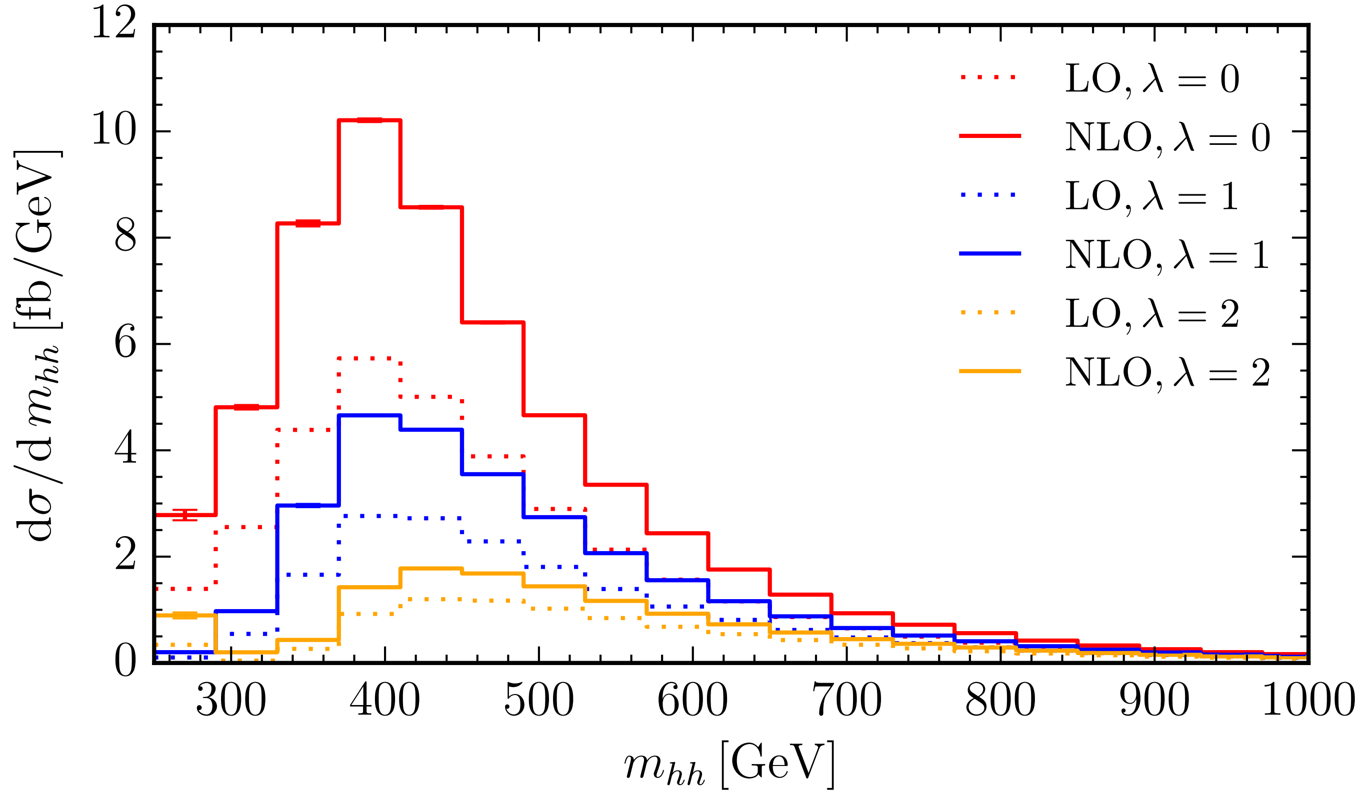
$\sqrt{s} = 14\,\mathrm{TeV}$



Lambda Variation

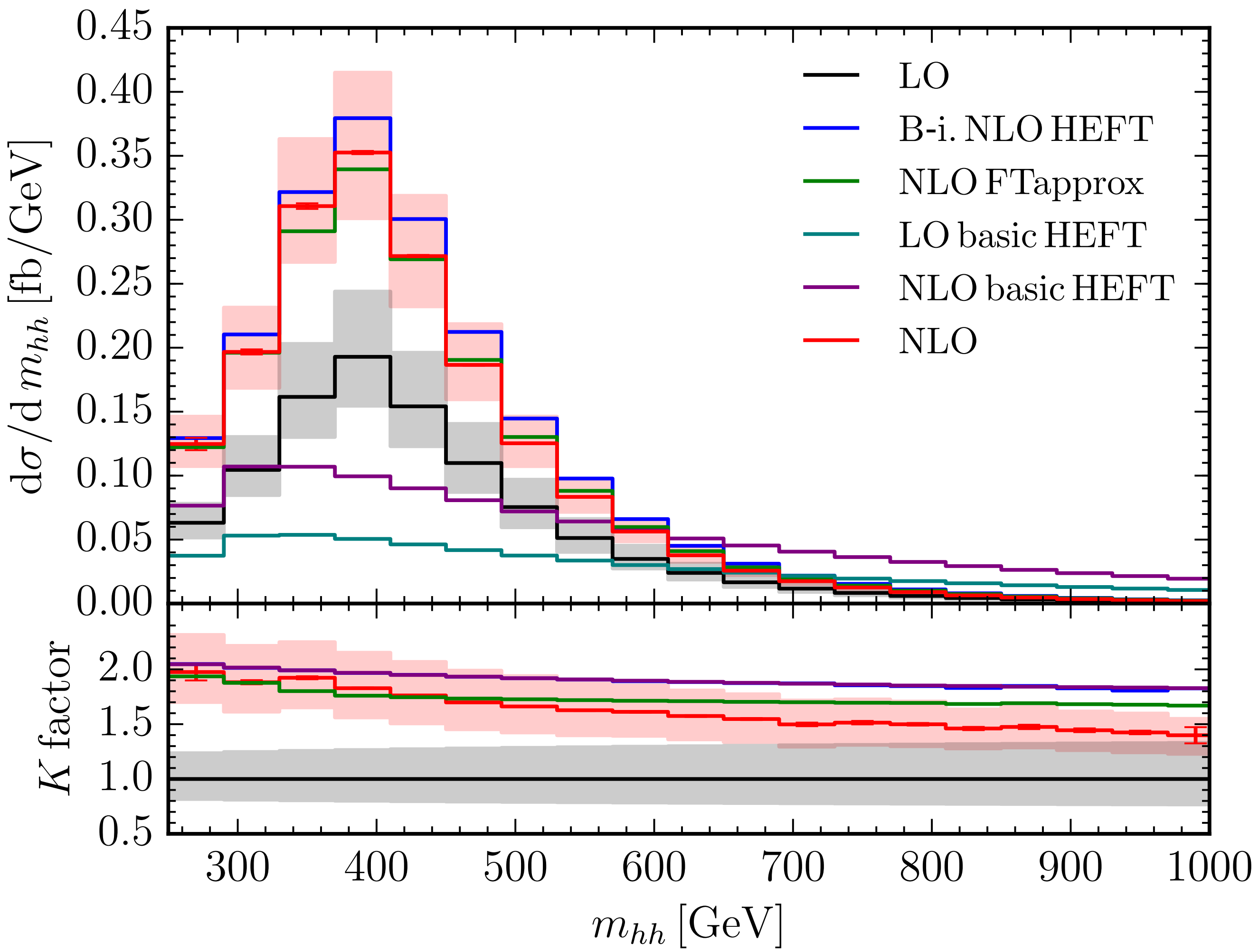
$\sqrt{s} = 100\,\mathrm{TeV}$

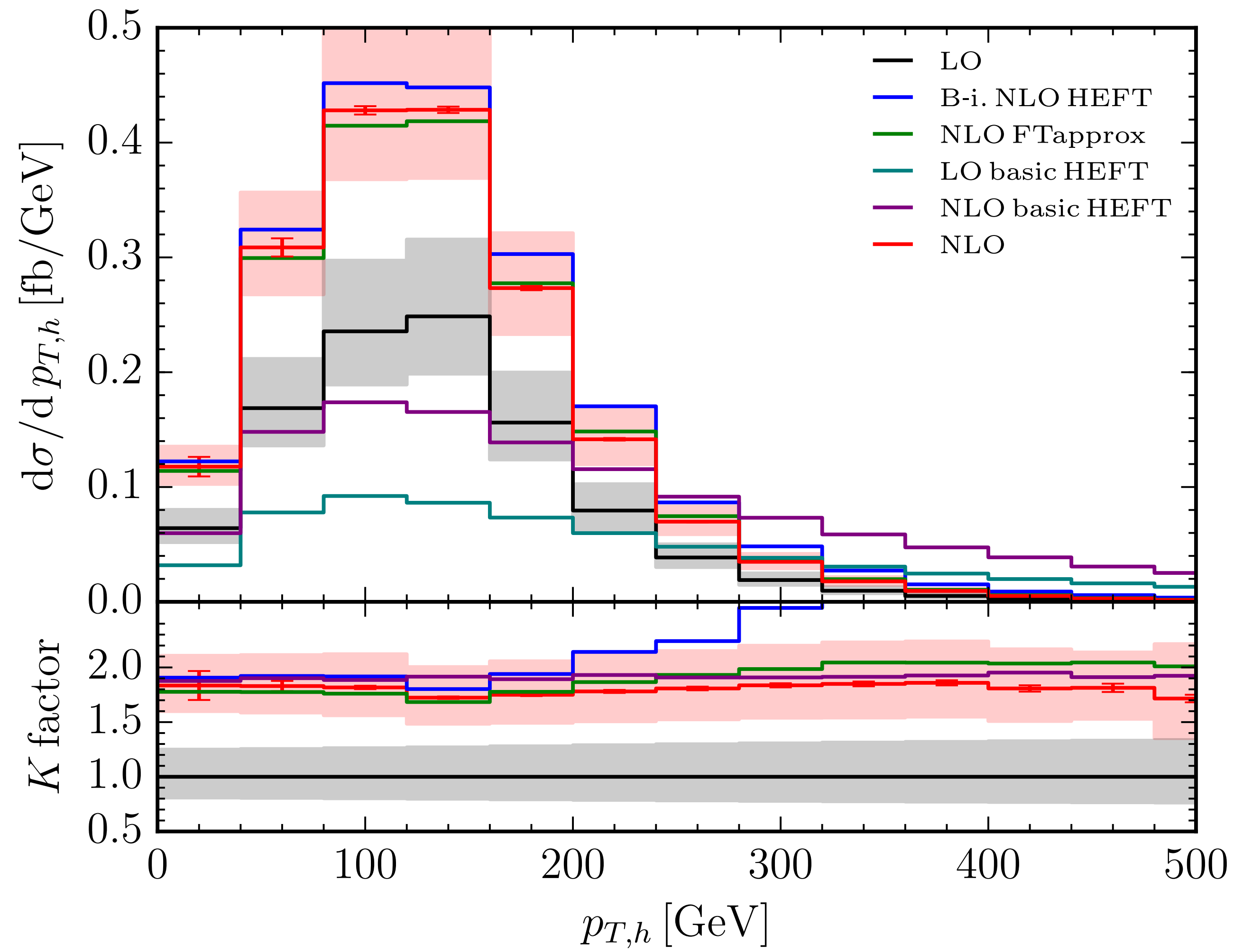




Lambda 0 x SM

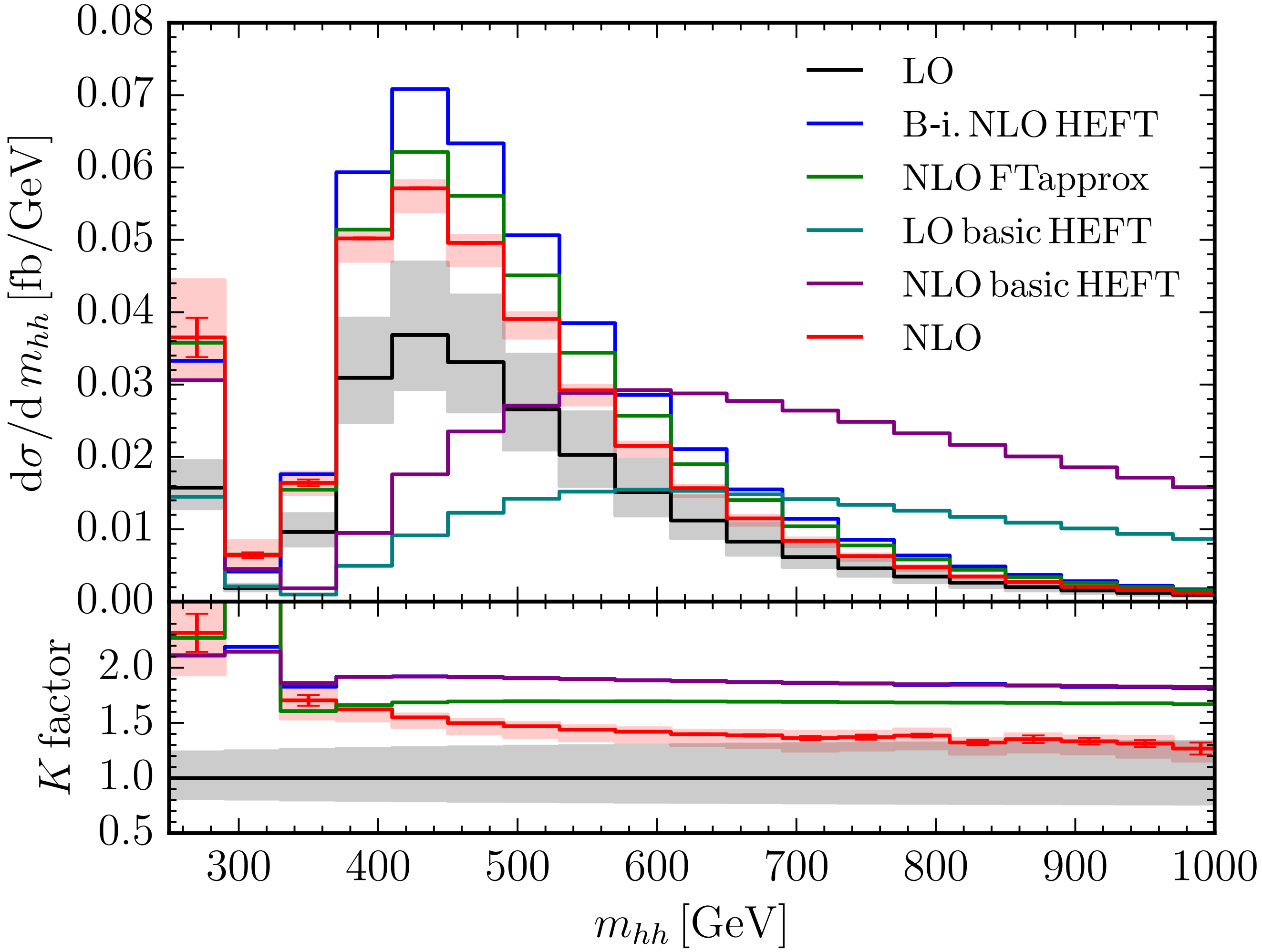
$\sqrt{s} = 14\,\mathrm{TeV}$

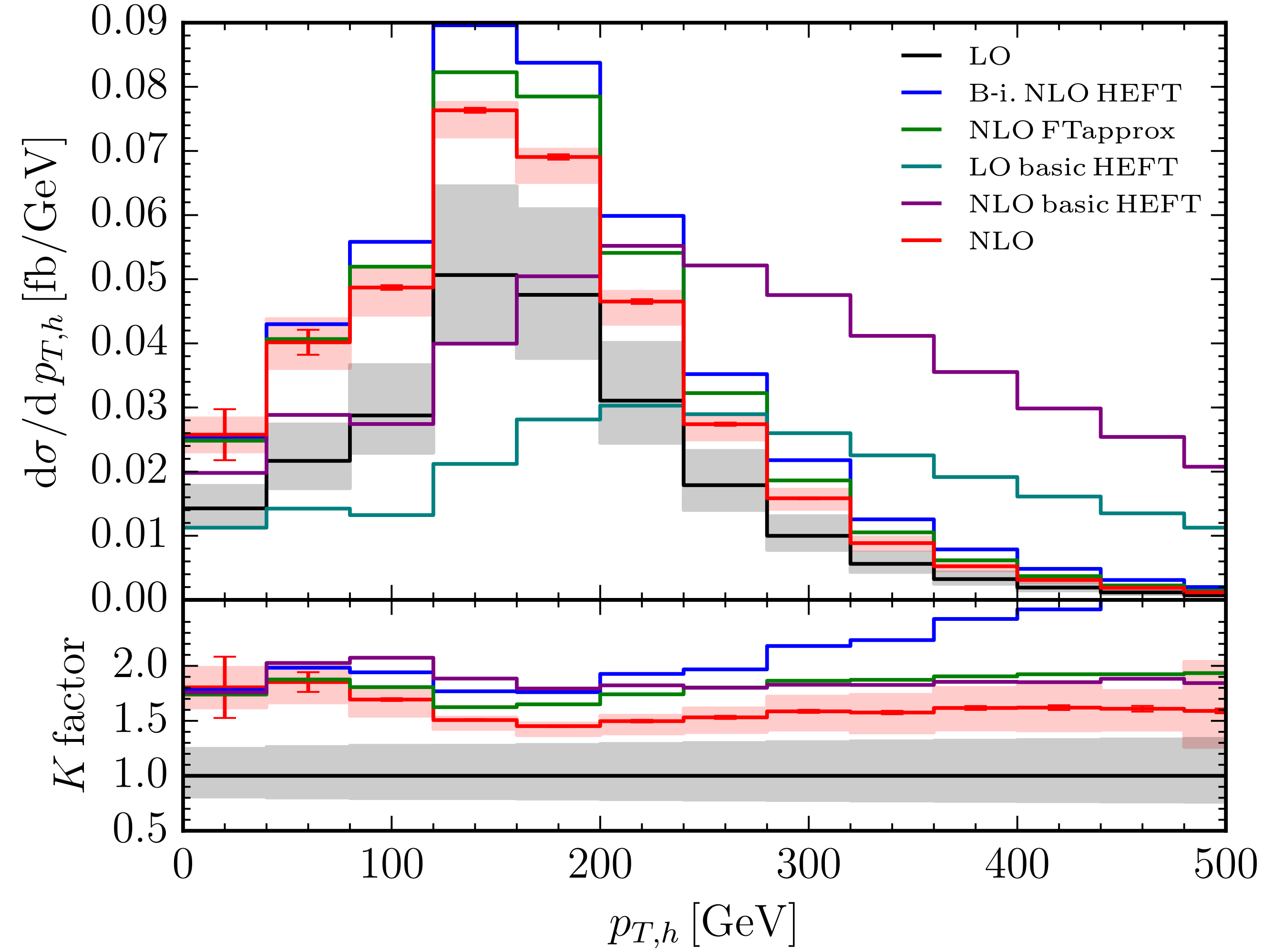




Lambda 2 x SM

