

MODIFIED GRAVITY: THEORY AND CONSTRAINTS

TESSA BAKER, UNIVERSITY OF OXFORD

OUTLINE



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- The landscape of modified gravity.



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- The landscape of modified gravity.
- Parameterised frameworks for testing GR.



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- The landscape of modified gravity.
- Parameterised frameworks for testing GR.
- Current and future constraints.





1. THE GRAVITY LANDSCAPE

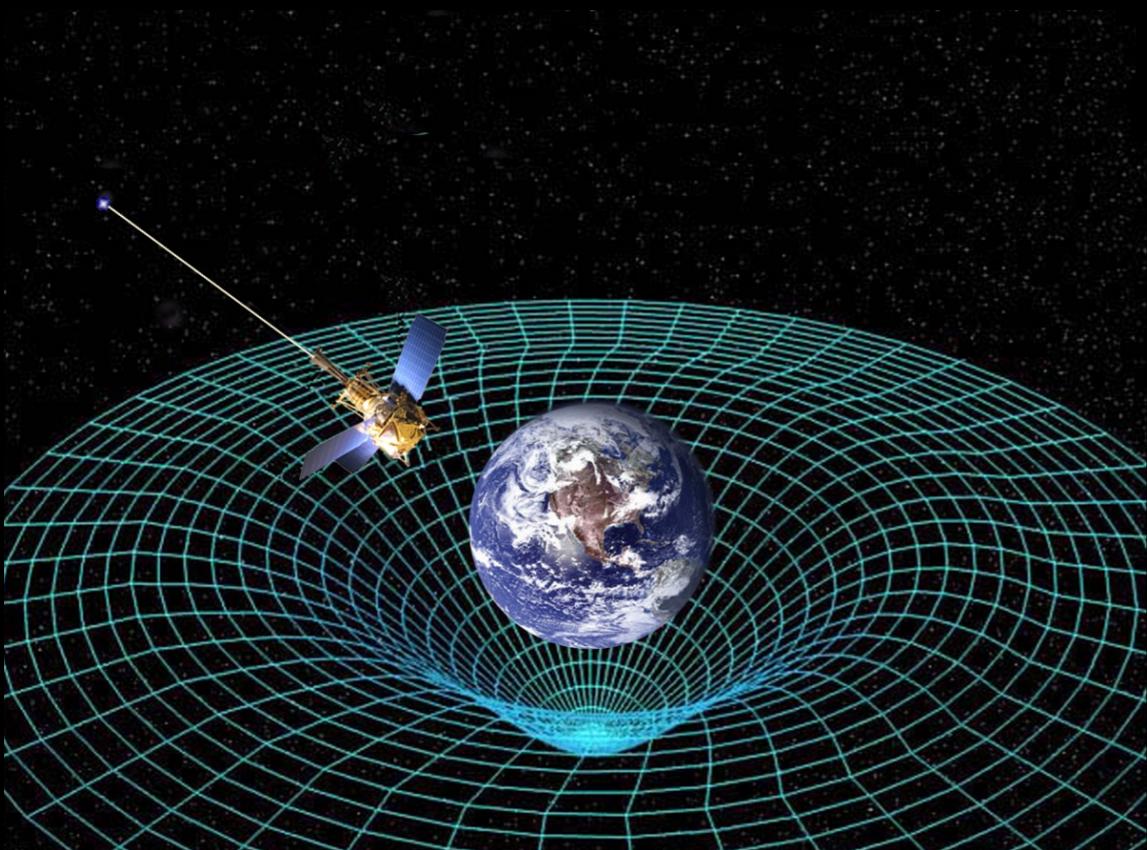
WHY TEST GRAVITY?

