Standard Model Parton Distributions at Very High Energies

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Work in collaboration with Christian Bauer and Nicolas Ferland, LBNL

1703.08562 = JHEP 08(2017)036

Motivation

- Electroweak corrections becoming essential
  - Fixed order adequate at present energies
  - Enhanced higher orders important for FCC
- SM may be valid up to much higher energies
  - Implications for cosmology and astrophysics
- Need full simulations of VHE interactions: parton shower event generators for full SM
  - First step: event generators need PDFs

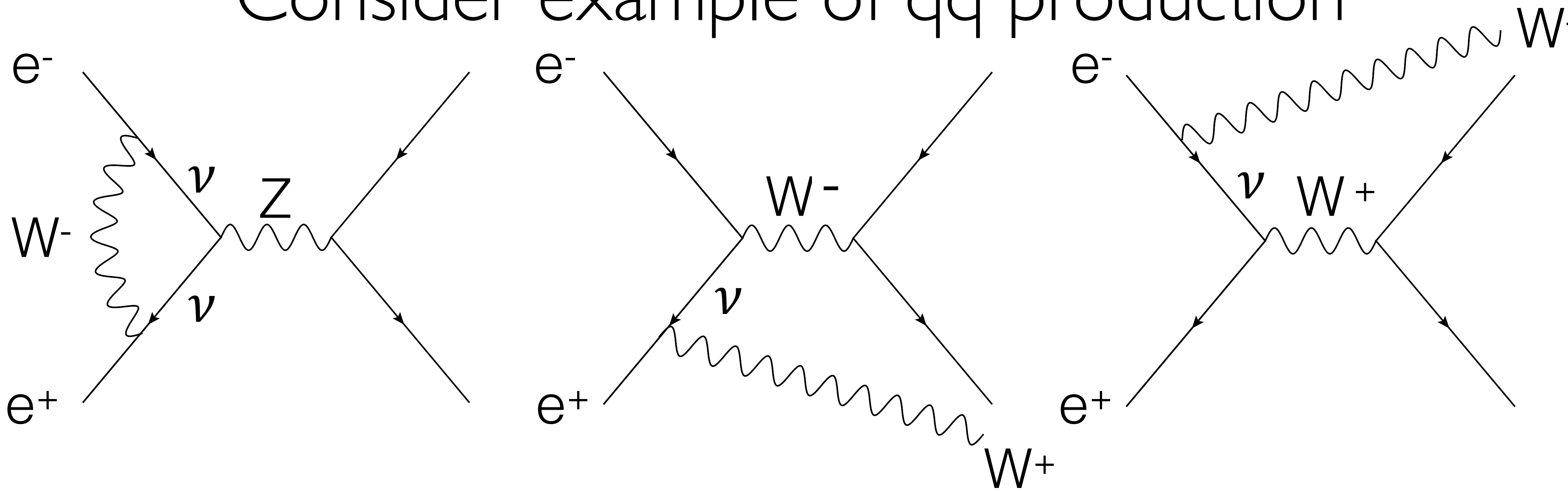
Outline

- Electroweak effects at high energies
  - Non-cancelling large logarithms
  - Sudakov factors
- SM parton distributions
  - DGLAP and double-log evolution
  - L-R and isospin asymmetries
- Lepton pair production
  - Matching to fixed order
- Conclusions and prospects

Electroweak Effects at High Energies

Electroweak effects: e^+e^-

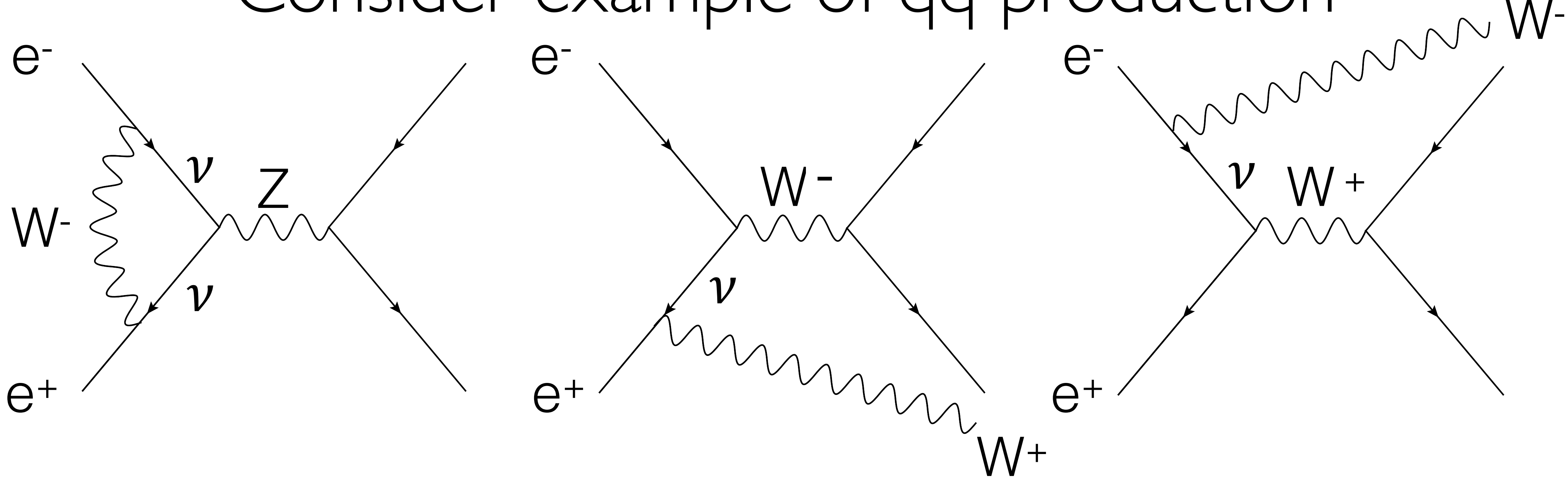
Consider example of $q\bar{q}$ production



- For massless bosons, IR divergences in each graph, cancel in inclusive sum over SU(2) multiplets
- For massive bosons, divergences become $\log(m_W^2/s)$, generally two per power of α_W

Electroweak effects: e^+e^-

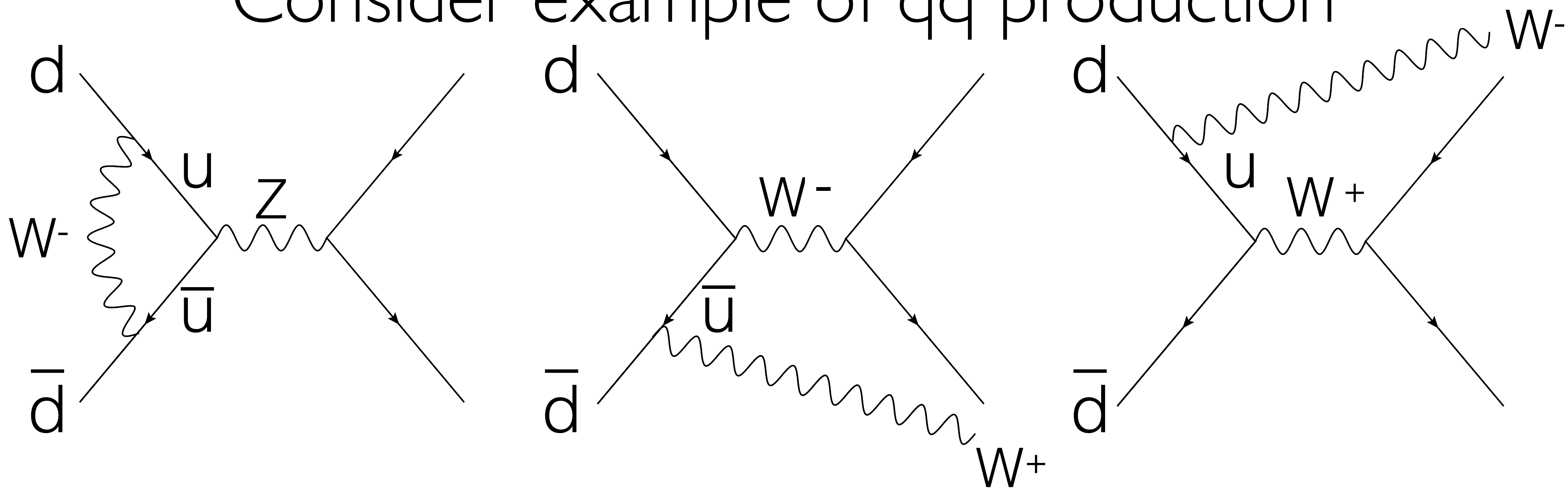
Consider example of $q\bar{q}$ production



- $\alpha_W \log^2(m_W^2/s)$ from each graph, cancel in inclusive sum over SU(2) multiplets
- But we don't have νν or eν colliders, so cancellation is incomplete

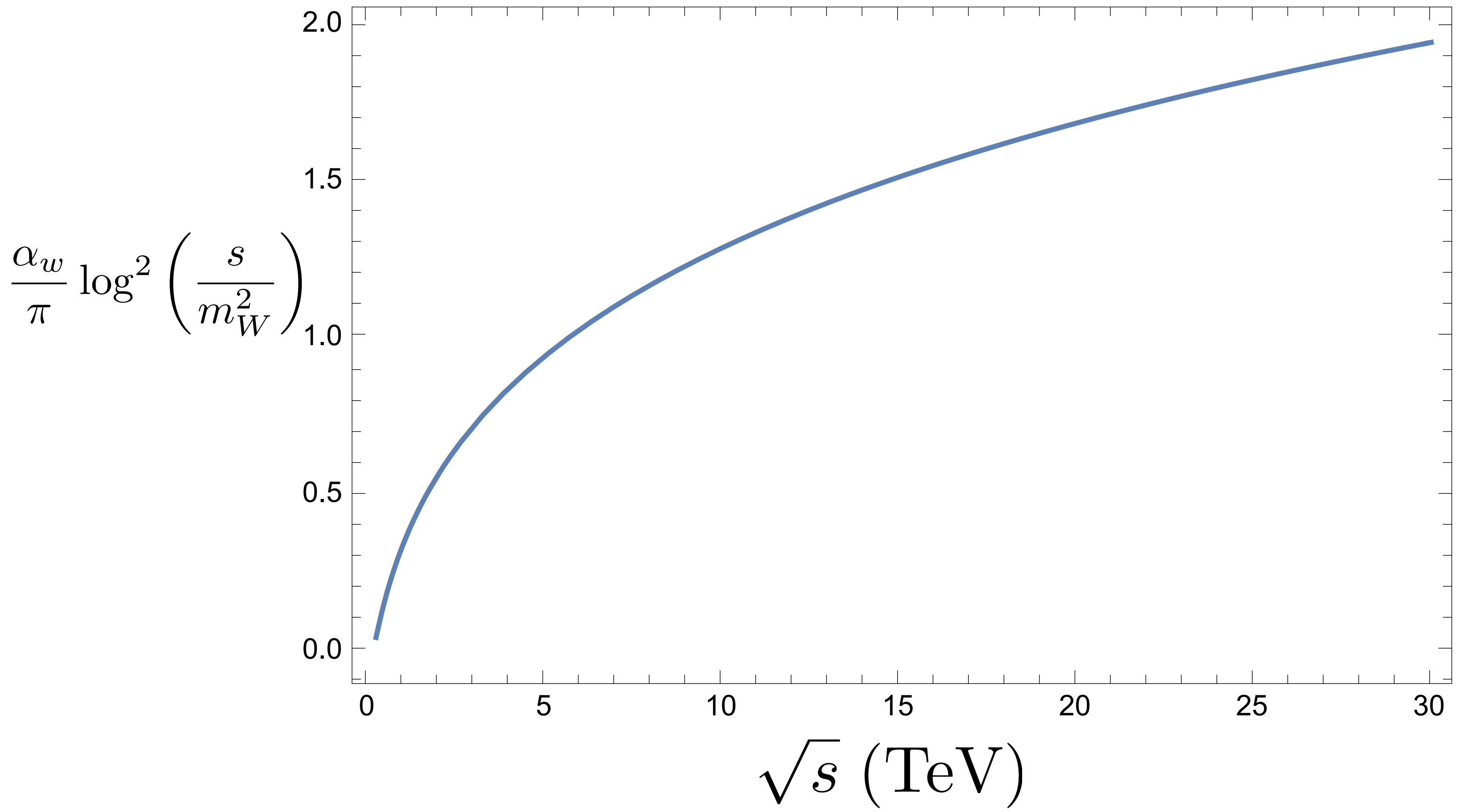
Electroweak effects: $q\bar{q}$

Consider example of $q\bar{q}$ production



- $\alpha_W \log^2(m_W^2/s)$ from each graph, cancel in inclusive sum over SU(2) multiplets
- In pp, u-quark PDF $\neq$ d-quark PDF, so cancellation is incomplete

Electroweak logarithms

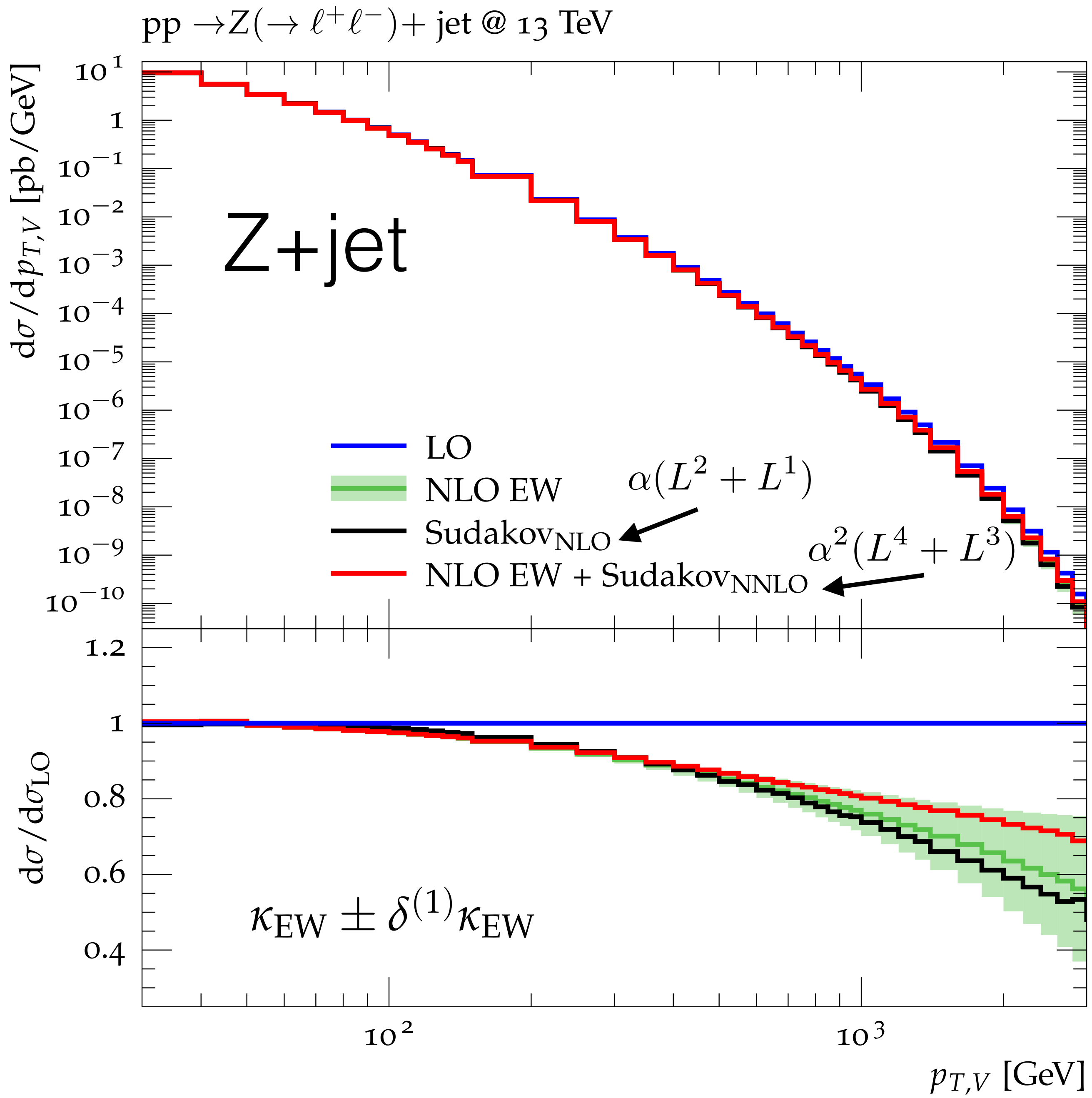


- Electroweak logs get large at high energy
- Virtual corrections exponentiate as Sudakov factor

$$\Delta_i(s) \sim \exp\left[-C_i \frac{\alpha_w}{\pi} \log^2\left(\frac{s}{m_W^2}\right)\right]$$

Electroweak effects in Z+jet

J Lindert, PSR2017



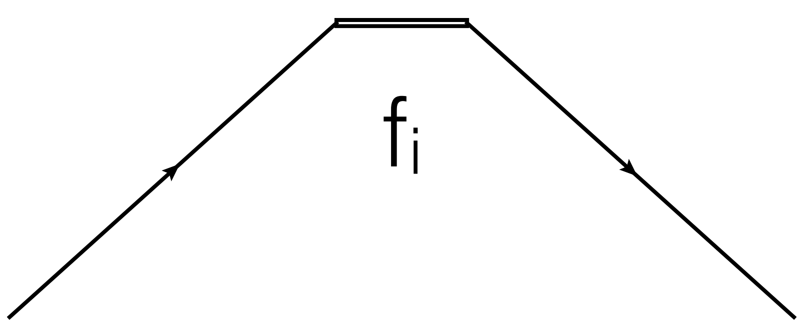
- ~25% at p_T~1 TeV

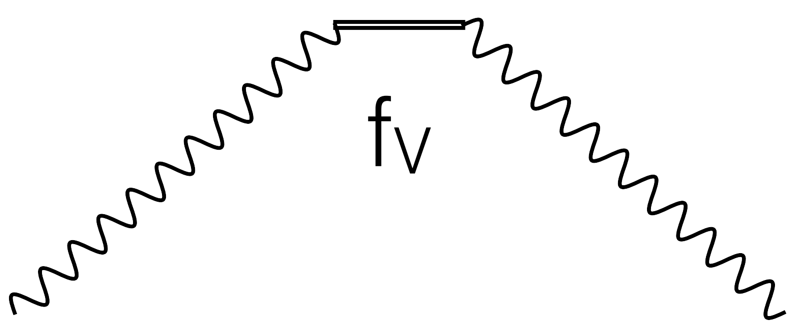
Parton Distribution Functions

PDFs as bilocal operator MEs

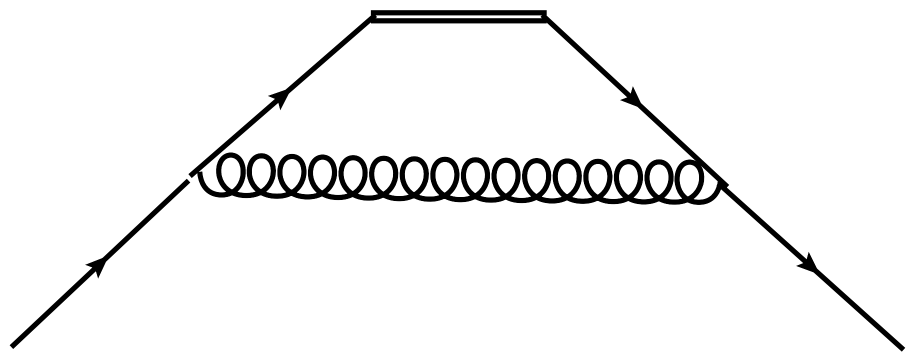
$$f_i(x) = x\int\frac{dy}{2\pi}\,e^{-i\,2x\bar{n}\cdot p\,y}\big\langle p\big|\,\bar{\psi}^{(i)}(y)\,\bar{\not{n}}\,\psi^{(i)}(-y)\big|p\big\rangle$$

$$f_V(x) = \frac{2}{\bar{n}\cdot p}\int\frac{dy}{2\pi}\,e^{-i\,2x\bar{n}\cdot p\,y}\,\bar{n}_\mu\bar{n}^\nu\big\langle p\big|\,V^{\mu\lambda}(y)V_{\lambda\nu}(-y)\big|p\big\rangle\Big|_{\text{spin avg.}}$$