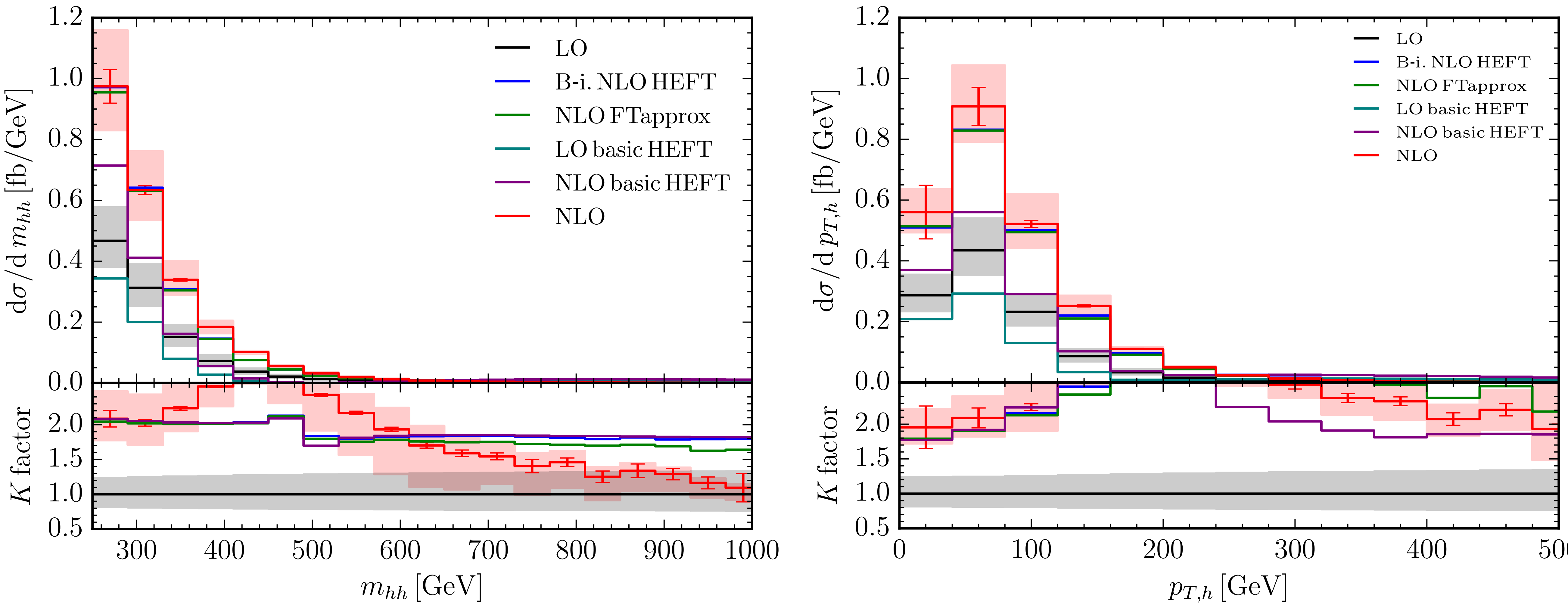
$\sqrt{s} = 14\,\mathrm{TeV}$





Lambda 5 x SM

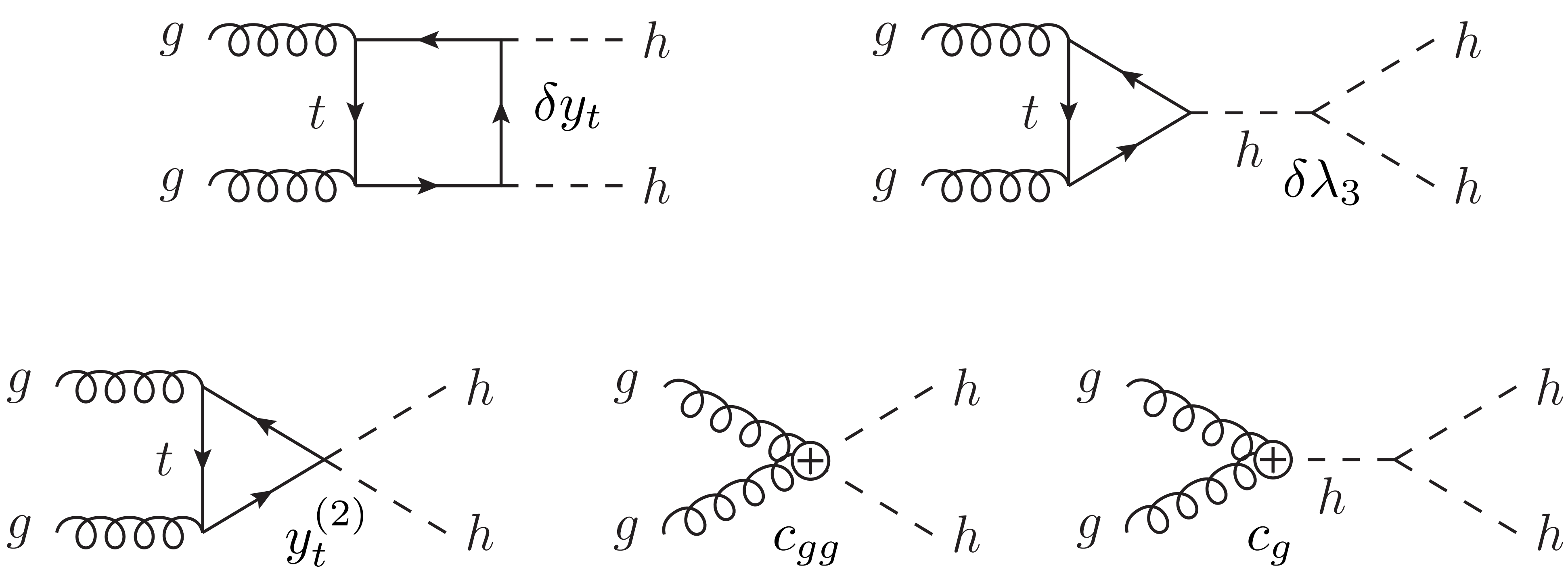
$\sqrt{s} = 14\,\mathrm{TeV}$



BSM EFT

Note: Just varying λ: one ``direction'' in EFT parameter space

Parametrise **non-resonant** new physics with EFT (5 parameters):



Azatov, Contino, Panico, Son 15;