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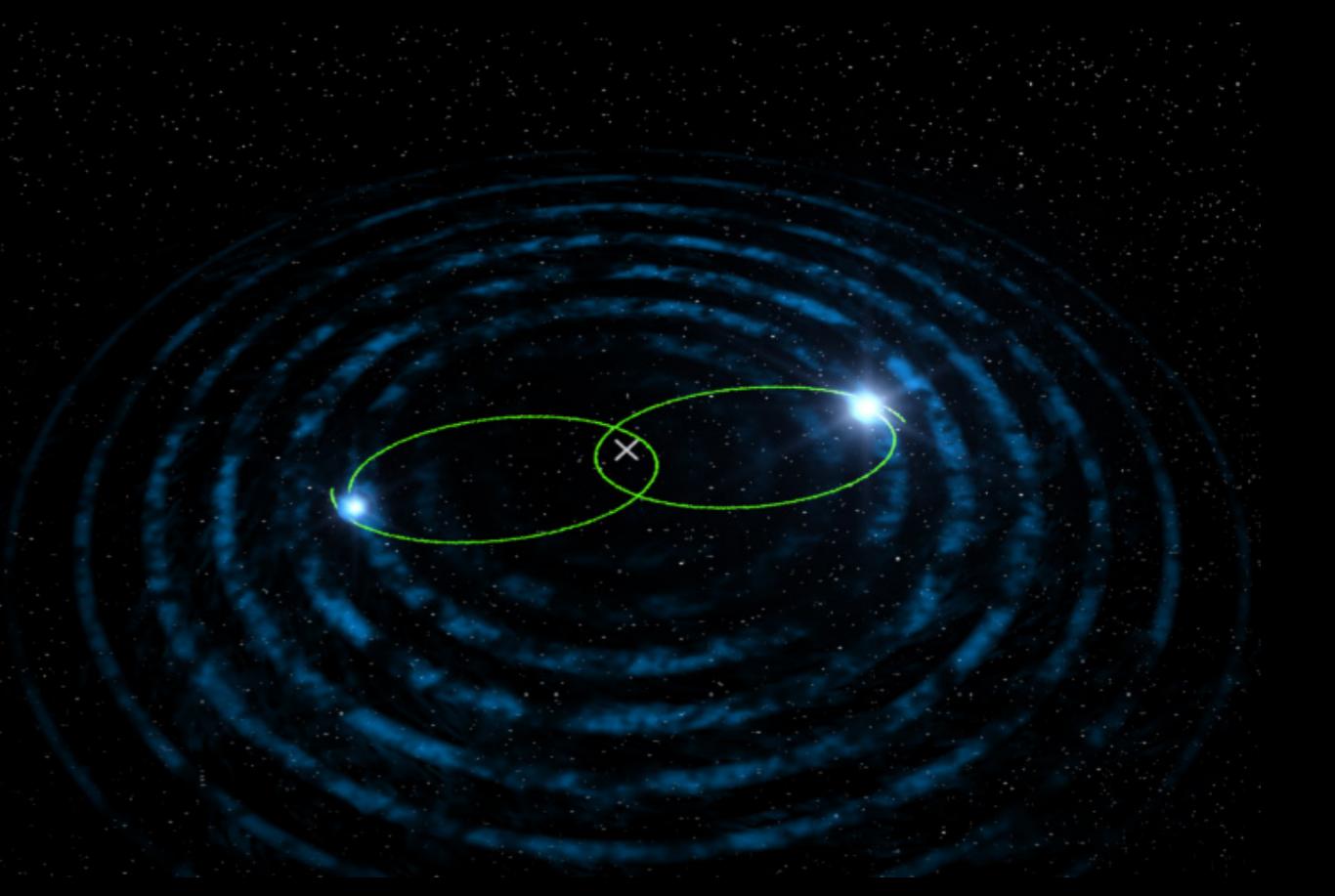
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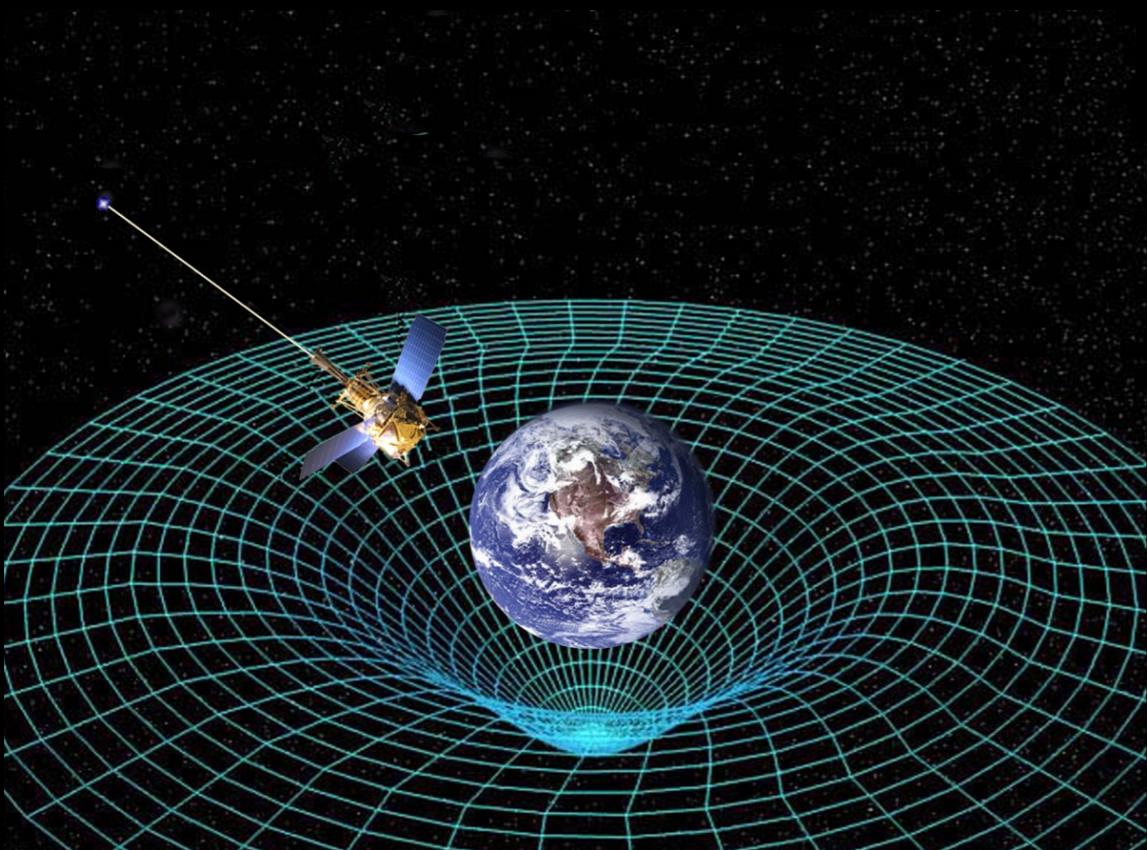


