NNLO phenomenology with Antenna Subtraction

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NNLOJET Collaboration

Outline

QCD NNLO Calculations

Goals:

Get a better understanding of the theory.

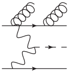
Reduce the theoretical uncertainty.

What does this entail:

- Introduce $\alpha_s(\mu)^2$ terms with respect to Leading Order.
- More complicated infrared cancellation.
- Very challenging both theoretically and technically.

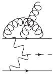
What do we mean by NNLO QCD

- Double Radiation matrix elements (M^0_{n+2})



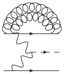
  - Implicit double unresolved singularities arise during phase space integration
  - Very computationally challenging

- Single Radiation one loop matrix elements (M^1_{n+1})



  - Explicit IR poles arising from loop integration
  - Single unresolved singularities arise during phase space integration

- Two loops matrix elements (M^2_n)



  - Only explicit IR poles arise, coming from loop integration
  - Very challenging analytically
  - Theoretical bottleneck of most NNLO calculations

Cross Section integrals

$$\hat{\sigma}_{NNLO} = \hat{\sigma}^{RR}_{NNLO} + \hat{\sigma}^{RV}_{NNLO} + \hat{\sigma}^{VV}_{NNLO}$$

Final result is finite, therefore singularities in the different phase space integral must cancel each other.

$$\hat{\sigma}^{RR}_{NNLO} = \int_{d\Phi_{n+2}} M^0_{n+2}$$

$$\hat{\sigma}^{RV}_{NNLO} = \int_{d\Phi_{n+1}} M^1_{n+1}$$

$$\hat{\sigma}^{VV}_{NNLO} = \int_{d\Phi_{n}} M^2_{n}$$

However, in order to numerically perform the phase space integral we need a finite integrand.

Subtraction

We can achieve a finite integrand by introducing functions to cancel these explicit singularities while not modifying the cross section.

$$\hat{\sigma}_{NNLO} = \int_{d\Phi_{n+2}} \left[d\hat{\sigma}^{RR}_{NNLO} - d\hat{\sigma}^{S}_{NNLO} \right] + \int_{d\Phi_{n+1}} \left[d\hat{\sigma}^{RV}_{NNLO} - d\hat{\sigma}^{T}_{NNLO} \right] + \int_{d\Phi_{n}} \left[d\hat{\sigma}^{VV}_{NNLO} - d\hat{\sigma}^{U}_{NNLO} \right]$$

$$0 = d\hat{\sigma}^{S}_{NNLO} + d\hat{\sigma}^{T}_{NNLO} + d\hat{\sigma}^{U}_{NNLO}$$

Control of singularities

There are many approaches in which the cancellation of the singularities can be achieved:

- dipole subtraction
- residue subtraction
- **antenna subtraction**
- phase space slicing
- sector decomposition
- n-jettiness
- projection to born

Antenna Subtraction

In singular limits, all QCD matrix elements factorise as universal singular functions[1] times a reduced matrix element.
Eg:

$$\lim_{s_{ij}\to 0} M^0_{n+1}\left(i,j,...n,n+1\right) = P^0_{ij} M^0_n\left(\tilde{ij},...,n\right)$$

We would like to obtain a set of functions such that:

- they contain all possible singularities (single and double)
- they are analytically integrable over the phase space of the unresolved parton(s)

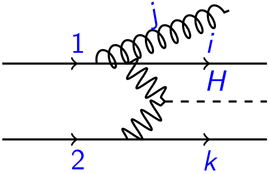
[1] which only depend on the flavour of the particles involved in the limit

Antenna Subtraction

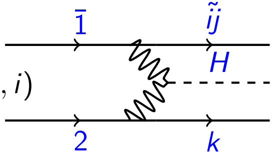
Antenna subtraction (AS) exploits the fact that matrix elements already contain all possible (single and double) singularities to create a set of functions which contain said limits and their correspondent integrations:

$$X_3^0(i,j,k) = \frac{M_3^0(i,j,k)}{M_2^0(\tilde{ij},\tilde{jk})} \qquad \mathcal{X}_3^0 = \int_{d\Phi_{ijk}} X_3^0(i,j,k)$$

