Azimuthal asymmetries in QCD hard scattering: infrared safe but divergent

Massimiliano Grazzini
University of Zurich

in collaboration with Stefano Catani and Hayk Sargsyan

Higgstools final meeting
Durham, september 13th 2017

Outline

- Introduction
- Azimuthal asymmetries
- Examples at fixed order
- Origin of the singular behaviour
- All order resummation

  - results for heavy-quark production
- Summary & Outlook

Introduction

Angular distributions of final-state particles in high-energy collisions are relevant observables to understand the dynamics of the underlying production mechanism

In the case of polarised collisions azimuthal correlations have been extensively studied

see eg. E.C. Aschenauer, U. D'Alesio, F. Murgia (2015)

In the case of unpolarised collisions the available studies focus on the Drell-Yan process

. see eg. M. Lambertsen and W. Vogelsang (2016)

Data available at the LHC

Activity within the framework of Transverse-Momentum Dependent (TMD) parton distributions

Linearly polarised gluons produce specific cos(2φ) and cos(4φ) modulation of the azimuthal dependence

. . P. J. Mulders and J. Rodrigues (2000); S.Catani, MG (2011)

. C. Pisano, D. Boer, S. J. Brodsky, M. G. A. Buffing and P. J. Mulders (2013)

Azimuthal asymmetries

We consider the hard scattering process $h_1(P_1) + h_2(P_2) \to F(\{p_3, p_4\}) + X$

The generic system F is composed by two 'particles' with momenta p_3 and p_4
invariant mass M and transverse momentum q_T
These 'particles' can either be pointlike or hadronic jets

Define $\varphi=\varphi_3-\varphi(q_T)$: azimuthal separation between p_3 and p_3+p_4

NOTE: we do not consider $\Delta\varphi=\varphi_3-\varphi_4$!

We mostly use the angles defined in the Collins-Soper rest frame of F

The following discussion is however independent on this choice and one can also use the angles ϕ defined in the CM frame of the colliding hadrons

We have in fact

$$\cos\varphi = \cos(\phi_3 - \phi(\boldsymbol{q_T})) + \mathcal{O}(q_T/M)$$

Azimuthal asymmetries

Consider $d\sigma/dq_T$ and $d\sigma/dq_T d\varphi$

Both these quantities are IR safe but the computation of azimuthal correlations can lead to divergences as $q_T \to 0$

Our main observation is that:

$$\frac{d\sigma}{dM^2 d\varphi} = \begin{cases} \text{finite at any fixed order (DY production)} \\ \text{divergent for any } \varphi \text{ at some fixed order (ttbar, Vj, jj, VV......)} \end{cases}$$

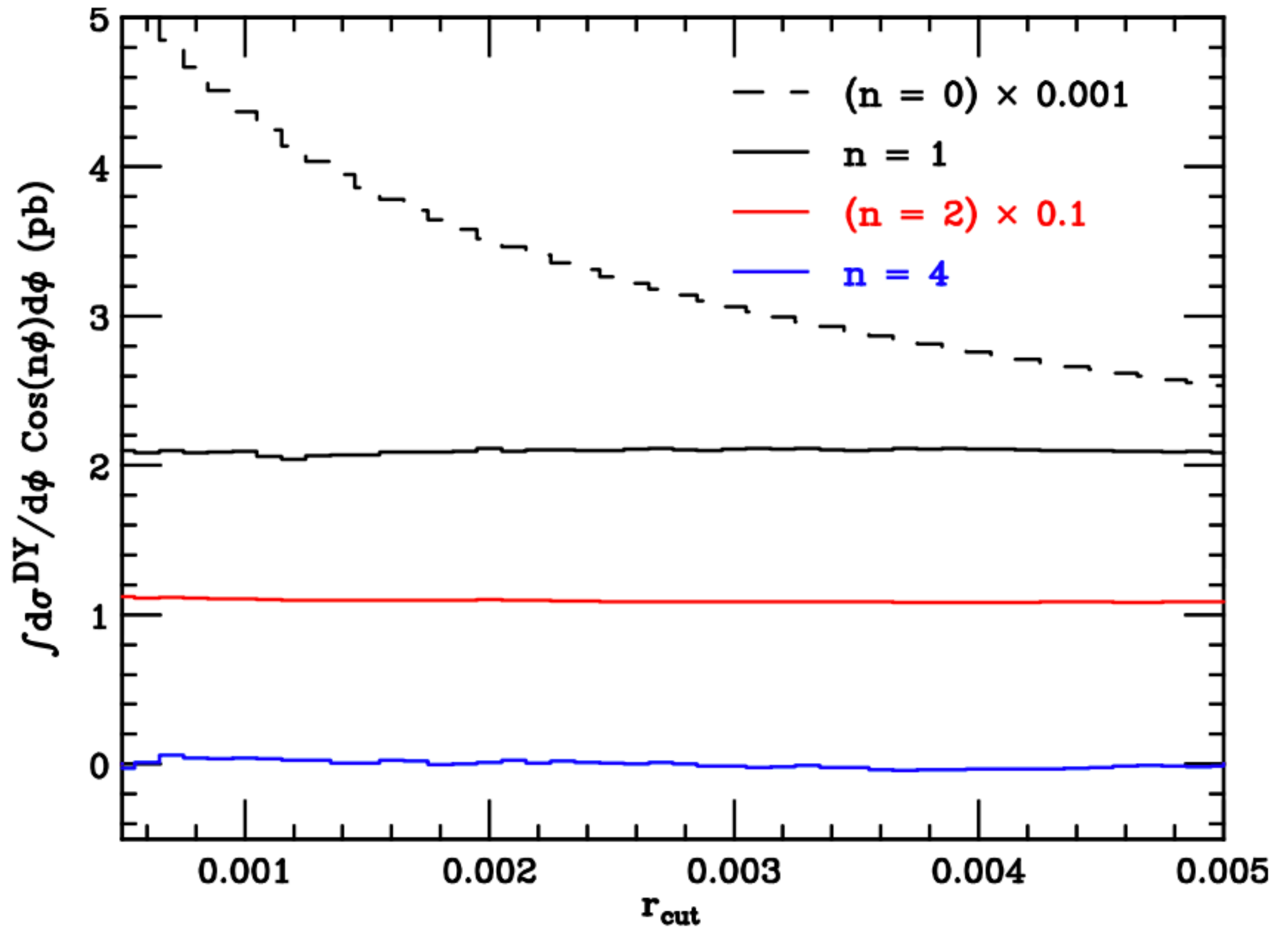
The source of the singularities are azimuthal correlations at small q_T

The case of Drell-Yan

Define harmonic components of azimuthally dependent cross sections

$$\frac{d\sigma_n}{dM^2} \equiv \int_0^{2\pi} d\varphi \; \cos(n\varphi) \int_0^{+\infty} dq_T^2 \, \frac{d\sigma}{dM^2\, dq_T^2\, d\varphi} \, \Theta(q_T - q_{\rm cut}) \qquad q_{\rm cut} = r_{\rm cut}\, M$$

We define here φ as the azimuthal angle of the electron in the Collins-Soper frame



In the DY case the cross section contains only four harmonics:

$$\cos(\varphi), \sin(\varphi), \cos(2\varphi), \sin(2\varphi)$$

Measured by ATLAS and CMS

The harmonics are finite (and small) for n≠0

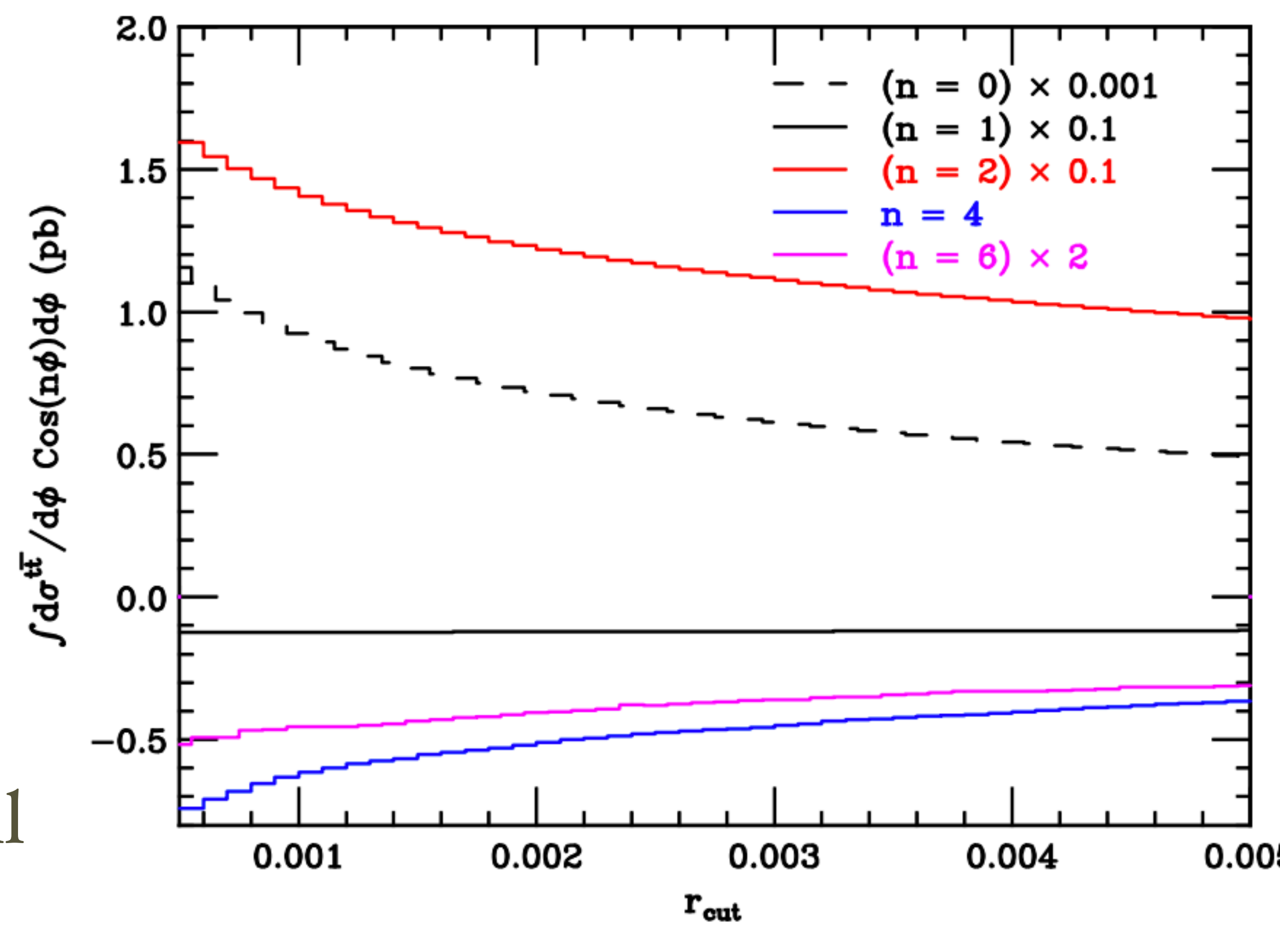
→ azimuthal correlations are present but they are finite as $q_T \to 0$