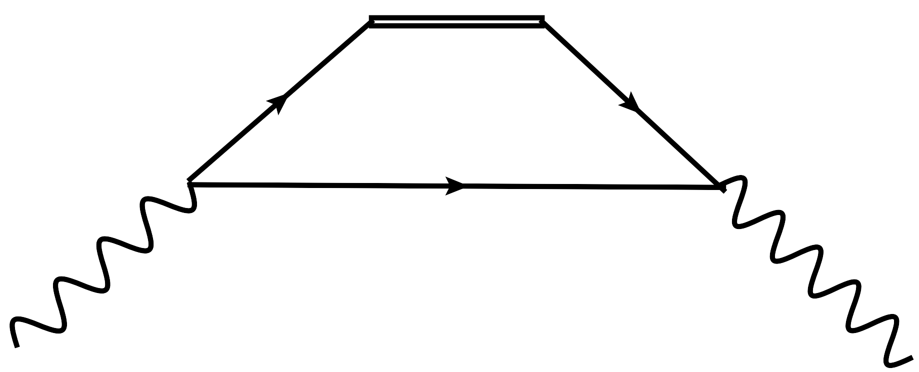




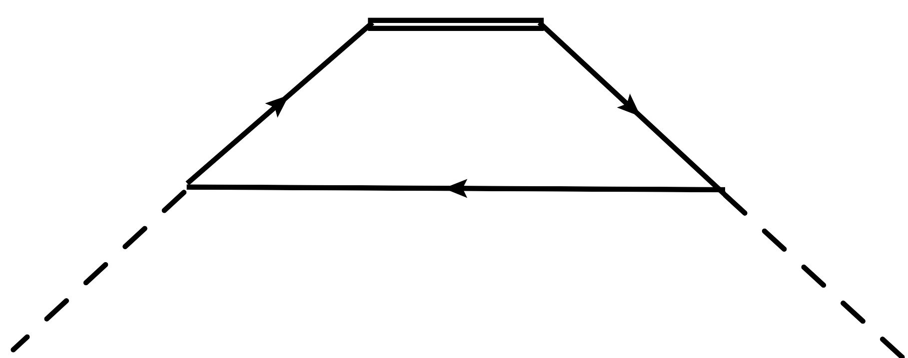
PDF Evolution



$$q\, d/dq\, f = P_{ff} \otimes f$$



$$q\, d/dq\, f = P_{fV} \otimes V$$

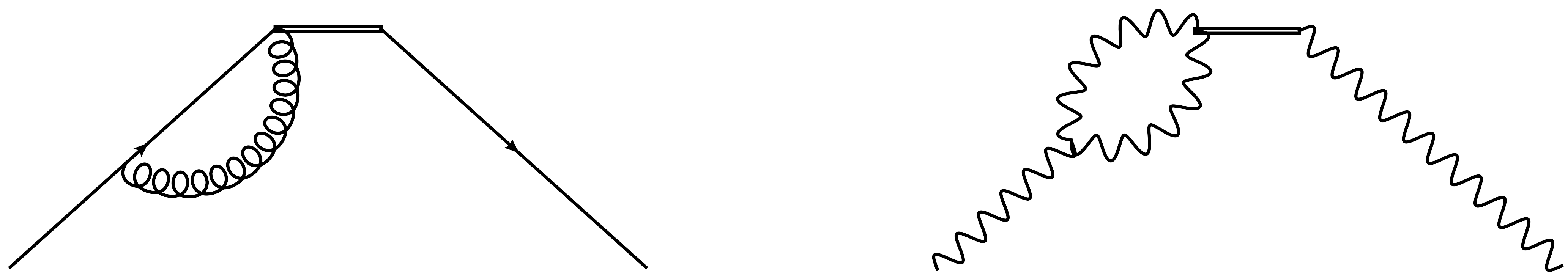


$$q\, d/dq\, f = P_{fH} \otimes H$$

Real and Virtual Contributions

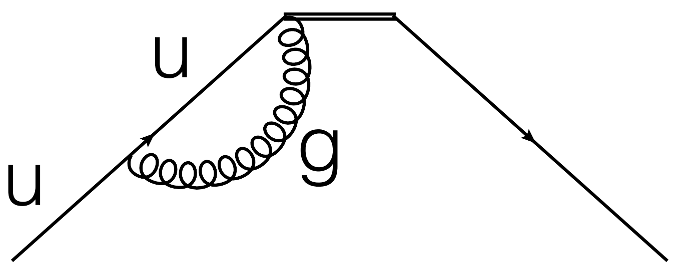
- Reals have loops from one side to the other

- Virtuals have loops on same side

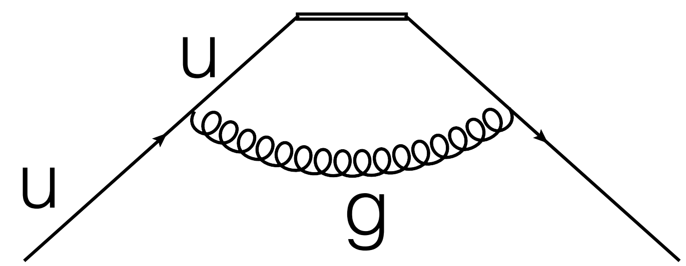


SU(3) Evolution (DGLAP)

- Consider evolution of u quark PDF



Virtual



Real

$$q\frac{\partial}{\partial q}f_u(x,q) = \frac{\alpha_3 C_F}{\pi}P_f^V(q)f_u(x,q)$$

$$P_f^V(q) = -\int_0^{1-\mu/q} \mathrm{d}z\, P_{ff}(z)$$

$\mu \to 0$

$$q\frac{\partial}{\partial q}f_u(x,q) = \frac{\alpha_3 C_F}{\pi}\int_x^{1-\mu/q} \mathrm{d}z\, P_{ff}(z) f_u(x/z,q)$$

$$P_{ff}(z) = \frac{1+z^2}{1-z}$$

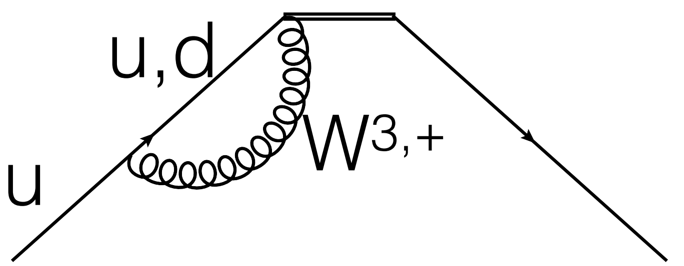
Combination

$$q\frac{\partial}{\partial q}f_u(x,q) = \frac{\alpha_3 C_F}{\pi}\int_0^{1-\mu/q} \mathrm{d}z\, P_{ff}(z)\left[f_u(x/z,q) - f_u(x)\right]$$

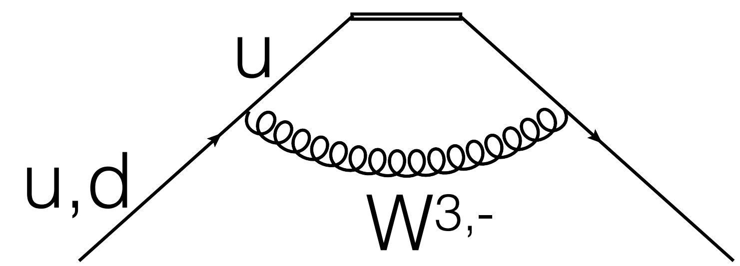
- z=1 singularity cancels ➡ single-log evolution

SU(2) Evolution

- Consider evolution of u_L quark PDF



Virtual



Real

$$q\frac{\partial}{\partial q} f_u(x,q) = \frac{\alpha_2 C_F}{\pi} P_f^V(q) f_u(x,q)$$

$$P_f^V(q) = -\int_0^{1-\mu/q} \mathrm{d}z\, P_{ff}(z)$$

$\mu \sim m_W$

$$q\frac{\partial}{\partial q} f_u(x,q) = \frac{\alpha_2 C_F}{\pi} \int_x^{1-\mu/q} \mathrm{d}z\, P_{ff}(z) \times \left[\frac{1}{3} f_u(x/z,q) + \frac{2}{3} f_d(x/z,q)\right]$$

Combination

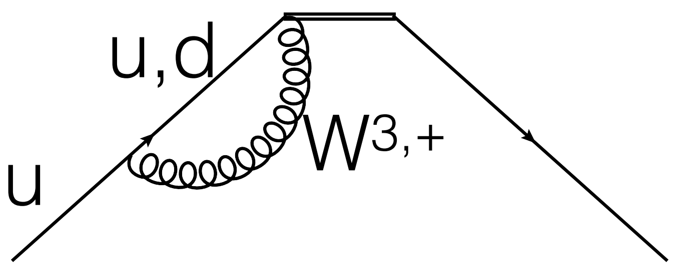
$$q\frac{\partial}{\partial q} f_u(x,q) = \frac{\alpha_2 C_F}{\pi} \int_0^{1-\mu/q} \mathrm{d}z\, P_{ff}(z) \left[\frac{1}{3} f_u(x/z,q) + \frac{2}{3} f_d(x/z,q) - f_u(x,q)\right]$$

- z=1 doesn't cancel ➞ double-log evolution

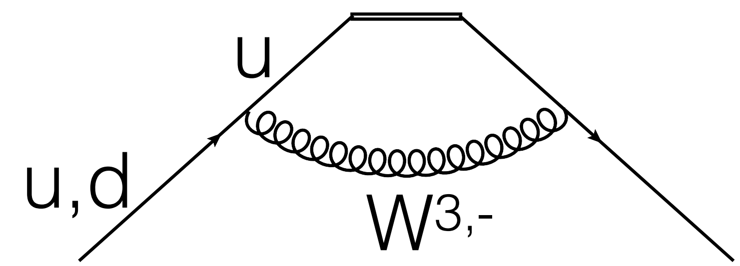
M Ciafaloni, P Ciafaloni, D Comelli, hep-ph/9809321, 0001142, 0111109, 0505047

SU(2) Evolution

- Consider evolution of u_L quark PDF



Virtual



Real

$C_F = 3/4$

$$q\frac{\partial}{\partial q} f_u(x,q) = \frac{\alpha_2 C_F}{\pi} P_f^V(q) f_u(x,q)$$

$$P_f^V(q) = -\int_0^{1-\mu/q} \mathrm{d}z\, P_{ff}(z)$$

$\mu \sim m_W$

$$q\frac{\partial}{\partial q} f_u(x,q) = \frac{\alpha_2 C_F}{\pi} \int_x^{1-\mu/q} \mathrm{d}z\, P_{ff}(z) \times \left[\frac{1}{3} f_u(x/z,q) + \frac{2}{3} f_d(x/z,q)\right]$$

Combination

$$\mathcal{L}_{q\bar{q}W} = -\frac{g_2}{2}(\bar{u},\bar{d})\begin{pmatrix} W^3 & \sqrt{2}\,W^+ \\ \sqrt{2}\,W^- & -W^3 \end{pmatrix}\begin{pmatrix} u \\ d \end{pmatrix}$$

$$q\frac{\partial}{\partial q} f_u(x,q) = \frac{\alpha_2 C_F}{\pi} \int_0^{1-\mu/q} \mathrm{d}z\, P_{ff}(z) \left[\frac{1}{3} f_u(x/z,q) + \frac{2}{3} f_d(x/z,q) - f_u(x,q)\right]$$

- z=1 doesn't cancel ➡ double-log evolution

M Ciafaloni, P Ciafaloni, D Comelli, hep-ph/9809321, 0001142, 0111109, 0505047

General Evolution

$$q\frac{\partial}{\partial q}f_i(x,q) = \sum_I \frac{\alpha_I(q)}{\pi}\left[P^V_{i,I}(q)\,f_i(x,q) + \sum_j C_{ij,I}\int_x^{z^{ij,I}_{\max}(q)} \mathrm{d}z\,P^R_{ij,I}(z) f_j(x/z,q)\right]$$

$$(I = 1,2,3,\ldots)$$

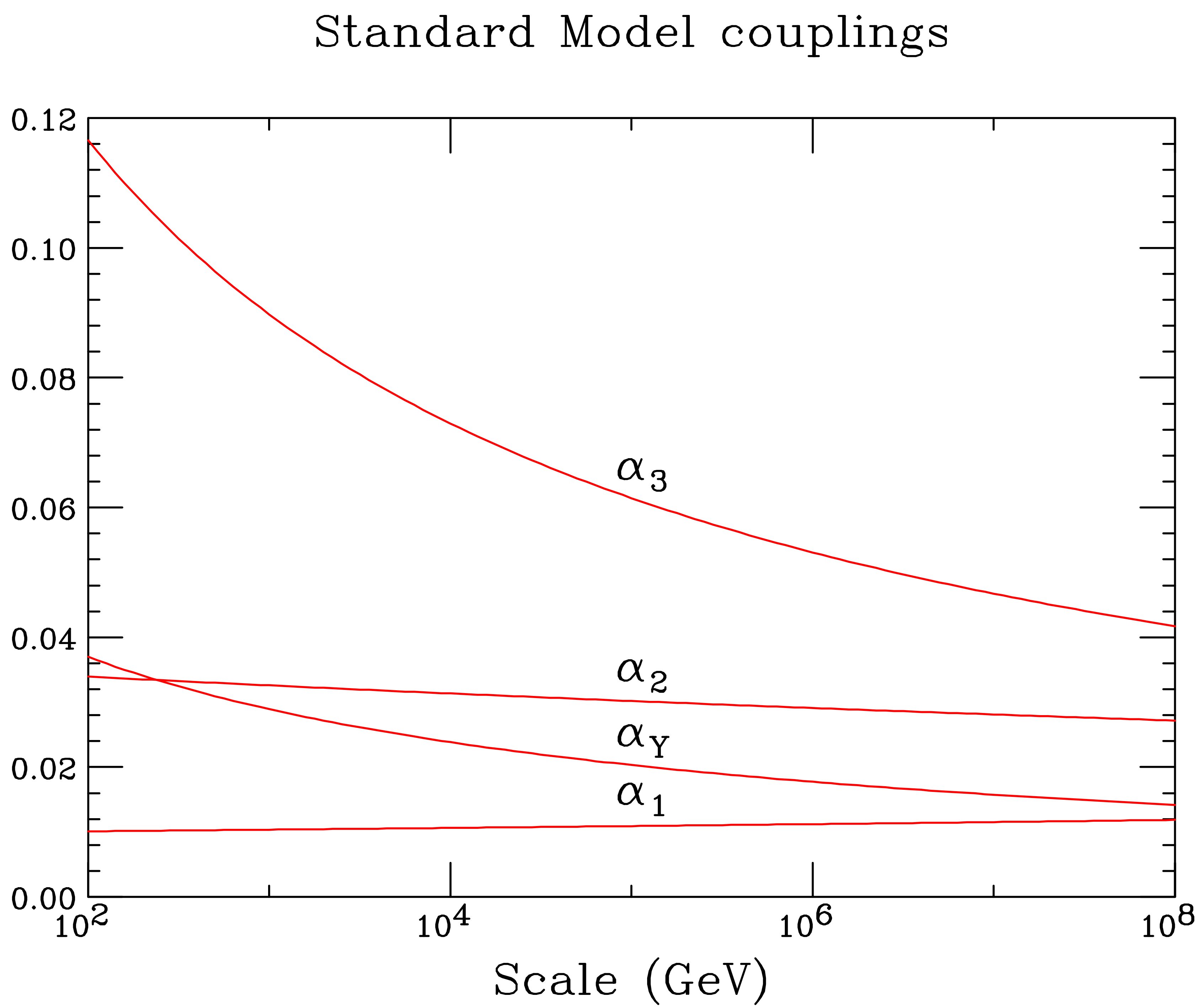
- $z^{ij,I}_{\max} = 1-\mu/q$ **with** $\mu = \mathcal{O}(m_V)$ **for** $j \to iV$
- Introduce Sudakov factor

$$\Delta_{i,I}(q) = \exp\left[\int_{q_0}^{q}\frac{\mathrm{d}q'}{q'}\frac{\alpha_I(q')}{\pi}P^V_{i,I}(q')\right]$$

- Then

$$\Delta_i(q)\,q\frac{\partial}{\partial q}\left[\frac{f_i(x,q)}{\Delta_i(q)}\right] = \sum_I \frac{\alpha_I(q)}{\pi}\sum_j C_{ij,I}P^R_{ij,I}\otimes f_j$$

Couplings


Standard Model couplings
0.12
0.10
0.08
0.06
0.04
0.02
0.00
10^2
10^4
10^6
10^8
Scale (GeV)
α_3
α_2
α_Y
α_1

SM Fermion Evolution

U(1): $$\left[\Delta_{f,1}\, q\frac{\partial}{\partial q}\frac{f_f}{\Delta_{f,1}}\right]_1 = \frac{\alpha_1}{\pi}Y_f^2\left[P^R_{ff,G}\otimes f_f + N_f P^R_{fV,G}\otimes f_B\right]$$