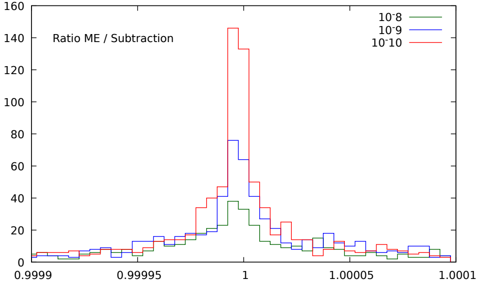
$$X_4^0(i,j,k,l) = \frac{M_4^0(i,j,k,l)}{M_2^0(\tilde{ijk},\tilde{jkl})} \qquad \mathcal{X}_4^0 = \int_{d\Phi_{ijkl}} X_4^0(i,j,k,l)$$



$$-X_3^0(1,j,i)$$



The subtraction term mimics the behaviour of the ME as we dive deeper into the limit.
This feature allows us to numerically integrate using known methods (eg: MC).

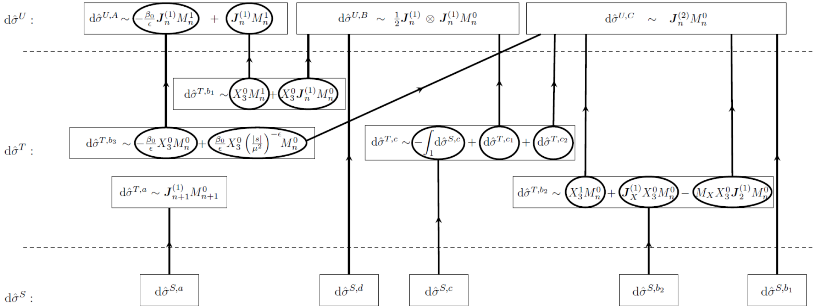


A word of warning

The previous example presented a well behaved single limit with a very good cancellation. This is not always the case:

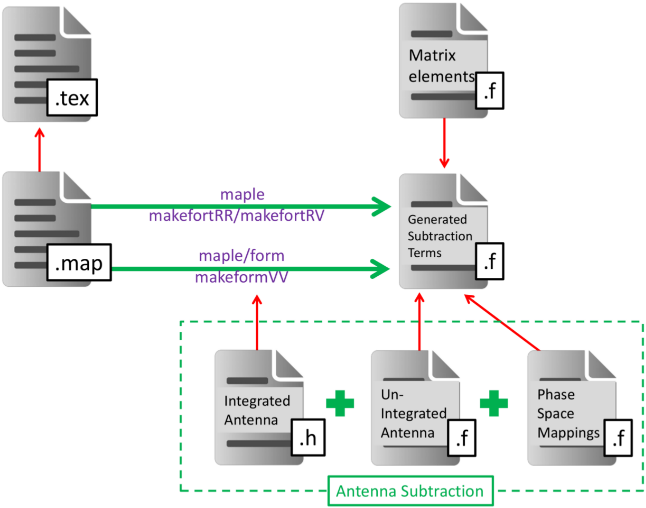
- Full cancellation is not always achieved without diving very deep into the singular limits.
  - Numerical instabilities.
  - Very computationally expensive problems.
- Even though all single and double singular limits can be constructed with our antennaes, double singular limits present a much more complicated structure.
  - This means we may introduce spurious limits.
  - The complexity of the subtraction terms for NNLO is much greater than NLO.
  - → Autogeneration for NNLO subtractions is still in the "to do" list.

Infrared Structure at NNLO



For more detail see: hep-ph/1301.4693

Autogeneration of code



NNLOJET

The antenna subtraction method has been successfully applied to a plethora of processes within NNLOJET.

- $pp \to H \to \gamma\gamma$ + 0,1,2 jets (1507.02850, 1601.04569, 1605.04295)
- $pp \to Z \to e^+e^-$ + 0,1,2 jets (1408.5325, 1604.04085, 1610.01843, 1708.00008)
- $pp \to$ dijets (1310.3993, 1611.01460, 1705.10271)
- $ep \to 2, 3$ jets (1606.03991, 1703.05977)
- e^+e^- annihilation (1709.01097)
- . . .