Images: NASA, Norbert Bartel.

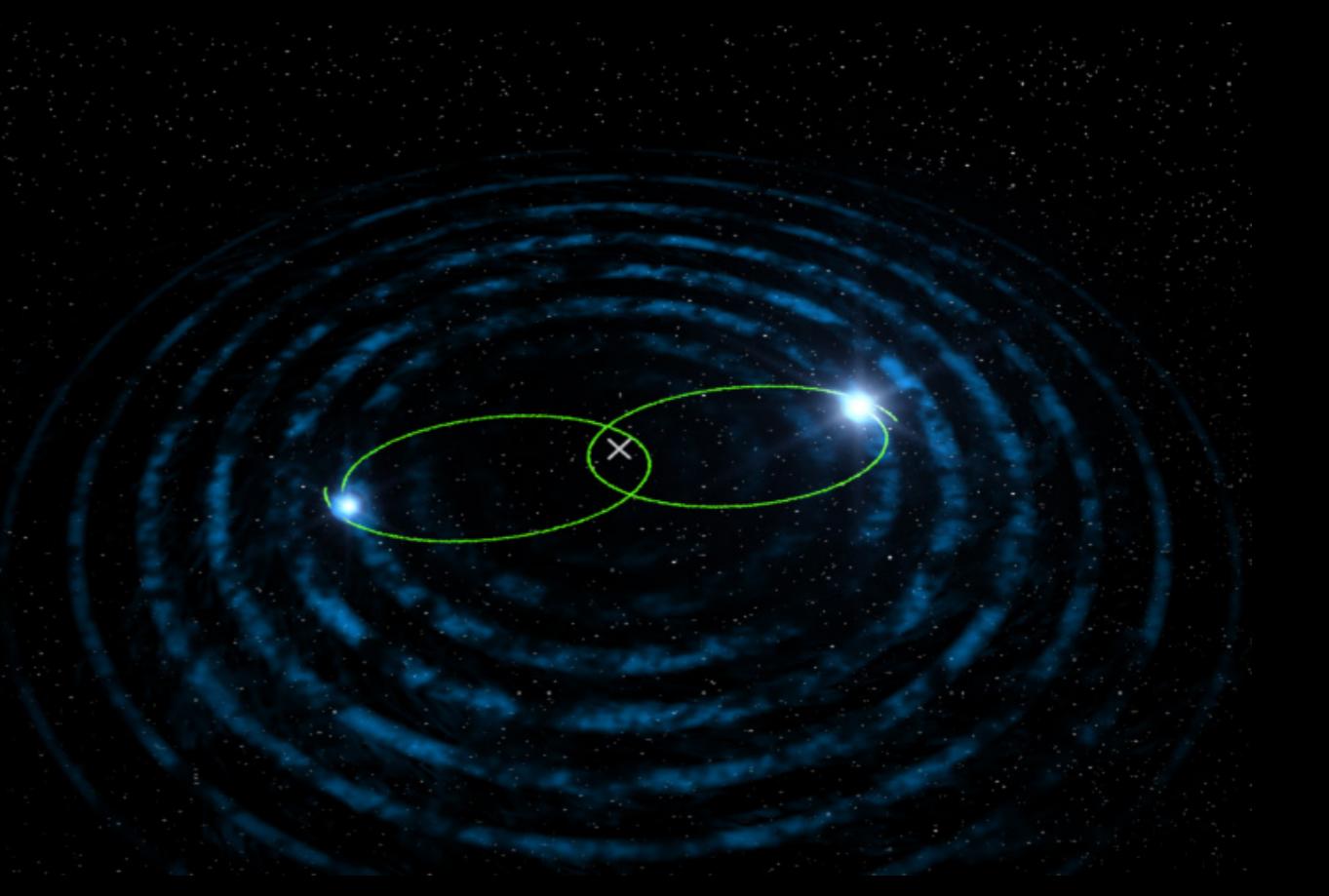


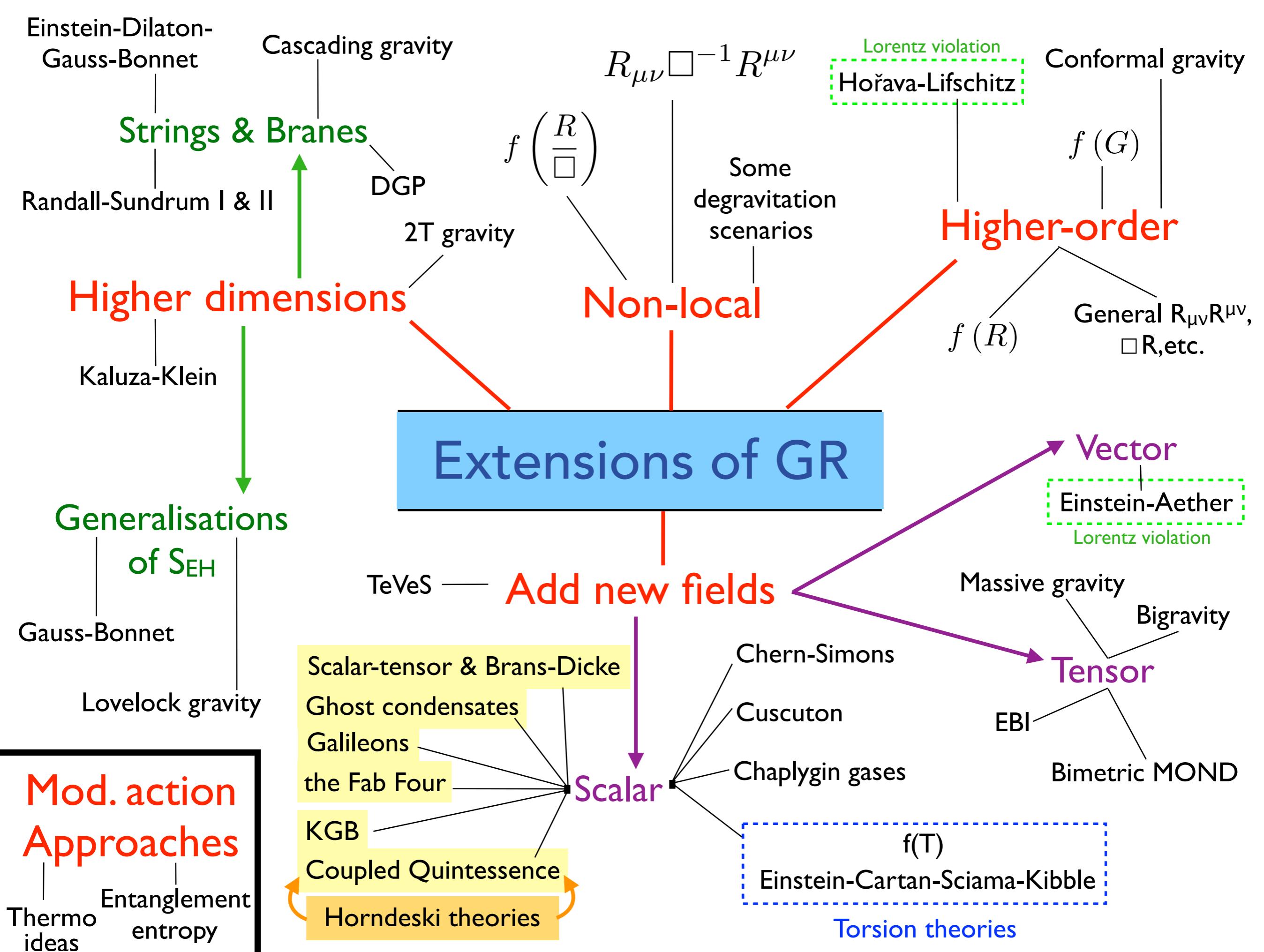
WHY TEST GRAVITY?

- Cosmic acceleration, i.e. dark energy.
- Cosmological scales \gg scales of precision tests.
- Fifth forces on astrophysical scales are poorly constrained.



Images: NASA, Norbert Bartel.