SU(2): $$\left[\Delta_{f_L,2}\, q\frac{\partial}{\partial q}\frac{f_{u_L}}{\Delta_{f_L,2}}\right]_2 = \frac{\alpha_2}{\pi}\left\{P^R_{ff,G}\otimes\left[\frac{f_{d_L}}{2}+\frac{f_{u_L}}{4}\right]\right.$$
$$\left. + N_f P_{fV,G}\otimes\left[\frac{f_{W^+}}{2}+\frac{f_{W^3}}{4}\right]\right\}$$

SU(3): $$\left[\Delta_{q,3}\, q\frac{\partial}{\partial q}\frac{f_q}{\Delta_{q,3}}\right]_3 = \frac{\alpha_3}{\pi}\left[C_F P^R_{ff,G}\otimes f_q + T_R P^R_{fV,G}\otimes f_g\right]$$

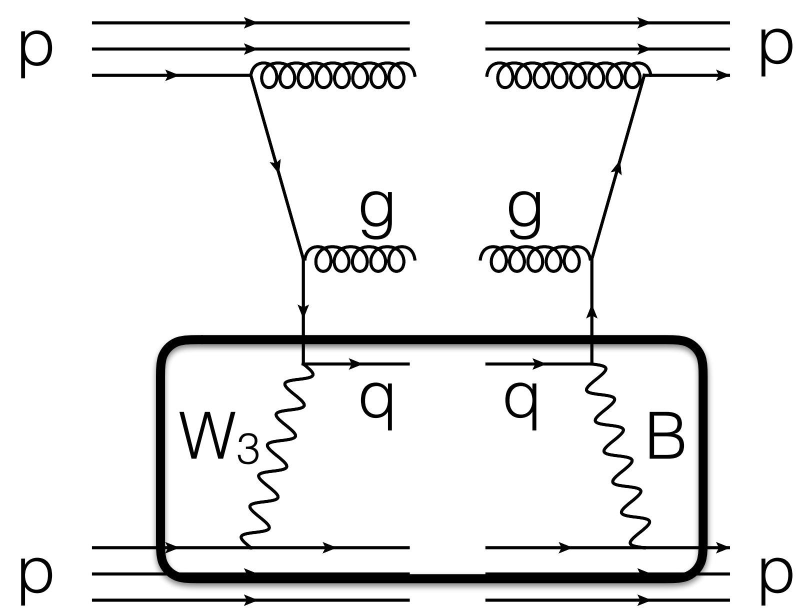
Yukawa: $$\left[\Delta_{q_L^3,Y}\, q\frac{\partial}{\partial q}\frac{f_{t_L}}{\Delta_{q_L^3,Y}}\right]_Y = \frac{\alpha_Y}{\pi}\left\{P^R_{ff,Y}\otimes f_{t_R} + N_C P_{fH,Y}\otimes f_{\bar{H}^0}\right\}$$

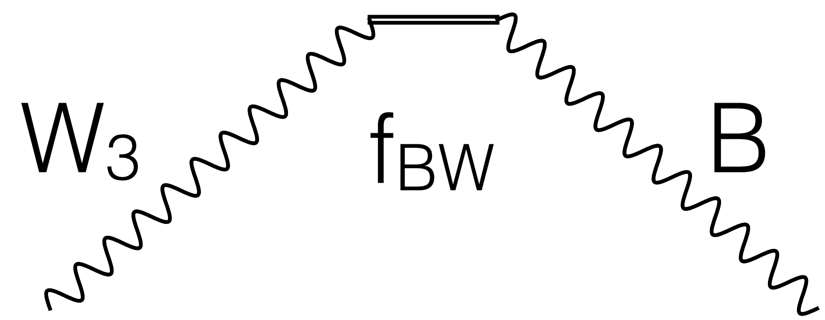
Mixed: $$\left[q\frac{\partial}{\partial q}f_{f_u}\right]_M = \frac{\alpha_M}{\pi}\frac{Y_f}{2}N_f P^R_{fV,G}\otimes f_{BW}$$

(N_f = 3 for quarks, 1 for leptons)

Mixed U(1)xSU(2) PDF

$$f_{BW}(x) = \frac{2}{\bar{n}\cdot p}\int \frac{dy}{2\pi}\, e^{-i\,2x\bar{n}\cdot p\,y}\,\bar{n}^{\mu}\bar{n}_{\nu}\langle p|\, B_{\mu\lambda}(y) W_3^{\lambda\nu}(-y)|p\rangle\Big|_{\text{spin avg.}} + \text{h.c.}$$





$$\alpha_M = \sqrt{\alpha_1\alpha_2}$$

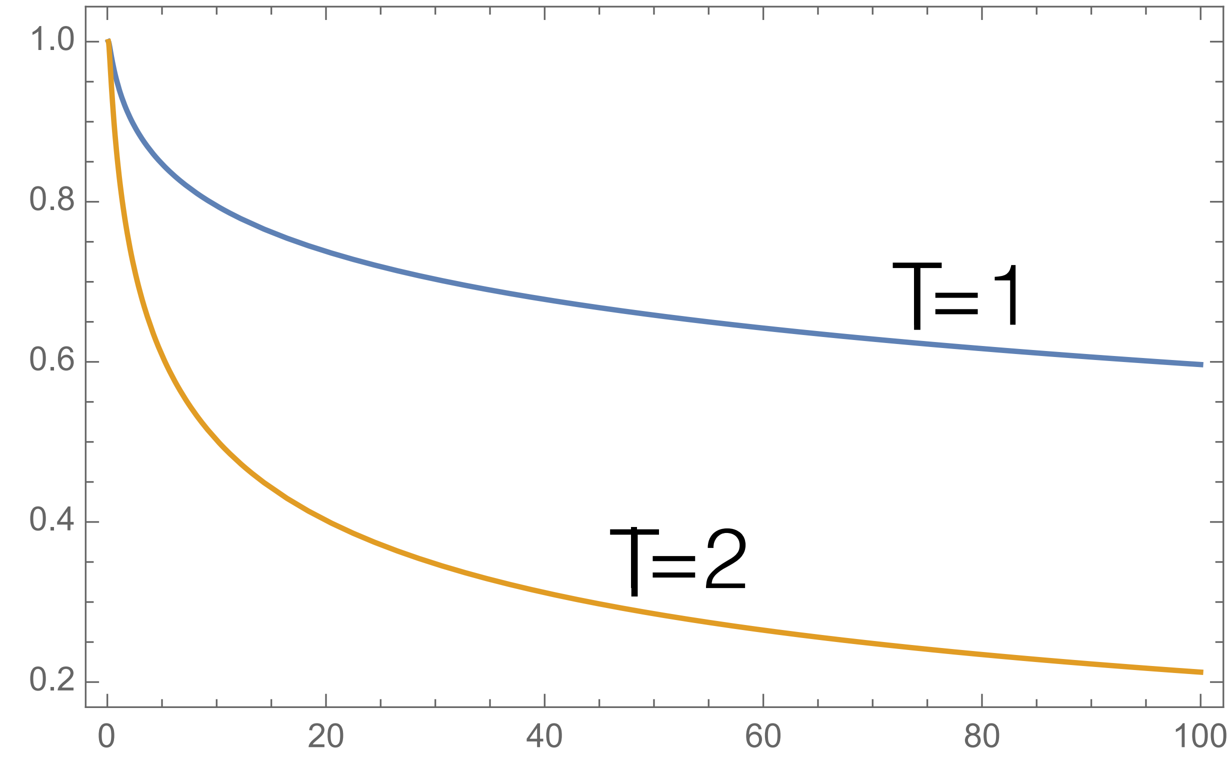
- Left-handed quarks have isospin and hypercharge, so they can generate f_{BW}
- This means in broken basis we have f_γ, f_Z and $f_{\gamma Z}$

Isospin (T) + CP PDFs

$$f_{q_L}^{0+} = \frac{1}{4}\left(f_{u_L} + f_{d_L} + f_{\bar{u}_L} + f_{\bar{d}_L}\right), \quad f_{q_L}^{0-} = \frac{1}{4}\left(f_{u_L} + f_{d_L} - f_{\bar{u}_L} - f_{\bar{d}_L}\right),$$
$$f_{q_L}^{1+} = \frac{1}{4}\left(f_{u_L} - f_{d_L} + f_{\bar{u}_L} - f_{\bar{d}_L}\right), \quad f_{q_L}^{1-} = \frac{1}{4}\left(f_{u_L} - f_{d_L} - f_{\bar{u}_L} + f_{\bar{d}_L}\right),$$
$$f_W^{0+} = \frac{1}{3}\left(f_{W^+} + f_{W^-} + f_{W^3}\right), \quad f_W^{1-} = \frac{1}{2}\left(f_{W^+} - f_{W^-}\right), \quad f_W^{2+} = \frac{1}{6}\left(f_{W^+} + f_{W^-} - 2f_{W^3}\right)$$

- Double logs only appear in $T \neq 0$ PDFs

$$f_i^{T\pm}(x,q) \sim \exp\left[-\frac{T(T+1)}{2}\frac{\alpha_2}{\pi}\log^2\left(\frac{q}{m_W}\right)\right]$$
$$\sim \left[\Delta_{i,2}(q)\right]^{T(T+1)/2C_i}$$



Counting PDFs

$\{T, \mathrm{CP}\}$	fields	
$\{0,+\}$	$2n_g \times q_R\,, n_g \times \ell_R\,, n_g \times q_L\,, n_g \times \ell_L\,, g\,, W\,, B\,, H$	19
$\{0,-\}$	$2n_g \times q_R\,, n_g \times \ell_R\,, n_g \times q_L\,, n_g \times \ell_L\,, H$	16
$\{1,+\}$	$n_g \times q_L\,, n_g \times \ell_L\,, BW, H$	8
$\{1,-\}$	$n_g \times q_L\,, n_g \times \ell_L\,, W, H$	8
$\{2,+\}$	W	1

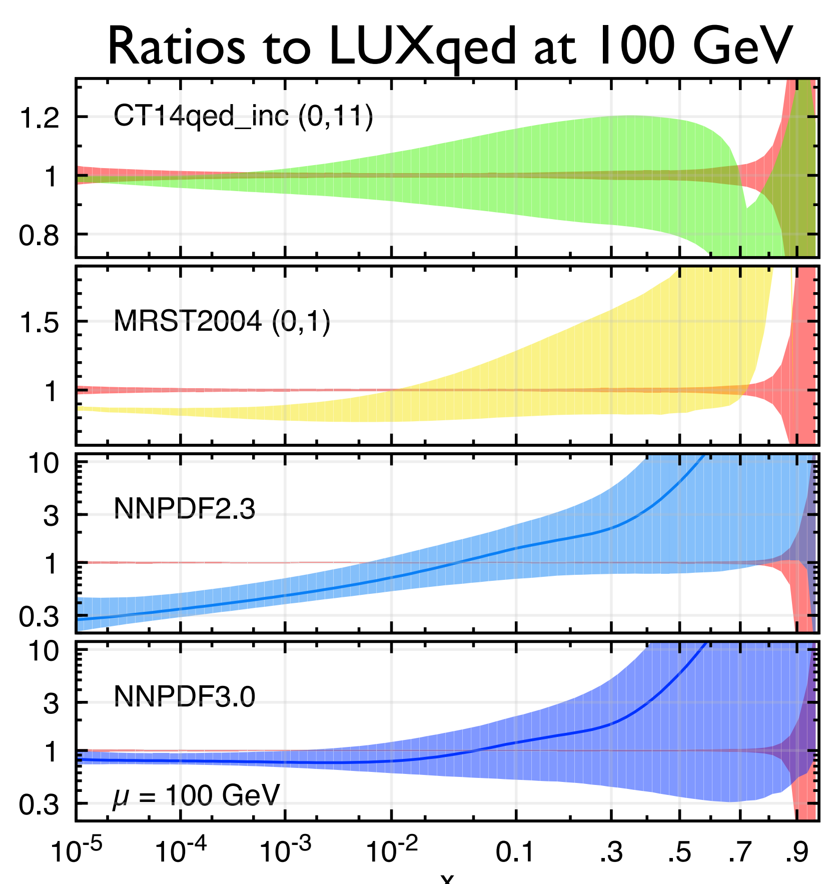
Counting PDFs

$\{T, \mathrm{CP}\}$	fields	
$\{0,+\}$	$2n_g \times q_R\,, n_g \times \ell_R\,, n_g \times q_L\,, n_g \times \ell_L\,, g\,, W\,, B\,, H$	19
$\{0,-\}$	$2n_g \times q_R\,, n_g \times \ell_R\,, n_g \times q_L\,, n_g \times \ell_L\,, H$	16
$\{1,+\}$	$n_g \times q_L\,, n_g \times \ell_L\,, BW, H$	8
$\{1,-\}$	$n_g \times q_L\,, n_g \times \ell_L\,, W, H$	8
$\{2,+\}$	W	1
		52

- 52 SM PDFs for unpolarised proton (36 distinct)
- Only those with same {T,CP} can mix
- Only {0,+} contribute to momentum
- Momentum conserved for each interaction

SMevol Implementation

- Input at 10 GeV: CT14qed partons with LUXqed photon
- SU(3)xU(1)$_{em}$ LO evolution (inc. leptons) up to 100 GeV
- SU(3)xSU(2)xU(1) LO evolution from 100 to 10^8 GeV
- Evolution due to Yukawa interaction of top quark
- Neglect all power-suppressed effects



SMevol: Bauer, Ferland, BW, 1703.08562

CT14: Schmidt, Pumplin, Stump, Yuan, 1509.02905

LUX: Manohar, Nason, Salam, Zanderighi, 1607.04266

Matching at 100 GeV

$$\begin{pmatrix} f_\gamma \\ f_Z \\ f_{\gamma Z} \end{pmatrix} = \begin{pmatrix} c_W^2 & s_W^2 & c_W s_W \\ s_W^2 & c_W^2 & -c_W s_W \\ -2c_W s_W & 2c_W s_W & c_W^2 - s_W^2 \end{pmatrix} \begin{pmatrix} f_B \\ f_{W_3} \\ f_{BW} \end{pmatrix}$$

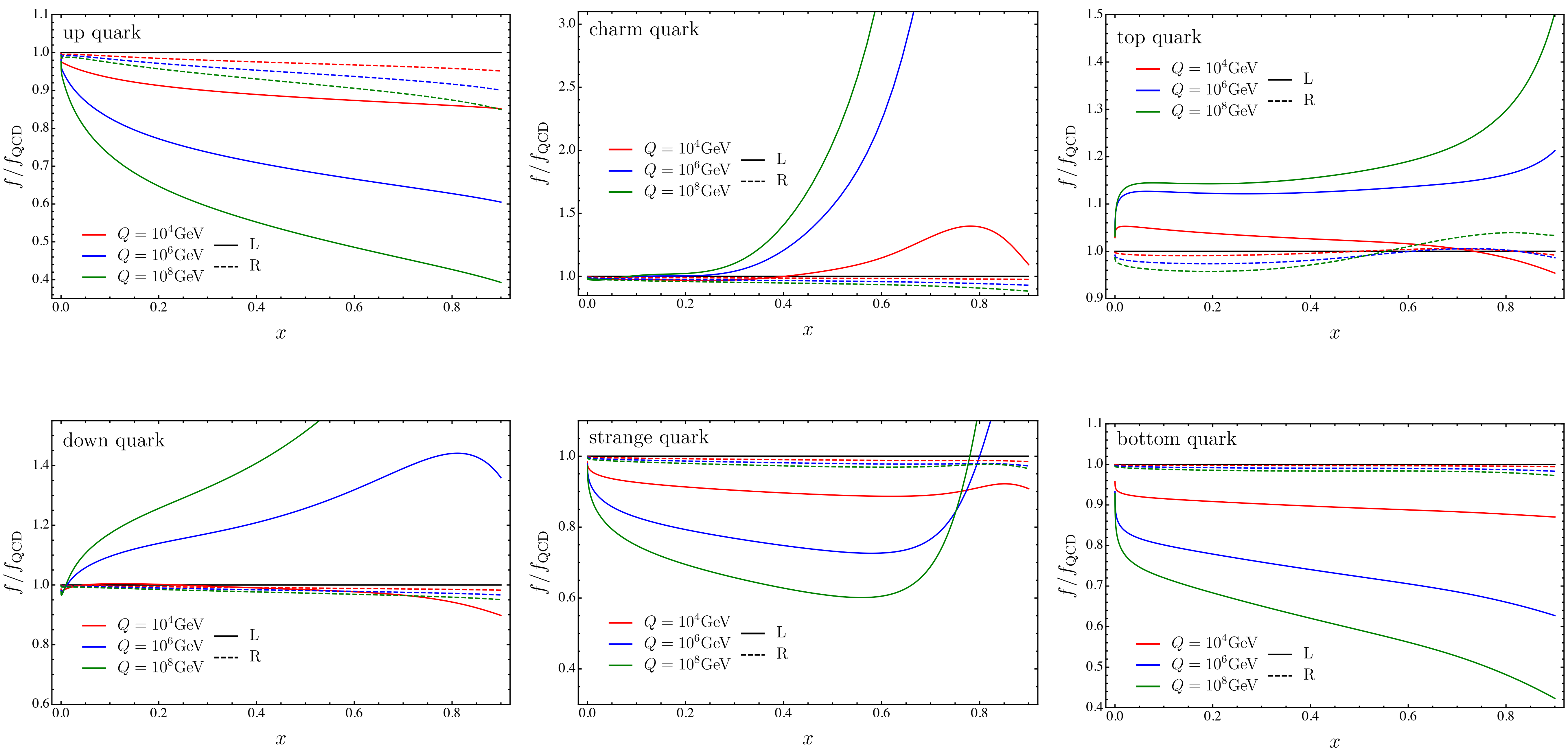
- At q=100 GeV: $f_\gamma \neq 0$, $f_Z = f_{\gamma Z} = 0$, hence

$$f_B = c_W^2 f_\gamma\,, \quad f_{W_3} = s_W^2 f_\gamma\,, \quad f_{BW} = 2c_W s_W f_\gamma$$