NNLOJET

An ever growing collaboration:
X. Chen, JCM, J. Currie, A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, M. Hoefer, A. Huss, I. Majer, T. Morgan, J. Niehues, J. Pires, D. Walker, J. Whitehead...

- $pp \to H \to \gamma\gamma$ + 0,1,2 jets (1507.02850, 1601.04569, 1605.04295)
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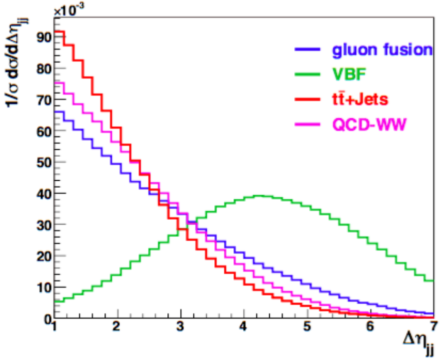
- . . .

Vector Boson Fusion Higgs production

Higgs production on Vector Boson Fusion (VBF) is a very important channel for Higgs phenomenology:

- At the LHC its production rate is second only to gluon fusion.
- Its very clean signature make this channel very easy to isolate.



State of the art

- Inclusive cross section known for a few years up to NNLO in the structure function approach. NNLO corrections believed to be small (around 1% from NLO at 13 TeV) (hep-ph/1003.4451, P. Bolzonoi, F. Maltoni, S. Moch, M. Zaro, 2010)
- Recent study suggest a NNLO correction for the total cross section greater than 5% when typical VBF cuts are applied (and even greater for differential distributions) (hep-ph/1506.02660, M. Cacciari, F. Dreyer, A. Karlberg, G. Salam, G. Zanderighi, 2015)
- The N^3LO cross section in the DIS approximation was recently obtained (hep-ph/1606.00840)

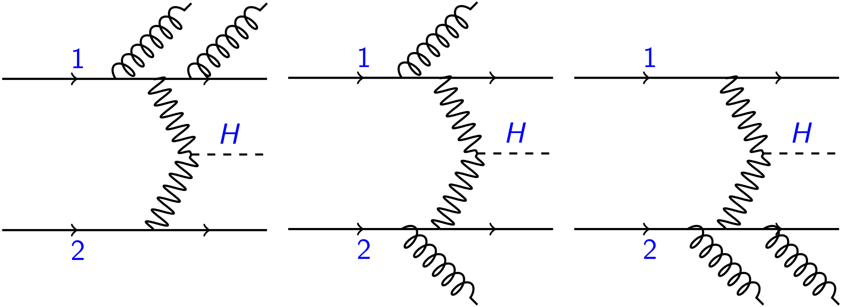
Vector Boson Fusion amplitudes

First of all, we need to define what are we calling "Vector Boson Fusion"

- Diagrams in which the vector boson is exchanged in the t channel
- Not including exchange of gluons between upper and lower legs
- Not including same flavour quark annihilation

These contributions are estimated to be negligible when VBF cuts are applied.

Example: VBF at RR



$\mathcal{M}_1$ $\mathcal{M}_2$ $\mathcal{M}_3$

These are some representative Double Real diagrams. We only include contributions $\mathcal{M}_i \mathcal{M}_j^*$ for $i = j$.

Example: RR subtraction term

This subtraction term corresponds to the Leading Colour contribution of some of the previous shown matrix elements squared.

An example of the contents of a .map file:

```
FN:=qqpC2g0WFHSs1(1,k,l,j,2,i,H):
XX:=
+ d30FF(i,l,k)*C1g0WFHs1(1,[k,l],j,2,[i,l],H) *JET23([i,l],[k,l],j)
+ qd30IF(1,k,l)*C1g0WFHs1([1],[k,l],j,2,i,H) *JET23(i,[k,l],j)
+ A40(1,k,l,i)*C0g0WFH([1],j,2,[i,l,k],H) *JET22([i,k,l],j)
- (1/2)*qA30IF(1,k,i)* qA30FI(2,l,j)* C0g0WFH([1],[j,l],[2],[i,k])*
JET22([i,k],[j,l])
- (1/2)*qA30FI(2,l,j)* qA30IF(1,k,i)* C0g0WFH([1],[j,l],[2],[i,k])*
JET22([i,k],[j,l])
```

