

# NLO corrections to $h \rightarrow WW/ZZ \rightarrow 4$ fermions in a Singlet Extension of the Standard Model

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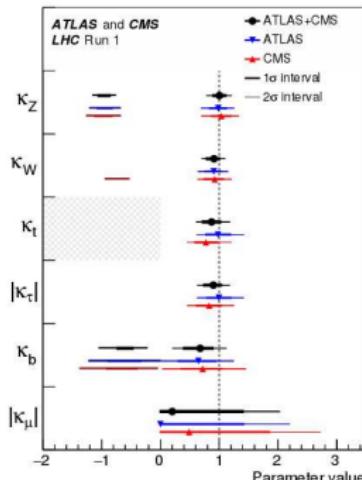
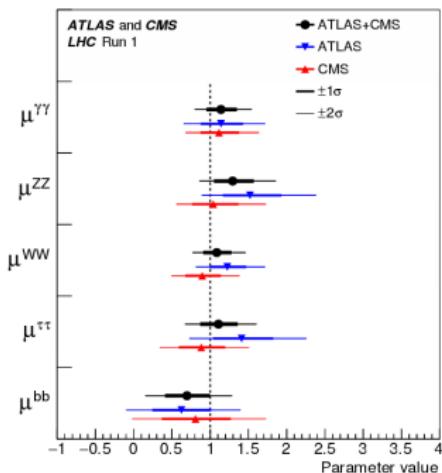
# Overview

- 1 Introduction
- 2 SESM basics
- 3 Renormalization of SESM
- 4 The decay  $h \rightarrow WW/ZZ \rightarrow 4 \text{ fermions}$
- 5 Numerical analysis



# Introduction

# Higgs measurements are compatible with SM predictions



$$\mu = \frac{\Gamma_{\text{exp}}}{\Gamma_{\text{SM}}} \propto \kappa_f$$

Diagram illustrating the relationship between the measured ratio  $\mu$  and the coupling constant  $\kappa_f$ . The diagram shows a vertex where a Higgs boson  $h$  decays into two fermions  $f$ , with the coupling proportional to  $\kappa_f$ .

$$\mu = \frac{\Gamma_{\text{exp}}}{\Gamma_{\text{SM}}} \propto \kappa_V$$

Diagram illustrating the relationship between the measured ratio  $\mu$  and the coupling constant  $\kappa_V$ . The diagram shows a vertex where a Higgs boson  $h$  decays into a gauge boson  $V$ , with the coupling proportional to  $\kappa_V$ .

but

- baryon asymmetry
- dark matter
- neutrino masses
- ...

$\Rightarrow$  SM cannot be the ultimate theory  
BSM precise predictions are required

the SM Higgs sector

$$\mathcal{L}_{\text{Higgs}}^{\text{SM}} = (D_\mu \Phi)^\dagger D^\mu \Phi + \mu^2 \Phi^\dagger \Phi - \frac{\lambda}{4} (\Phi^\dagger \Phi)^2$$

is working well, we consider

$$\mathcal{L}_{\text{Higgs}} = \mathcal{L}_{\text{Higgs}}^{\text{SM}} + \text{???}$$

how?

- take advantage of the “Higgs portal”  $\mu^2 \Phi^\dagger \Phi$
- add a real scalar singlet
- write all the interactions (compatible with symmetries)

→ simplest Higgs sector extension



# Intro

singlet used in the literature for

- dark matter

extra symmetry  
vanishing singlet VEV

[Silveira, Zee, 1985]

[McDonald, 1994]

[Burgess, Pospelov, Veldhuis, 2001]

[Davoudiasl, Kitano, Li, Murayama, 2005]

[Barger et al., 2008]

[Fischer, Van der Bij, 2014]

- hidden sector SB

extra symmetry  
non-vanishing VEVs

[Datta, Raychaudhuri, 1997]

[Patt, Wilczek, 2008]

[Pruna, Robens, 2013]

- baryon asymmetry

[Profumo, Ramsey-Musolf, Shaughnessy, 2007]

[Barger et al., 2007]



## SESM in a nutshell

# SESM in a nutshell

recipe:

- most general  $\mathbb{Z}_2$ - and gauge-invariant scalar Lagrangian

$$\mathcal{L}_{\text{Higgs}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) + \frac{1}{2} (\partial_\mu \sigma) (\partial^\mu \sigma) - V(\Phi, \sigma)$$

$$V(\Phi, \sigma) = -\mu_2^2 \Phi^\dagger \Phi + \frac{\lambda_2}{4} (\Phi^\dagger \Phi)^2 + \lambda_{12} \sigma^2 \Phi^\dagger \Phi - \mu_1^2 \sigma^2 + \lambda_1 \sigma^4$$

- EWSB on both scalar fields

$$\Phi = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}[v_2 + h_2 + i\phi^0] \end{pmatrix}, \quad \sigma = v_1 + h_1$$

- covariant derivative

$$D_\mu = \partial_\mu - ig_2 I_w^a W_\mu^a + ig_1 \frac{Y_w}{2} B_\mu$$



# SESM in a nutshell

after EWSB

$$V = -t_2 h_2 - t_1 h_1 + \frac{1}{2} (h_2, h_1) \mathcal{M}_{\text{Higgs}} \begin{pmatrix} h_2 \\ h_1 \end{pmatrix} + \dots$$

with **non-diagonal** mass matrix  $\mathcal{M}_{\text{Higgs}}$

$$\mathcal{M}_{\text{Higgs}} = \begin{pmatrix} v_1^2 \lambda_{12} + \frac{3v_2^2 \lambda_2}{4} - \mu_2^2 & 2v_1 v_2 \lambda_{12} \\ 2v_1 v_2 \lambda_{12} & v_2^2 \lambda_{12} + 12v_1^2 \lambda_1 - 2\mu_1^2 \end{pmatrix}$$

⇒  $h_2, h_1$  have non-diagonal propagators!

$$h_2 \text{---} h_2 = \frac{i}{k^2 - M_{22}^2}, \quad h_1 \text{---} h_1 = \frac{i}{k^2 - M_{11}^2}, \quad h_2 \text{---} h_1 \neq 0$$

rotate about an angle  $\alpha$  to get “mass-basis”  $h, H$

$$\begin{pmatrix} h \\ H \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} h_2 \\ h_1 \end{pmatrix}$$



# SESM in a nutshell

requiring diagonal Higgs propagators

$$h \text{---} h = \frac{i}{k^2 - M_h^2}, \quad H \text{---} H = \frac{i}{k^2 - M_H^2}, \quad h \text{---} H = 0$$