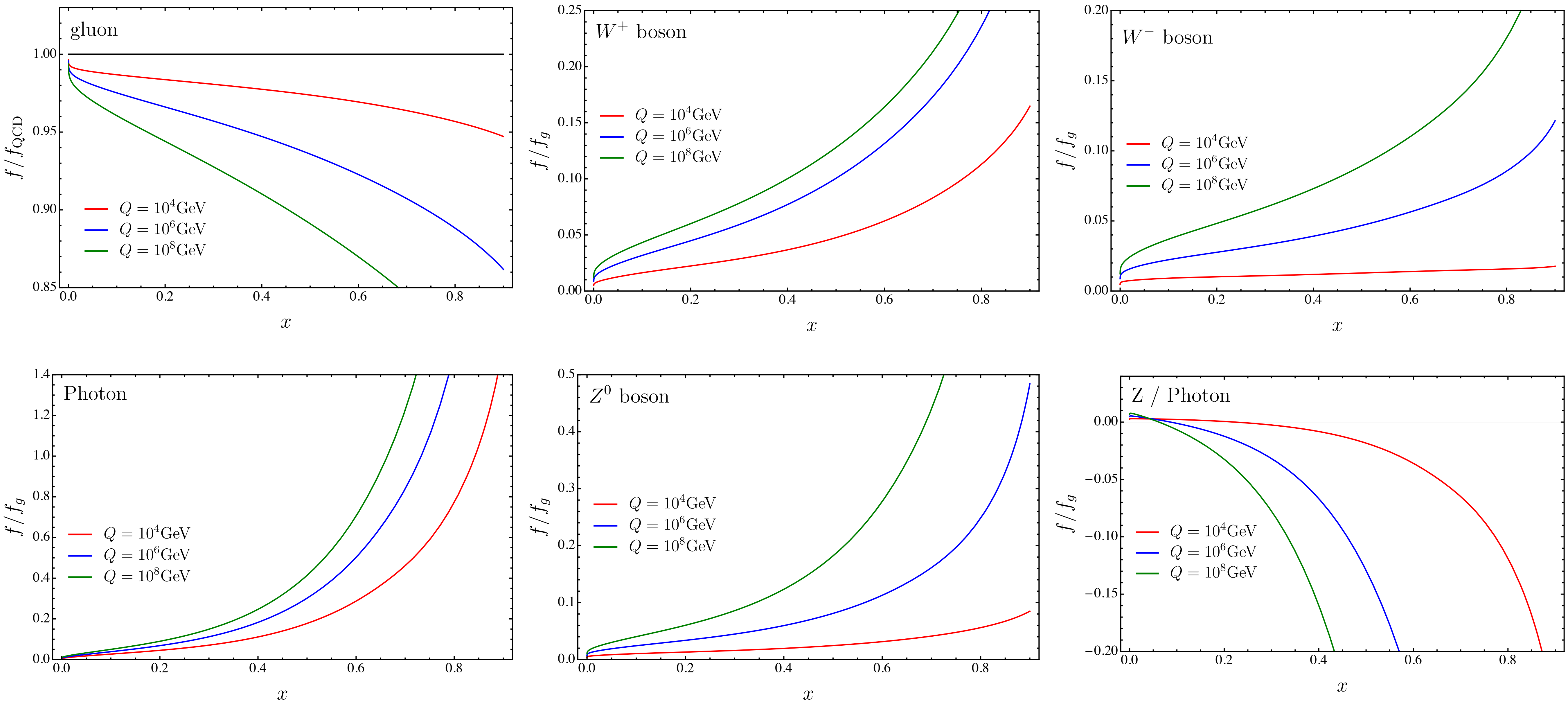
- Project back on f_γ, f_Z and $f_{\gamma Z}$ at higher scales
- $f_W = f_H = 0$ at $q \leq 100$ GeV
- $f_t = 0$ at $q \leq m_t(m_t) = 163$ GeV

Results

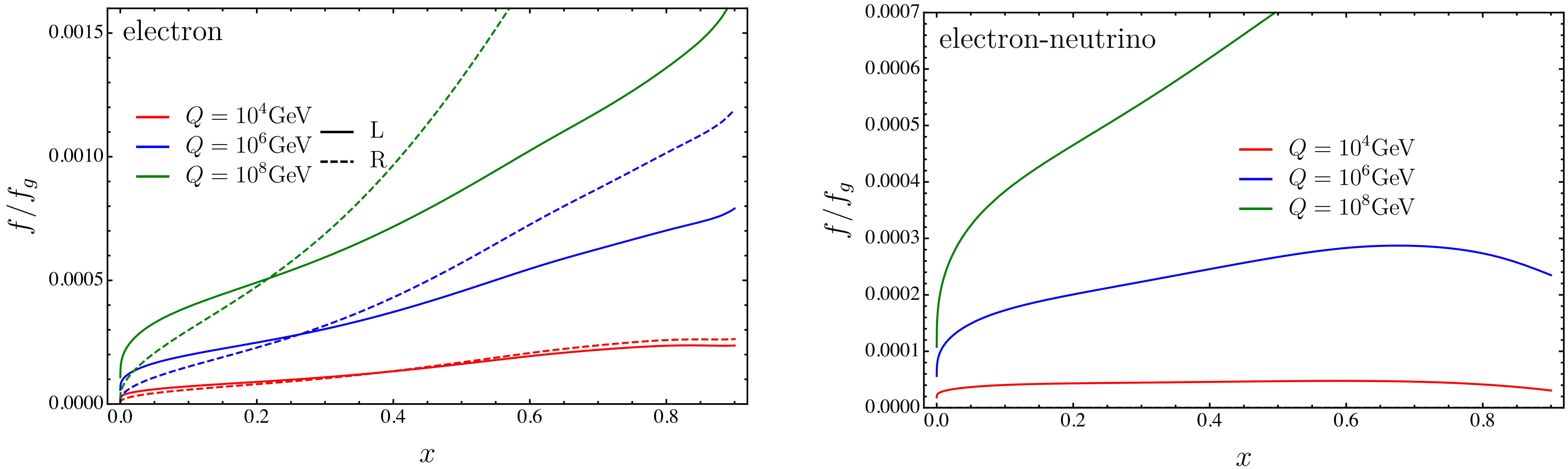
Quarks relative to QCD



Bosons relative to gluon

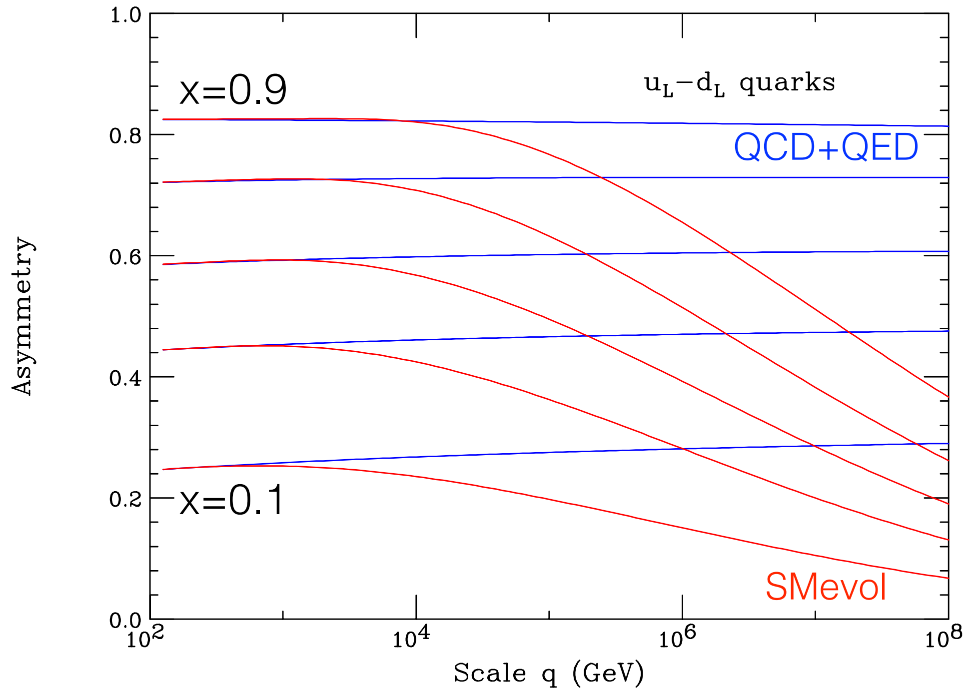


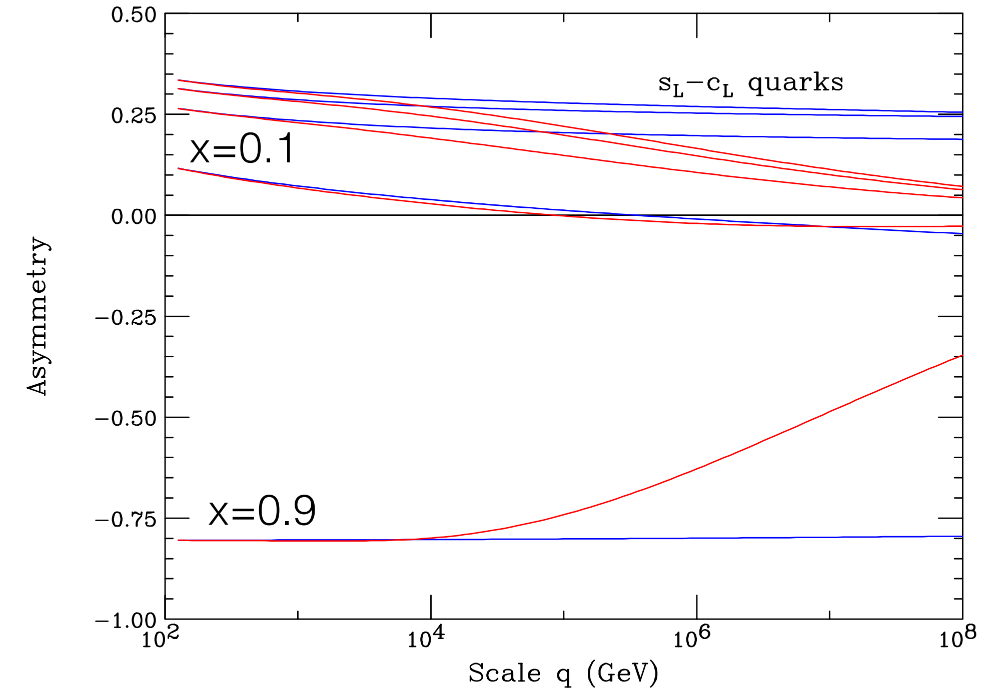
Leptons relative to gluon

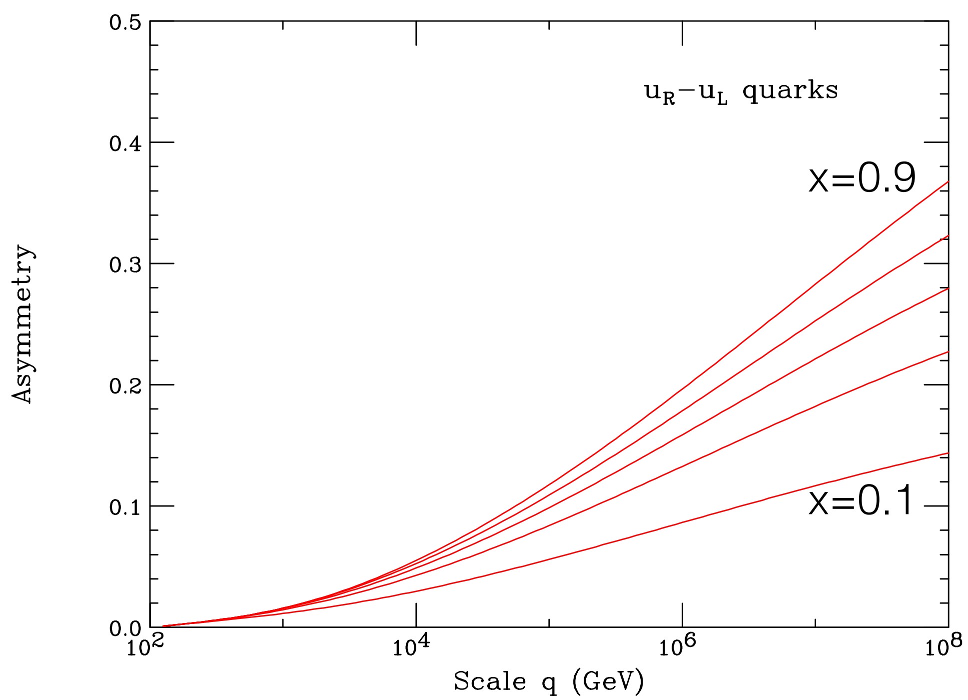


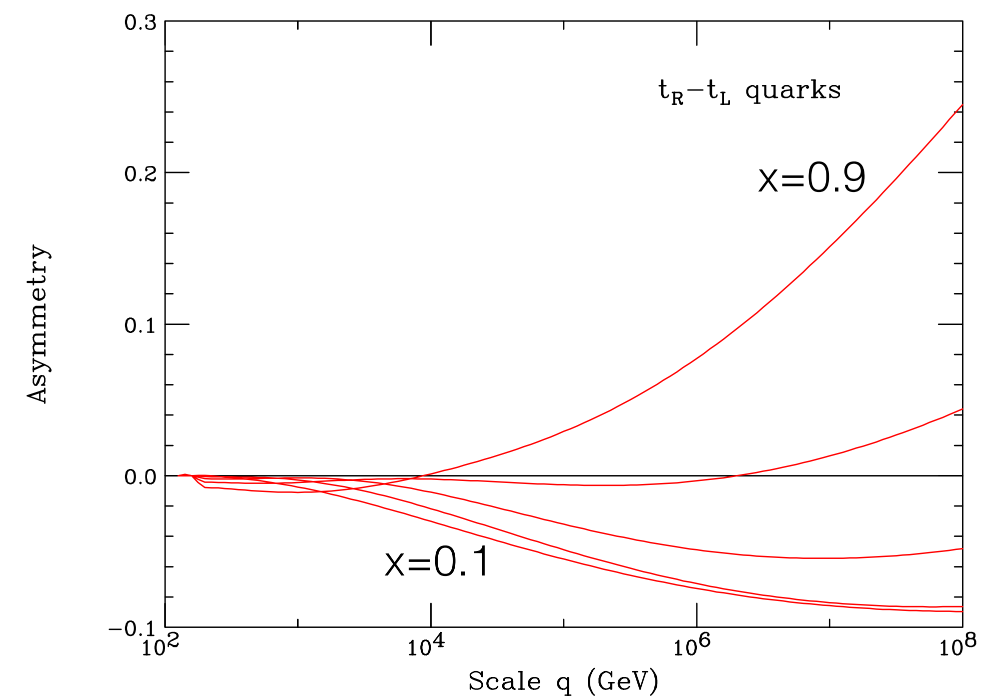
- Masses neglected ➡ all generations equal