⋮

Example: RR subtraction term

$d\hat{\sigma}^{S,a}$

$d\hat{\sigma}^{S,b_1}$

$d\hat{\sigma}^{S,d}$

```
FN:=qqpC2g0WFHSs1(1,k,l,j,2,i,H):
XX:=
+d30FF(i,l,k)*C1g0WFHs1(1,[k,l],j,2,[i,l],H)
*JET23([i,l],[k,l],j)
+ qd30IF(1,k,l)*C1g0WFHs1([1],[k,l],j,2,i,H)
*JET23(i,[k,l],j)
+ A40(1,k,l,i)*C0g0WFH([1],j,2,[i,l,k],H)
*JET22([i,k,l],j)
- (1/2)*qA30IF(1,k,i)* qA30FI(2,l,j)*
C0g0WFH([1],[j,l],[2],[i,k])* JET22([i,k],[j,l])
- (1/2)*qA30FI(2,l,j)* qA30IF(1,k,i)*
C0g0WFH([1],[j,l],[2],[i,k])* JET22([i,k],[j,l])
```

⋮

Example: RV subtraction term

```
FN:=qqpC1g1WFHT(1,k,j,2,i,H):
XX:=
 -(1/2)*calD30FF(i,k)* C1g0WFHs1(1,k,j,2,i,H)
*JET23(i,k,j)
-(1/2)*calqD30IF(1,k)* C1g0WFHs1(1,k,j,2,i,H)
*JET23(i,k,j)
 +qA30IF(1,k,i)*(
+delta(1-x1)*delta(1-x2)*C0g1WFH([1],j,2,[i,k],H)
+calqA30IF([1],[i,k])*C0g0WFH([1],j,2,[i,k],H)
+calqA30FI(2,j)*C0g0WFH([1],j,2,[i,k],H)
)*JET22(j,[i,k])
```

$d\hat{\sigma}^{T,a}$

$d\hat{\sigma}^{T,b_1}$

⋮

VBF Results at NLO

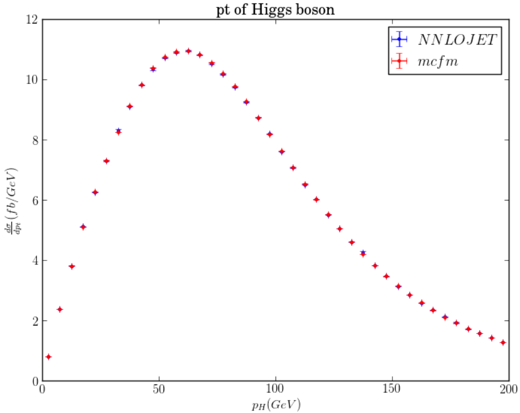


Figure: Comparison between our code and MCFM at NLO. Transverse momentum of the Higgs

VBF Results at NLO

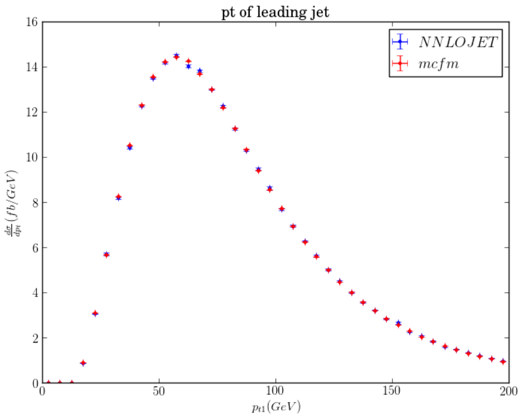


Figure: Comparison between our code and MCFM and NLO. Transverse momentum of the hardest jet

VBF+J at NLO

The first step for a test of NNLO result is to test the +1J component of the NNLO calculation. We achieve this by asking the jet algorithm of the following components for one extra jet.

$$\begin{aligned}\hat{\sigma}_{+J,NLO} = & \int_{d\Phi_{n+2}} \left[d\hat{\sigma}^{RR}_{NNLO} - d\hat{\sigma}^{S,a}_{NNLO} \right] J_4^3 \\ & + \int_{d\Phi_{n+1}} \left[d\hat{\sigma}^{RV}_{NNLO} - d\hat{\sigma}^{T,a}_{NNLO} \right] J_3^3\end{aligned}$$