The case of $t\bar{t}$

Define harmonic components of azimuthally dependent cross sections

$$\frac{d\sigma_n}{dM^2} \equiv \int_0^{2\pi} d\varphi \;\cos(n\varphi)\; \int_0^{+\infty} dq_T^2 \,\frac{d\sigma}{dM^2\, dq_T^2\, d\varphi}\,\Theta(q_T - q_{\rm cut})$$

We define here φ as the azimuthal angle of the top quark in the Collins-Soper frame



Here the cross section receives contribution from harmonics of arbitrary n

At variance with the DY case here all even harmonics are divergent

Singular azimuthal correlations

At small transverse momenta QCD radiative corrections are dynamically enhanced

The general structure of the NLO cross section at small q_T is

less singular terms

$$\frac{d\sigma^{NLO}}{dM^2 d^2\boldsymbol{q_T}} \propto \delta^{(2)}(\boldsymbol{q_T}) + \alpha_S \left\{ \left(\underbrace{a_2 \left[\frac{1}{q_T^2} \ln\left(\frac{M^2}{q_T^2}\right)\right]_+ + a_1 \left[\frac{1}{q_T^2}\right]_+}_{\text{customary large logarithmic terms}} + a_0\, \delta^{(2)}(\boldsymbol{q_T}) + \boxed{\frac{a_{\rm corr}(\hat{\boldsymbol{q}}_{\boldsymbol{T}})}{q_T^2}} \right) + \dots \right\}$$

customary large logarithmic terms

azimuthal correlation term

The coefficients a_1 and a_2 are independent on the direction of $\mathbf{q_T}$ while a_{corr} does depend on it

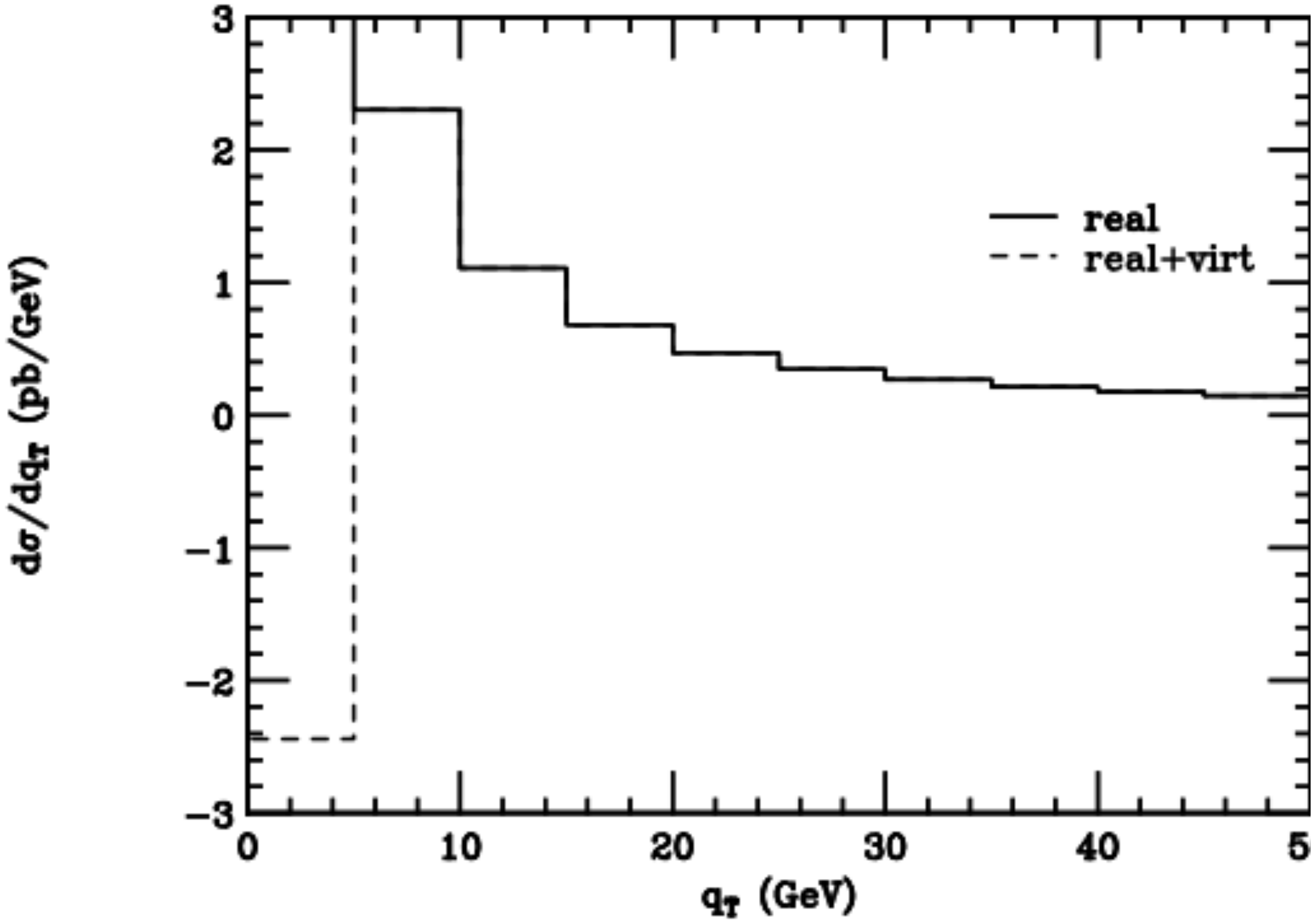
The azimuthal correlations are absent in the DY case (a_{corr} vanishes)

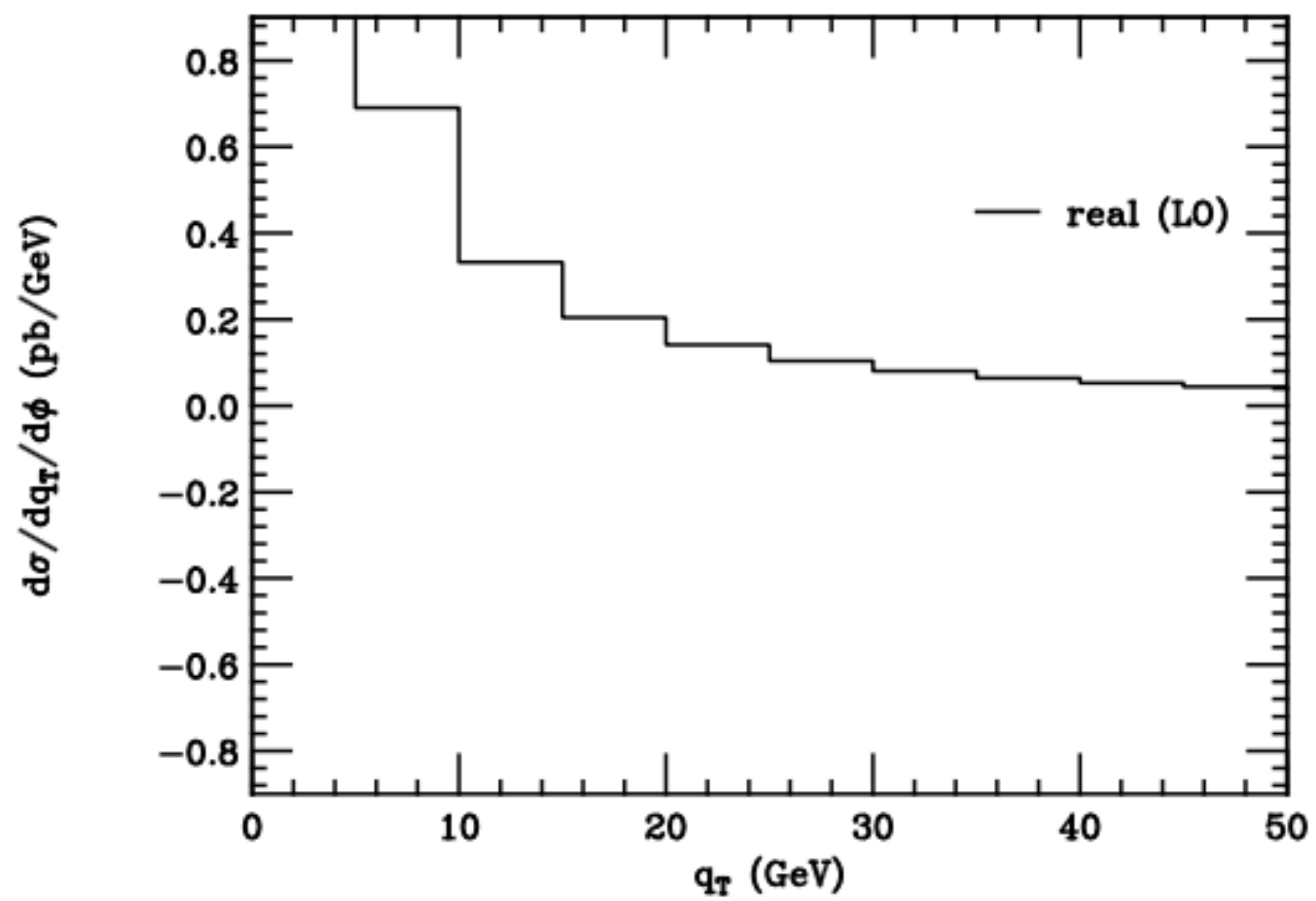
The azimuthal correlations of course vanish when we consider azimuthally integrated cross sections

The singularity driven by a_{corr} cannot be cancelled by the virtual !