Asymmetries $(f_i-f_j)/(f_i+f_j)$

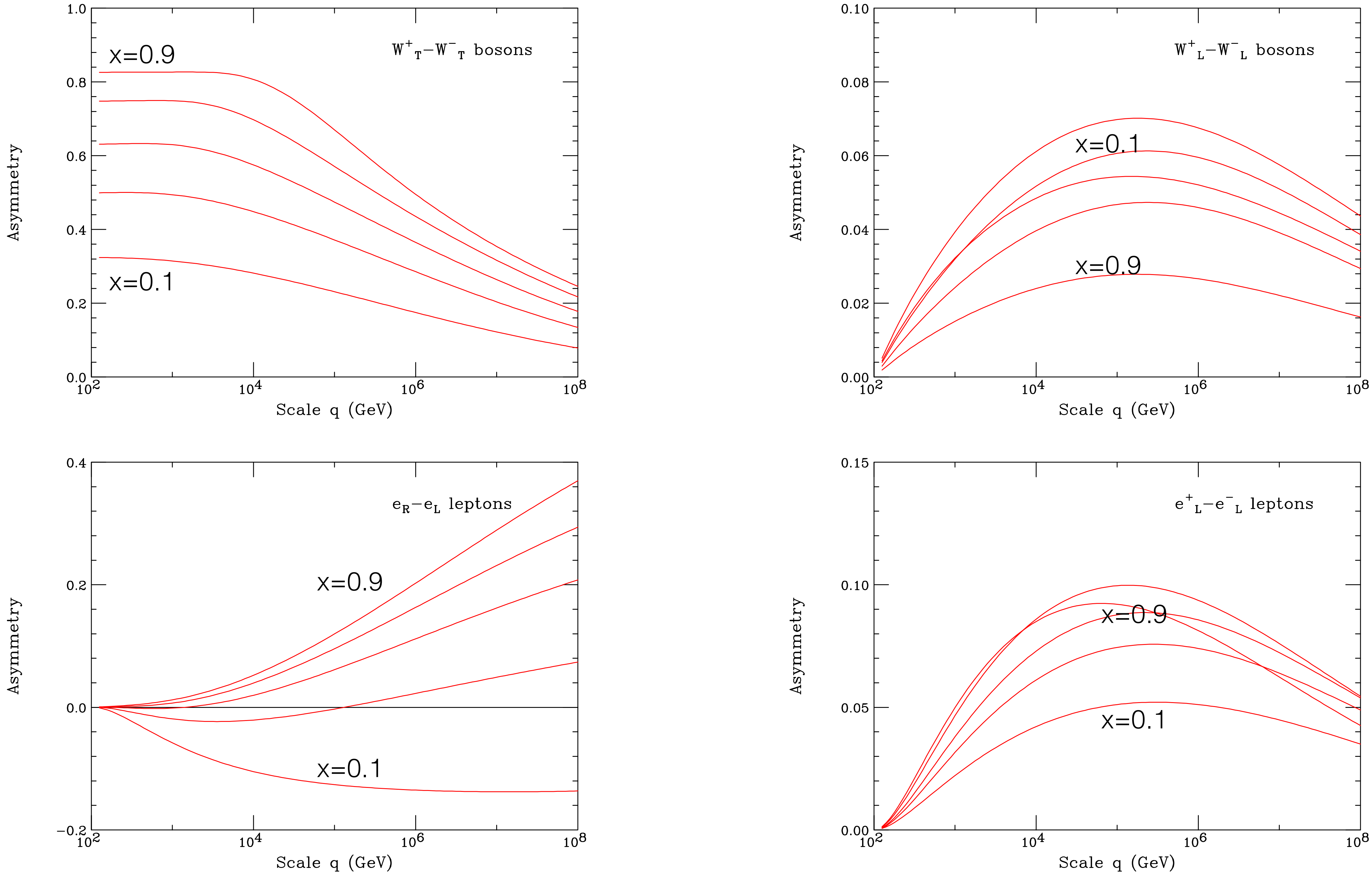






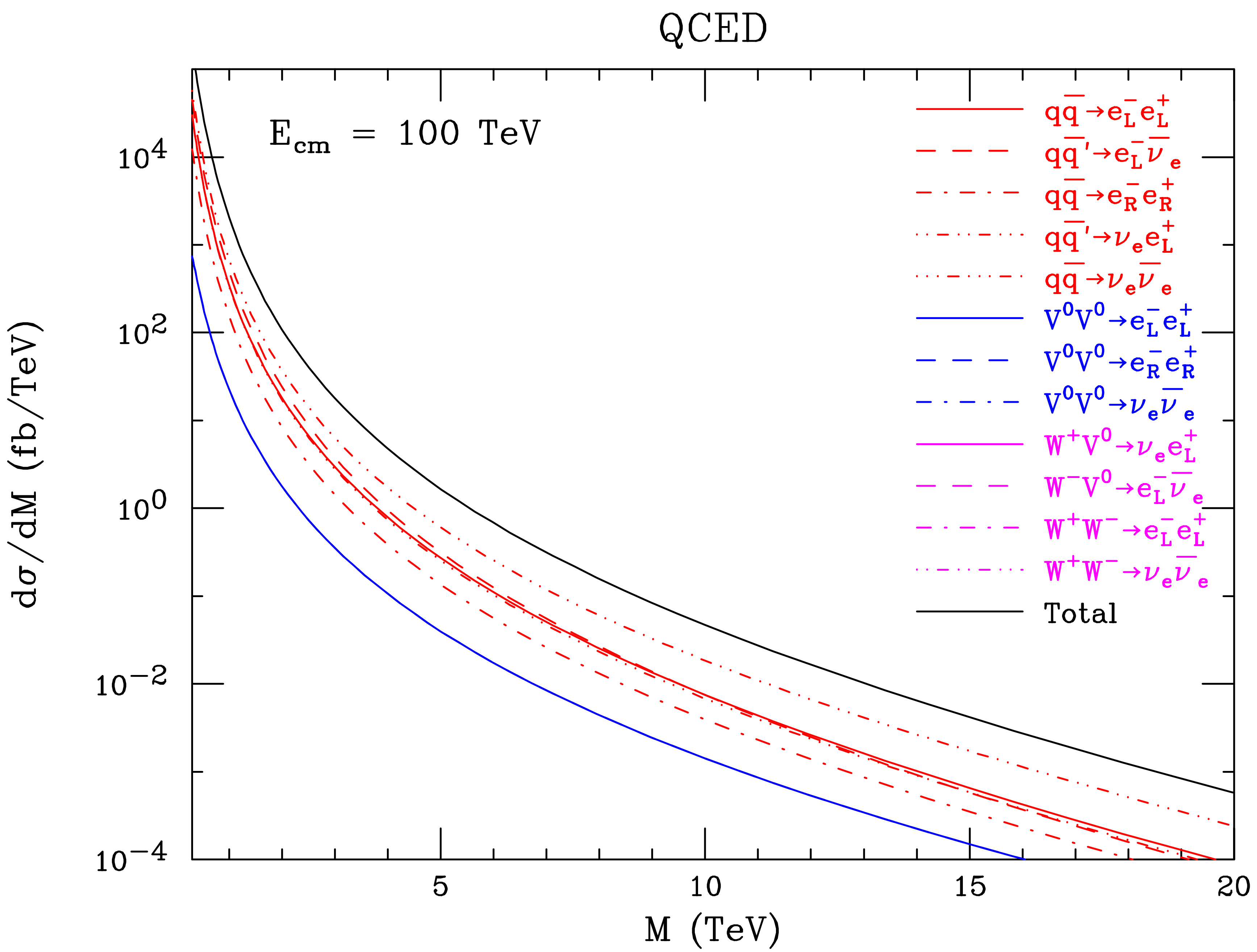


Asymmetries $(f_i-f_j)/(f_i+f_j)$



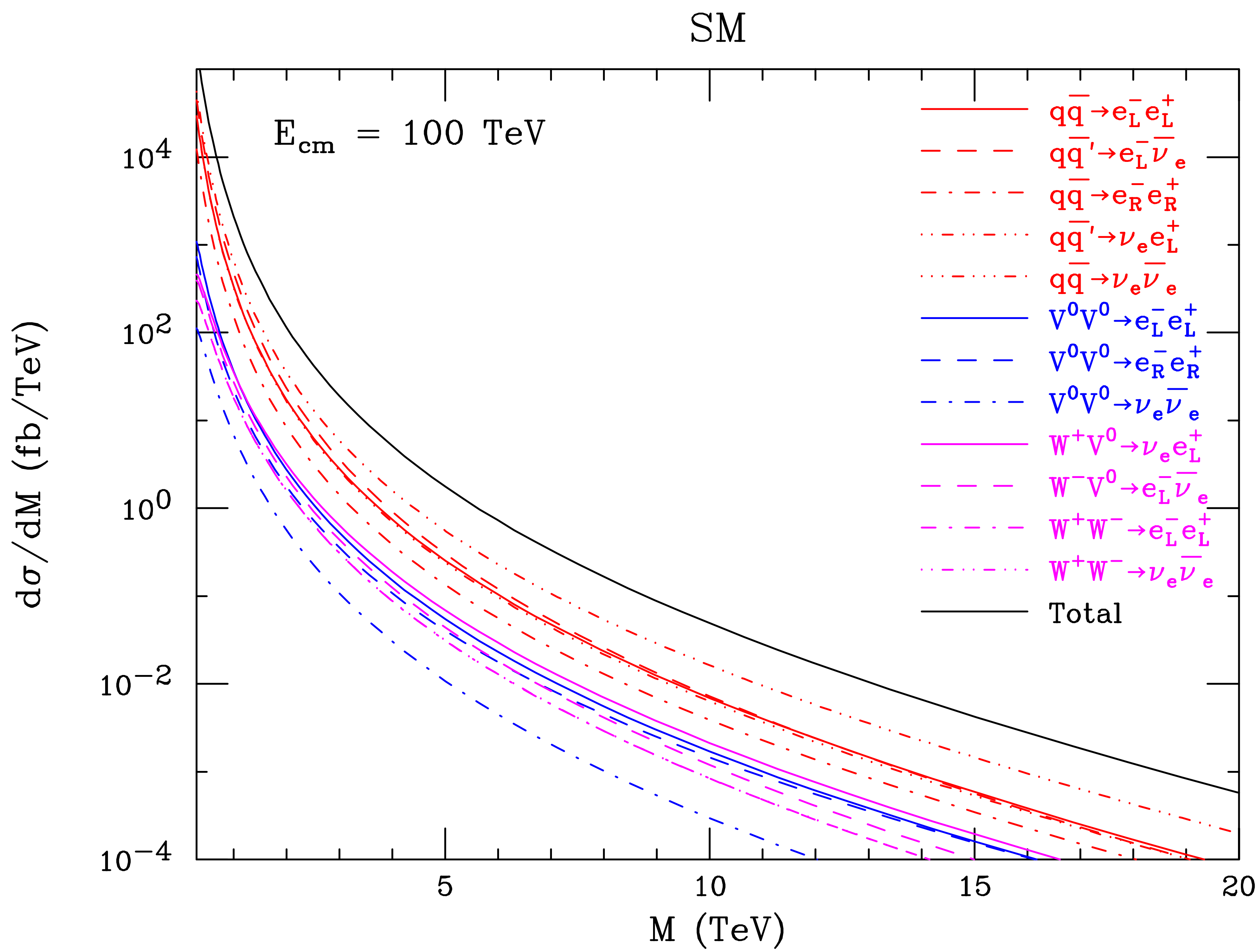
Lepton Pair Production at 100 TeV Collider

Lepton Pair Production



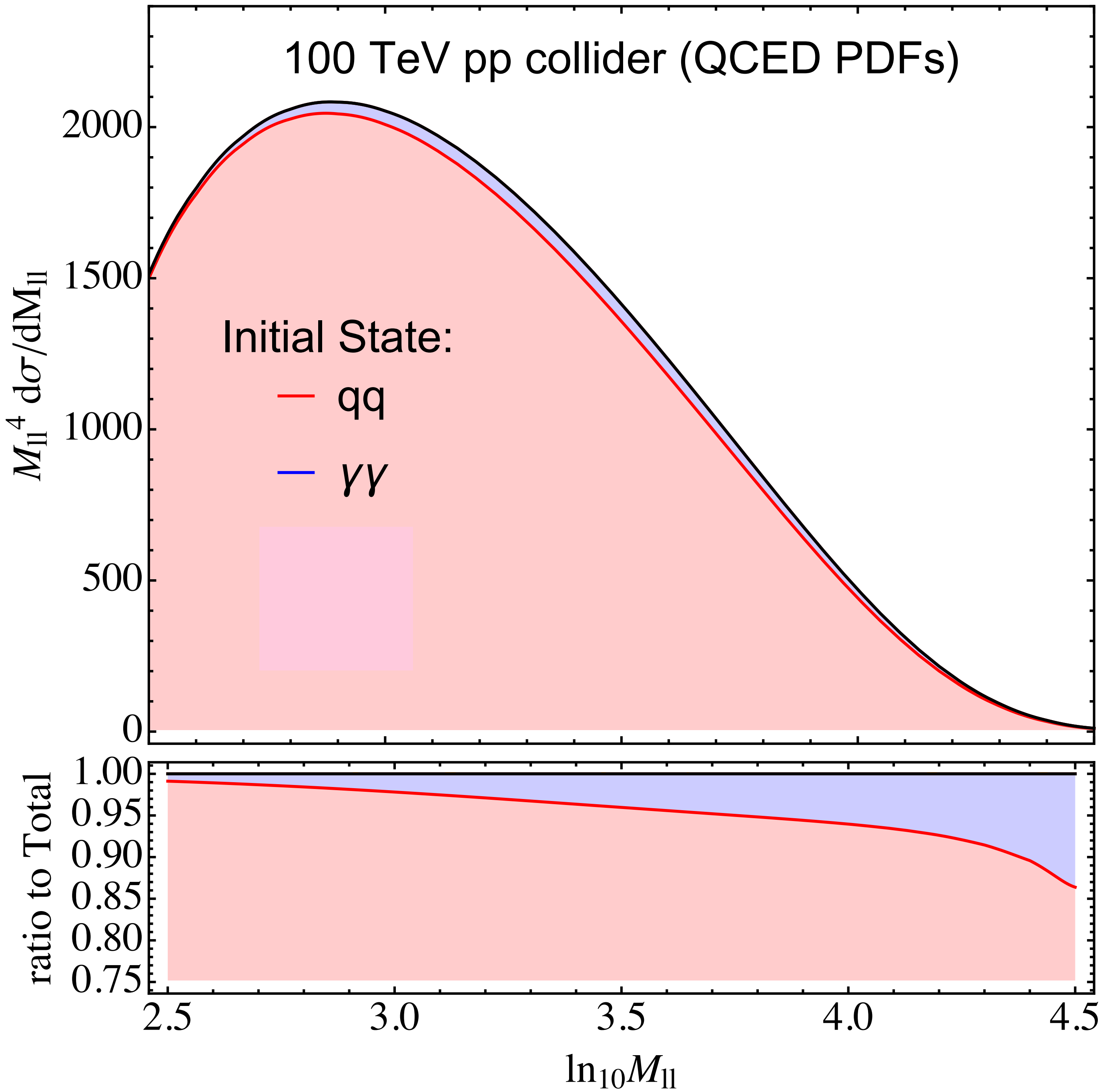
- QCED = SU(3)xU(1)$_{em}$

Lepton Pair Production



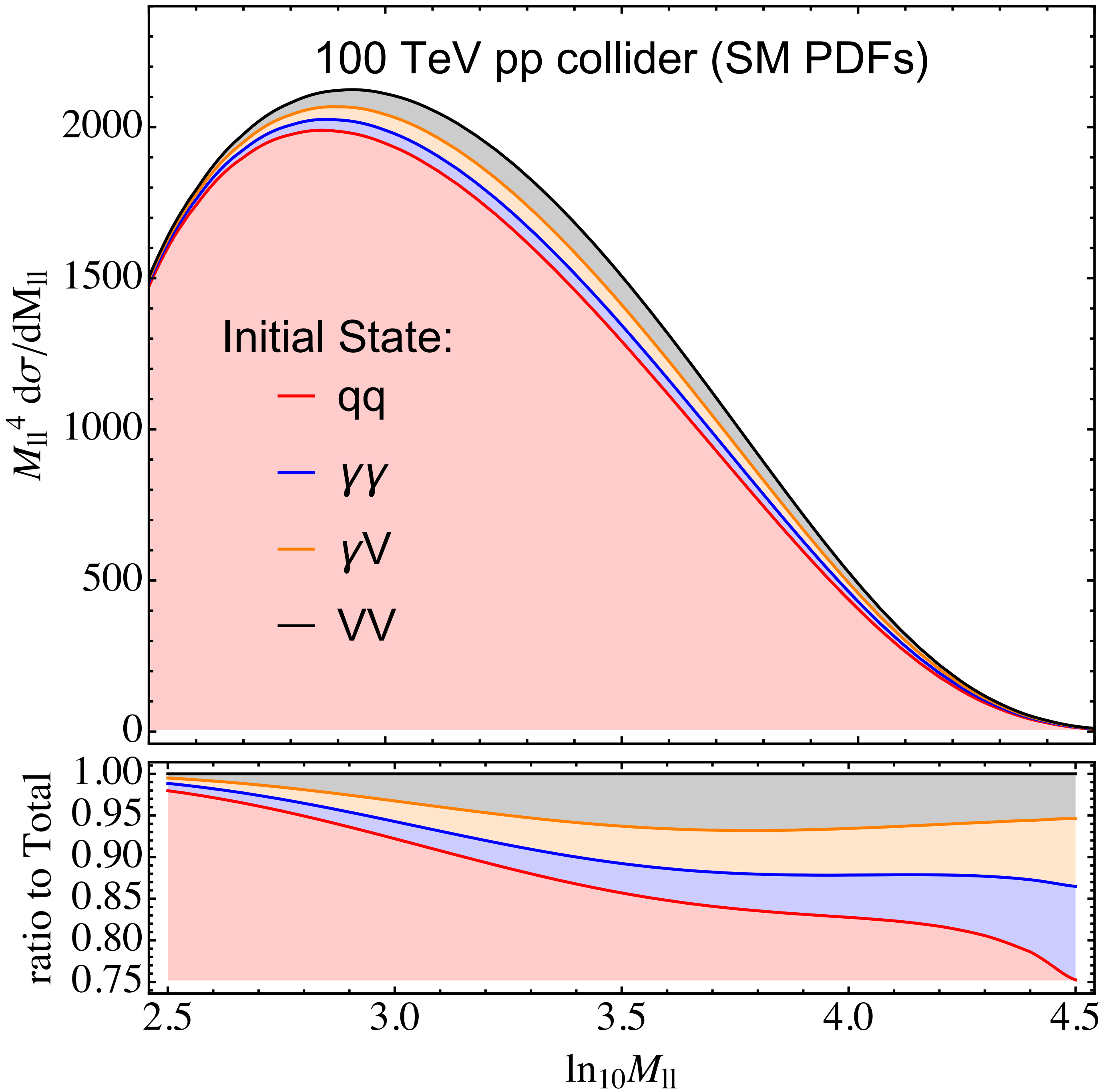
- $SM = SU(3) \times SU(2)_L \times U(1)_Y$

Lepton Pair Production



- QCED = SU(3)xU(1)$_{em}$

Lepton Pair Production



- SM = SU(3)xSU(2)$_L$xU(1)$_Y$

Matching to Fixed Order

Matching to O(α) EW

C Bauer, N Ferland, BW, in preparation

$$q\frac{\partial}{\partial q} f_i^{\rm SM}(x,q) = \sum_I \frac{\alpha_I(q)}{\pi}\left[P^V_{i,I}(q)\, f_i^{\rm SM}(x,q) + \sum_j C_{ij,I}\int_x^{z^{ij,I}_{\rm max}(q)} dz\, P^R_{ij,I}(z) f_j^{\rm SM}(x/z,q)\right]$$

- **Define** $f_i^{\rm SM}(x,q) = f_i^{\rm QCED}(x,q) + g_i(x,q) + \mathcal{O}(\alpha^2)$

$$q\frac{\partial}{\partial q} g_i(x,q) = \frac{\alpha_3(q)}{\pi}\left[P^V_{i,3}(q)\, g_i(x,q) + \sum_j C_{ij,I}\int_x^1 dz\, P^R_{ij,3}(z) g_j(x/z,q)\right]$$
$$+ \sum_{I\in 1,2,M} \frac{\alpha_I(q)}{\pi}\left[P^V_{i,I}(q)\, f_i^{\rm QCED}(x,q) + \sum_j C_{ij,I}\int_x^{z^{ij,I}_{\rm max}(q)} dz\, P^R_{ij,I}(z) f_j^{\rm QCED}(x/z,q)\right]$$
$$- \frac{\alpha_{\rm em}(q)}{\pi}\left[P^V_{i,{\rm em}}(q)\, f_i^{\rm QCED}(x,q) + \sum_j C_{ij,{\rm em}}\int_x^1 dz\, P^R_{ij,{\rm em}}(z) f_j^{\rm QCED}(x/z,q)\right]$$

Matching to O(α) EW

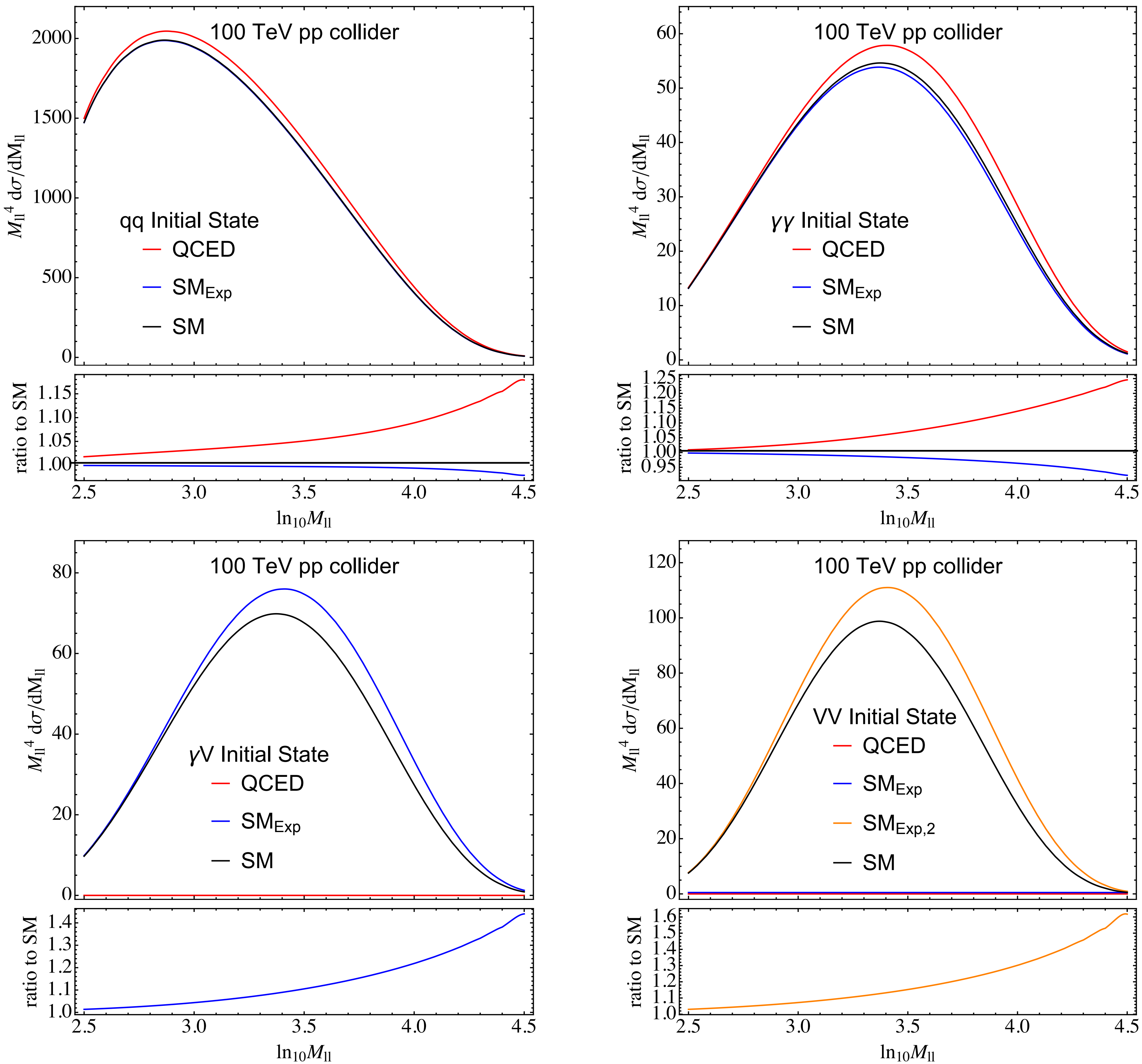
$$f_i^{\rm SM}(x,q) = f_i^{\rm QCED}(x,q) + g_i(x,q) + \mathcal{O}(\alpha^2)$$

$$\sigma_{ij}^{\rm QCED} = f_i^{\rm QCED} \otimes \hat{\sigma}_{ij} \otimes f_j^{\rm QCED}\,, \quad \sigma_{ij}^{\rm SM} = f_i^{\rm SM} \otimes \hat{\sigma}_{ij} \otimes f_j^{\rm SM}$$

$$\sigma_{ij}^{\rm SMexp} = \sigma_{ij}^{\rm QCED} + f_i^{\rm QCED} \otimes \hat{\sigma}_{ij} \otimes g_j + g_i \otimes \hat{\sigma}_{ij} \otimes f_j^{\rm QCED}$$