(Cluster analysis) Dall'Osso, Dorigo, Gottardo, Oliveira, Tosi, Goertz 15;
+ Carvalho, Manzano, Dorigo, Gouzevich 16;
(B.I. HEFT) Gröber, Mühlleitner, Spira, Streicher 15;
Buchalla et al. 15;

12 representative ``clusters''

Top-quark Width Effects

Total XS @ LO: reduced by 2% by including top-quark width

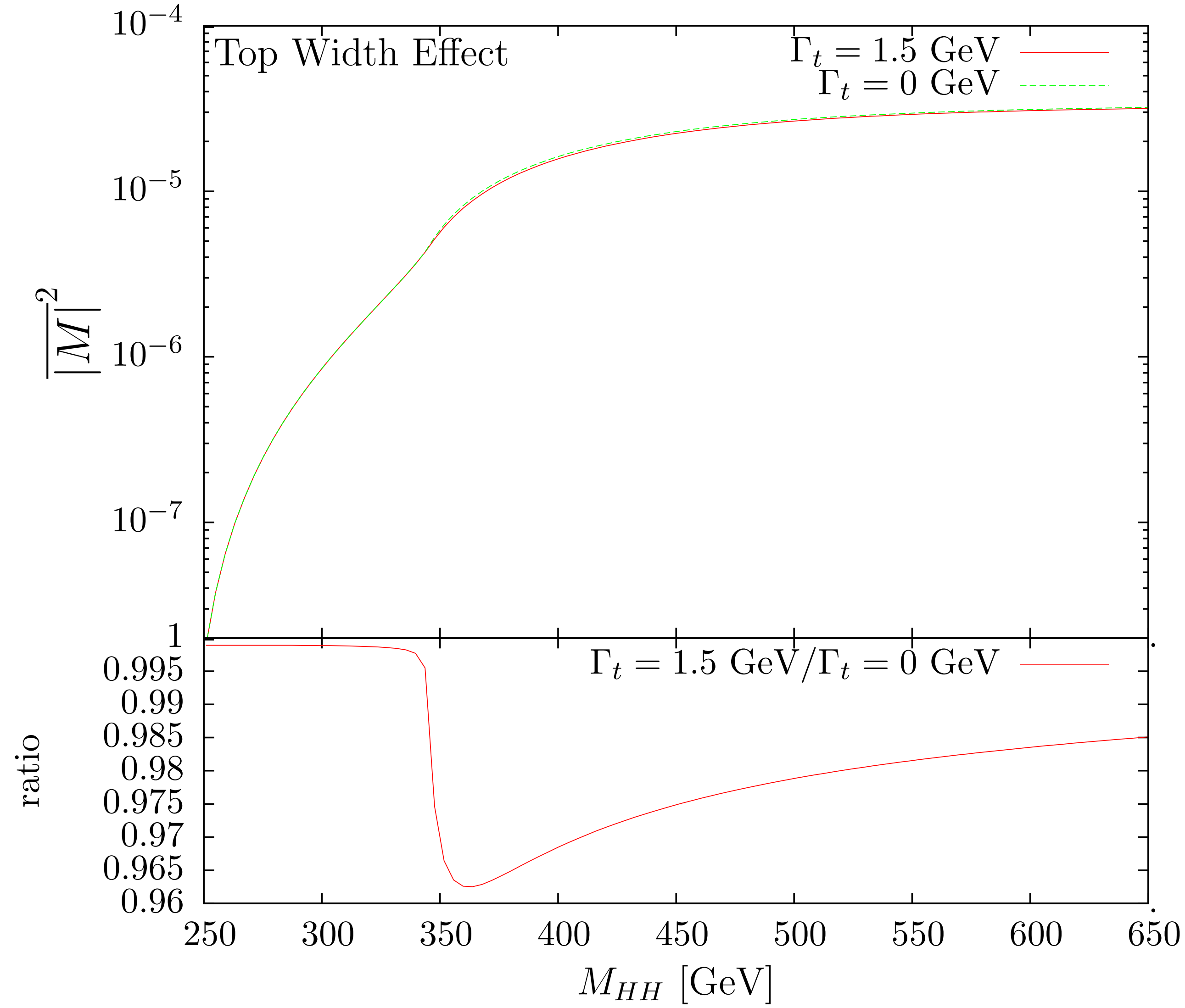
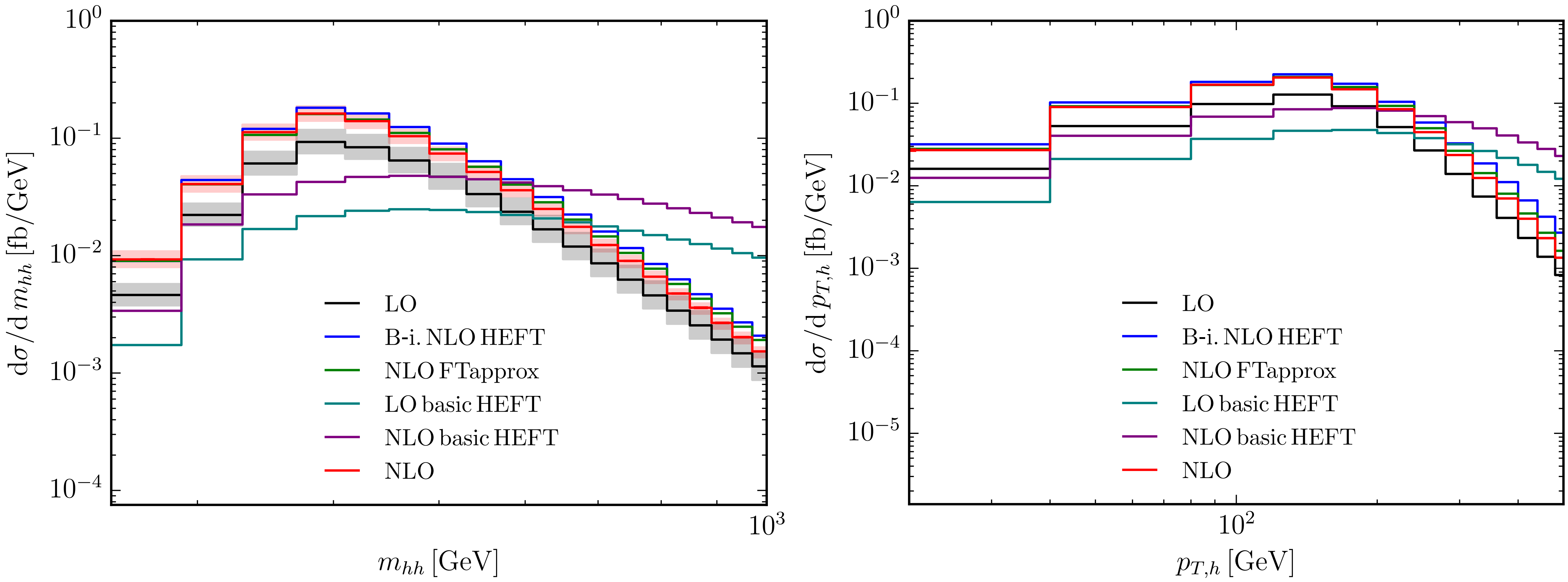