⇒ inversion relations

$$\mu_2^2 = \frac{1}{2v_2} [3c_\alpha t_h + 3t_H s_\alpha + c_\alpha M_h^2 (v_2 c_\alpha - v_1 s_\alpha) + M_H^2 s_\alpha (v_1 c_\alpha + v_2 s_\alpha)]$$

$$\mu_1^2 = \frac{1}{4v_1} [M_h^2 s_\alpha (v_1 s_\alpha - v_2 c_\alpha) + c_\alpha M_H^2 (v_1 c_\alpha + v_2 s_\alpha) + 3c_\alpha t_H - 3t_h s_\alpha]$$

$$\lambda_2 = \frac{2}{v_2^3} [v_2 (c_\alpha^2 M_h^2 + M_H^2 s_\alpha^2) + c_\alpha t_h + t_H s_\alpha]$$

$$\lambda_1 = \frac{1}{8v_1^3} [v_1 (c_\alpha^2 M_H^2 + M_h^2 s_\alpha^2) + c_\alpha t_H - t_h s_\alpha]$$

$$v_1 = \frac{c_\alpha s_\alpha}{2\lambda_{12} v_2} (M_H^2 - M_h^2)$$



# SESM in a nutshell

requiring diagonal Higgs propagators

$$h \text{---} h = \frac{i}{k^2 - M_h^2}, \quad H \text{---} H = \frac{i}{k^2 - M_H^2}, \quad h \text{---} H = 0$$

⇒ mass terms

$$M_h^2 = \frac{1}{4}v_2^2\lambda_2 + 4v_1^2\lambda_1 \pm \sqrt{(2v_1v_2\lambda_{12})^2 + \frac{1}{16}(16v_1^2\lambda_1 - v_2^2\lambda_2)^2}$$

$$M_H^2 = \frac{1}{4}v_2^2\lambda_2 + 4v_1^2\lambda_1 \mp \sqrt{(2v_1v_2\lambda_{12})^2 + \frac{1}{16}(16v_1^2\lambda_1 - v_2^2\lambda_2)^2}$$

sign choice such that

$$M_H^2 > M_h^2$$



# SESM in a nutshell

## gauge- and fermion-scalar interactions

light Higgs

$$\begin{array}{c} f \\ \diagup \\ h \\ \diagdown \\ f \end{array}, \quad \begin{array}{c} h \\ \diagup \\ V \\ \diagdown \end{array} \propto c_\alpha$$

heavy Higgs

$$\begin{array}{c} f \\ \diagup \\ H \\ \diagdown \\ f \end{array}, \quad \begin{array}{c} H \\ \diagup \\ V \\ \diagdown \end{array} \propto s_\alpha$$

## multi-scalar interactions

$$\begin{aligned} V \supset & c_{hhh}h^3 + c_{hhH}h^2H + c_{hHH}hH^2 + c_{HHH}H^3 + c_{\phi\phi\phi\phi}\left(2\phi^+\phi^- + (\phi^0)^2\right)^2 \\ & + c_{hhhh}h^4 + c_{hhhH}h^3H + c_{hhHH}h^2H^2 + c_{hHHH}hH^3 + c_{HHHH}H^4 \\ & + [c_{h\phi\phi}h + c_{H\phi\phi}H + c_{hh\phi\phi}h^2 + c_{hH\phi\phi}hH + c_{HH\phi\phi}H^2]\left(2\phi^+\phi^- + (\phi^0)^2\right) \end{aligned}$$

## SESM parameters

$$\{M_h, \textcolor{blue}{M_H}, M_W, M_Z, e, \lambda_{12}, \alpha, m_f, t_h, \textcolor{blue}{t_H}\}$$

# Renormalization of SESM

# Renormalization transformations

bare parameters do not have physical meaning

- scalar sector

$$M_{h,0}^2 = M_h^2 + \delta M_h^2 \quad t_{h,0} = t_h + \delta t_h$$

$$M_{H,0}^2 = M_H^2 + \delta M_H^2 \quad t_{H,0} = t_H + \delta t_H$$

$$\lambda_{12,0} = \lambda_{12} + \delta\lambda_{12} \quad \alpha_0 = \alpha + \delta\alpha$$

$$\begin{pmatrix} h_0 \\ H_0 \end{pmatrix} = \begin{pmatrix} 1 + \frac{1}{2}\delta Z_{hh} & \frac{1}{2}\delta Z_{hH} \\ \frac{1}{2}\delta Z_{Hh} & 1 + \frac{1}{2}\delta Z_{HH} \end{pmatrix} \begin{pmatrix} h \\ H \end{pmatrix}$$

- standard transformations for gauge and fermion sectors



# Renormalization conditions

as far as possible, make use of OS renormalization conditions

$M_h, M_H, M_W, M_Z, m_f$   
are the physical (OS) masses

clear prescription:

- renormalized masses equal physical masses
- residues of the OS propagators are not changed by one-loop corrections
- OS physical fields do not mix on their respective mass shells



$$\begin{aligned} & \delta M_h^2, \delta M_H^2, \delta M_W^2, \delta M_Z^2, \delta m_f \\ & \delta Z_{hh}, \delta Z_{HH}, \delta Z_{hH}, \delta Z_{Zh} \\ & \delta Z_W, \delta Z_{ZZ}, \delta Z_{\gamma\gamma}, \delta Z_{\gamma Z}, \delta Z_f \end{aligned}$$



# Renormalization conditions

OS renormalization not always possible

no natural choice for  
 $\alpha, \lambda_{12}$

possibilities:

- relate parameters to a physical process, **but**
  - no observations so far
  - large corrections to other observables
- minimal subtraction schemes

our choice  $\Rightarrow \overline{\text{MS}}$

$$\left( \frac{h}{Z} \cancel{\times} \frac{Z}{Z} + \frac{h}{Z} \bullet \frac{Z}{Z} \right) \Bigg|_{\text{UV}} \stackrel{!}{=} 0 \quad \left( \frac{h}{Z} \cancel{\times} \frac{h}{h} + \frac{h}{Z} \bullet \frac{h}{h} \right) \Bigg|_{\text{UV}} \stackrel{!}{=} 0$$



# Tadpole renormalization

two schemes considered

renorm. tadpoles  $t_h = t_H = 0$

- ignore explicit tadpoles



- gauge-dependent  $\delta t_h, \delta t_H$  in counterterms

- bare parameters potentially gauge-dependent

gauge-dependent contributions cancel in OS scheme

bare tadpoles  $t_{h,0} = t_{H,0} = 0$

[Fleischer, Jegerlehner, 1980] [Actis et al., 2006]

- include everywhere



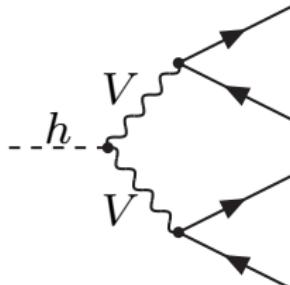
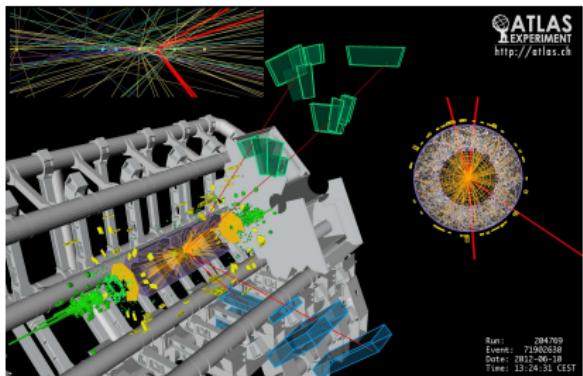
- technical variants can be used (e.g. shift VEVs)