Singular azimuthal correlations

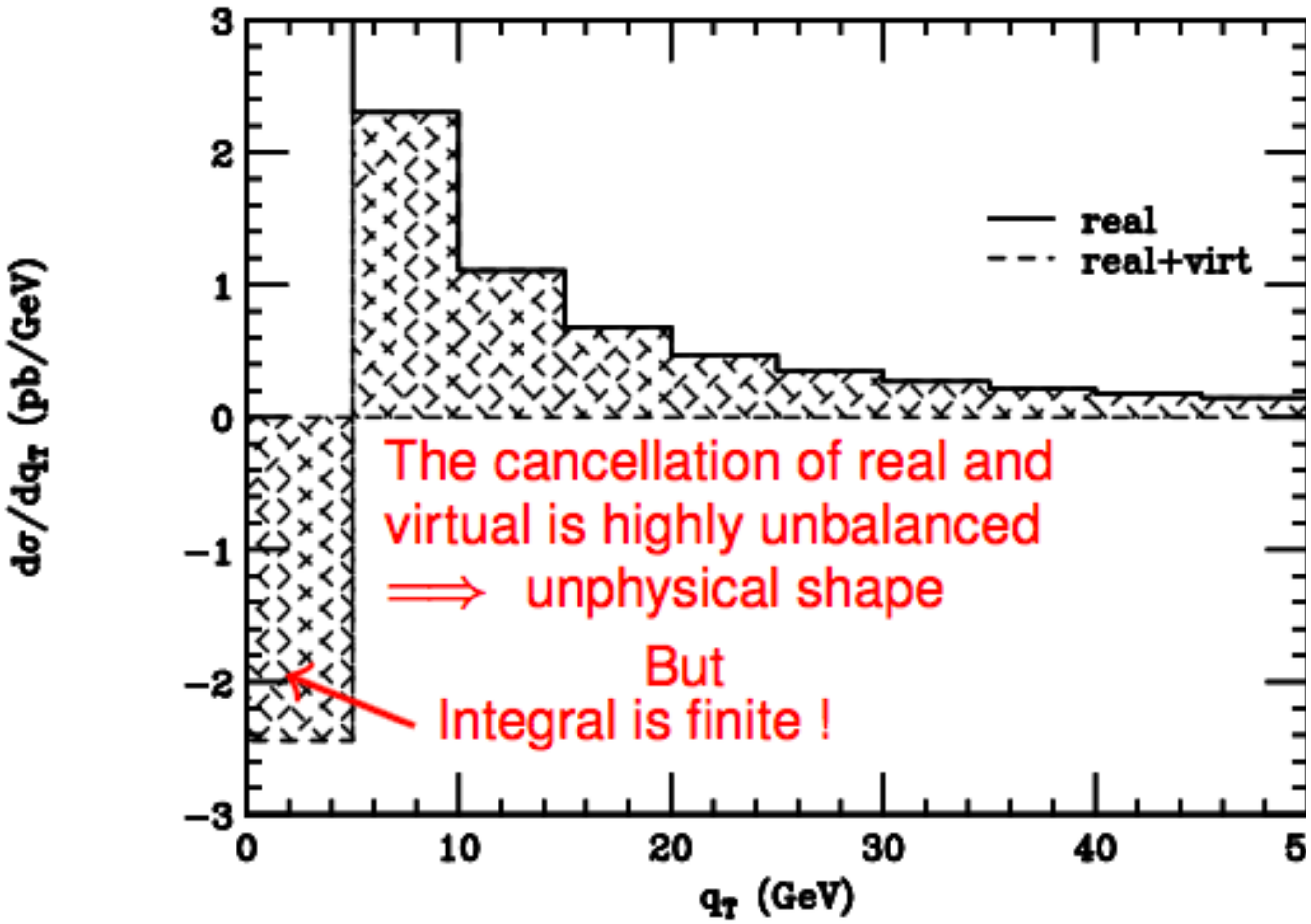
Compare $d\sigma/dq_T$ and $d\sigma/dq_T d\varphi$ when azimuthal correlations are present

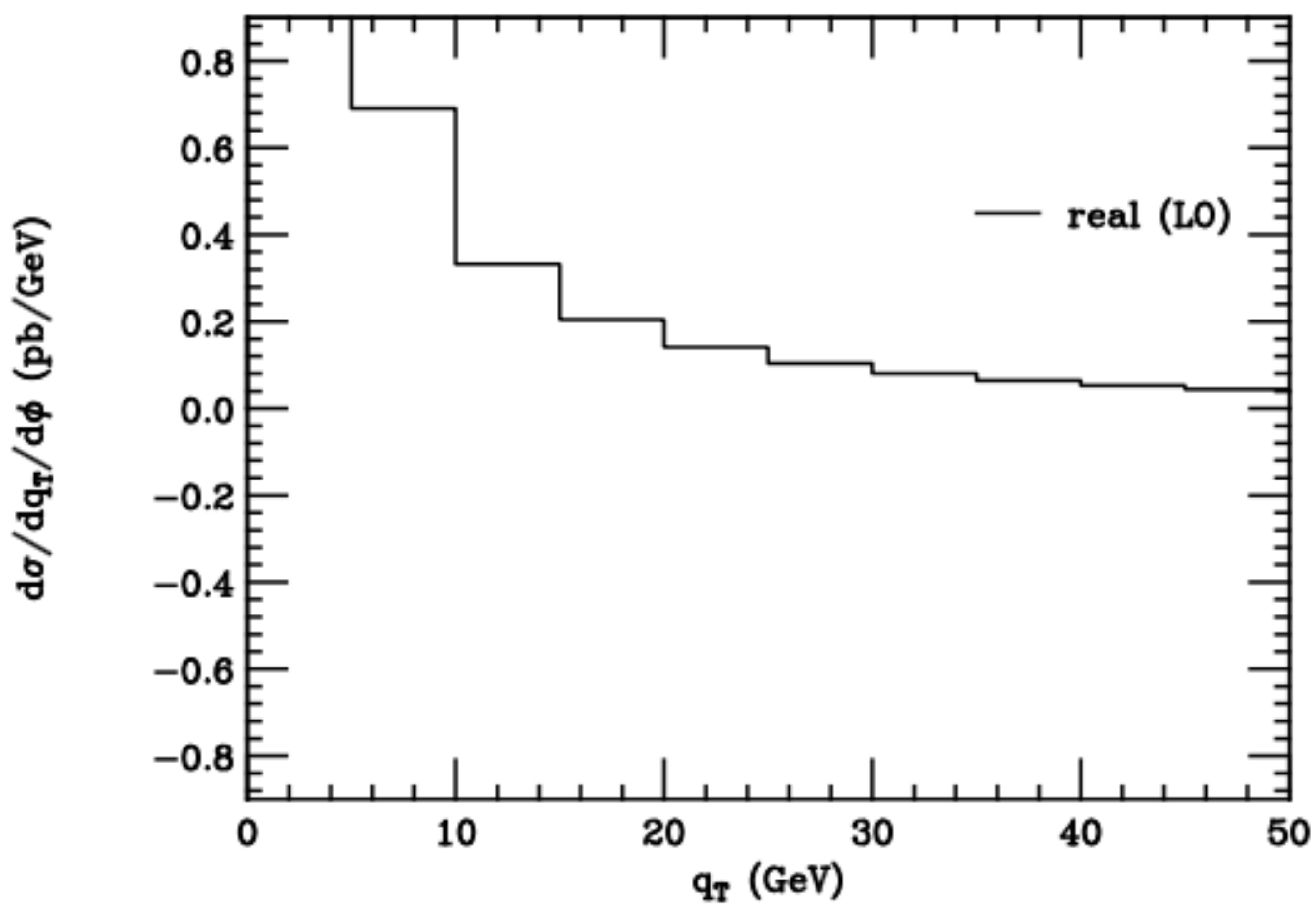




Singular azimuthal correlations

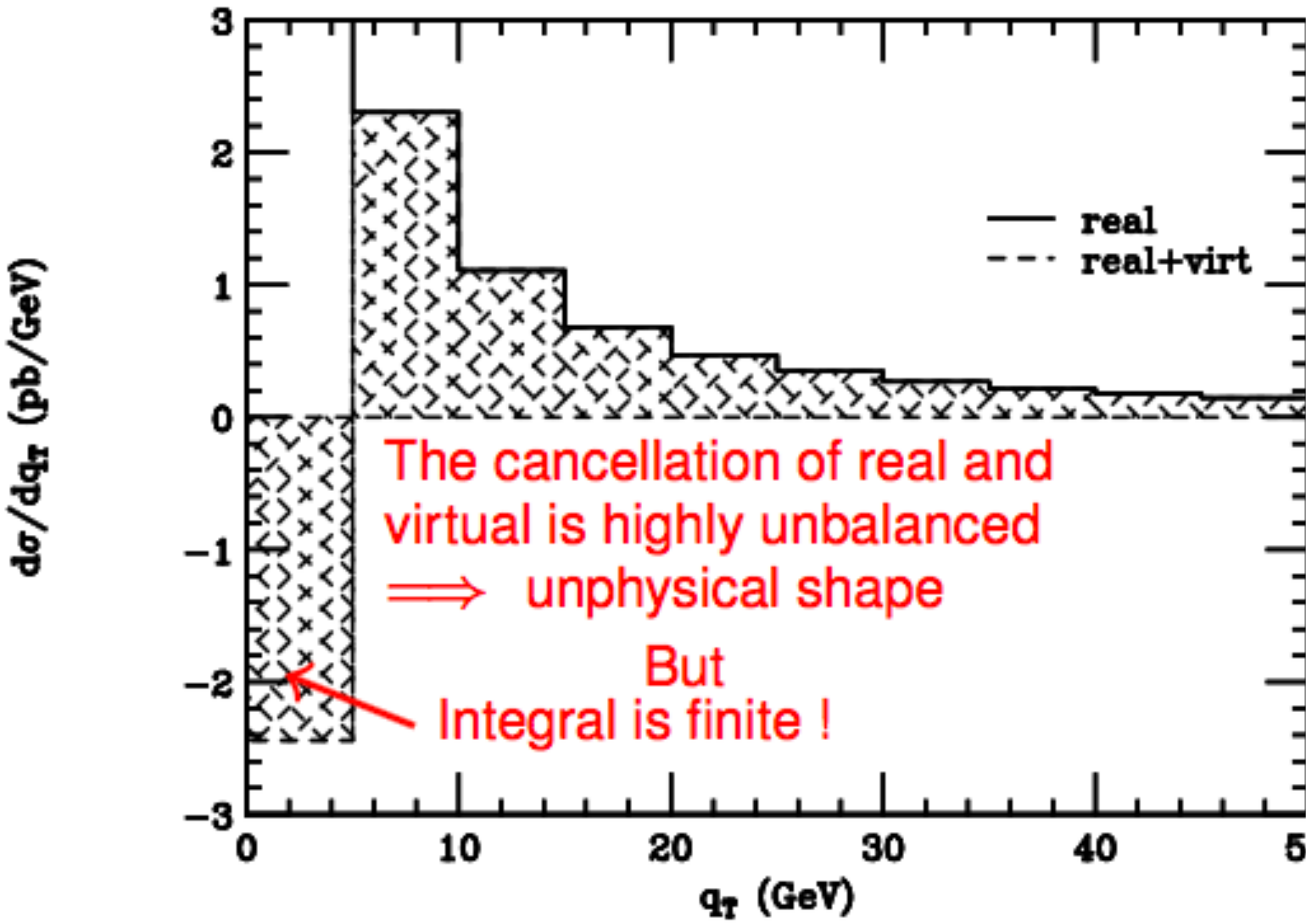
Compare $d\sigma/dq_T$ and $d\sigma/dq_T d\varphi$ when azimuthal correlations are present