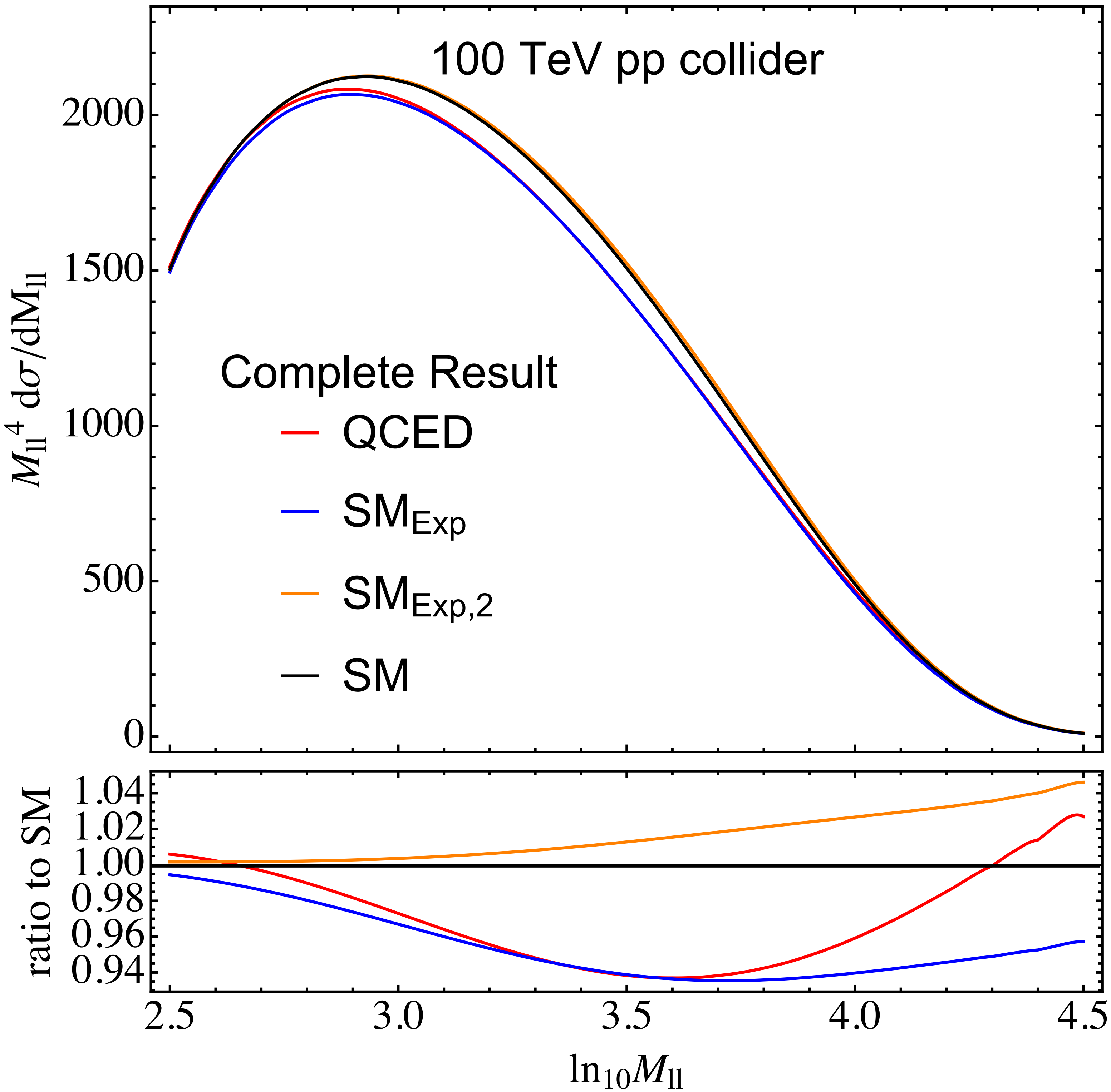
- Define $\sigma_{ij}^{\rm SMexp2} = \sigma_{ij}^{\rm SMexp}$ when $\sigma_{ij}^{\rm SMexp} \neq 0$, else
  $\sigma_{ij}^{\rm SMexp2} = g_i \otimes \hat{\sigma}_{ij} \otimes g_j$ (e.g. WW fusion)
- Then $\sigma_{ij}^{\rm SM} - \sigma_{ij}^{\rm SMexp2}$ is resummation of (IS) HO logs

Preliminary Results



- SM-SM$_{Exp(2)}$ is extra HO contribution
- Large HO contributions to VBF

Preliminary Results



- Overall HO contribution is smaller (few %)

Conclusions and Prospects

- Rich SM structure inside the proton
  - 52 parton distributions (36 distinct)
- Symmetries restored double-logarithmically, distinct left and right-handed PDFs
  - Onset of large effects around 10 TeV
  - Significant for ~100 TeV collider
  - Matching to FO almost ready
- Next step: complete SM event generator
  - Electroweak jets, ISR, MET, …

Thanks for listening!

Backup

PDFs and Parton Luminosity

- Factorization

$$\sigma_{pp\to X}(s) = \sum_{i,j}\int_0^1 \frac{\mathrm{d}x_1}{x_1}\frac{\mathrm{d}x_2}{x_2} f_i(x_1,q) f_j(x_2,q)\hat{\sigma}_{ij\to X}(x_1 x_2 s, q)$$

- Momentum sum rule

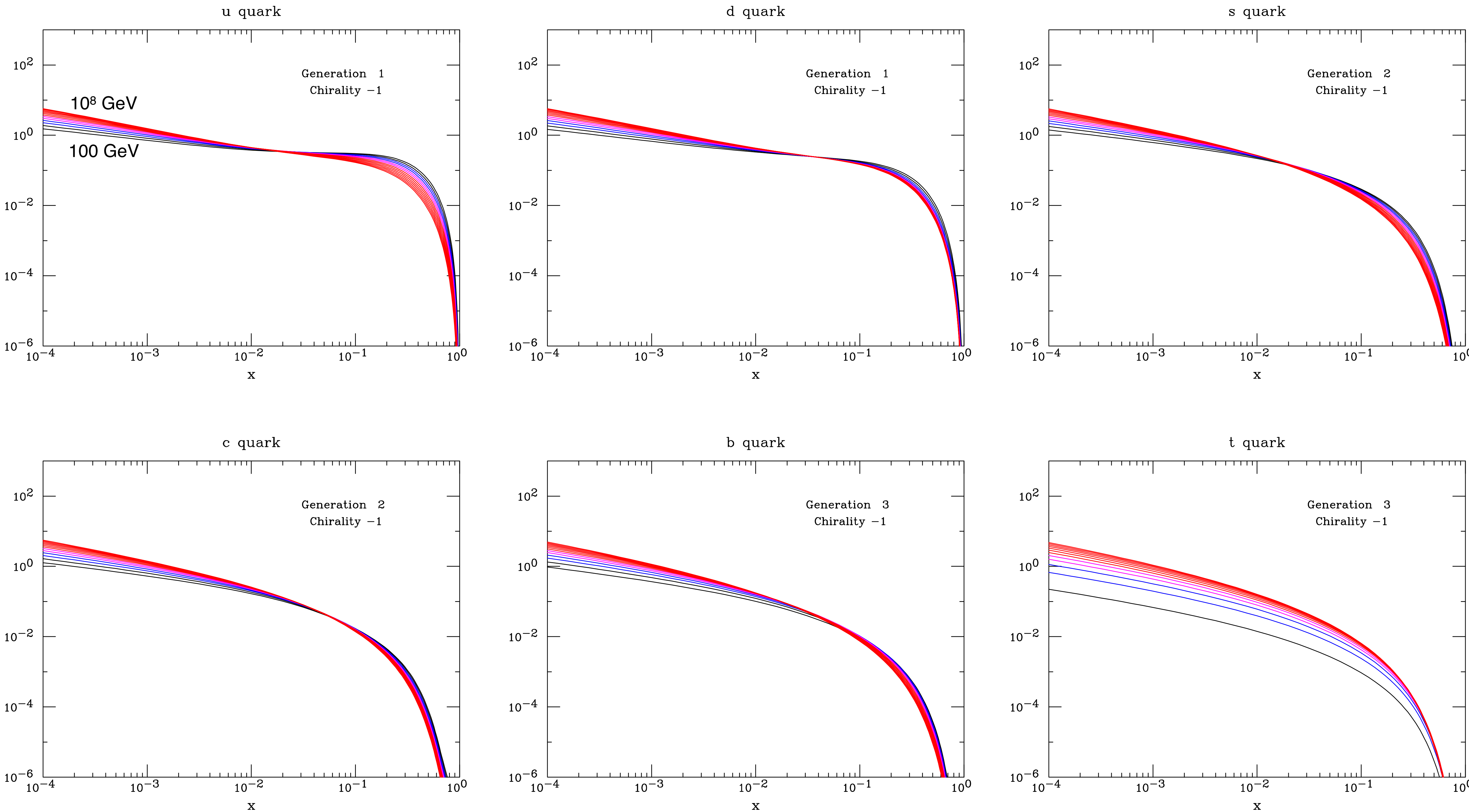
$$\sum_i \int_0^1 \mathrm{d}x\, f_i(x,q) = 1$$

- Luminosity

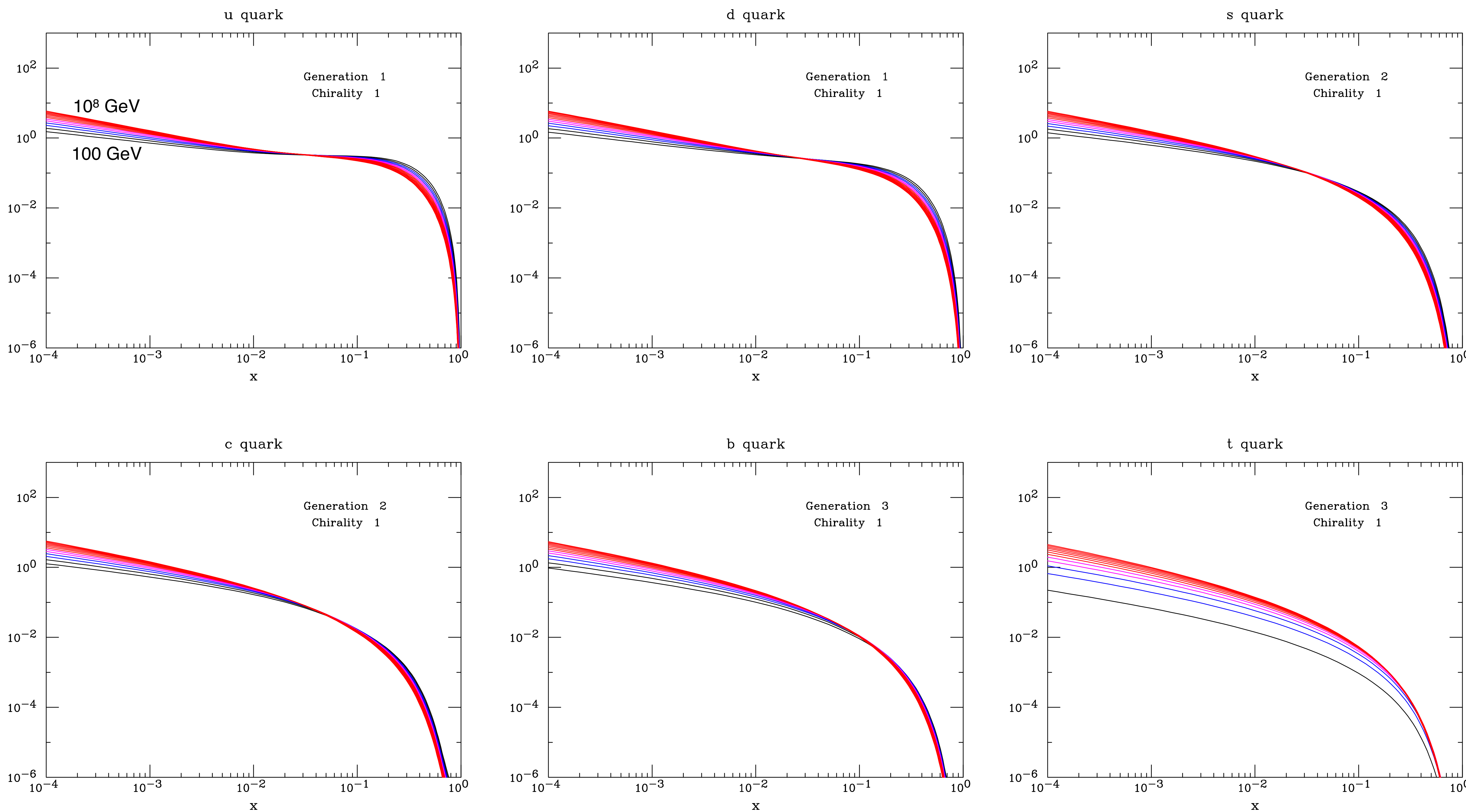
$$\frac{\mathrm{d}\mathcal{L}_{ij}}{\mathrm{d}M^2} = \int_0^1 \frac{\mathrm{d}x_1}{x_1}\frac{\mathrm{d}x_2}{x_2} f_i(x_1,M) f_j(x_2,M)\, \delta(M^2 - x_1 x_2 s)$$

$$\sigma_{pp\to X}(s) = \sum_{i,j}\int_0^s \mathrm{d}M^2 \frac{\mathrm{d}\mathcal{L}_{ij}}{\mathrm{d}M^2}\hat{\sigma}_{ij\to X}(M^2, M)$$

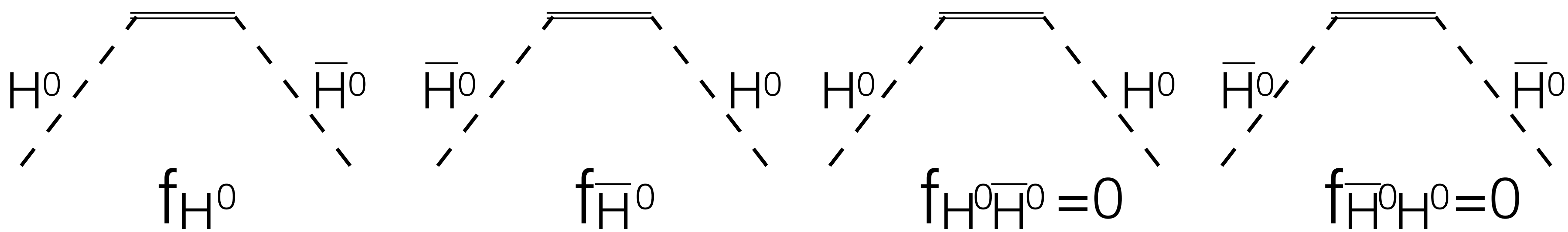
Left-handed quarks



Right-handed quarks



Higgs PDFs



$$H^0 = \frac{1}{\sqrt{2}}(h - iZ_L)\,, \quad \bar{H}^0 = \frac{1}{\sqrt{2}}(h + iZ_L)$$