Figure 3: Top width effect on the one-loop (Born) matrix element squared for $gg \to HH$. The results for $\Gamma_t = 0$ and 1.5 GeV are shown along with the corresponding ratio.

Maltoni, Vryonidou, Zaro 14

Scaling

$\sqrt{s} = 14\,\mathrm{TeV}$



Higgs Self Coupling Constraints

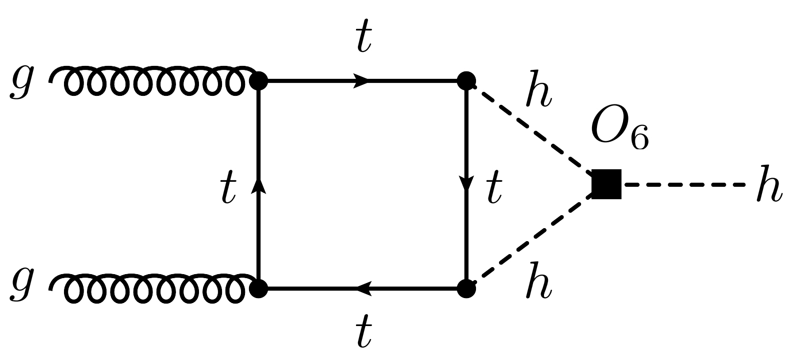
HH extremely challenging to measure: 95% Exclusion $\lesssim 30 \cdot \sigma_{\mathrm{SM}}$

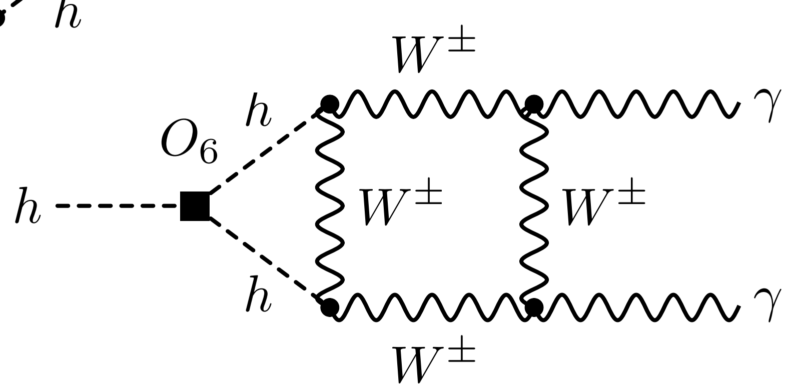
ATLAS PRD 94 (052002) 2016; CMS PRD 94 (052012) 2016

Several other promising ideas to obtain competitive/ complementary limits on deviation of self coupling from SM:

Electroweak corrections to single H production (also VBF, VH)

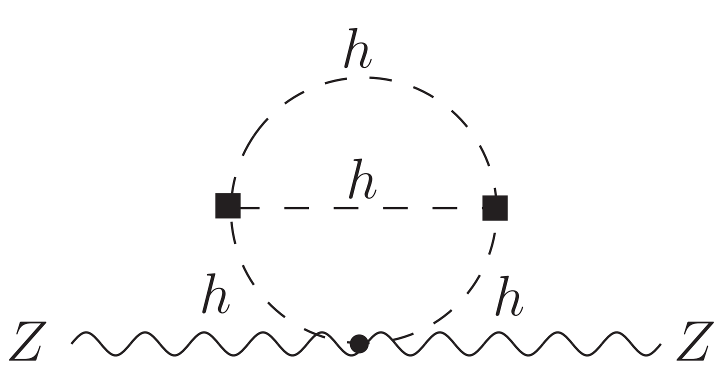
Gorbahn, Haisch 16;
Bizoń, Gorbahn, Haisch, Zanderighi 16;
Degrassi, Giardino, Maltoni, Pagani 16;





Modification of precision EW observables (EW oblique corrections) S, T

Degrassi, Fedele, Giardino 17;
Kribs, Maier, Rzehak, Spannowsky, Waite 17;



Details of Calculation

Form Factor Decomposition

Expose tensor structure: $\mathcal{M} = \epsilon^1_\mu \epsilon^2_\nu \mathcal{M}^{\mu\nu}$

Form Factors (Contain integrals)

$$\boxed{\mathcal{M}^{\mu\nu} = F_1(\hat{s}, \hat{t}, m_h^2, m_t^2, D) T_1^{\mu\nu} + F_2(\hat{s}, \hat{t}, m_h^2, m_t^2, D) T_2^{\mu\nu}}$$

(Tensor) Basis, built from external momenta & metric

Choose:

$$\mathcal{M}^{++} = \mathcal{M}^{--} = -F_1$$
$$\mathcal{M}^{+-} = \mathcal{M}^{-+} = -F_2$$

Self-coupling diagrams are 1PR by cutting a scalar propagator
By angular momentum conservation they contribute only to F_1

$$T_1^{\mu\nu} = g^{\mu\nu} - \frac{p_2^\mu p_1^\nu}{p_1 \cdot p_2} \qquad p_T^2 = \frac{ut - m_H^4}{s}$$

$$T_2^{\mu\nu} = g^{\mu\nu} + \frac{m_H^2 p_2^\mu p_1^\nu}{p_T^2 p_1 \cdot p_2} - \frac{2 p_1 \cdot p_3 p_2^\mu p_3^\nu}{p_T^2 p_1 \cdot p_2} - \frac{2 p_2 \cdot p_3 p_3^\mu p_1^\nu}{p_T^2 p_1 \cdot p_2} + \frac{2 p_3^\mu p_3^\nu}{p_T^2}$$

Glover, van der Bij 88

Form Factor Decomposition (II)

Construct Projectors:

No Integrals

$$P_j^{\mu\nu} = \sum_{i=1}^{2} B_{ji}(\hat{s}, \hat{t}, m_H^2, d) T_i^{\mu\nu}$$

Same Basis as amplitude

Such that:

$$P_{1\mu\nu}\mathcal{M}^{\mu\nu} = F_1$$

$$P_{2\mu\nu}\mathcal{M}^{\mu\nu} = F_2$$

Separately calculate the contraction of each projector with $\mathcal{M}^{\mu\nu}$

Projectors (CDR $D = 4 - 2\epsilon$):

$$P_1^{\mu\nu} = \frac{1}{4}\frac{D-2}{D-3}T_1^{\mu\nu} - \frac{1}{4}\frac{D-4}{D-3}T_2^{\mu\nu}$$

$$P_2^{\mu\nu} = -\frac{1}{4}\frac{D-4}{D-3}T_1^{\mu\nu} + \frac{1}{4}\frac{D-2}{D-3}T_2^{\mu\nu}$$