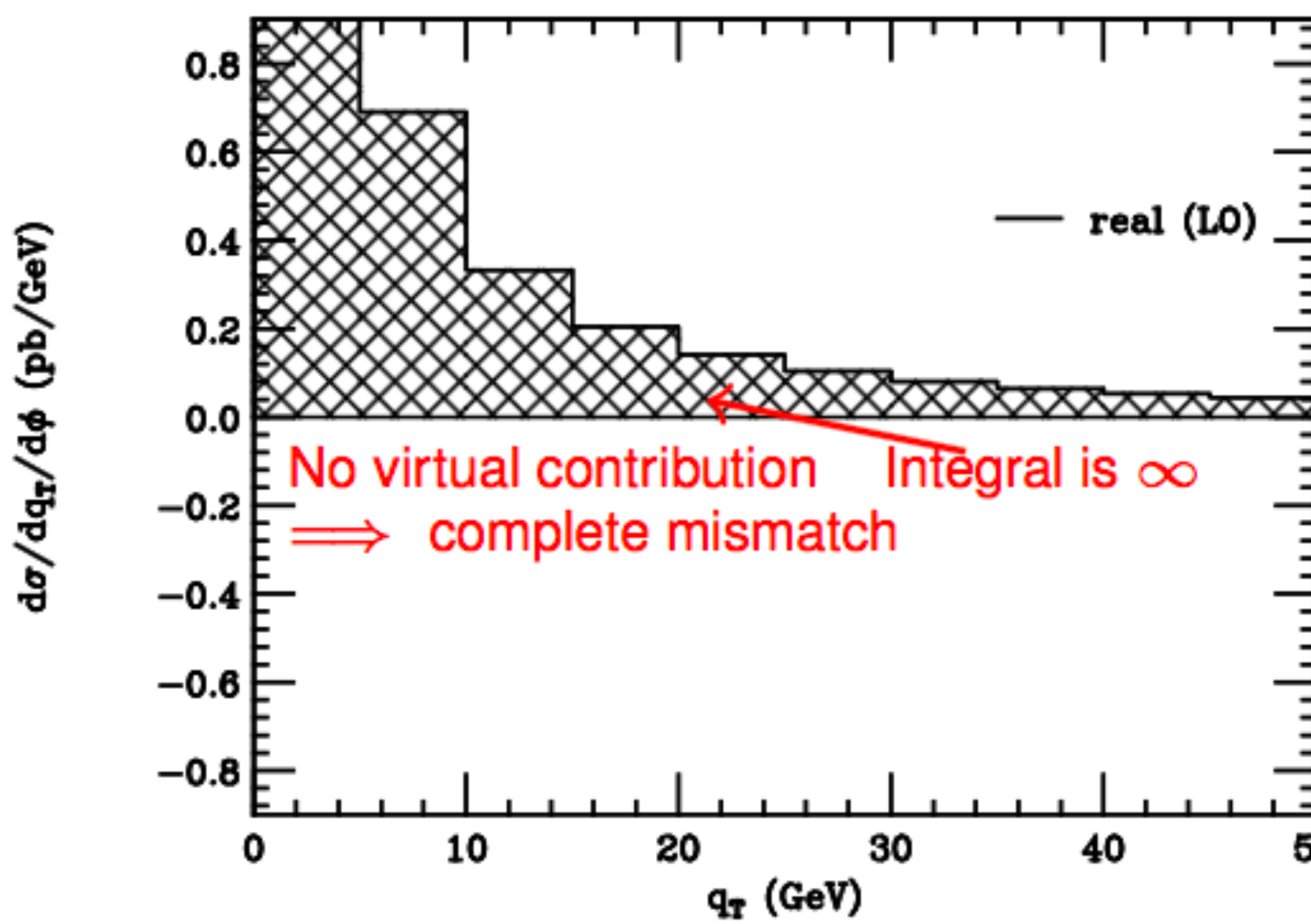




Singular azimuthal correlations

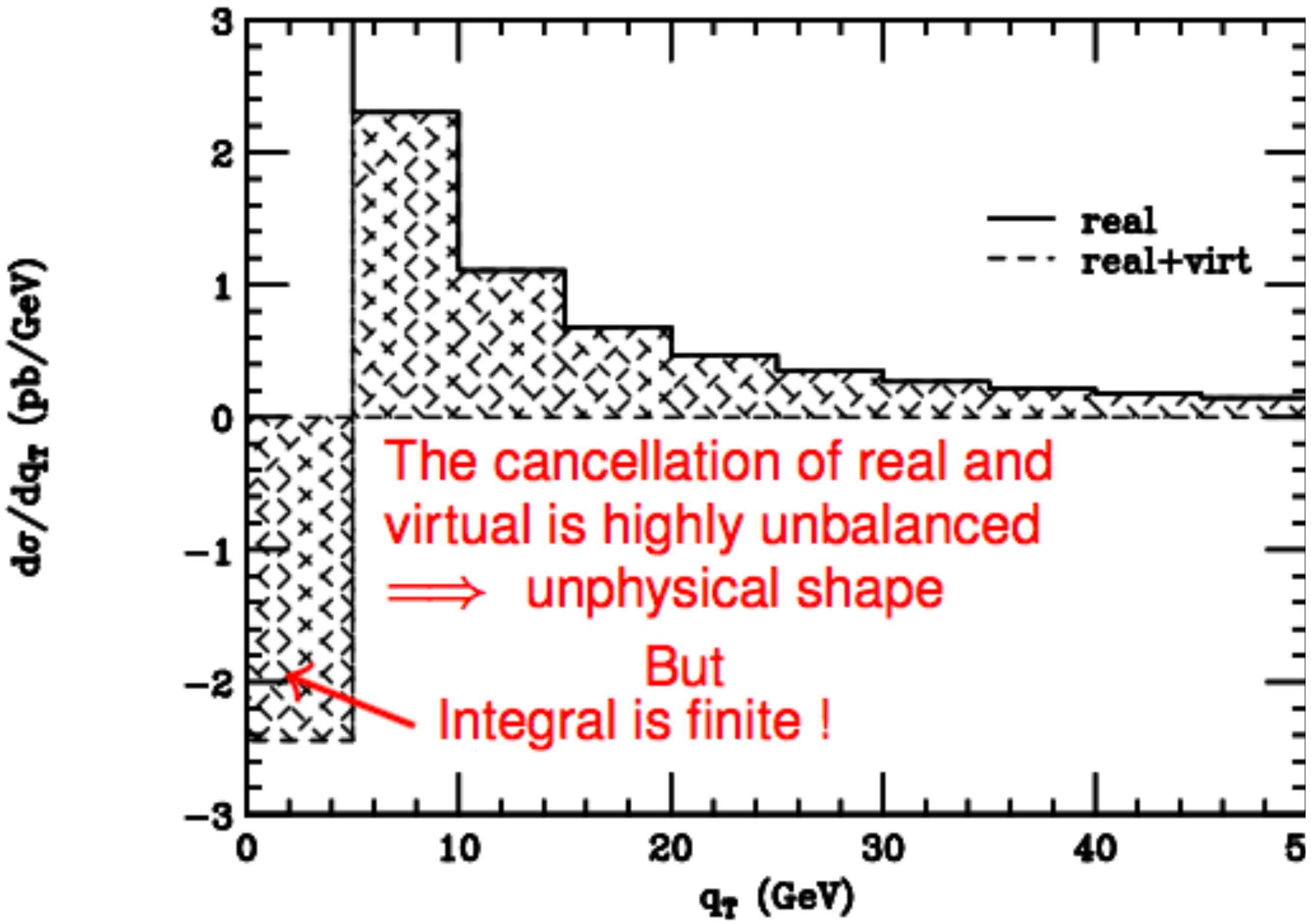
Compare $d\sigma/dq_T$ and $d\sigma/dq_T d\varphi$ when azimuthal correlations are present

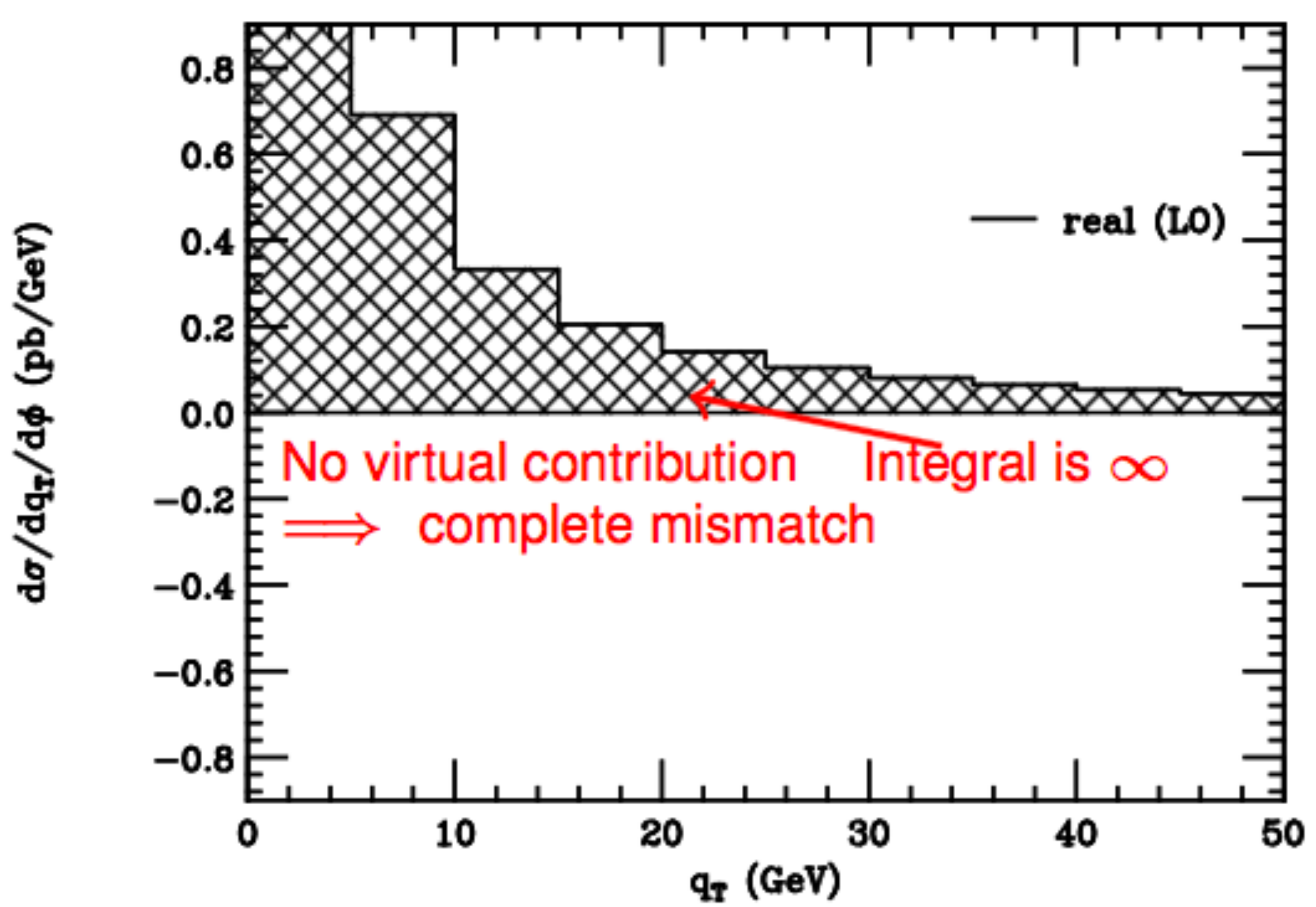




Singular azimuthal correlations

Compare $d\sigma/dq_T$ and $d\sigma/dq_T d\varphi$ when azimuthal correlations are present





Although IR safe the azimuthal correlation is divergent !