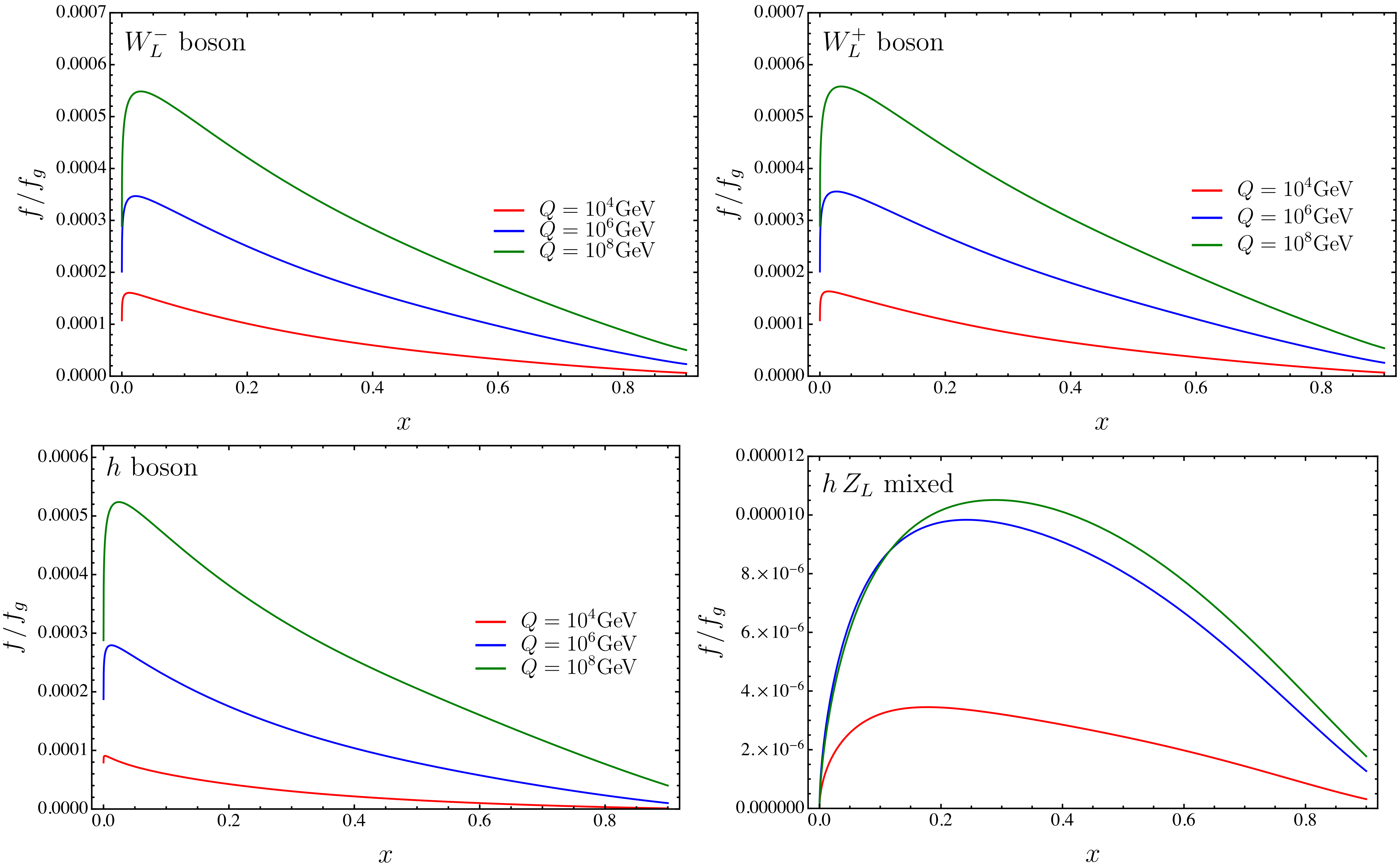
$$h = \frac{1}{\sqrt{2}}\left(H^0 + \bar{H}^0\right)\,, \quad Z_L = \frac{i}{\sqrt{2}}\left(H^0 - \bar{H}^0\right)$$

- Hence, neglecting power-suppressed symmetry breaking effects

$$\boxed{f_h = f_{Z_L} = \frac{1}{2}\left(f_{H^0} + f_{\bar{H}^0}\right)}$$

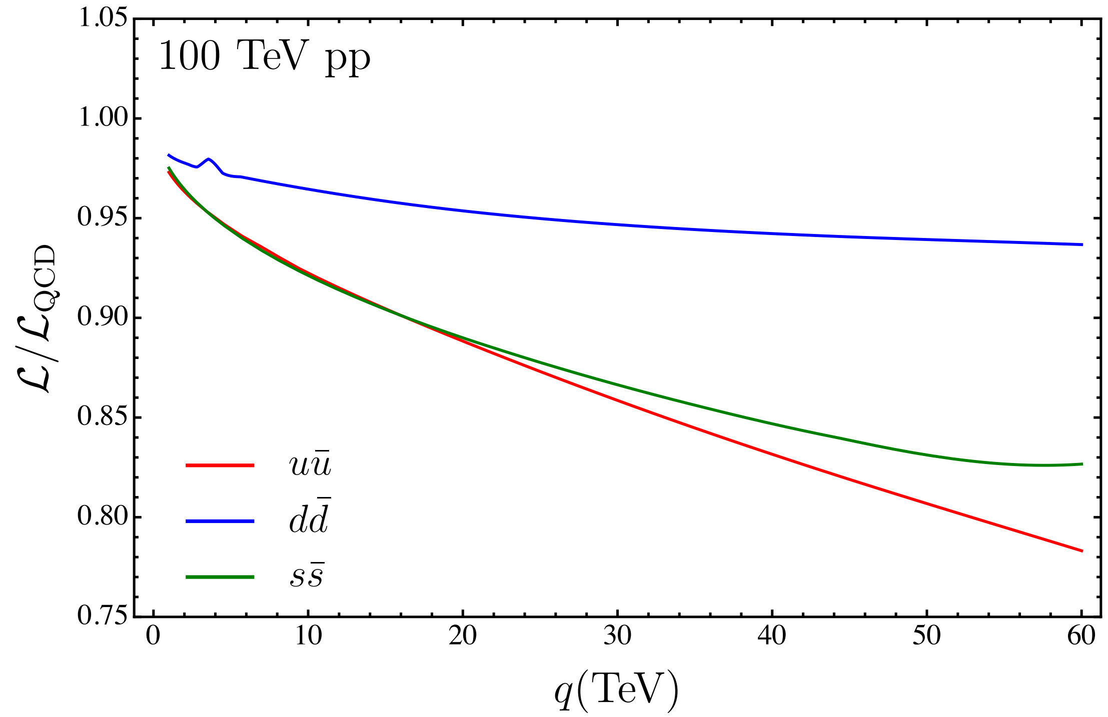
$$f_{hZ_L} \equiv \frac{1}{2}\left(f_{H^0} - f_{\bar{H}^0}\right)$$