⇒ gauge-independent relations between  
ren. parameters and observables

$h \rightarrow WW/ZZ \rightarrow 4 \text{ fermions}$



# $h \rightarrow WW/ZZ \rightarrow 4 \text{ fermions}$



- most promising channel for precise Higgs measurements @LHC
- implemented in **Prophecy4f**
  - SM [Bredenstein et al., 2006]
  - THDM [Altenkamp et al., 2017]
  - object of this talk → SESM

A Monte Carlo generator for a  
Proper description of the  
Higgs decay into 4 fermions

# NLO corrections

similarly to THDM case, presented in Turin [Dittmaier, 3<sup>rd</sup> HT Annual Meeting]

Survey of Feynman diagrams for NLO corrections to  $h \rightarrow WW/ZZ \rightarrow 4f$

Lowest order:   $= \sin(\beta - \alpha) \mathcal{M}_{SM,LO}$

Typical one-loop diagrams: # diagrams =  $\mathcal{O}(200-400)$

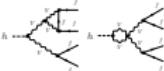
pentagons



boxes



vertices



self-energies



+ counterterms

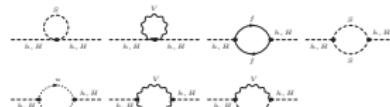
+ tree graphs with real gluon or photons



Silvia Eikenstaedt, THDM and corrections to  $h \rightarrow WW/ZZ \rightarrow 4f$  HiggsTools, Annual Meeting, Turin, May 2017 - 18

Generic diagrams for  $hh$ ,  $hH$ ,  $HH$  self-energies

$\hookrightarrow$  external wave-function renormalization +  $hH$  mixing



$S = h, H, A_0, H^\pm, G_0, G^\pm$

Generic diagrams with internal heavy Higgs bosons  $H$ ,  $A_0$ ,  $H^\pm$

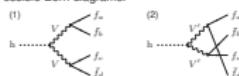


Silvia Eikenstaedt, THDM and corrections to  $h \rightarrow WW/ZZ \rightarrow 4f$  HiggsTools, Annual Meeting, Turin, May 2017 - 18

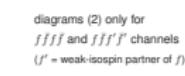
Classification of QCD corrections

Possible Born diagrams:

(1)



(2)



diagrams (2) only for  
 $ffff$  and  $fff'f'$  channels  
( $f'$  = weak-isospin partner of  $f$ )

Classification of QCD corrections into four categories: (typical diagrams shown)



(d) only QCD correction without  
universal scaling  $\propto s_{\beta-\alpha}$  from  $\mathcal{M}_{SM}$



(c)   
(d)

(b,c,d) = corrections to interferences (only for  $q\bar{q}q\bar{q}$  and  $q\bar{q}\gamma\gamma$  channels)



Silvia Eikenstaedt, THDM and corrections to  $h \rightarrow WW/ZZ \rightarrow 4f$  HiggsTools, Annual Meeting, Turin, May 2017 - 17

- $|\mathcal{M}_{LO}|^2$ , QCD and real emission rescaled by  $c_\alpha^2$  wrt SM
- singlet changes EW loops
- CTs are consistently taken into account



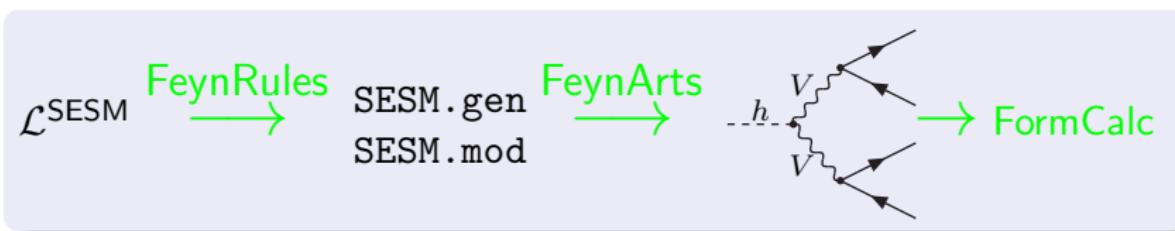
- loop corrections

FeynRules Feynman rules generation

FeynArts diagram generation

FormCalc algebraic simplification

→ complex mass scheme for vector boson resonances [Denner et al., 2005]



- real corrections

- dipole subtraction for IR singularities

[Catani, Seymour, 1996] [Dittmaier, 1999] [Dittmaier et al., 2008]

- multi-channel MC integration within Prophecy4f

# Numerical analysis

# Input parameters

$$\{M_h, M_H, M_W, M_Z, e, \lambda_{12}, \alpha, m_i\}$$

## SM parameters

- [ATLAS, CMS, 2015]  $\rightarrow M_h$
- [HXSWG, 2016]  $\rightarrow M_W^{\text{OS}}, M_Z^{\text{OS}}, \Gamma_W^{\text{OS}}, \Gamma_Z^{\text{OS}}, m_f, G_\mu, \alpha_s$

## BSM parameters restricted by

- perturbativity

$$4|\lambda_1| \lesssim 4\pi, \quad \frac{|\lambda_2|}{4} \lesssim 4\pi, \quad 2|\lambda_{12}| \lesssim 4\pi$$

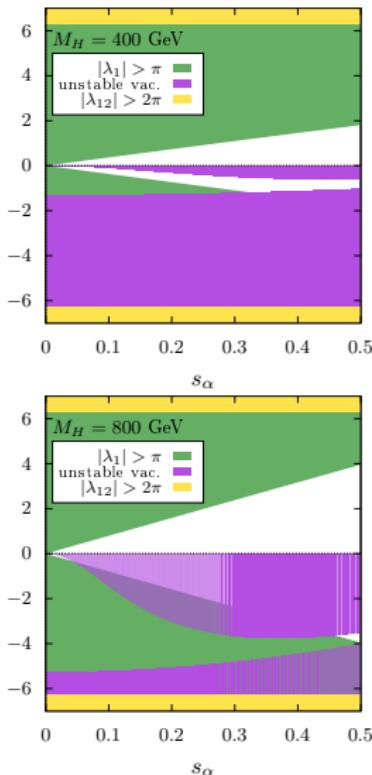
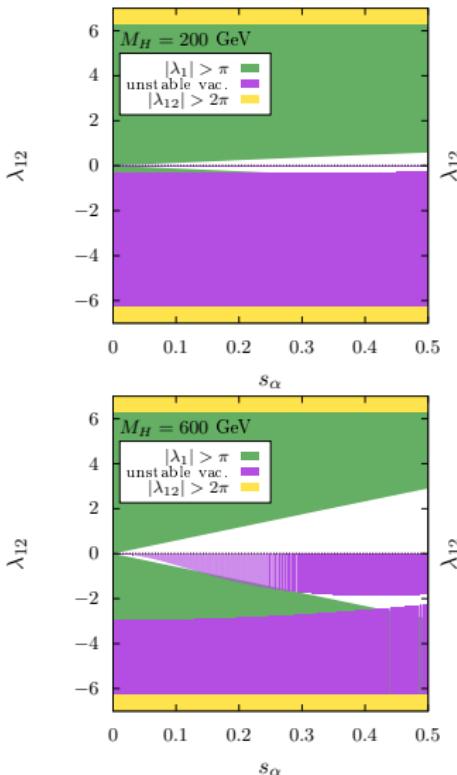
- vacuum stability  $\mu_2^2, \mu_1^2 > 0$