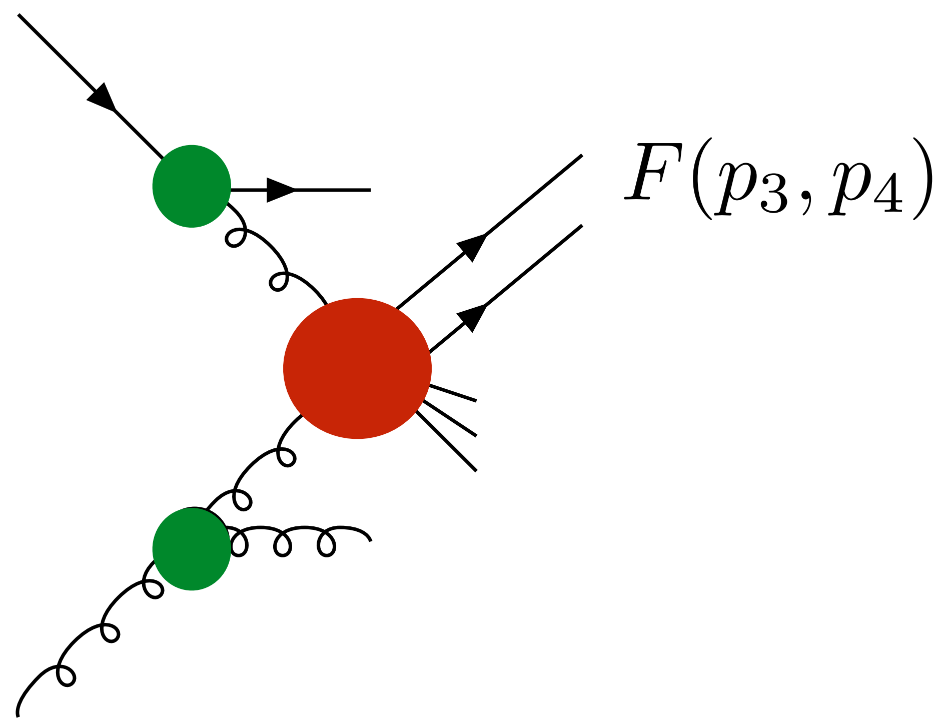
Divergences arise from a single phase space point at q_T=0

Origin of the singular behaviour

Singular azimuthal correlations have two distinctive physical origins:

1) Collinear radiation from initial state colliding gluons

S.Catani, MG (2011)



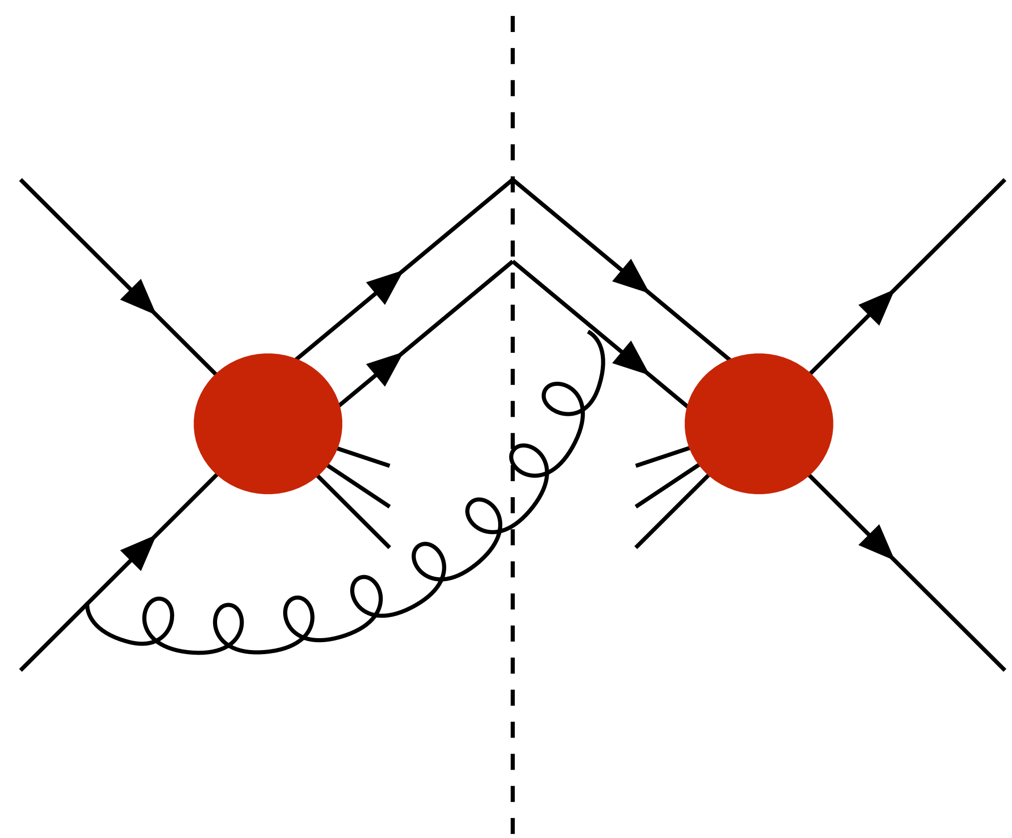
azimuthal correlations are induced by the customary spin correlations in gluon splitting processes (absent in fermion splitting due to helicity conservation)

➡ responsible for singularities only in the n=2 and n=4 harmonics

Origin of the singular behaviour

Singular azimuthal correlations have two distinctive physical origins:

2) Soft wide-angle radiation in the case of final states containing coloured particles



these contributions are explicitly known in the case in which F is a ttbar pair

➡ they are responsible for singularities with arbitrary n

S.Catani, A.Torre, MG (2013)

In the $t\bar{t}$ case only even harmonics are divergent at NLO
(expected not to hold beyond NLO)

Origin of the singular behaviour

Based on the previous discussion we can conclude that azimuthal correlations will have divergences starting from some perturbative order if the final state system F produced at Born level by $c_1 c_2 \to F$ and

1) **at least one of the initial state colliding partons c_1 or c_2 is a gluon**

2) **at least one of the final state particles is coloured**

It is important to note that one of these two conditions is sufficient to produce divergences

- $t\bar{t}$ production: both conditions fulfilled ($gg \to t\bar{t}$ contributes at Born level and the final state is coloured)
- Similarly divergences are expected for F=Vj, F=jj
- F=$\gamma\gamma$, WW and ZZ lead to divergences due to the gg fusion subprocess starting at N^{3}LO

A further example: Z+jet

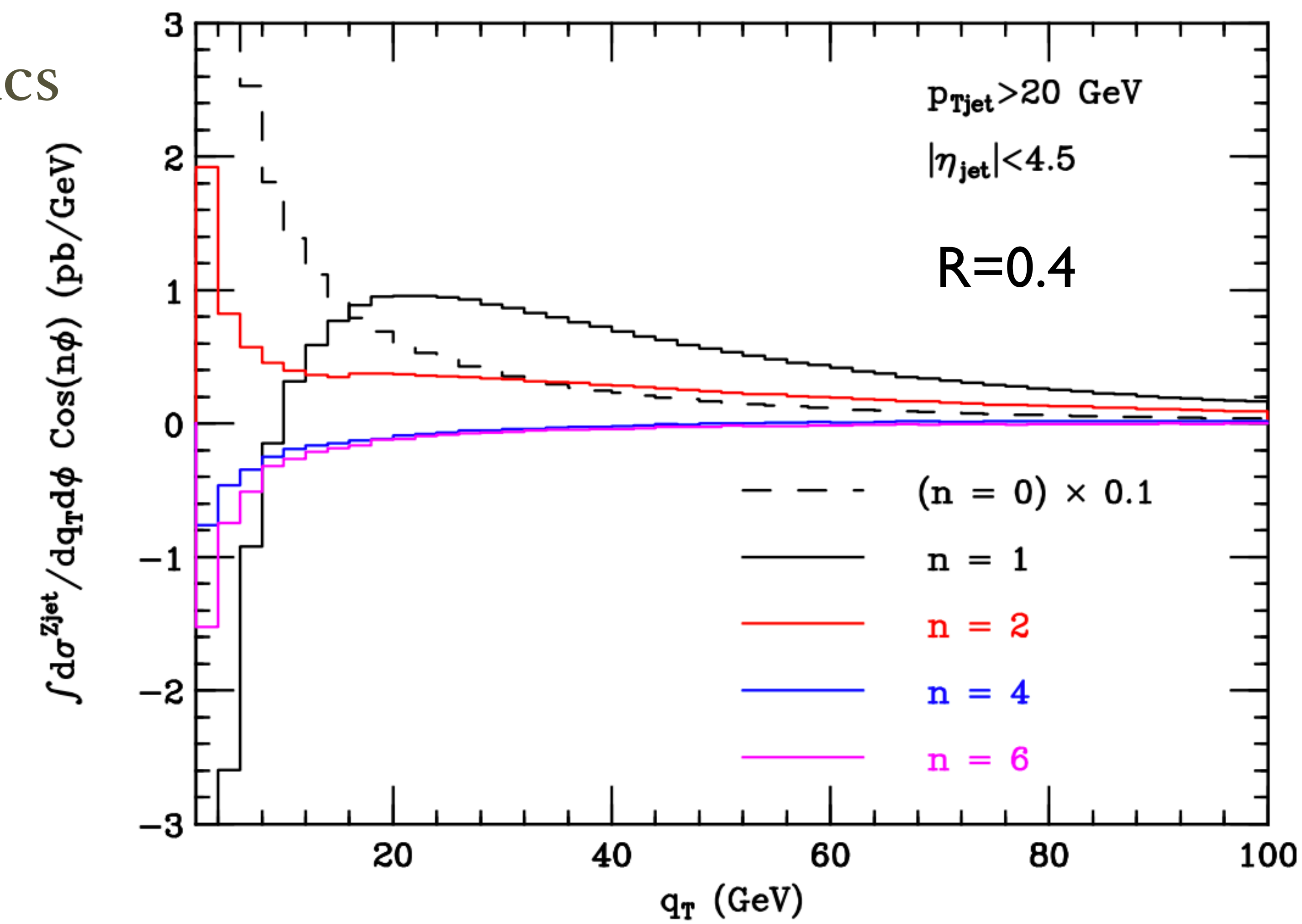
As a further example we consider the case F=Zj in pp collisions at $\sqrt{s}$=8 TeV