VBF+J at NLO

We compare the inclusive cross section with VBFNLO[2], but no agreement was found.

$$\sigma^{VBFNLO} = 457 \pm 2 \ fb$$
$$\sigma^{NNLOJET} = 423 \pm 2 \ fb$$

Not even for particular channels:

$$\sigma^{VBFNLO}_{qq,W} = 180.3 \pm 0.7 \ fb$$
$$\sigma^{NNLOJET}_{qq,W} = 157.2 \pm 0.4 \ fb$$

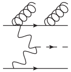
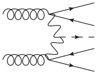
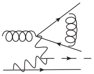
[2] Published version, 2.7

Checks of the calculation

In order to check the validity of our calculation we perform several checks that test different pieces of the calculation. Some of these checks include:

- Scale dependent terms
- Spike plots (ME / Subtraction)
- Pointwise comparison of the ME
- Cancellation of IR poles
- Integration of the ME at tree level
- Layer checks
- Comparison of different Subtraction Schemes

Pointwise comparison of the ME

Process	NNLOJET	Madgraph	Ratio
	5.1406085025982153E-014	5.1406085025982185E-014	1.0000000000000007
	4.0982703031871614E-017	4.0982703031871552E-017	1.00000000000000163
	-5.9293779081794126E-011	-5.9293779081799606E-011	0.99999999999990763

Layer checks

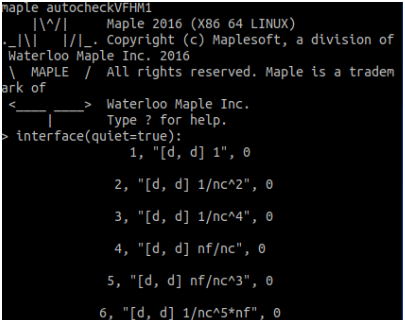
The automatisation of the code allows for some checks to be carried over during code creation.

- The antennae introduced at RR or RV levels must cancel against the integrated antennae at RV or VV.
- This check can be performed analytically using the same .map files as input and extracting information directly from the NNLOJET program.
- It also ensures that QCD, parton ordering, and symmetry factors are consistent between layers.

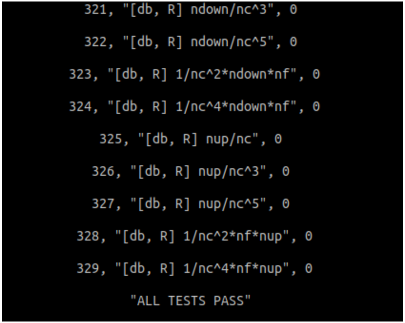
Layer checks

We want to ensure the following equality holds:

$$d\hat{\sigma}^{S}_{NNLO} + d\hat{\sigma}^{T}_{NNLO} + d\hat{\sigma}^{U}_{NNLO} = 0$$



```
maple autocheckVFHM1
    |\^/|     Maple 2016 (X86 64 LINUX)
._|\|   |/|_. Copyright (c) Maplesoft, a division of
 Waterloo Maple Inc. 2016
 \  MAPLE  /  All rights reserved. Maple is a tradem
ark of
 <____ ____>  Waterloo Maple Inc.
      |       Type ? for help.
> interface(quiet=true):
                1, "[d, d] 1", 0

              2, "[d, d] 1/nc^2", 0

              3, "[d, d] 1/nc^4", 0

              4, "[d, d] nf/nc", 0

             5, "[d, d] nf/nc^3", 0

            6, "[d, d] 1/nc^5*nf", 0
```



```
321, "[db, R] ndown/nc^3", 0

322, "[db, R] ndown/nc^5", 0

323, "[db, R] 1/nc^2*ndown*nf", 0

324, "[db, R] 1/nc^4*ndown*nf", 0

325, "[db, R] nup/nc", 0

326, "[db, R] nup/nc^3", 0

327, "[db, R] nup/nc^5", 0

328, "[db, R] 1/nc^2*nf*nup", 0

329, "[db, R] 1/nc^4*nf*nup", 0

"ALL TESTS PASS"
```