$$-\frac{c_\alpha^2 M_H^2 + M_h^2 s_\alpha^2}{2v_2^2} < \lambda_{12} < -\frac{c_\alpha^2 s_\alpha^2 (M_H^2 - M_h^2)^2}{2v_2^2 (c_\alpha^2 M_h^2 + M_H^2 s_\alpha^2)} \quad \text{or} \quad \lambda_{12} > 0$$

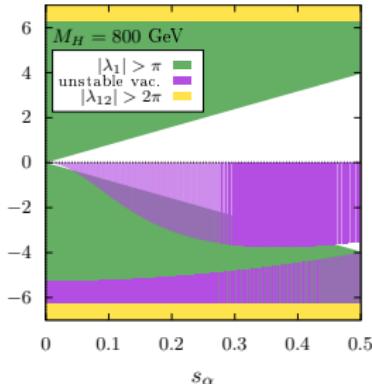
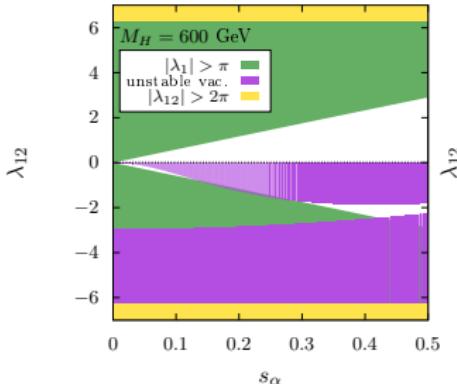


# Input parameters

Perturbativity and vacuum stability constraints



perturbativity  
matters for  
small  $M_H$



vacuum stab.  
important for  
higher  $M_H$

$\lambda_{12} < 0$   
mostly  
excluded

considered scenarios from [HXSWG, 2016] [Robens, Stefaniak 2016]



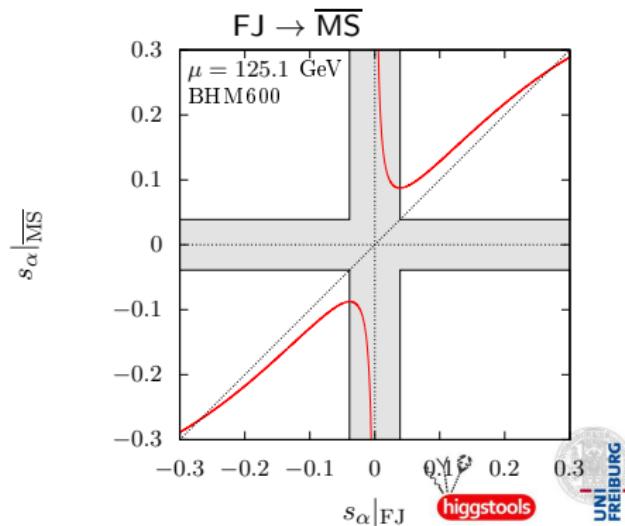
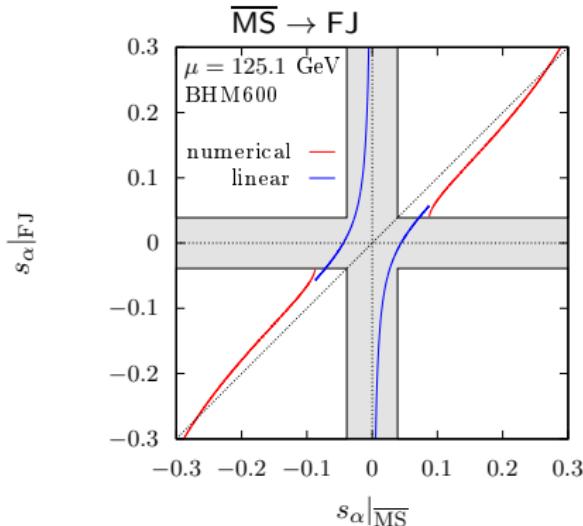
# Input parameters

## Scheme conversion

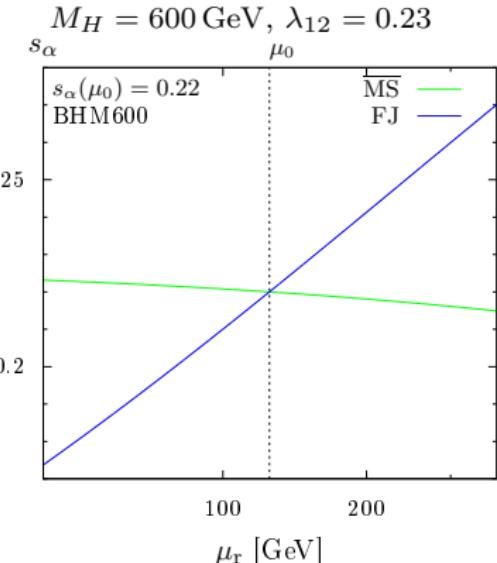
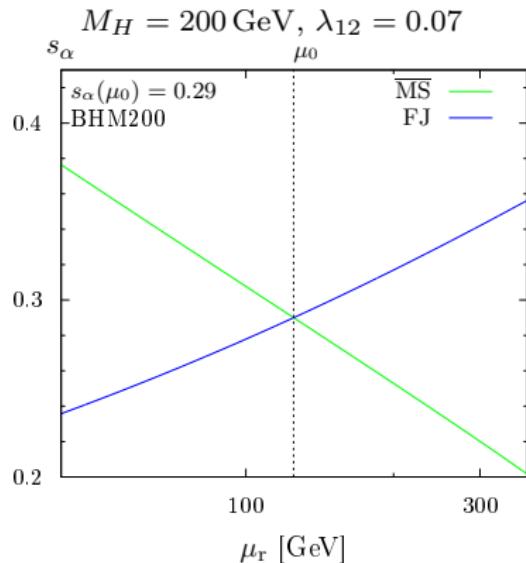
in order to compare results in different renormalization schemes

$$p_0 = p^{\overline{\text{MS}}} + \delta p^{\overline{\text{MS}}}(p^{\overline{\text{MS}}}) = p^{\text{FJ}} + \delta p^{\text{FJ}}(p^{\text{FJ}})$$

example for  $M_H = 600 \text{ GeV}$ ,  $\lambda_{12} = 0.23$



# Running of $s_\alpha$



- explains LO behavior of  $\Gamma^{h \rightarrow 4f}$  scale dependence
- for consistency, running of  $\lambda_{12}$  taken into account