We define $\varphi = \phi(\mathbf{p}_{Tjet}) - \phi(\mathbf{q}_T)$

We consider the lowest order contribution to the n=1,2,4,6 harmonics

- n=2: both (soft and collinear) sources of singular azimuthal correlations are present

- n=1,4,6: only soft correlations are present at this order



➡ We expect a singular behaviour $d\sigma_n/dq_T \propto 1/q_T$

The numerical results are consistent with these expectations

Resummation

We first review the case in which there are no divergent azimuthal correlations

The transverse-momentum resummation formalism has been developed in the eighties

Y.Dokshitzer, D.Diakonov, S.I.Troian (1978)
G. Parisi, R. Petronzio (1979)
G. Curci, M.Greco, Y.Srivastava(1979)
J. Collins, D.E. Soper, G. Sterman (1985)

As it is customary in QCD resummations one has to work in a conjugate space in order to allow the kinematics of multiple gluon emission to factorize

In this case, to exactly implement momentum conservation, the resummation has to be performed in impact parameter b-space

$$\delta^{(2)}(\mathbf{p}_T - \mathbf{p}_{T1} - ....\mathbf{p}_{Tn}) \quad \rightarrow \quad e^{i\mathbf{b}\cdot\mathbf{p}_T} \prod_{i=1}^{n} e^{-i\mathbf{b}\cdot\mathbf{p}_{Ti}}$$

The resummed cross section is then obtained by inverse Fourier transformation from a resummed form factor

Resummation

The resummed form factor does not depend on the direction of **b**

➡ the Fourier transform turns into a Bessel transform

$$\frac{d\sigma^{(\mathrm{res})}_{\mathrm{az.av.}}}{dM^2 dq_T^2} = \int_0^{+\infty} db\, b\, J_0(bq_T)\, \Sigma^{(\mathrm{res})}_{\mathrm{az.av.}}(M,b)$$

The behaviour of the Bessel function at small bq_T is $J_0(bq_T) \approx 1 + O(bq_T)$

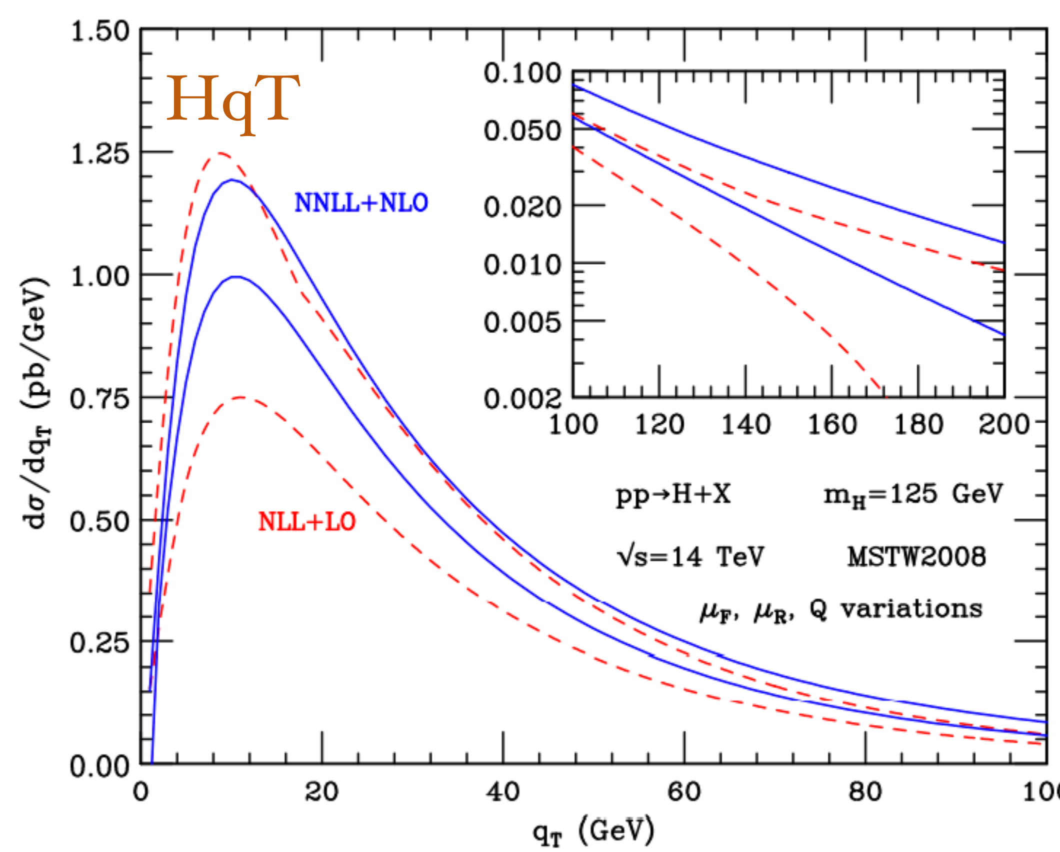
The resummed form factor at large b is strongly damped (Sudakov suppression)

G. Parisi, R. Petronzio (1979)

➡ at small q_T we have

$$\frac{d\sigma^{(\mathrm{res})}_{\mathrm{az.av.}}}{dM^2 dq_T^2} \propto \text{const.}$$

Since $d\sigma/dq_T = 2q_T\, d\sigma/dq_T^2$ we have the customary kinematical peak in the low q_T region



Resummation

We now consider the resummation of the singular azimuthal correlations

The prototype of this resummation has been first studied for heavy-quark pairs

S.Catani, A.Torre, MG (2013)

We consider the q_T dependence of the n-harmonic

$$\frac{d\sigma_n}{dM^2 dq_T^2} \equiv \int_0^{2\pi} d\varphi \;\cos(n\varphi)\; \frac{d\sigma}{dM^2\, dq_T^2\, d\varphi}$$

Also in this case the projection on the n-harmonic allows us to transform the Fourier into a Bessel transformation and we get the n-order Bessel function

$$\frac{d\sigma_n^{(\mathrm{res})}}{dM^2 dq_T^2} = \int_0^{+\infty} db\; b\; J_n(bq_T)\; \Sigma_n^{(\mathrm{res})}(M,b)$$

The leading logarithmic behaviour of the resummed form factor is the same appearing in the azimuthally averaged case

→ The Sudakov suppression at large-b is such that the small-q_T behaviour of the resummed cross section is driven by the one of J_n

Resummation

Since $J_n(bq_T) \approx O((bq_T)^n)$ we have $$\frac{d\sigma_n^{(\mathrm{res})}}{dM^2 dq_T^2} \propto q_T^n$$

The small-q_T behaviour of the resummed cross section is integrable for any n=1,2,3...: highly non-trivial result of resummation

It is interesting to contrast the impact of resummation in the two cases

- In the case of the azimuthally averaged cross section the effect of resummation is