Integral Families

Can rewrite tensor integrals/scalar products as inverse propagators

#scalar products

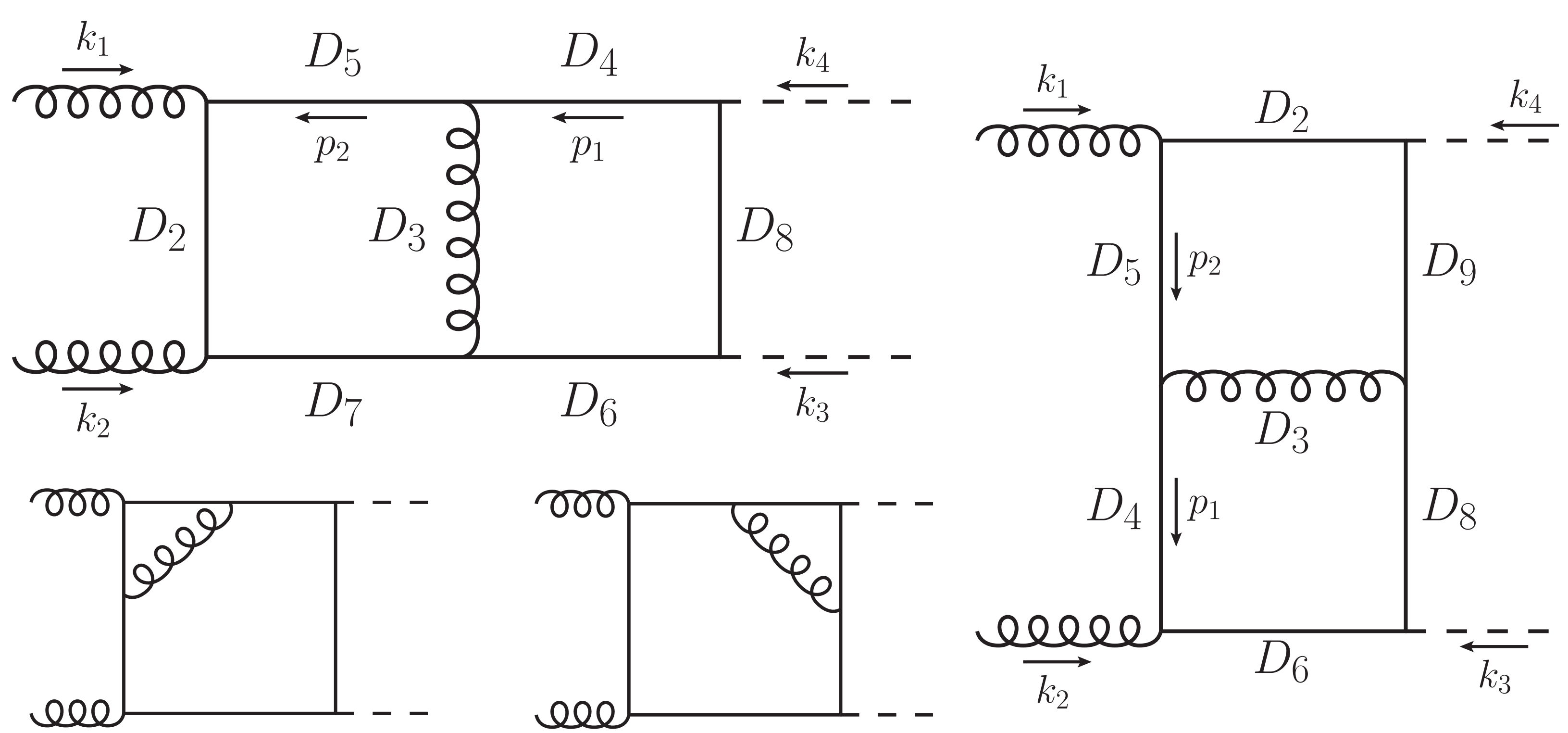
$$S = \frac{l(l+1)}{2} + lm \qquad \begin{array}{l} l = 2 \\ m = 3 \end{array} \quad \begin{array}{l} \textbf{\#loops} \\ \textbf{\#l.i. external momenta} \end{array} \quad \Rightarrow \quad S = 9$$

Introduce Integral Families with 9 propagators

$$I^{\mathrm{fam}_j}_{\nu_1,\ldots,\nu_9} = \int \mathrm{d}^d p_1 \int \mathrm{d}^d p_2 \frac{1}{D_1^{\nu_1} D_2^{\nu_2} \cdots D_9^{\nu_9}} \qquad \nu_i \in \mathbb{Z}$$

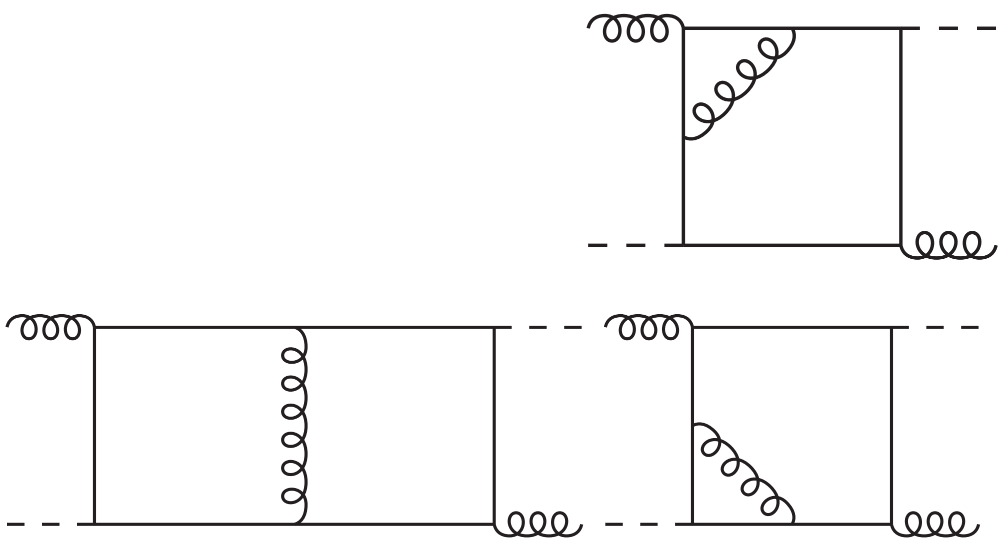
Planar family 1:

$$\begin{aligned}
D_1 &= p_1^2 - m_t^2 \\
D_2 &= p_2^2 - m_t^2 \\
D_3 &= (p_1 - p_2)^2 \\
D_4 &= (p_1 + k_1)^2 - m_t^2 \\
D_5 &= (p_2 + k_1)^2 - m_t^2 \\
D_6 &= (p_1 - k_2)^2 - m_t^2 \\
D_7 &= (p_2 - k_2)^2 - m_t^2 \\
D_8 &= (p_1 - k_2 - k_3)^2 - m_t^2 \\
D_9 &= (p_2 - k_2 - k_3)^2 - m_t^2
\end{aligned}$$

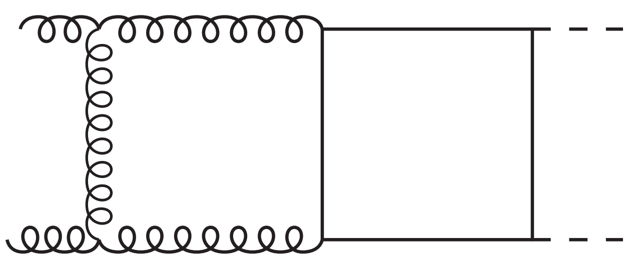


Integral Families (II)

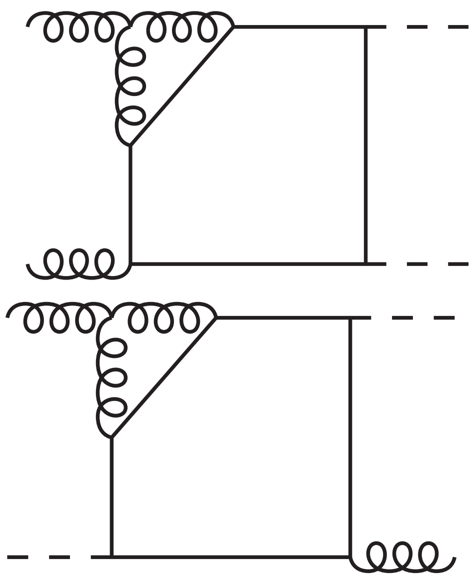
Planar Family 2



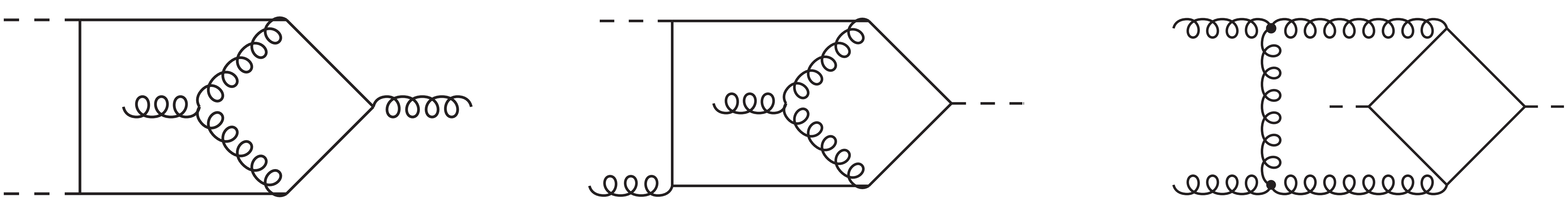
Planar Family 3



Planar Families 4/5



3 non-planar families:



Integral Reduction

Practically, 2-loop reduction with 4 scales $(\hat{s}, \hat{t}, m_T^2, m_H^2)$ and 4 inverse propagators is challenging

Simplification: Fix $m_T = 173\,\mathrm{GeV}, m_H = 125\,\mathrm{GeV}$

Price: Many arbitrary precision integers in reduction (slow)
Can not vary masses in result

Planar Integrals (145+83 crossed)
Reduction with `REDUZE 2`
von Manteuffel, Studerus 12

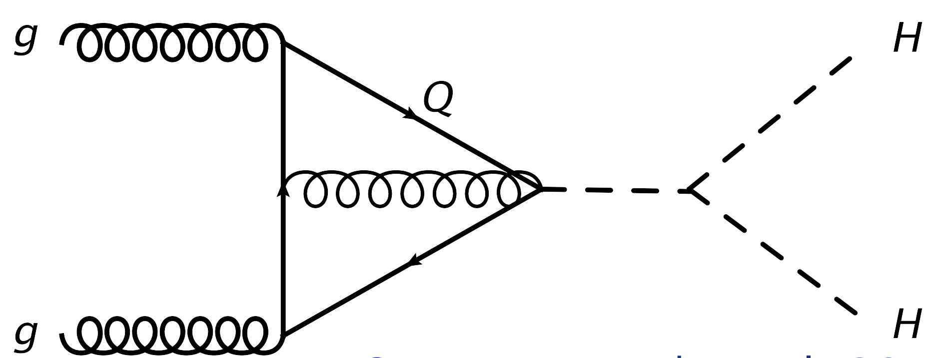
Non-Planar Integrals (70+29 crossed)
Computed mostly without reduction

Integrals	1-loop	2-loop
Direct	63	9865
+ Symmetries	21	1601
+ IBPs	8	~260-270 Currently: 327

For reduced integrals we choose a Finite Basis using `REDUZE`
Panzer 14; von Manteuffel, Panzer, Schabinger 15