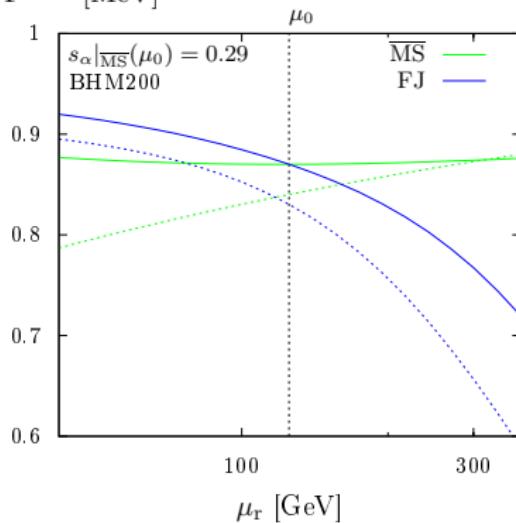


# Scale dependence of $\Gamma^{h \rightarrow 4f}$

LO (dashed) vs. NLO (solid)

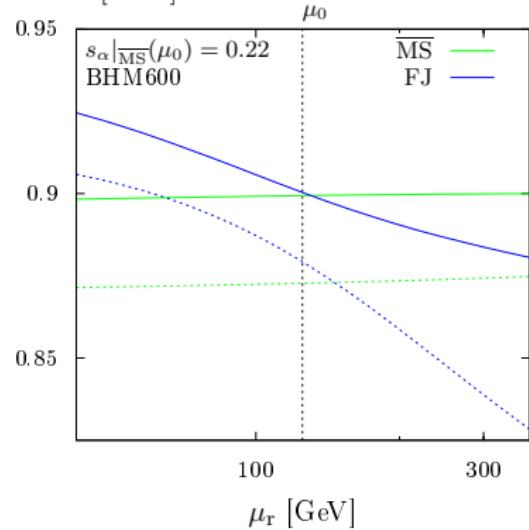
$$M_H = 200 \text{ GeV}, s_\alpha = 0.29, \lambda_{12} = 0.07$$

$$\Gamma^{h \rightarrow 4f} [\text{MeV}]$$



$$M_H = 600 \text{ GeV}, s_\alpha = 0.22, \lambda_{12} = 0.23$$

$$\Gamma^{h \rightarrow 4f} [\text{MeV}]$$

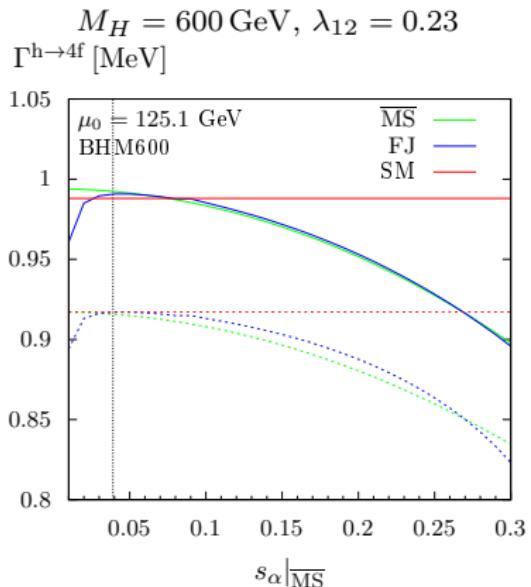
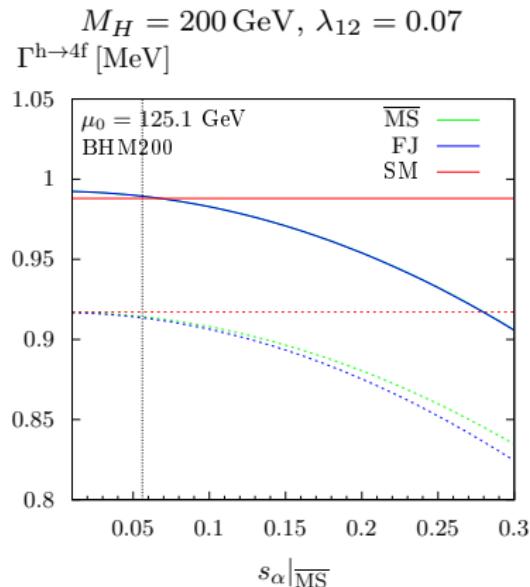


- more pronounced scale and scheme dependence at LO
- $\mu_r = M_h$  appropriate renormalization scale



# Mixing angle dependence of $\Gamma^{h \rightarrow 4f}$

LO (dashed) vs. NLO (solid)



- behavior mostly driven by  $c_\alpha^2$  factor
- $\Delta_{\text{SM}}$  typically reduced by NLO contributions
- $\Delta_{\text{SM}} \lesssim 1\%(5\%)$  for  $s_\alpha < 0.1(0.2)$

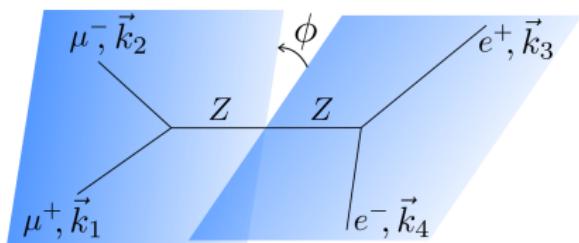
\* spurious effects in LO driven by NLO parameter conversion