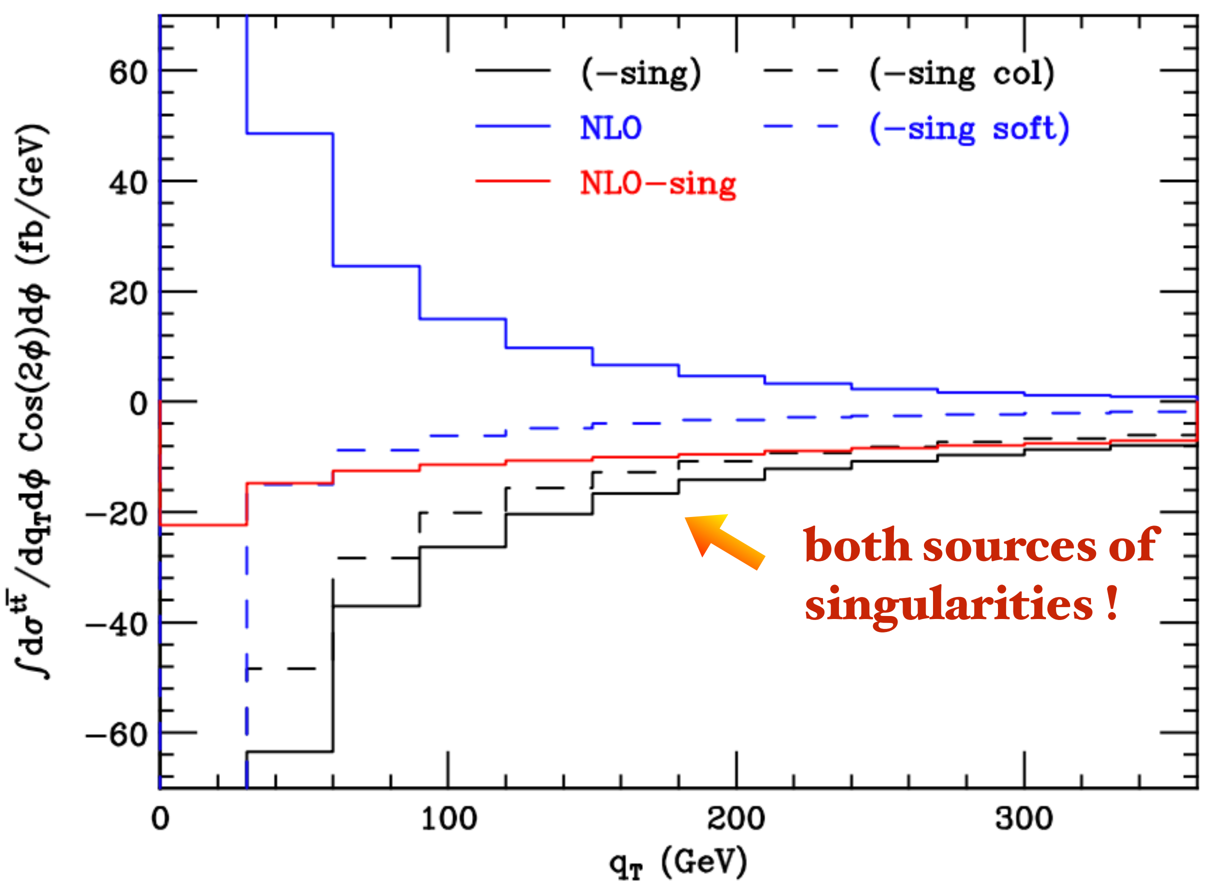
$$1/q_T^2 \to \mathrm{const}$$

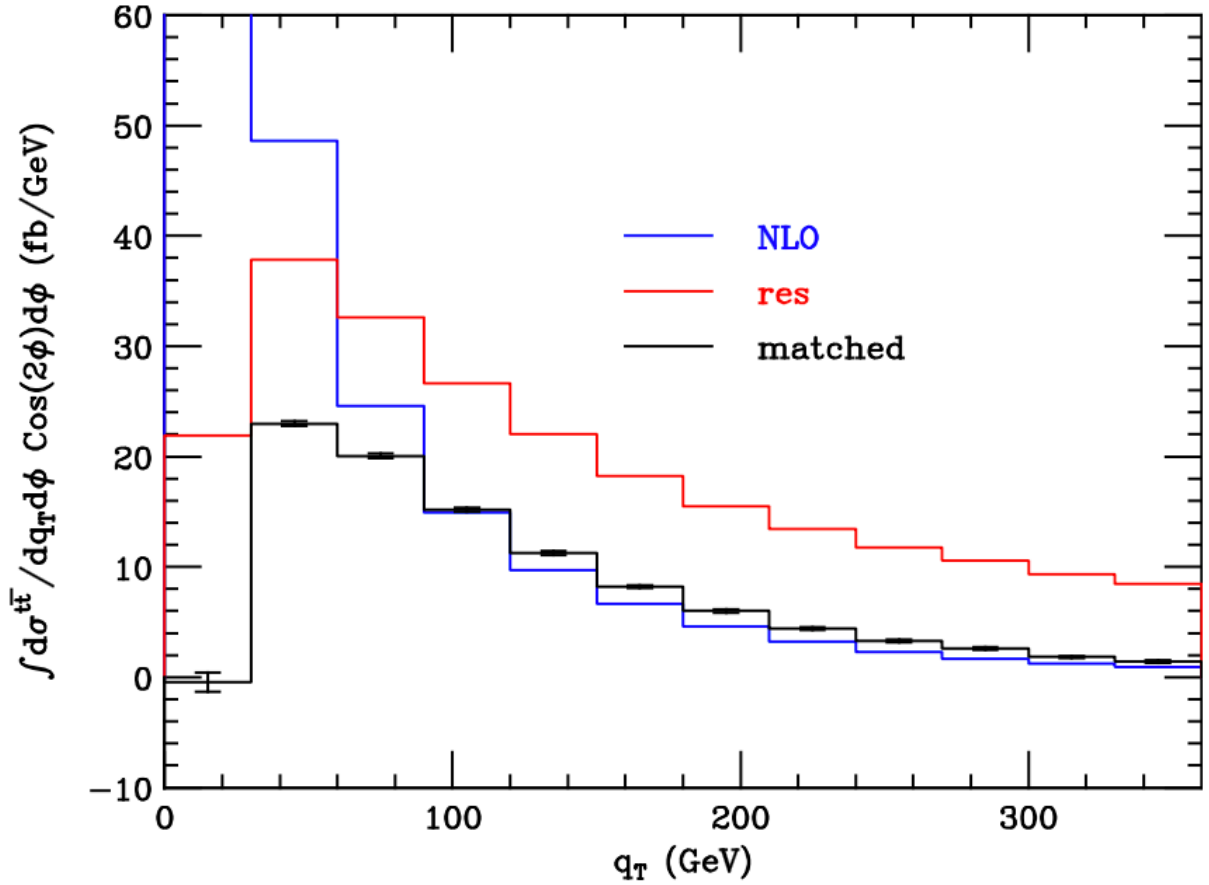
- In the case of the n-harmonic we get

$$1/q_T^2 \to q_T^n$$

→ The effect of resummation for azimuthal correlations is even more substantial and the shape of the resummed spectrum is expected to be significantly different

Results for $t\bar{t}$ at NLL+NLO (n=2)





The resummed result is peaked in the region 30 GeV < q_T < 60 GeV

→ Consistent with suppression at small q_T expected from $(q_T)^3$ behaviour

The matching contributes substantially also at small q_T

The integrated n=2 harmonic is $\sigma^{t\bar{t}}_{n=2} = 3$ pb → about 1/75 of the total NLO cross section

Effective LO prediction within resummed PT

Summary

- Angular distributions are relevant observables to understand the dynamics of hard scattering processes
- We have considered azimuthal asymmetries in the inclusive hadronic production of generic high mass systems composed by two particles
- We have shown that despite the IR safety of these observables their fixed order QCD computation can lead to divergences
- Examples of processes with fixed-order divergences are heavy-quark production, associated production of vector bosons and a jet, dijet and diboson production (DY instead features no singular asymmetries)
- The divergences originate from singular collinear correlations in gg initiated processes and in wide-angle soft gluon correlations in processes with coloured particles in the final state

  → Complete mismatch between virtual and real contributions

Summary

- We have shown quantitative results for the lowest order azimuthal harmonics for Drell-Yan, ttbar and Zj production
- We have discussed the resummed structure of azimuthal correlations by contrasting it with the case of azimuthally averaged cross sections
- Resummation tames the divergent behaviour by producing a powerlike behaviour of the spectrum at low q_T
- We have presented quantitative results for the resummed n=2 harmonic in the case of $t\bar{t}$ production
- In the case of azimuthal harmonics the resummation allows us to obtain effective lowest order predictions

Outlook

- The results I have presented set the stage for applications to a variety of hadron collider processes including vector boson plus jet and dijet production
- Interesting applications can arise also in QED
- Indeed although the DY process is free of QCD singular azimuthal correlations the presence of charged leptons in the final state produces singular correlations in QED due to large angle soft photon emissions
- QED divergences are predicted also in e^+e^- collisions

→ for example in the $e^+e^- \to \mu^+\mu^- + X$ or in the $\gamma\gamma \to \mu^+\mu^- + X$ processes

Backup

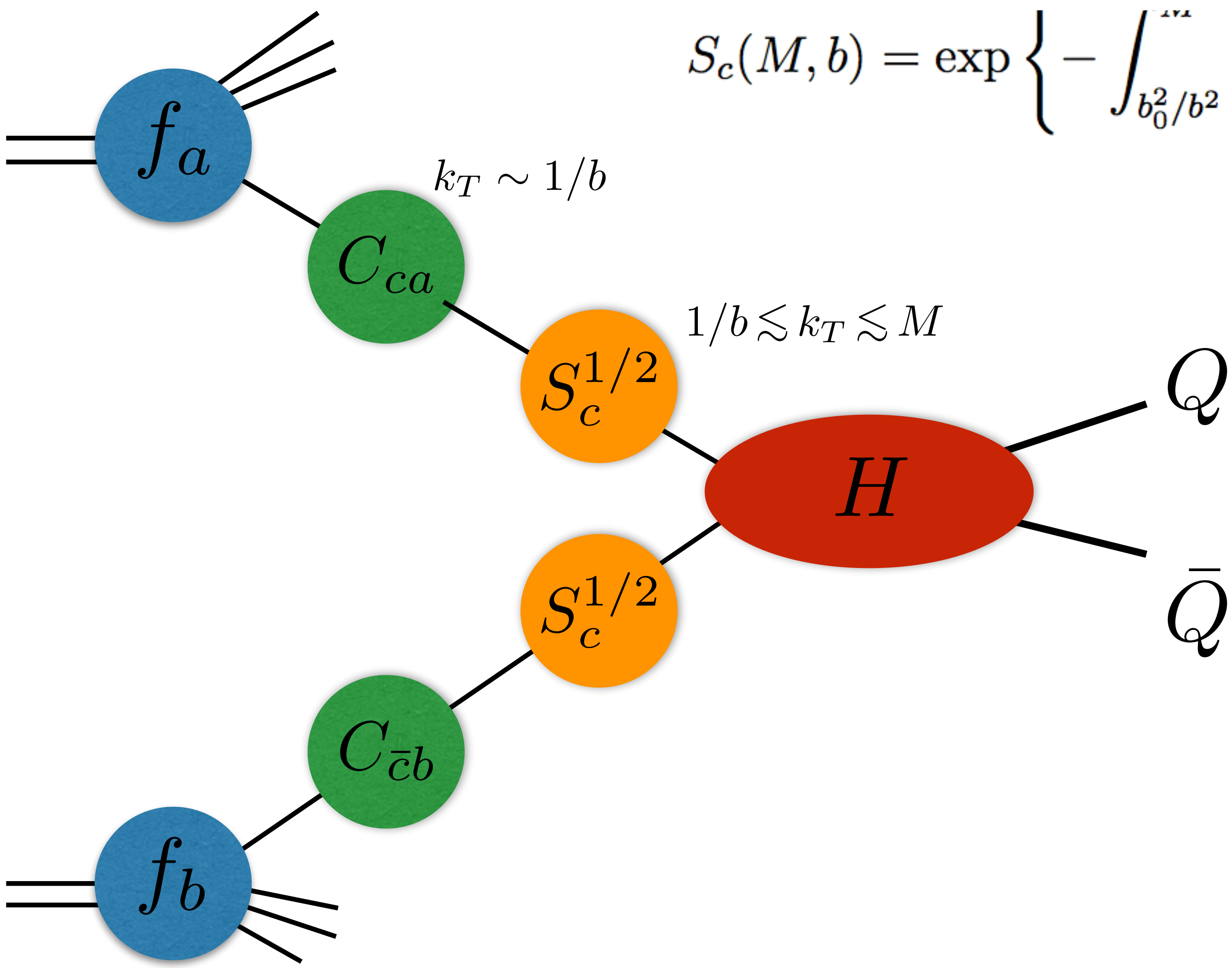
Universal resummation formula

J.Collins, D.Soper, G.Sterman (1984)
S.Catani, D. de Florian, MG (2000); S.Catani, MG (2010)

$$\frac{d\sigma_F^{(\mathrm{sing})}(p_1,p_2;\mathbf{q_T},M,y,\mathbf{\Omega})}{d^2\mathbf{q_T}\;dM^2\;dy\;d\mathbf{\Omega}} = \frac{M^2}{s}\sum_{c=q,\bar{q},g}\left[d\sigma^{(0)}_{c\bar{c},F}\right]\int\frac{d^2\mathbf{b}}{(2\pi)^2}\;e^{i\mathbf{b}\cdot\mathbf{q_T}}\;S_c(M,b)$$

$$\times\sum_{a_1,a_2}\int_{x_1}^{1}\frac{dz_1}{z_1}\int_{x_2}^{1}\frac{dz_2}{z_2}\;\left[H^F C_1 C_2\right]_{c\bar{c};a_1a_2}\;f_{a_1/h_1}(x_1/z_1,b_0^2/b^2)\;f_{a_2/h_2}(x_2/z_2,b_0^2/b^2)$$

$$S_c(M,b) = \exp\left\{-\int_{b_0^2/b^2}^{M^2}\frac{dq^2}{q^2}\left[A_c(\alpha_S(q^2))\;\ln\frac{M^2}{q^2}+B_c(\alpha_S(q^2))\right]\right\}$$