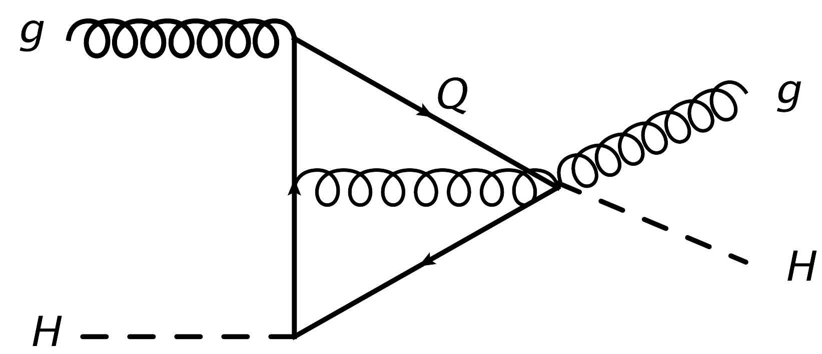
Master Integrals

Known Analytically:



Spira, Djouadi et al. 93, 95;
Bonciani, P. Mastrolia 03,04;
Anastasiou, Beerli et al. 06;

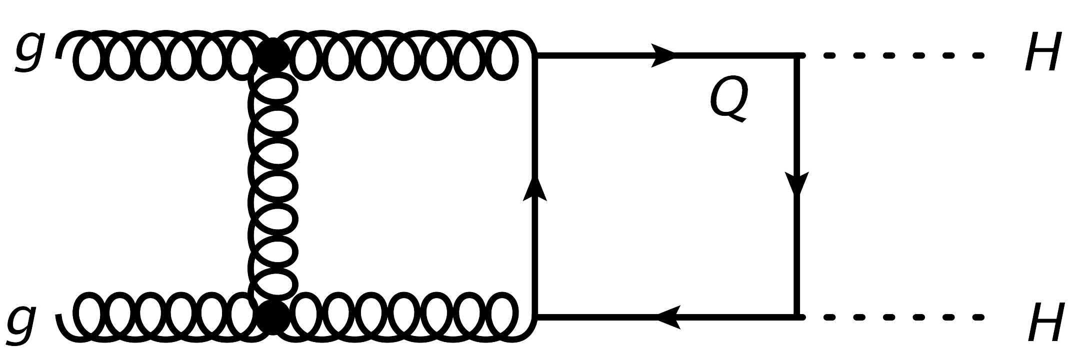
3-point, 1 off-shell leg
HPLs



Gehrmann, Guns, Kara 15

3-point, 2 off-shell legs
Generalized HPLs, 12 Letters

Numeric Evaluation:



Up to 4-point,
4 scales s, t, m_T^2, m_H^2
`SecDec`

Slide: Matthias Kerner

Finite Basis

Always possible to pick finite basis of integrals, rewrite integrals using:

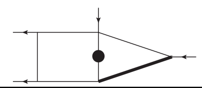
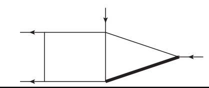
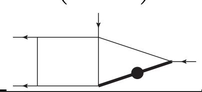
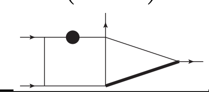
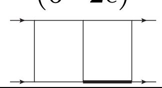
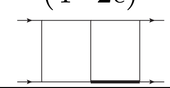
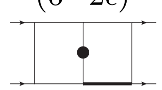
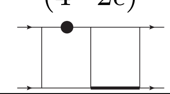
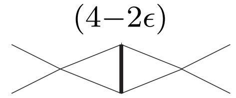
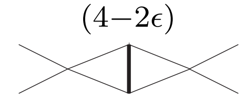
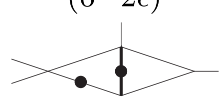
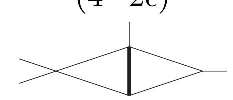
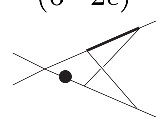
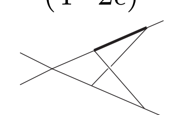
- Dimension Shifts Tarasov 96; Lee 10
- Dots

Panzer 14; von Manteuffel, Panzer, Schabinger 15

Two-loop EW-QCD Drell-Yan

von Manteuffel, Schabinger 17

Finite Basis... Conventional...

$(6-2\epsilon)$  (s,t)	201 s	2.34×10^{-4}	$(4-2\epsilon)$  (s,t)	384 s	8.12×10^{-4}
$(6-2\epsilon)$  (s,t)	150 s	4.83×10^{-4}	$(4-2\epsilon)$  (s,t)	56538 s	1.67×10^{-2}
$(6-2\epsilon)$  (s,t)	280 s	1.00×10^{-3}	$(4-2\epsilon)$  (s,t)	214135 s	8.29×10^{-3}
$(6-2\epsilon)$  (s,t)	294 s	1.21×10^{-3}	$(4-2\epsilon)$  (s,t)	3484378 s	30.9
$(4-2\epsilon)$  (s)	91 s	3.76×10^{-4}	$(4-2\epsilon)$  (s)	87 s	3.76×10^{-4}
$(6-2\epsilon)$  (s)	17 s	5.15×10^{-4}	$(4-2\epsilon)$  (s)	20 s	1.95×10^{-4}
$(6-2\epsilon)$  (s)	119 s	2.32×10^{-3}	$(4-2\epsilon)$  (s)	118 s	2.12×10^{-3}
Total/Max:	3995 s	5.84×10^{-3}	Total/Max:	5136862 s	30.9

Rel. Err.

Huge decrease in time to numerically integrate and relative error

Rank 1 Shifted Lattices

$\mathcal{O}(n^{-1})$ algorithm for numerical integration: Review: Dick, Kuo, Sloan 13

$$I_s[f] \equiv \int_{[0,1]^s} \mathrm{d}^s x f(\vec{x})$$

$$f : \mathbb{R}^s \to \mathbb{C}$$

$$I_s[f] \approx \bar{Q}_{s,n,m}[f] \equiv \frac{1}{m} \sum_{k=1}^{m} \frac{1}{n} \sum_{i=0}^{n-1} f\left(\left\{ \frac{i\vec{z}}{n} + \vec{\Delta}_k \right\}\right)$$

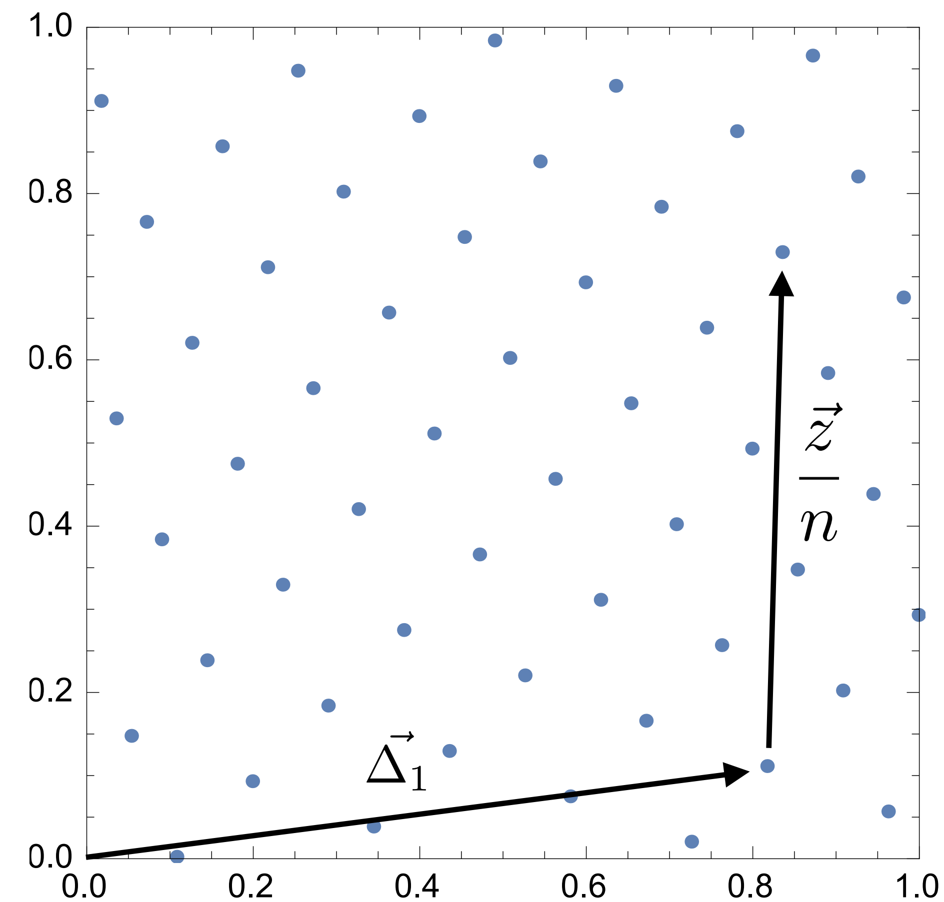
$\vec{z}$ - Generating vec.

$\vec{\Delta}_k$ - Random shift vec.