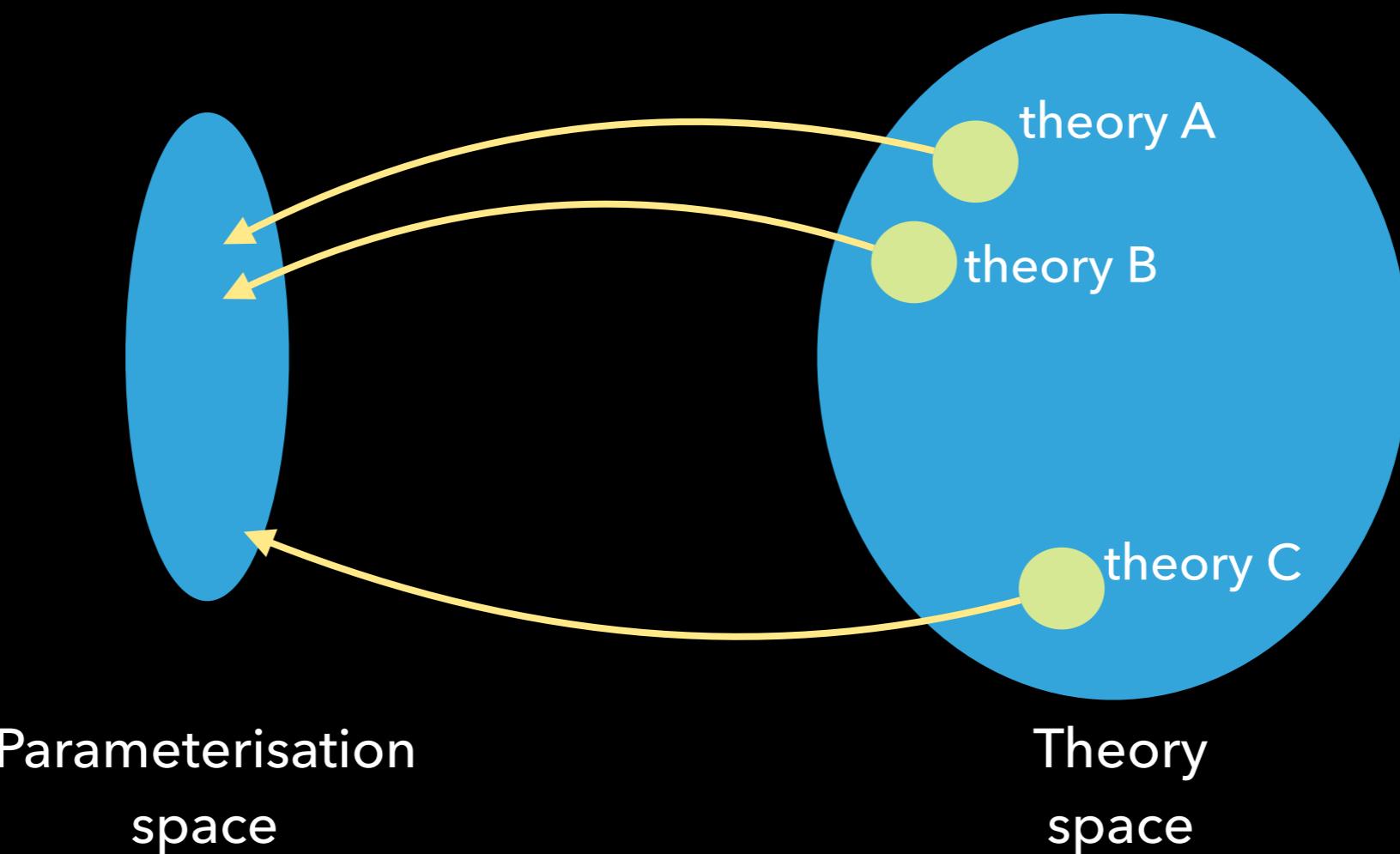






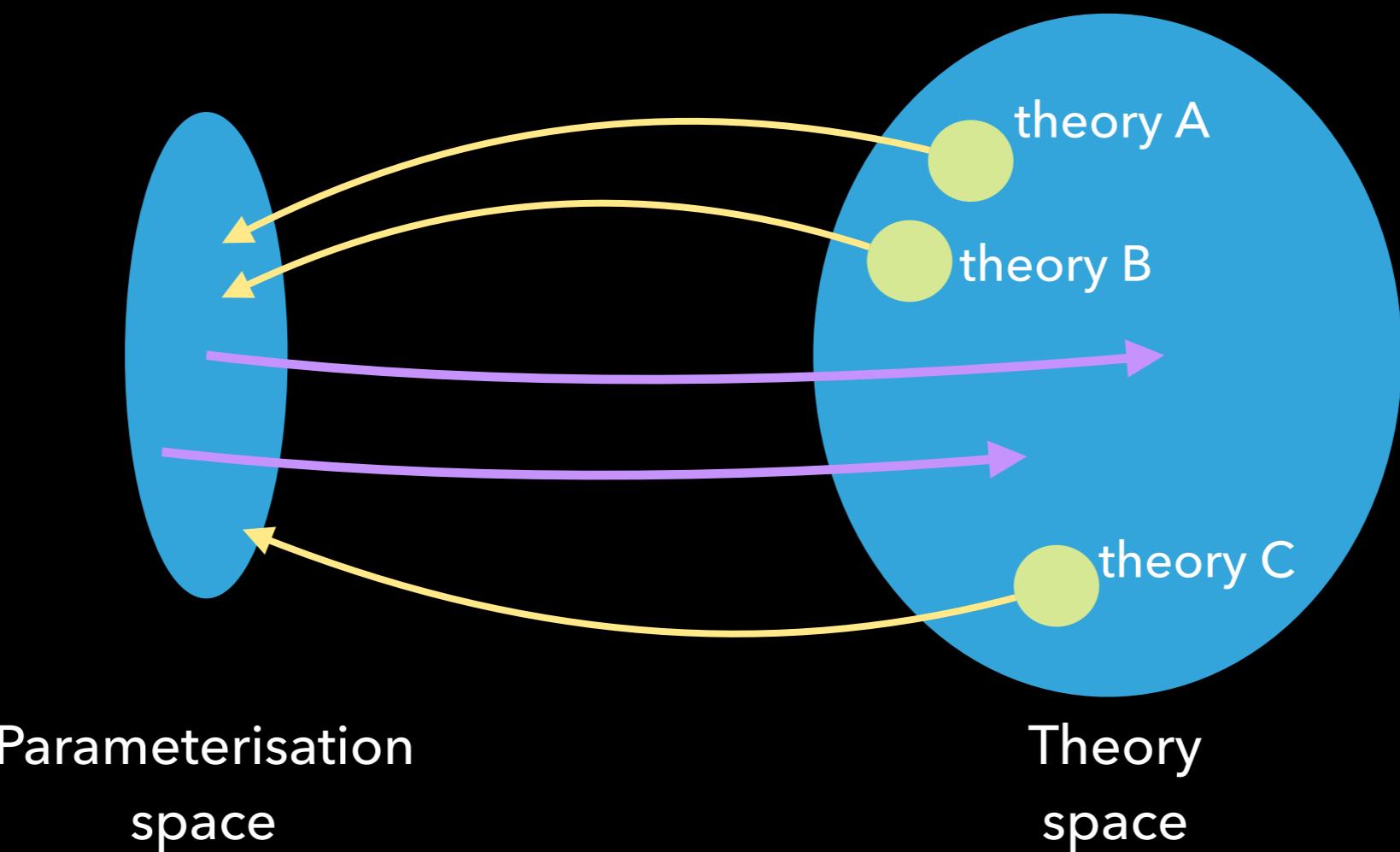
2. PARAMETERISED METHODS FOR TESTING GR