Comparison of different subtraction schemes

The automatised nature of NNLOJET means integrating a process with a different subtraction scheme require little tweaking.
Indeed, since we are looking at a NLO process, we can check whether we get the same result if we integrate using Catani-Seymour dipoles.
It amounts to substitutions of the form:

$$X_3^0(i,j,k) \to D_{ij;k} + D_{jk;i}$$

Since they will contain the same limits

$$\begin{gathered} X_3^0(i,j,k) \ni (i \parallel j), (j \parallel k) \\ D_{ij;k} \ni (i \parallel j) \\ D_{jk;i} \ni (j \parallel k) \end{gathered}$$

Catani-Seymour vs Antenna subtraction

First we "calibrate" our implementation of Catani Seymour (CS) testing the implementation with **VBF at NLO**.(for this process we have agreement with both VBFNLO and mcfm).

$$\sigma_{qq}^{CS} = 40.50 \pm 0.17 \ fb$$
$$\sigma_{qq}^{AS} = 40.28 \pm 0.20 \ fb$$

Ie, the result is independent of the subtraction scheme. Now we are ready to test this in some of the channels we don't achieve agreement with for **VBF + J at NLO**.

$$\sigma_{qq}^{CS} = 35.9 \pm 0.6 \ fb$$
$$\sigma_{qq}^{AS} = 35.4 \pm 0.7 \ fb$$

Catani-Seymour: NNLOJET vs Sherpa

Since Sherpa implements Catani-Seymour in an automatic way, we can separately test our Real-Subtraction ($\sigma^R - \sigma^S$) and Integrated Subtraction (σ^T) integrations (here we zero the Virtual Matrix Elements, $\sigma^V = 0$).
Once again we make sure we compare the same things by calibrating our implementation of CS.
VBF at NLO:

$$\sigma_{qq}^{NNLOJET,R-S} = 13.65 \pm 0.02 \ fb \qquad \sigma_{qq}^{NNLOJET,T} = 93.11 \pm 0.18 \ fb$$
$$\sigma_{qq}^{Sherpa,R-S} = 13.60 \pm 0.03 \ fb \qquad \sigma_{qq}^{Sherpa,T} = 93.24 \pm 0.09 \ fb$$

It agrees where we already had agreement.

Catani-Seymour: NNLOJET vs Sherpa

Since Sherpa implements Catani-Seymour in an automatic way, we can separately test our Real-Subtraction ($\sigma^R - \sigma^S$) and Integrated Subtraction (σ^T) integrations (here we zero the Virtual Matrix Elements, $\sigma^V = 0$).

VBF + J at NLO:

$$\sigma_{qq}^{NNLOJET,R-S} = -5.19 \pm 0.04 \; fb \qquad \sigma_{qq}^{NNLOJET,T} = 31.67 \pm 0.08 \; fb$$

$$\sigma_{qq}^{Sherpa,R-S} = -5.34 \pm 0.15 \; fb \qquad \sigma_{qq}^{Sherpa,T} = 31.78 \pm 0.08 \; fb$$

It also agrees.

Current state

Some questions still remain unanswered:

- Are we actually comparing the same things?
- Is the bug on our side of the calculation?

The next step is to compare the NNLO calculation against the DIS inclusive approach:

- Being fully inclusive in the RR phase space is challenging numerically
- Integration times of more than a week in a 16 cores machine

Conclusions

- NNLO calculations have much more complicated IR cancellations than NLO
- We need to take great care to ensure reliable answers:
  - Automate as much as possible
  - Validate against external code and resutls as much as possible.
- VBF validated at NLO, but still not at NNLO, searching for any possible remaining bug.

Thanks!

Vector Boson Fusion: VBF Cuts

In order to enhance the signal, we impose the following cuts to our results:

- One outgoing jet in each hemisphere ($y_1 y_2 < 0$, where $y_i =$ rapidity of the ith-hardest jest).
- Furthermore, we ask the two hardest jets to have a $\Delta y > 4.5$.
- We apply a cut in the dijet invariant mass of 600 GeV.
- Finally we ask the two hardest jets p_T to be above 25 GeV.