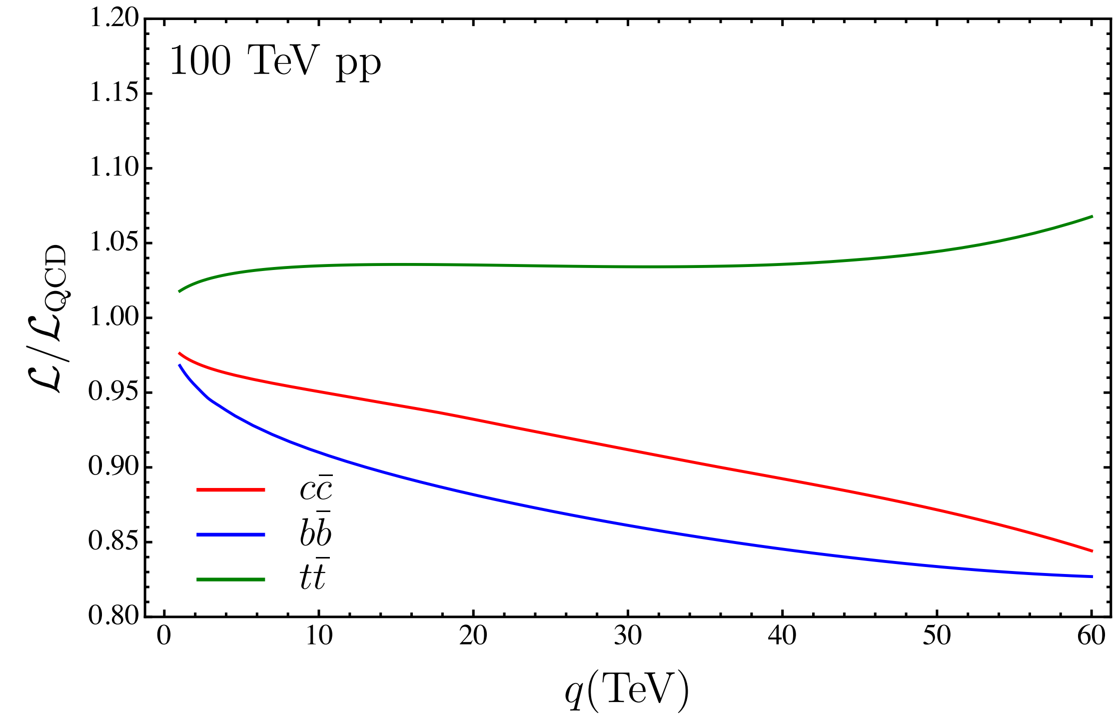
Higgs relative to gluon



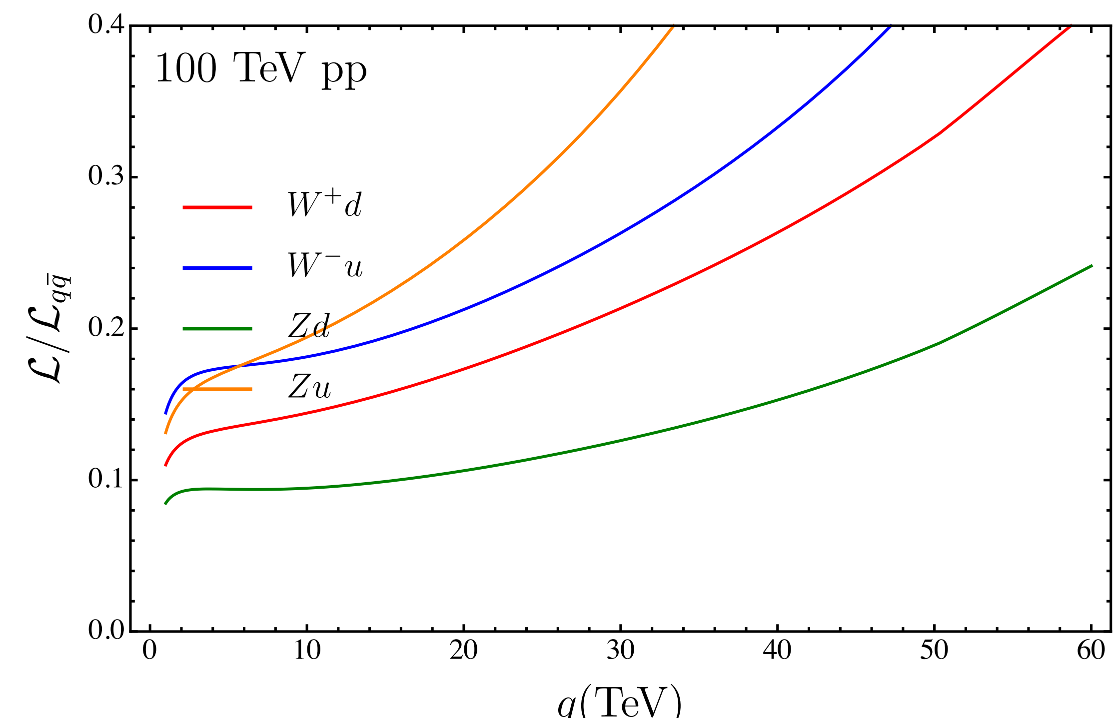
Luminosities at 100 TeV

- $q\bar{q}$ relative to QCD

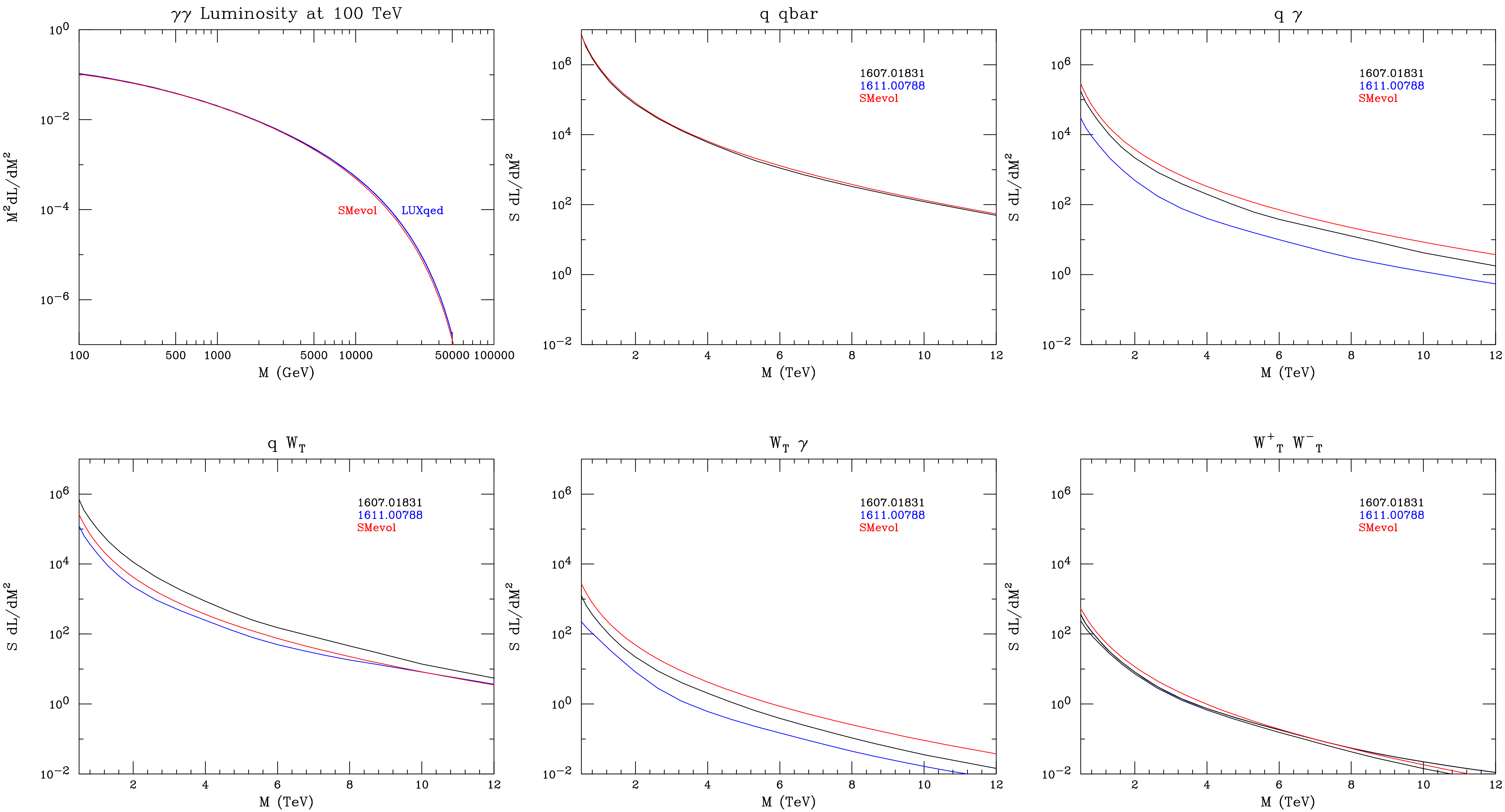




- Vq relative to $u\bar{u}+d\bar{d}$



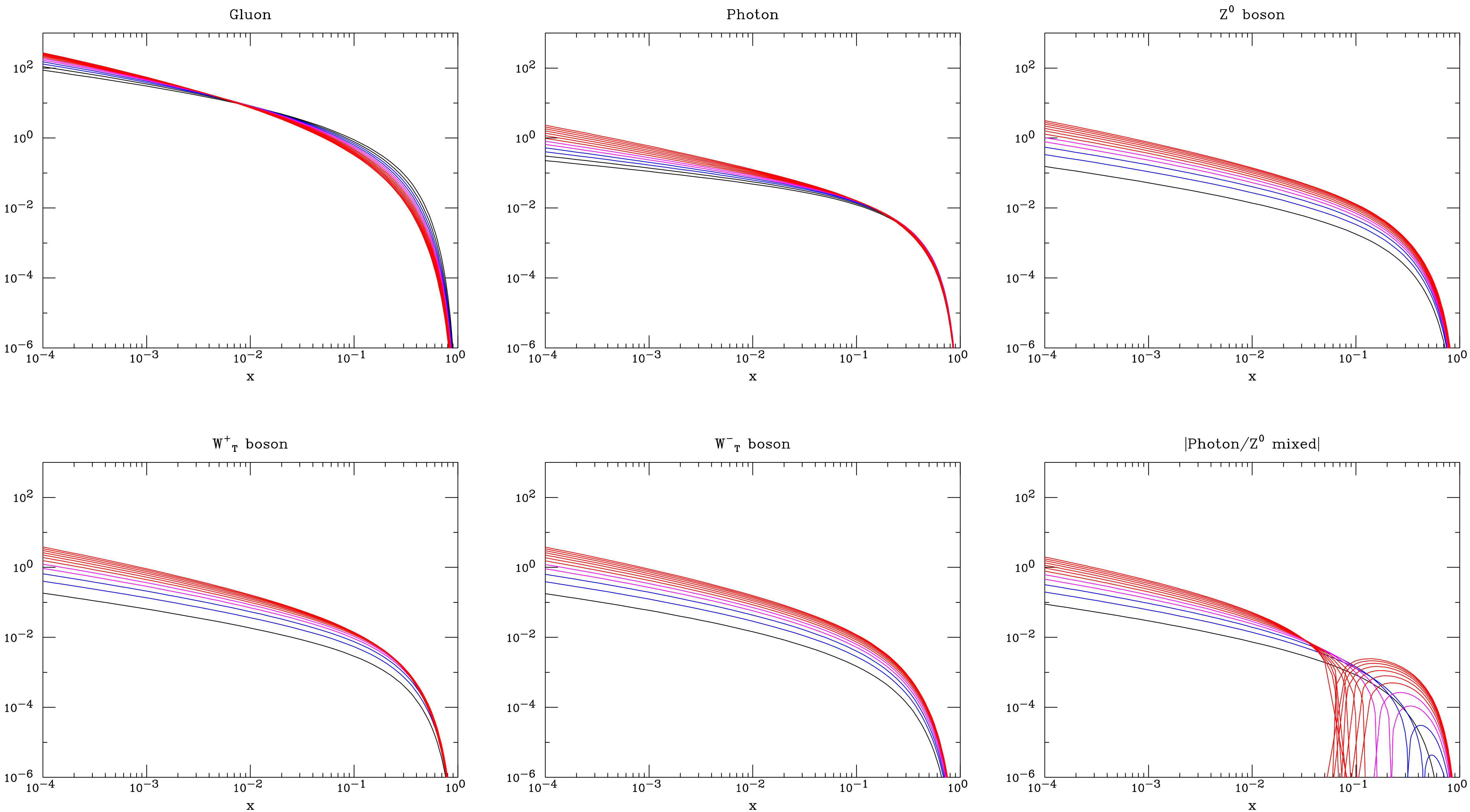
Luminosities at 100 TeV



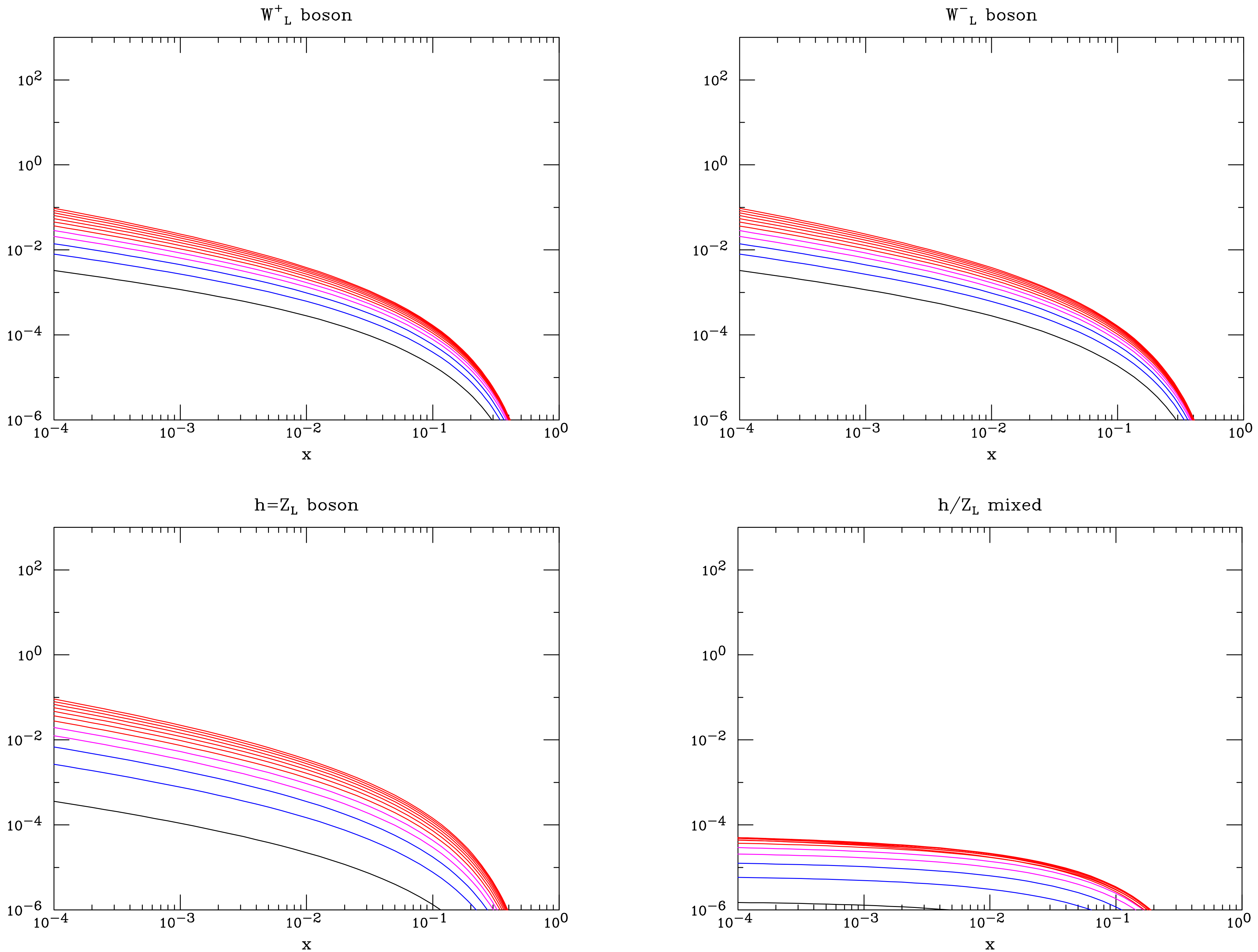
Mangano et al., 1607.01831

Chen, Han, Tweedie, 1611.00788

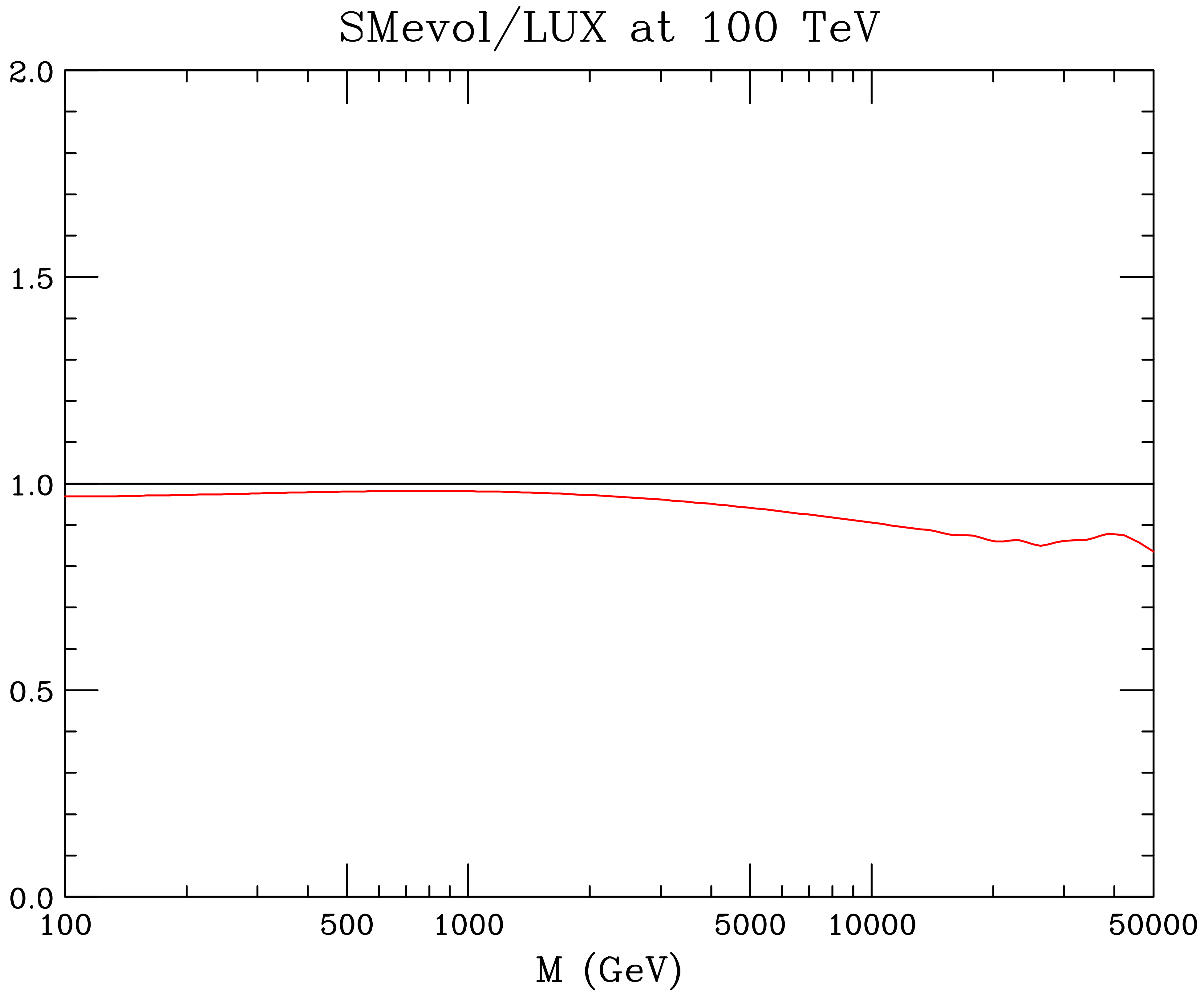
Gauge boson PDFs



Higgs PDFs

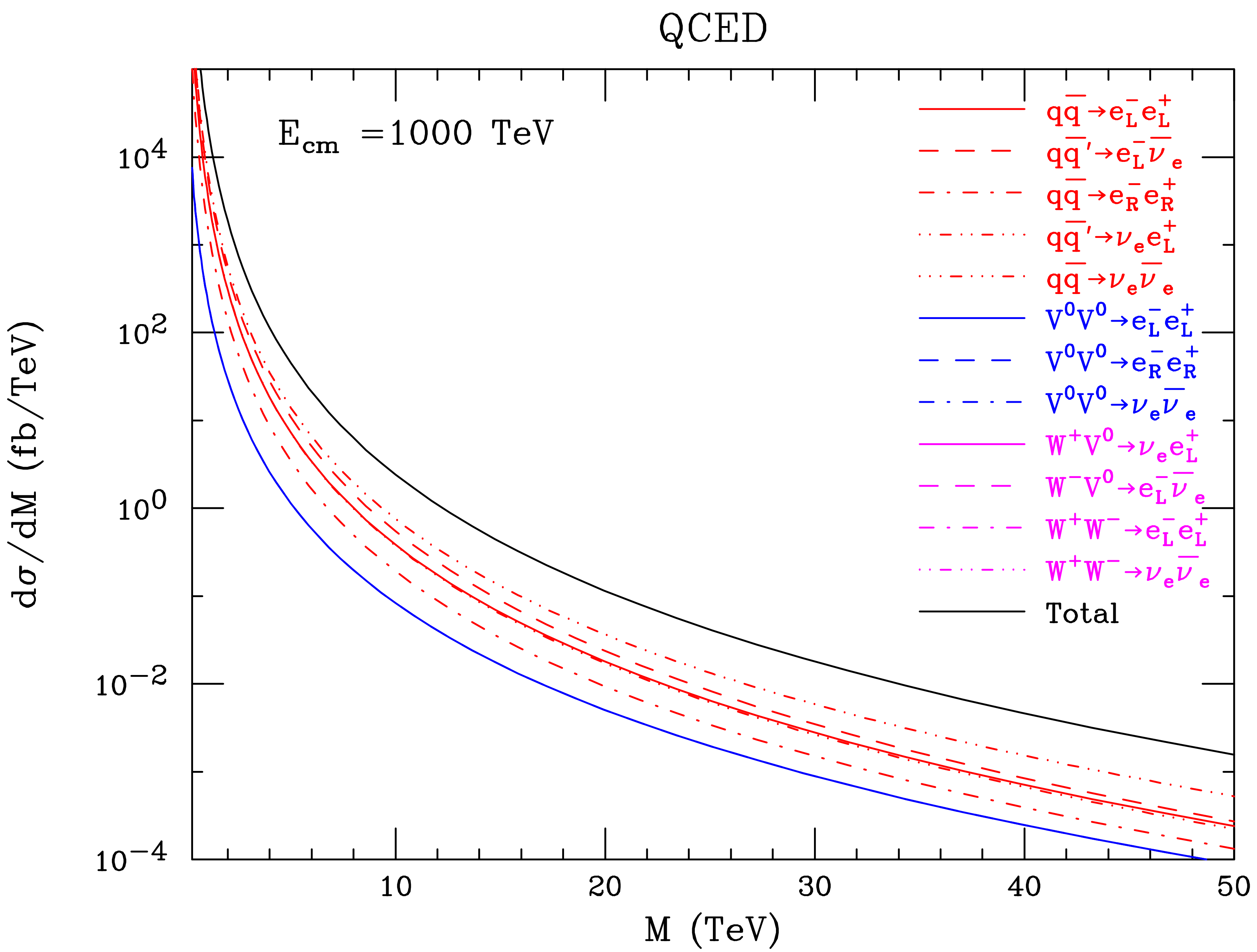

W+L boson
W−L boson
h=ZL boson
h/ZL mixed
10^2
10^0
10^−2
10^−4
10^−6
10^−4
10^−3
10^−2
10^−1
10^0
x

SMevol/LUX $\gamma\gamma$ luminosity



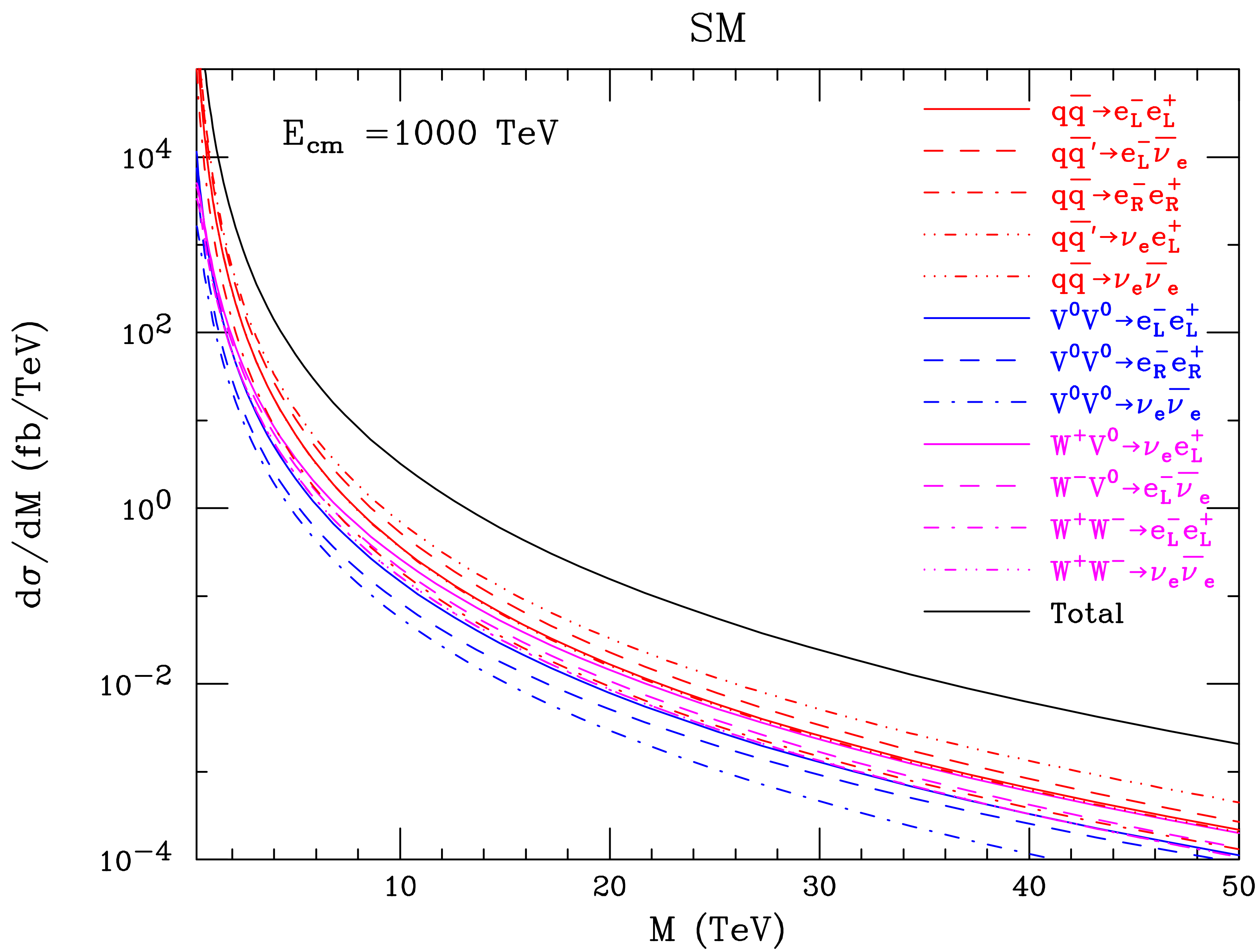
PeV Collider!

Lepton Pair Production



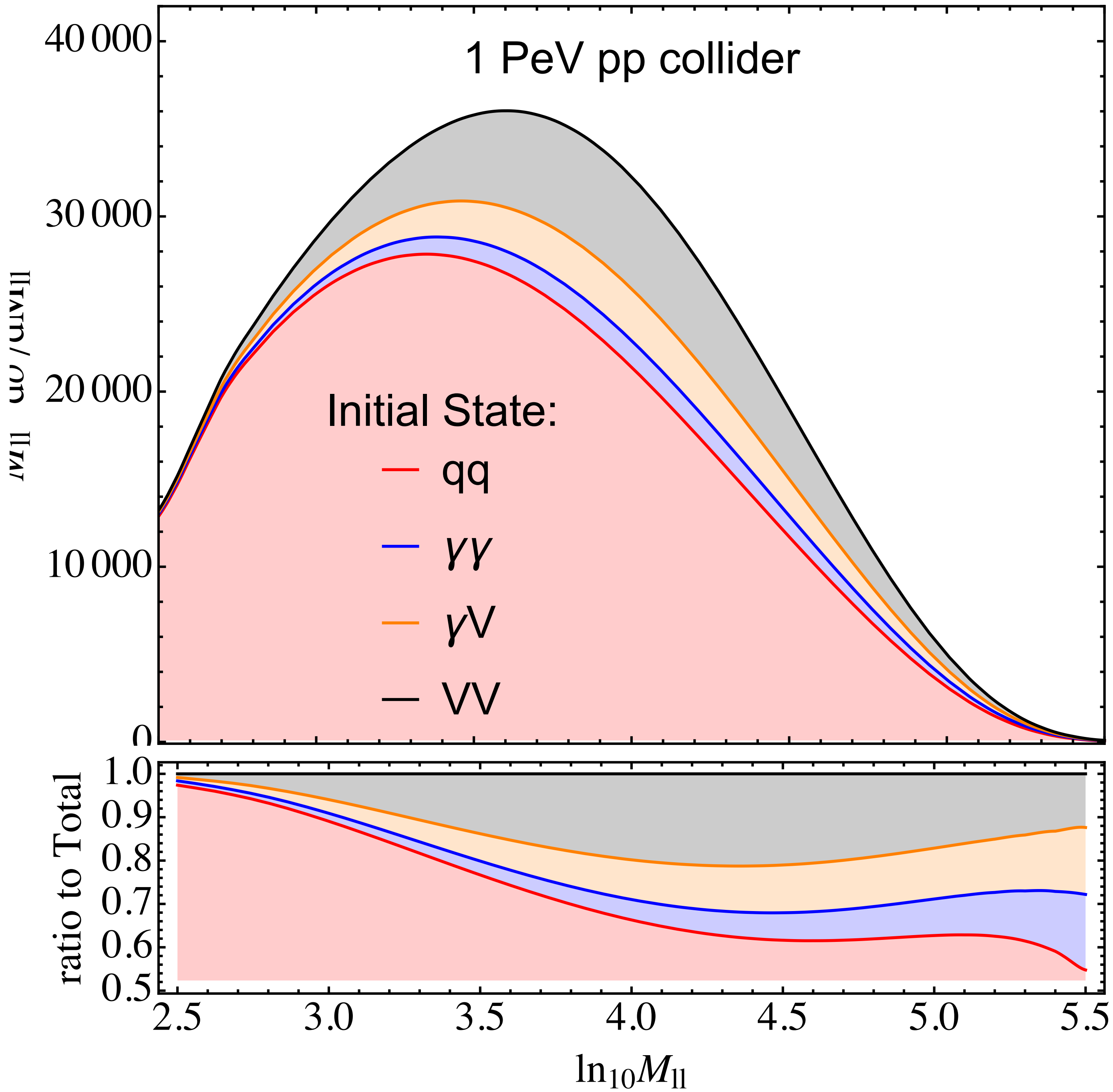
- QCED = SU(3)xU(1)$_{em}$

Lepton Pair Production



- SM = SU(3)xSU(2)$_L$xU(1)$_Y$

Lepton Pair Production



- SM = SU(3)xSU(2)$_L$xU(1)$_Y$

Preliminary Results

- Large HO contributions to VBF (note scales)