PARAMETERISING GRAVITY



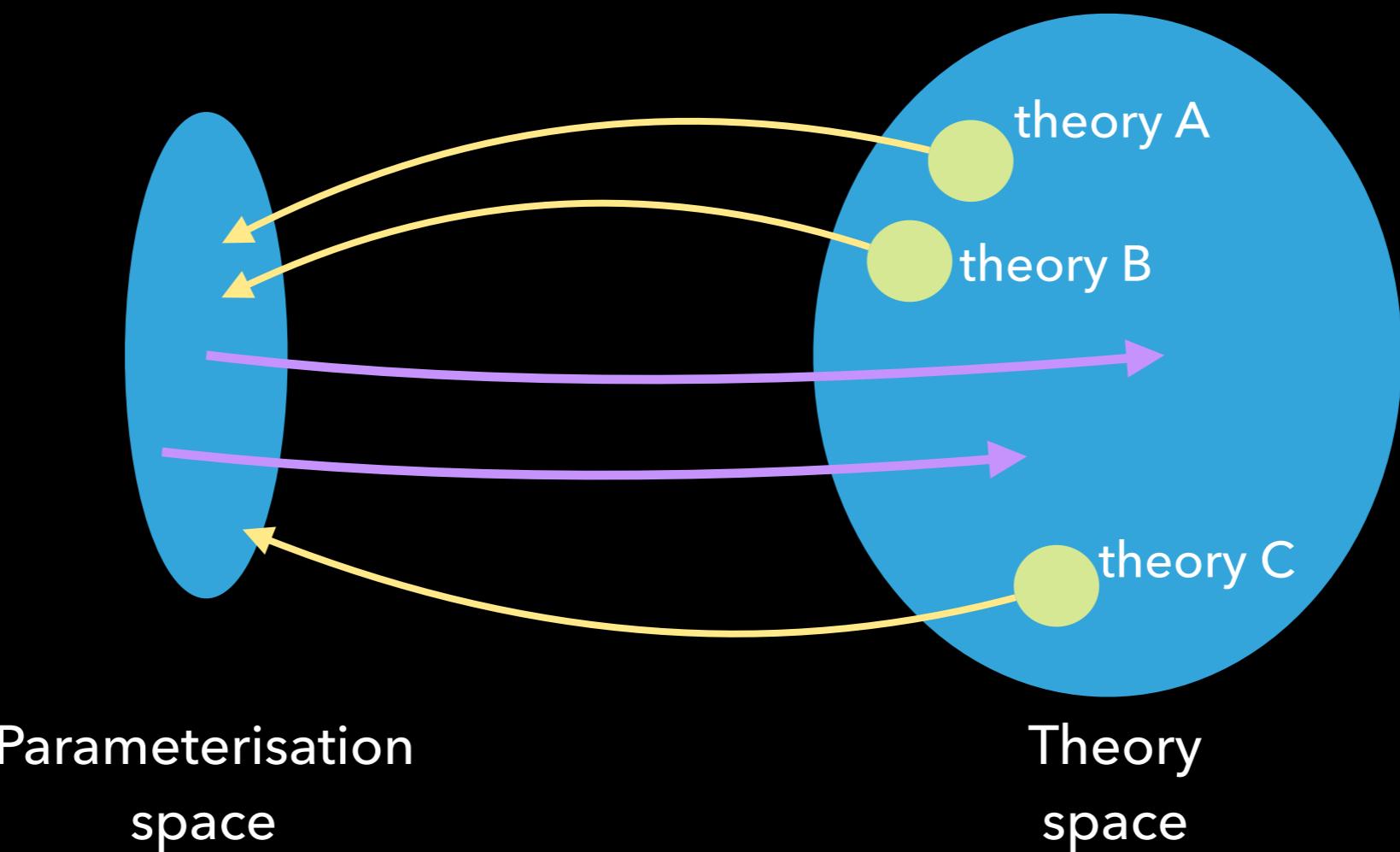
Gleyzes, Langlois & Vernizzi (2014)
Lagos, Baker ++ (2016)