# Differential distributions

possible generation of distributions for

- invariant masses
- angles

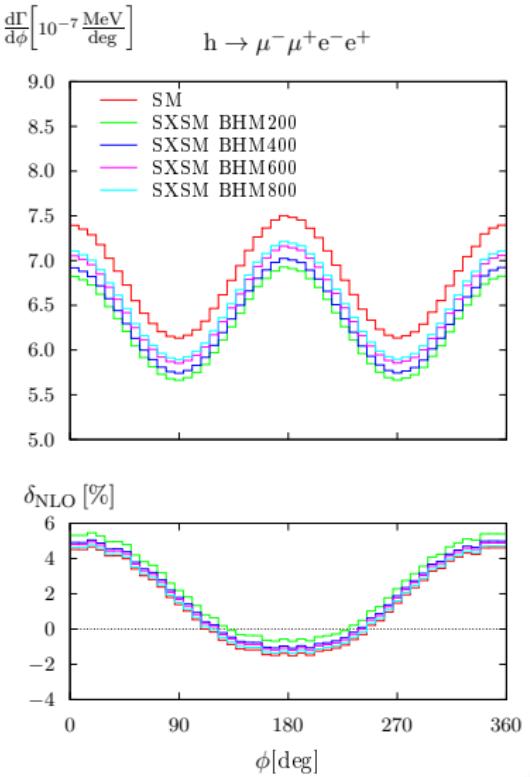
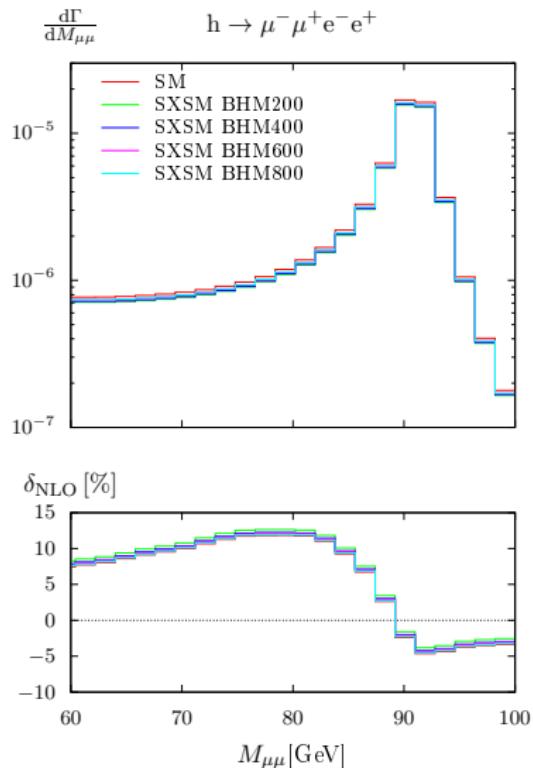


$$\cos \theta_{Z\mu} = \frac{\vec{k}_{2,Z} \cdot (\vec{k}_{3,Z} + \vec{k}_{4,Z})}{|\vec{k}_{2,Z}| |\vec{k}_{3,Z} + \vec{k}_{4,Z}|}$$

$$\cos \phi_{\mu e, T} = \frac{\vec{k}_{2,T} \cdot \vec{k}_{3,T}}{|\vec{k}_{2,T}| |\vec{k}_{3,T}|}$$

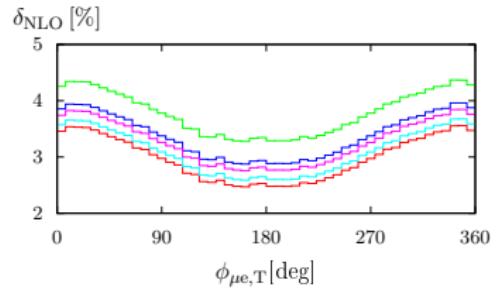
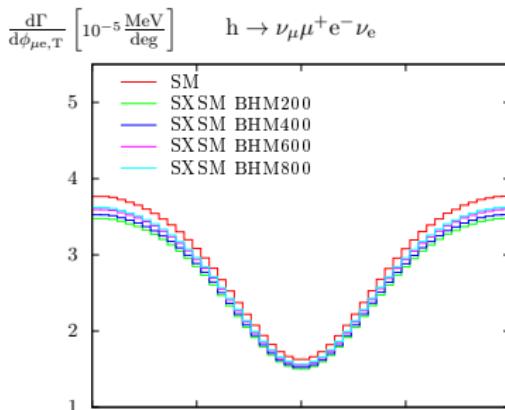
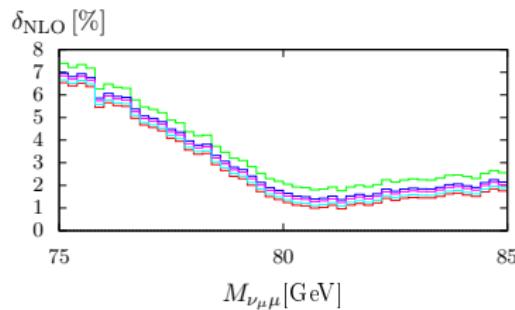
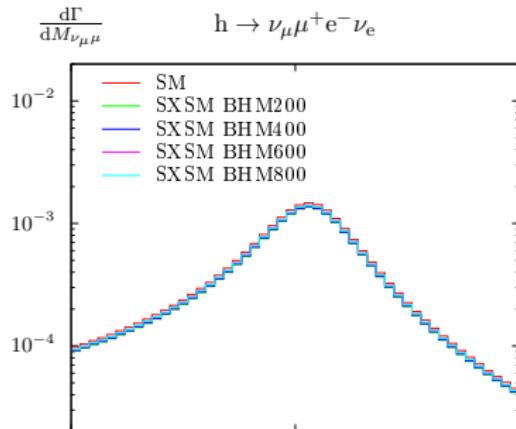
for all four-light-fermion final states

# NLO leptonic distributions



- constant offset in  $\delta_{\text{NLO}}$  wrt SM distributions
- similar behavior and magnitude in the FJ scheme

# NLO leptonic distributions



- constant offset in  $\delta_{\text{NLO}}$  wrt SM distributions
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# Conclusions

## SESM

- offers interesting phenomenology, despite its simplicity
  - renormalization performed treating tadpoles within two schemes
  - FeynArts model file for one-loop calculations produced
  - computed matrix elements for the decay  $h \rightarrow WW/ZZ \rightarrow 4f$
- $h \rightarrow WW/ZZ \rightarrow 4f$  results*
- four benchmark scenarios considered,  $M_H \in [200 - 800] \text{ GeV}$
  - scheme conversion: sizable effects, become larger when approaching non-perturbative regions
  - renormalization group equations solved for  $\overline{\text{MS}}$  parameters
  - scale and scheme dependence reduced in NLO results
  - $\Delta_{\text{SM}} \lesssim 5\%$  for the decay width in the proposed scenarios
  - no further distortion wrt SM in differential distributions