$\{\}$ - Fractional part

n - # Lattice points

m - # Random shifts

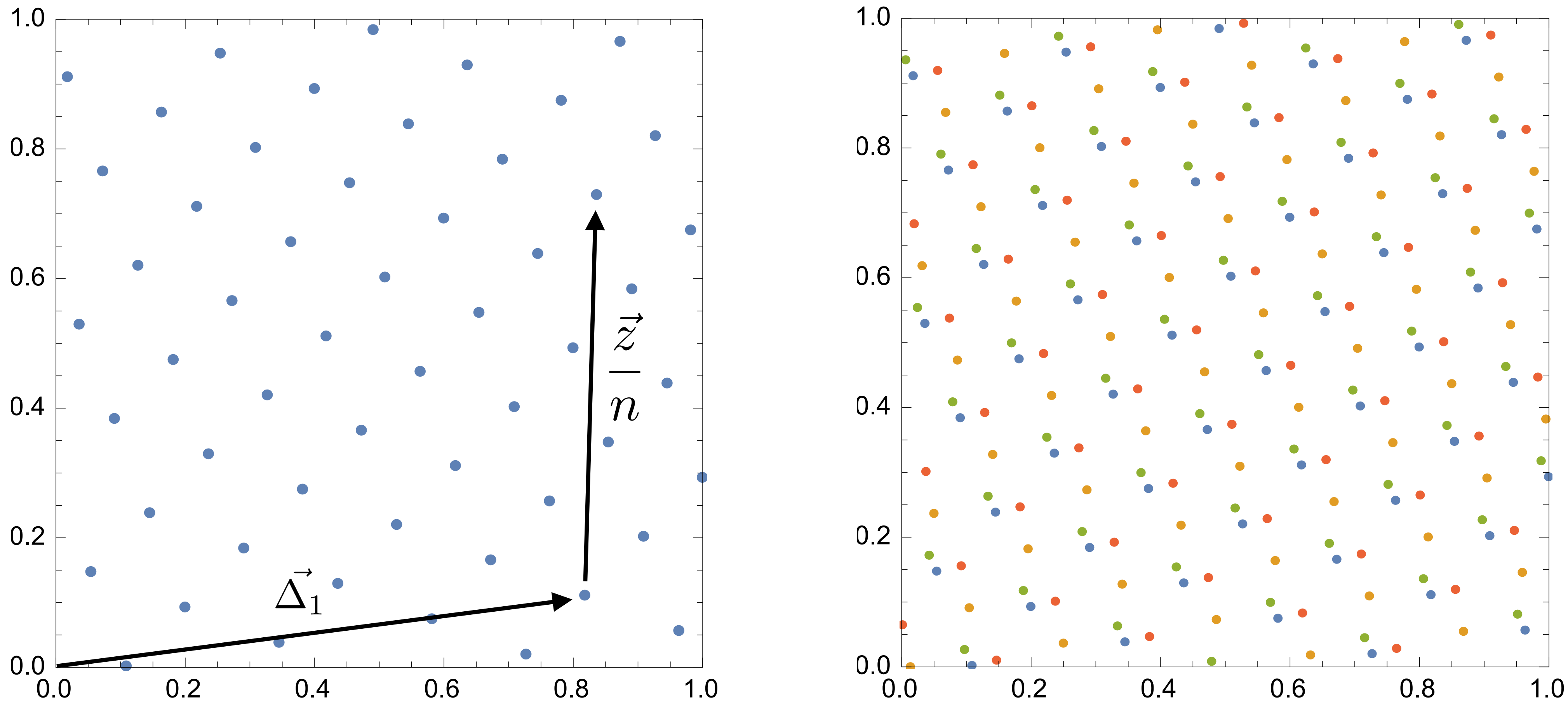


Generating vector $\vec{z}$ precomputed for a **fixed** number of lattice points, chosen to minimise worst-case error Nuyens 07

Rank 1 Shifted Lattices (II)

Unbiased error estimate computed from random shifts:

$$\mathrm{Var}[\bar{Q}_{s,n,m}[f]] \approx \frac{1}{m(m-1)} \sum_{k=1}^{m} (Q_{s,n,k} - \bar{Q}_{s,n,m})^2$$