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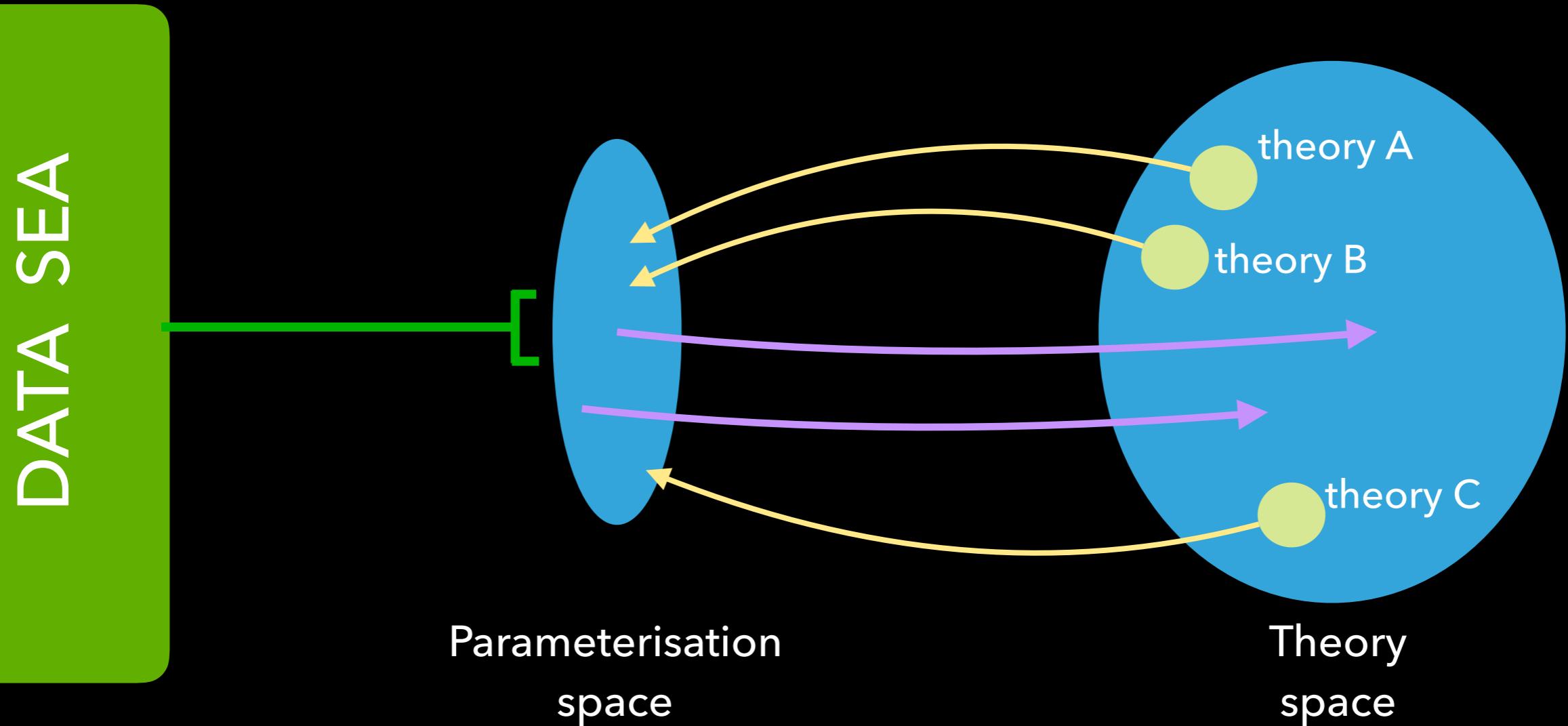
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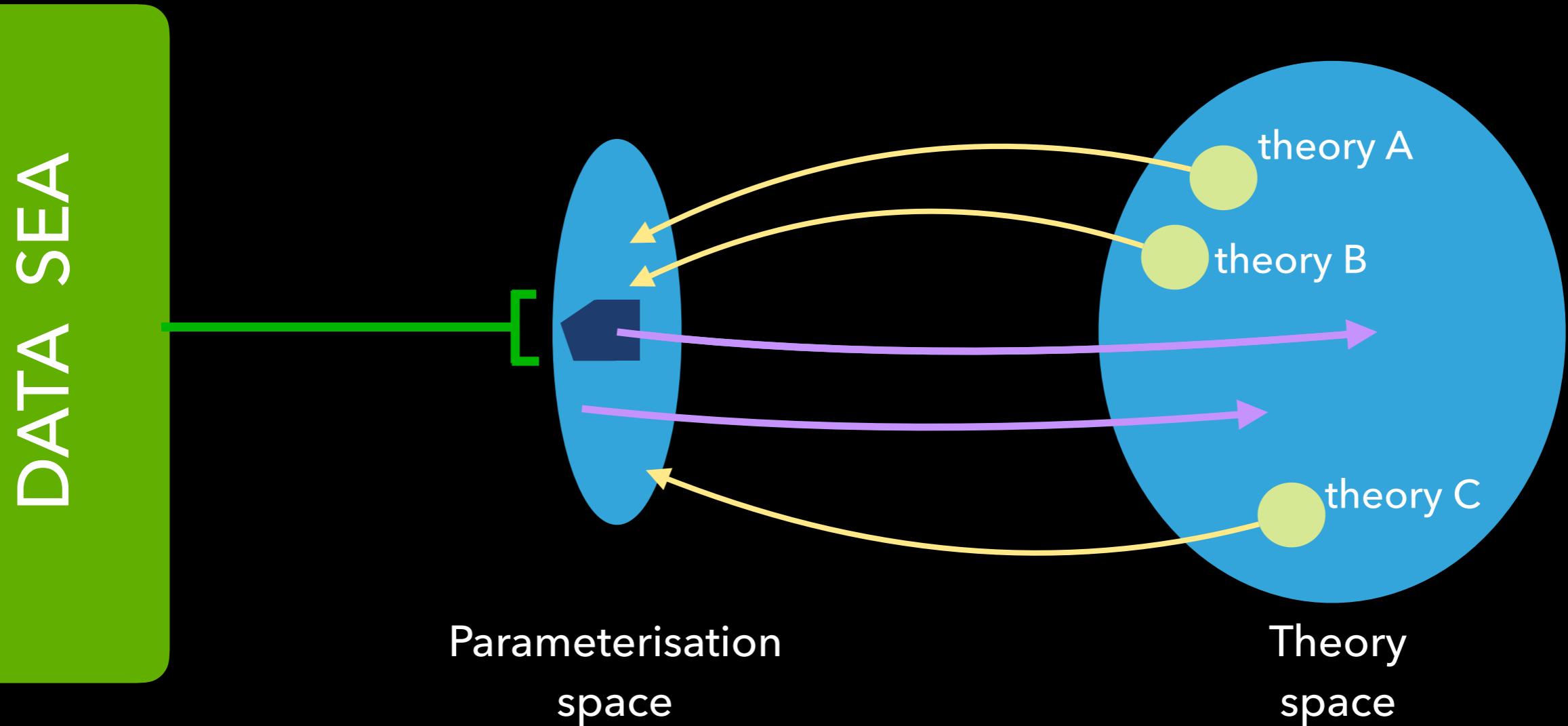
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EFT-STYLE APPROACHES