# Conclusions

coming soon

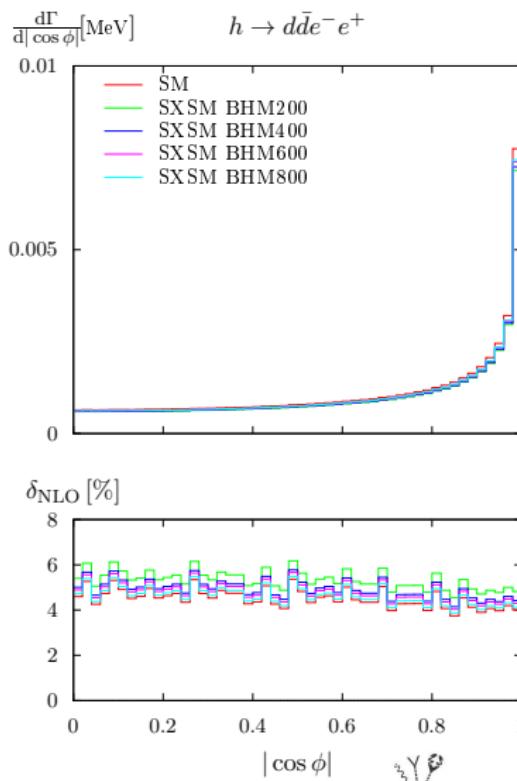
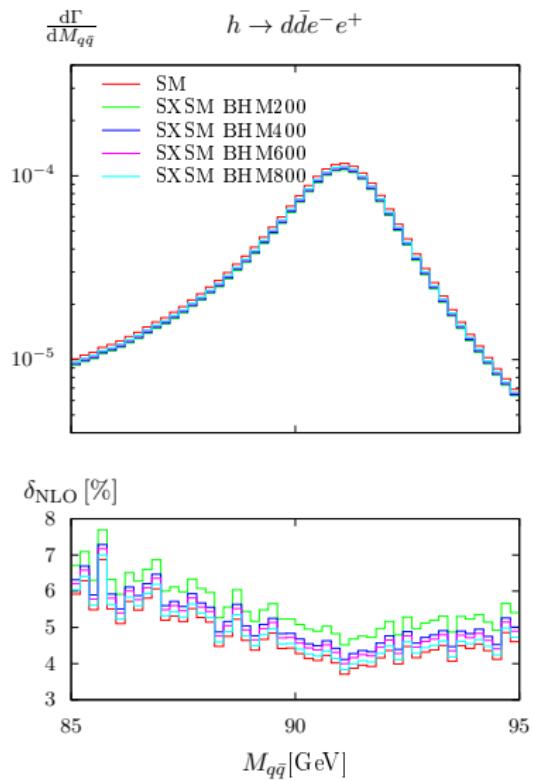
- Prophecy4f version including SESM implementation
- paper in preparation



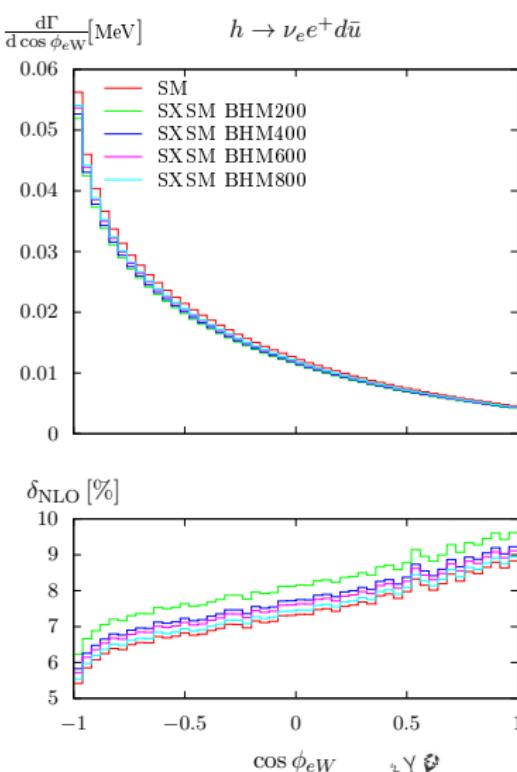
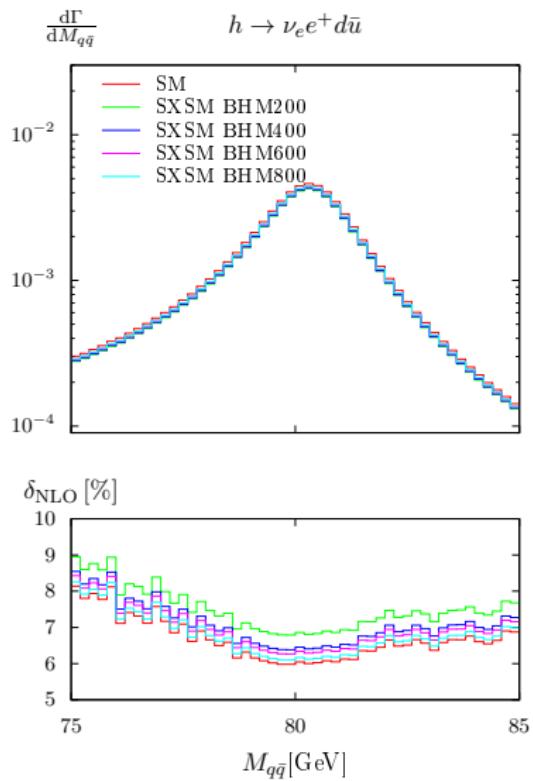
# Backup



# NLO semileptonic distributions



# NLO semileptonic distributions



# Input parameters

## SM parameters

- [ATLAS, CMS, 2015]

$$M_h = 125.09 \pm 0.21 \text{ (stat.)} \pm 0.11 \text{ (syst.)} \text{ GeV} \approx 125.1 \text{ GeV}$$

- [HXSWG, 2016]

$$M_W^{\text{OS}} = 80.385 \text{ GeV} \quad \Gamma_W^{\text{OS}} = 2.085 \text{ GeV}$$

$$M_Z^{\text{OS}} = 91.1876 \text{ GeV} \quad \Gamma_Z^{\text{OS}} = 2.4952 \text{ GeV}$$

$$m_e = 0.510998928 \text{ MeV} \quad m_\mu = 105.6583715 \text{ MeV} \quad m_\tau = 1776.82 \text{ MeV}$$

$$m_u = 0.1 \text{ GeV} \quad m_c = 1.51 \text{ GeV} \quad m_t = 172.5 \text{ GeV}$$

$$m_d = 0.1 \text{ GeV} \quad m_s = 0.1 \text{ GeV} \quad m_b = 4.92 \text{ GeV}$$

$$G_\mu = 1.1663787 \cdot 10^{-5} \text{ GeV}^{-2} \quad \alpha_s = 0.118$$



# Input parameters

internally

- pole masses from

$$M_V = \frac{M_V^{\text{OS}}}{\sqrt{1 + (\Gamma_V^{\text{OS}}/M_V^{\text{OS}})^2}}$$

- pole decay widths  $\Gamma_W$  and  $\Gamma_Z$  calculated from the experimental input, taking into account  $\mathcal{O}(\alpha_{\text{em}})$  corrections and using real masses
- $G_\mu$ -scheme (large corrections shifted to lowest order)

$$\alpha_{\text{em}} = \frac{\sqrt{2}G_\mu M_W^2}{\pi} \left( 1 - \frac{M_W^2}{M_Z^2} \right)$$



# Input parameters

## Benchmark scenarios

scenarios taken from [HXSWG, 2016] [Robens, Stefaniak 2016]

Scenario	$M_H$ [GeV]	$\sin \alpha$	$\lambda_{12}$
BHM200	200	0.29	0.07
BHM400	400	0.26	0.17
BHM600	600	0.22	0.23
BHM800	800	0.2	0.26



# Baryon asymmetry of the universe

three basic ingredients

- CP violation
- baryon number violation
- departure from thermal equilibrium (otherwise CPT would assure compensation between processes increasing and decreasing baryon number)