C coefficients embody collinear radiation at scale 1/b

S_c embodies soft and flavour conserving collinear radiation in the region

H^F includes hard radiation at scales $k_T \sim M$

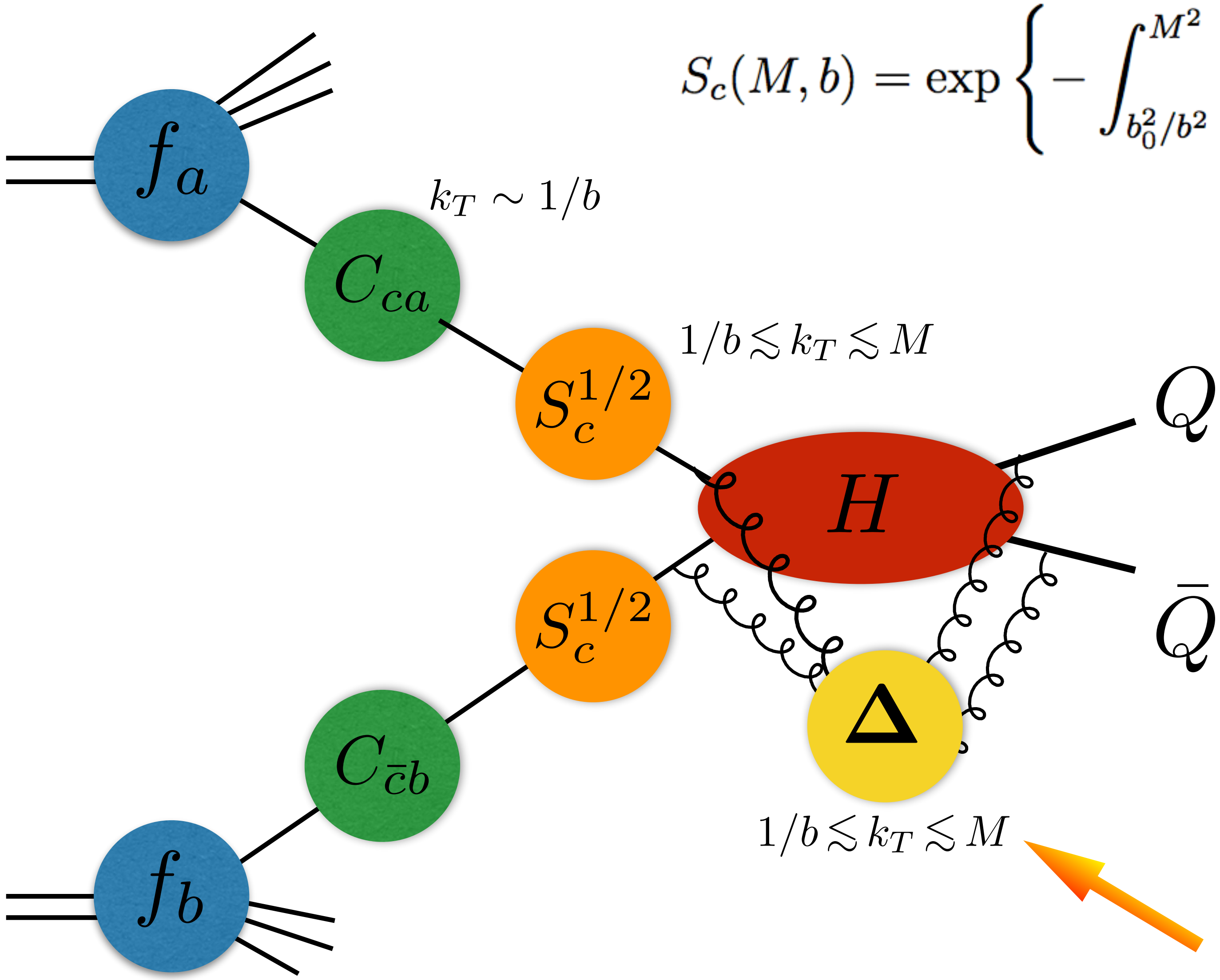
The case of heavy-quark production

S.Catani, A.Torre, MG (2014)
(see also Lin Yang et al. (2013)

$$\frac{d\sigma^{(\mathrm{sing})}(P_1, P_2; \mathbf{q_T}, M, y, \mathbf{\Omega})}{d^2\mathbf{q_T}\, dM^2\, dy\, d\mathbf{\Omega}} = \frac{M^2}{2P_1\cdot P_2} \sum_{c=q,\bar{q},g} \left[d\sigma^{(0)}_{c\bar{c}}\right] \int \frac{d^2\mathbf{b}}{(2\pi)^2}\; e^{i\mathbf{b}\cdot\mathbf{q_T}}\; S_c(M,b)$$

$$\times \sum_{a_1,a_2} \int_{x_1}^{1} \frac{dz_1}{z_1} \int_{x_2}^{1} \frac{dz_2}{z_2}\; \left[(\mathbf{H}\,\mathbf{\Delta})\, C_1 C_2\right]_{c\bar{c};a_1a_2}\; f_{a_1/h_1}(x_1/z_1, b_0^2/b^2)\; f_{a_2/h_2}(x_2/z_2, b_0^2/b^2)$$

$$S_c(M,b) = \exp\left\{ -\int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2} \left[A_c(\alpha_S(q^2))\; \ln\frac{M^2}{q^2} + B_c(\alpha_S(q^2)) \right] \right\}$$



C coefficients embody collinear radiation at scale 1/b

S_c embodies soft and flavour conserving collinear radiation in the region

H^F includes hard radiation at scales $k_T \sim M$

Additional radiative factor of purely soft origin (starts to contribute at NLL)

The case of heavy-quark production

Main issue: the heavy quarks carry non abelian colour charge

➡ soft radiation at large angles with respect to the colliding partons

$$(\mathbf{H}\,\mathbf{\Delta})_{c\bar{c}} = \frac{\langle\, \widetilde{\mathcal{M}}_{c\bar{c}\to Q\bar{Q}} \,|\, \mathbf{\Delta} \,|\, \widetilde{\mathcal{M}}_{c\bar{c}\to Q\bar{Q}} \,\rangle}{\alpha_S^2(M^2)\; |\mathcal{M}^{(0)}_{c\bar{c}\to Q\bar{Q}}(p_1, p_2; p_3, p_4)|^2}$$

$|\, \widetilde{\mathcal{M}}_{c\bar{c}\to Q\bar{Q}} \,\rangle$ subtracted virtual amplitude

$$\mathbf{\Delta}(\mathbf{b}, M; y_{34}, \phi_3) = \mathbf{V}^\dagger(b, M; y_{34})\;\; \mathbf{D}(\alpha_S(b_0^2/b^2); \phi_{3b}, y_{34})\; \mathbf{V}(b, M; y_{34})$$

$$\mathbf{V}(b, M; y_{34}) = \overline{P}_q\; \exp\left\{ -\int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2}\, \mathbf{\Gamma}_t(\alpha_S(q^2); y_{34}) \right\}$$

$\alpha_S^n L^m$ terms $n \geq m$

soft anomalous dimension

$\mathbf{\Gamma}_t^{(1)}$ and $\mathbf{\Gamma}_t^{(2)}$ directly related to singular structure of $|\mathcal{M}_{c\bar{c}\to Q\bar{Q}}\rangle$

M.Neubert et al. (2009)

$\mathbf{D}(\alpha_S; \phi_{3b}, y_{34})$ embodies azimuthal correlations at scale 1/b

$$\langle\, \mathbf{D}(\alpha_S; \phi_{3b}, y_{34})\rangle_{\text{av.}} = 1$$

Resummation coefficients

We have explicit computed all the first order resummation coefficients

$$\mathbf{\Gamma}_t^{(1)}(y_{34}) = -\frac{1}{4}\Bigg\{ (\mathbf{T}_3^2+\mathbf{T}_4^2)\,(1-i\pi) + \sum_{\substack{i=1,2\\ j=3,4}} \mathbf{T}_i\cdot\mathbf{T}_j \,\ln\frac{(2p_i\cdot p_j)^2}{M^2m^2} + 2\,\mathbf{T}_3\cdot\mathbf{T}_4\left[\frac{1}{2v}\ln\left(\frac{1+v}{1-v}\right) - i\pi\left(\frac{1}{v}+1\right)\right]\Bigg\} \; .$$

$$\mathbf{D}^{(1)}(\phi_{3b},y_{34}) = (\mathbf{T}_3^2+\mathbf{T}_4^2)\left[\frac{c_{3b}\,\mathrm{arcsinh}\,(c_{3b})}{\sqrt{1+c_{3b}^2}} - \frac{1}{2}\ln\left(\frac{m_T^2}{m^2}\right)\right] - (\mathbf{T}_3+\mathbf{T}_4)^2\left(\mathrm{arcsinh}^2\,(c_{3b}) + \frac{1}{2}\mathrm{Li}_2\left(-\frac{\mathbf{p_T^2}}{m^2}\right)\right) + \frac{1}{2v}\,\mathbf{T}_3\cdot\mathbf{T}_4\,(L_{34}^{\varphi}-L_{34})$$

$$L_{34}^{\varphi} = \mathrm{Sign}(c_{3b})\Big[L_\xi\left(\xi(c_{3b},\alpha_{34}),\alpha_{34}\right) - L_\xi\left(\xi(-c_{3b},\alpha_{34}),\alpha_{34}\right)\Big]$$

$$L_\xi(\xi,\alpha) = \frac{1}{2}\ln^2\frac{\xi(1+\xi)}{\alpha+\xi} - \ln^2\frac{\xi}{\alpha+\xi} + 2\left[\mathrm{Li}_2(-\xi) - \mathrm{Li}_2\left(\frac{\alpha+\xi}{\alpha-1}\right) + \ln(\alpha+\xi)\ln(1-\alpha)\right]$$

$$\xi(c,\alpha) = \left(c+\sqrt{1+c^2}\right)\left(c+\sqrt{\alpha+c^2}\right) \quad , \quad \alpha_{34} = \frac{2\sqrt{1-v^2}}{1-\sqrt{1-v^2}}\,c_{3b}^2$$