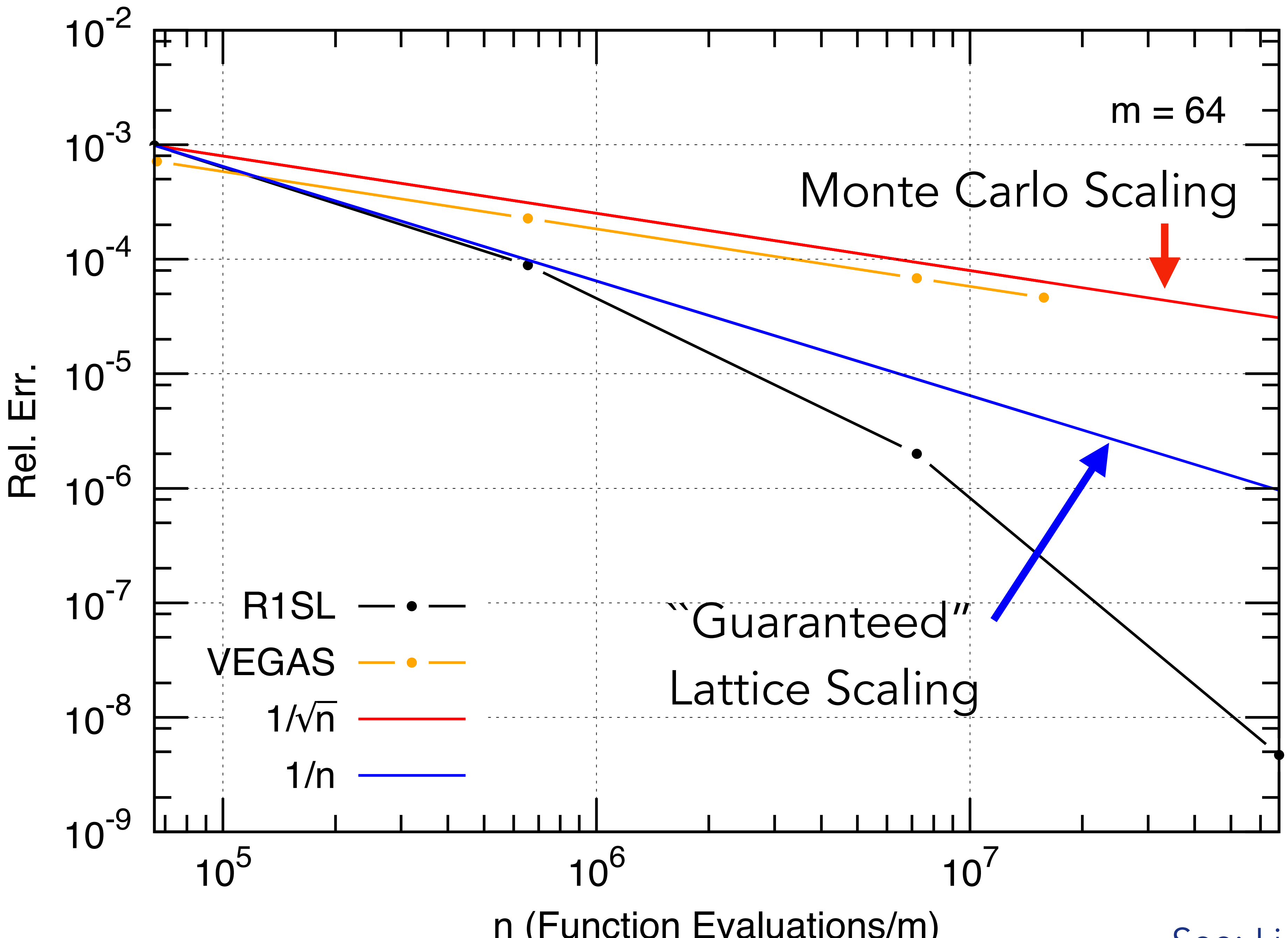
1 Shift

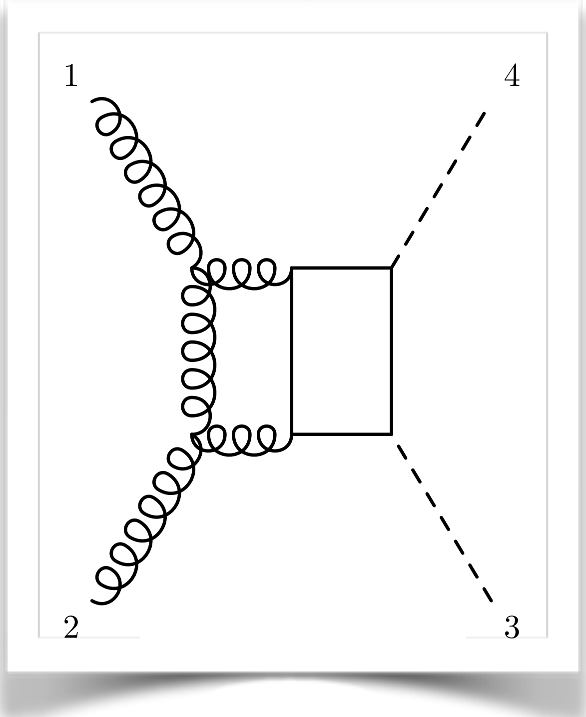
4 Shifts

Typically 10-50 shifts, production run: 20 shifts

R1SL: Algorithm Performance

Example: Rel. Err. of one sector of sector decomposed loop integral





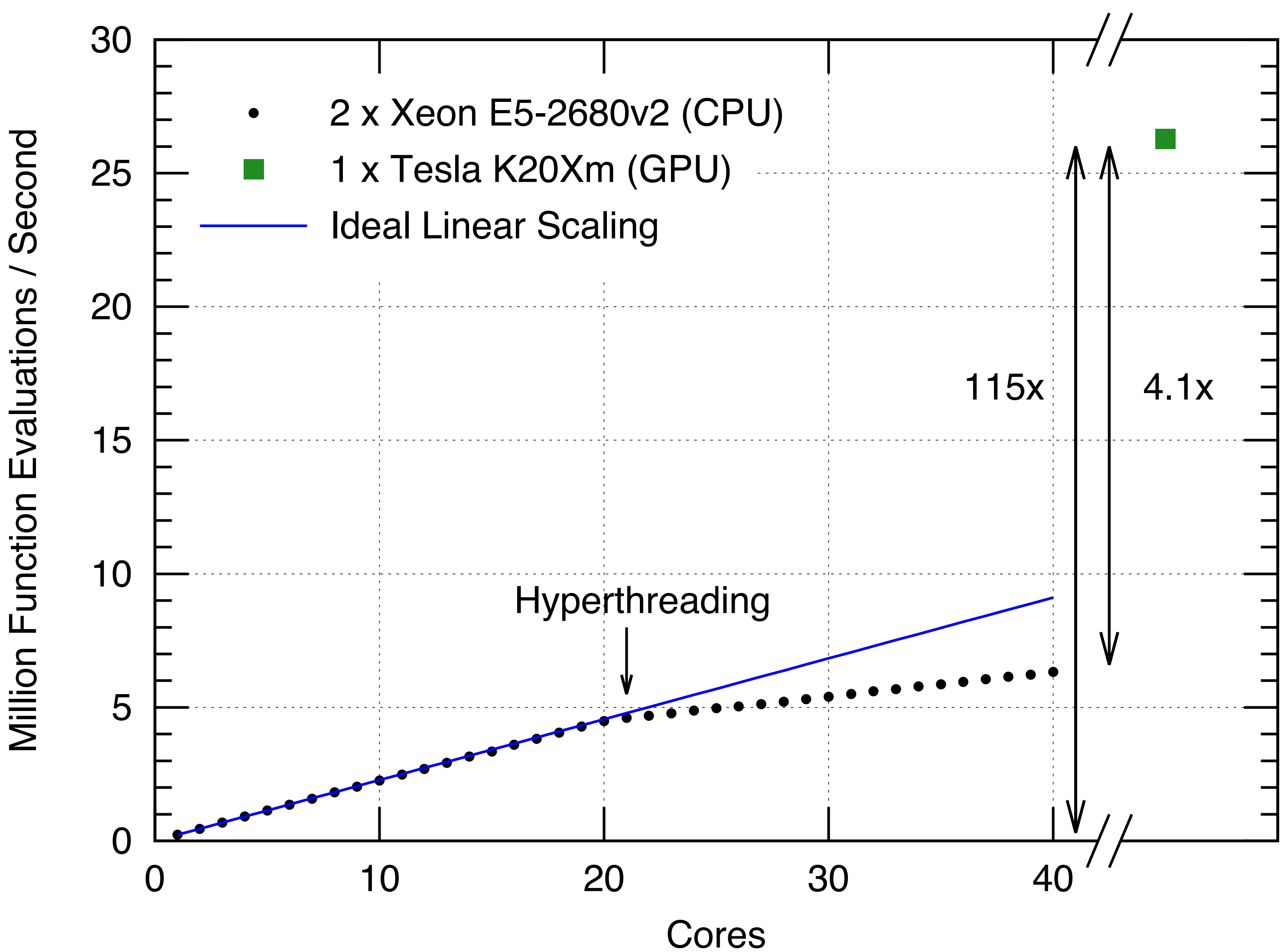
6 dimensional numerical integral

See: Li, Wang, Yan, Zhao 15

R1SL: Implementation Performance

Accuracy limited primarily by number of function evaluations

Implemented in OpenCL 1.1 for CPU & GPU, generate points on GPU/ CPU core, sum blocks of points (reduce memory usage/transfers)



2 CPUs (20 Cores + HT)

1 GPU

n	CPU (s)	GPU(s)	C/G
655357	6.63	1.60	4.1
7208951	72.3	16.4	4.4
67264993	674.2	152.2	4.4

Amplitude Structure

Factor dimensionful parameter M^2 out of integrals s.t. they depend on dimensionless ratios:

#prop. in denom. **#prop. in num.**

$$I_{r,s}(\hat{s},\hat{t},m_h^2,m_t^2) = (M^2)^{-L\epsilon}(M^2)^{2L-r+s} I_{r,s}\left(\frac{\hat{s}}{M^2},\frac{\hat{t}}{M^2},\frac{m_h^2}{M^2},\frac{m_t^2}{M^2}\right)$$

$\overline{\mathrm{MS}}$ scheme strong coupling a and OS top-quark mass:

$$\mathrm{F} = a\mathrm{F}^{(1)} + a^2(\delta Z_A + \delta Z_a)\mathrm{F}^{(1)} + a^2 \delta m_t^2 \mathrm{F}^{ct,(1)} + a^2 \mathrm{F}^{(2)} + O(a^3)$$

$$\mathrm{F}^{(1)} = \left(\frac{\mu_R^2}{M^2}\right)^{\epsilon}\left[b_0^{(1)} + b_1^{(1)}\epsilon + b_2^{(1)}\epsilon^2 + \mathcal{O}(\epsilon^3)\right]$$ ← 1-loop

$$\mathrm{F}^{ct,(1)} = \left(\frac{\mu_R^2}{M^2}\right)^{\epsilon}\left[c_0^{(1)} + c_1^{(1)}\epsilon + \mathcal{O}(\epsilon^2)\right]$$ ← Mass Counter-Terms

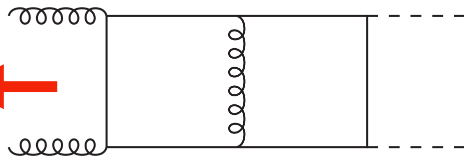
$$\mathrm{F}^{(2)} = \left(\frac{\mu_R^2}{M^2}\right)^{2\epsilon}\left[\frac{b_{-2}^{(2)}}{\epsilon^2} + \frac{b_{-1}^{(2)}}{\epsilon} + b_0^{(2)} + \mathcal{O}(\epsilon)\right]$$ ← 2-loop

Red terms contain integrals, computed numerically at each PS point, not re-evaluated for scale variations

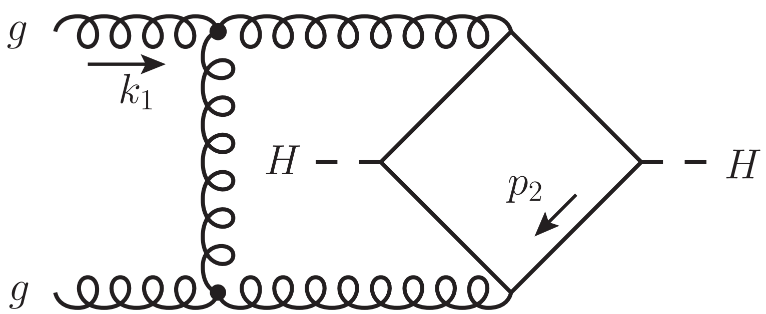
Amplitude Evaluation

Contributing integrals: $\sqrt{\hat{s}} = 327.25\,\mathrm{GeV}, \sqrt{-\hat{t}} = 170.05\,\mathrm{GeV}, M^2 = \hat{s}/4$

integral	value	error	time [s]
...			
F1_011111110_ord0	(0.484, 4.96e-05)	(4.40e-05, 4.23e-05)	11.8459
...			
N3_111111100_k1p2k2p2_ord0	(0.0929, -0.224)	(6.32e-05, 5.93e-05)	235.412
N3_111111100_1_ord0	(-0.0282, 0.179)	(8.01e-05, 9.18e-05)	265.896
N3_111111100_k1p2k1p2_ord0	(0.0245, 0.0888)	(5.06e-05, 5.31e-05)	282.794
N3_111111100_k1p2_ord0	(-0.00692, -0.108)	(3.05e-05, 3.05e-05)	433.342



$$I(s,t,m_t^2,m_h^2) = -\left(\frac{\mu^2}{M^2}\right)^{2\varepsilon} \Gamma(3+2\epsilon) M^{-4} \left(\frac{A_{-2}}{\epsilon^2} + \frac{A_{-1}}{\epsilon^1} + A_0 + \mathcal{O}(\epsilon)\right)$$



Sector Decomposition

sector	integral value	error	time [s]	#points
5	(-1.34e-03, 2.00e-07)	(2.38e-07, 2.69e-07)	0.255	1310420
6	(-1.58e-03, -9.23e-05)	(7.44e-07, 5.34e-07)	0.266	1310420
...				
41	(0.179, -0.856)	(1.10e-05, 1.22e-05)	29.484	79952820
42	(0.359, -1.308)	(1.40e-06, 1.58e-06)	80.24	211436900
44	(0.0752, -1.185)	(5.44e-07, 6.76e-07)	99.301	282904860

Slide:
Matthias Kerner
(LL 2016)

Experimental Prospects

Current Experimental Limits

Decay Ch.	B.R.	95% Excl.	Analysis ($\left[fb^{-1}\right], \sqrt{s}$ [TeV])
$b\bar{b}b\bar{b}$	33%	$< 29 \cdot \sigma_{\rm SM}$	ATLAS-CONF-2016-017 (3.2,13)
			ATLAS-CONF-2016-049 (13.3,13)
$b\bar{b}WW$	25%	–	–
$b\bar{b}\tau\tau$	7.3%	$< 200 \cdot \sigma_{\rm SM}$	CMS PAS HIG-16-012 (2.7,13)
			CMS PAS HIG-16-028 (12.9,13)
			CMS PAS HIG-15-013 (18.3,8)
$b\bar{b}ZZ$	3.0%	–	–
$WW\tau\tau$	2.71%	–	–
$WWZZ$	1.13%	–	–
$b\bar{b}\gamma\gamma$	0.26%	$< 3.9pb$	ATLAS-CONF-2016-004 (3.2,13)
		$< 74 \cdot \sigma_{\rm SM}$	CMS-HIG-13-032 (19.7,8)
$\gamma\gamma\gamma\gamma$	0.001%	–	–
$b\bar{b}VV(\to l\nu l\nu)$	1.23%	$400 \cdot \sigma_{\rm SM}$	CMS PAS HIG-16-024 (2.3,13)
$\gamma\gamma WW^*(\to l\nu jj)$	–	$< 25pb$	ATLAS-CONF-2016-071 (13.3,13)
Comb Ch.	–	$< 70 \cdot \sigma_{\rm SM}$	ATLAS arXiv:1509.04670v2 (20.3,8)

Warning: Last Updated August 2016

Future Experimental Prospects

HL-LHC (14 TeV)

ATLAS+CMS bbγγ + bbττ: Expected significance 1.9 sigma
CERN-LHCC-2015-10

ATLAS bbγγ: Signal significance 1.3 sigma ATL-PHYS-PUB-2014-019

ATLAS bbττ: Signal significance 0.6 sigma ATL-PHYS-PUB-2015-046

FCC (100 TeV)

This rate is expected to provide a clear signal in the $HH \to (b\bar{b})(\gamma\gamma)$ channel and to allow determination of λ_{3H} with an accuracy of $30-40\%$ with a luminosity of 3 ab^{-1}, and of $5-10\%$ with a luminosity of 30 ab^{-1} [497–499]. A rare decay channel which is potentially interesting is $HH \to (b\bar{b})(ZZ) \to (b\bar{b})(4l)$, with a few expected signal events against $\mathcal{O}(10)$ background events at 3 ab^{-1} [500].

arXiv:1607.01831