	SM	SESM
CP violation	✓	✓
B violation <sup>1</sup>	✓	✓
first order EWPT		✓

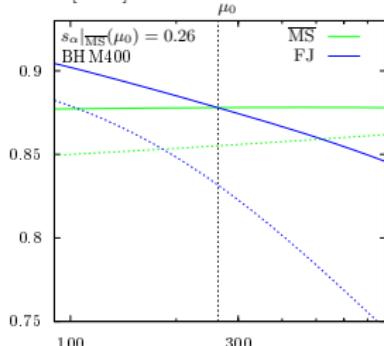
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<sup>1</sup>non-perturbative effect

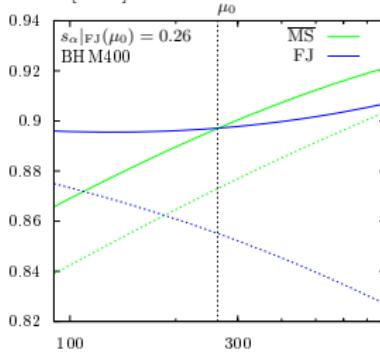
# Scale choice

## BHM400

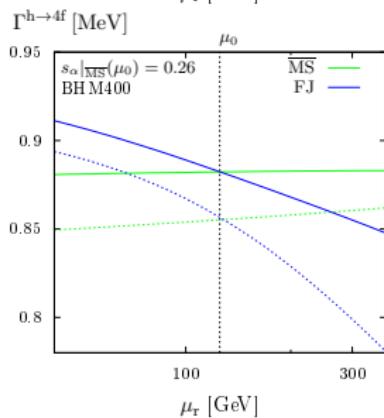
$\Gamma^{h \rightarrow 4f}$  [MeV]



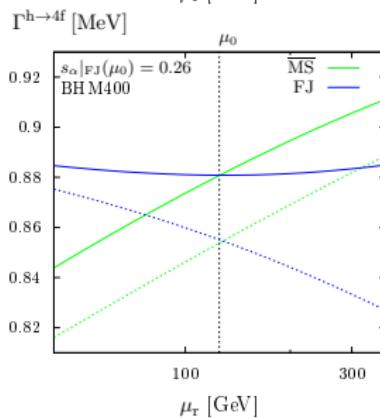
$\Gamma^{h \rightarrow 4f}$  [MeV]



$\Gamma^{h \rightarrow 4f}$  [MeV]



$\Gamma^{h \rightarrow 4f}$  [MeV]



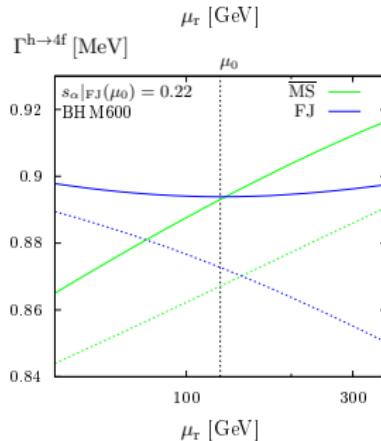
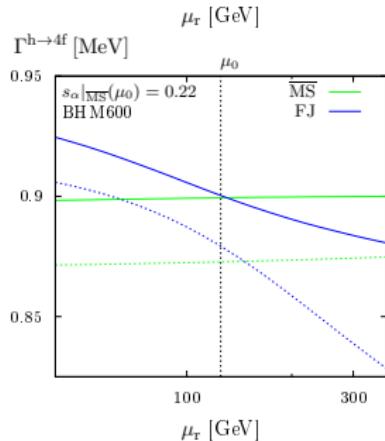
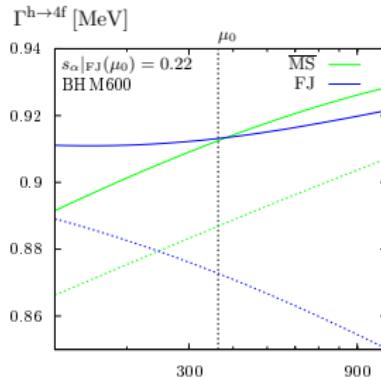
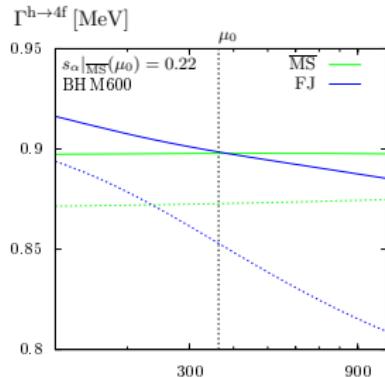
$$\mu_0 = \frac{M_h + M_H}{2}$$

$$\mu_0 = M_h$$



# Scale choice

## BHM600



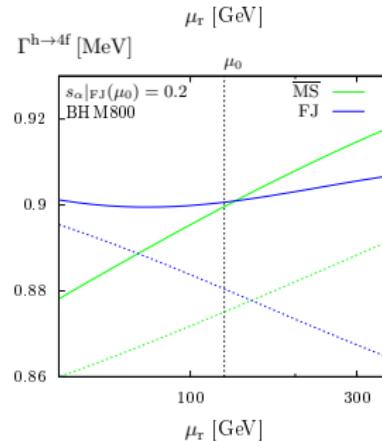
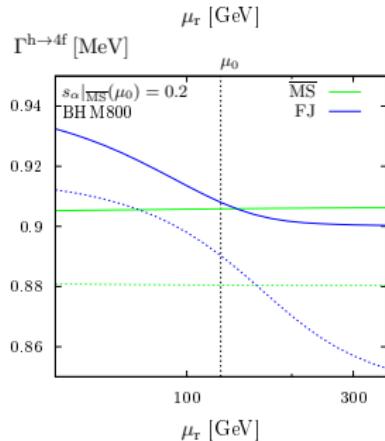
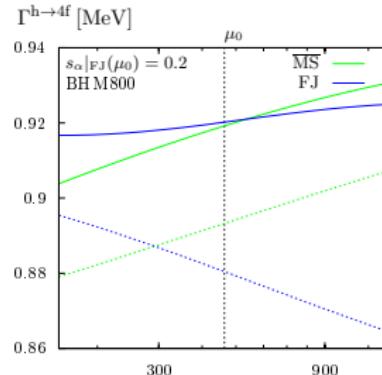
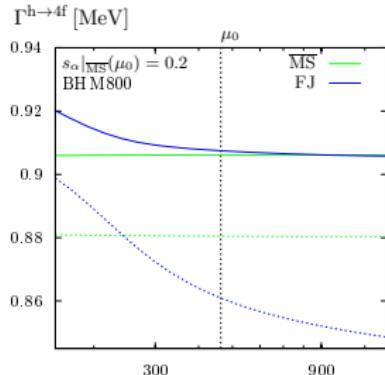
$$\mu_0 = \frac{M_h + M_H}{2}$$

$$\mu_0 = M_h$$



# Scale choice

## BHM800



$$\mu_0 = \frac{M_h + M_H}{2}$$

$$\mu_0 = M_h$$