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$$\vec{\Theta} = \left(g_{\mu\nu}, \phi, \vec{A}, q_{\mu\nu} \right)$$

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The equation shows a vector-like field $\vec{\Theta}$ composed of four components: $g_{\mu\nu}$, ϕ , \vec{A} , and $q_{\mu\nu}$. A blue arrow points from the first component, $g_{\mu\nu}$, to the label 'metric'. A blue bracket under the last three components, ϕ , \vec{A} , and $q_{\mu\nu}$, is labeled 'new fields'.

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2. Taylor expand the gravitational Lagrangian:

$$L \simeq \bar{L} + L_{\Theta_a} \delta\Theta_a + \frac{1}{2} L_{\Theta_a \Theta_b} \delta\Theta_a \delta\Theta_b + \dots$$

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 $\frac{\partial L}{\partial \Theta_a}$

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↓ [Gives us linearised
 $\frac{\partial L}{\partial \Theta_a}$ grav. field equations.]

EFT-STYLE APPROACHES

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Gives us linearised
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Symmetries (e.g. linear diff invariance) fix most of the $L_{\Theta_a \Theta_b}$ coefficients.

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Symmetries (e.g. linear diff invariance) fix most of the $L_{\Theta_a \Theta_b}$ coefficients.

Those remaining are the true parameters of the EFT $\Rightarrow \alpha_i(t)$

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$\alpha_H(t)$: disformal symmetries of the metric.

THE ALPHA PARAMETERS

Model Class		α_K	α_B	α_M	α_T
ΛCDM		0	0	0	0
quintessence	[1, 2]	$(1 - \Omega_m)(1 + w_X)$	0	0	0
k-essence/perfect fluid	[45, 46]	$\frac{(1-\Omega_m)(1+w_X)}{c_s^2}$	0	0	0
kinetic gravity braiding	[47–49]	$\frac{m^2(n_m+\kappa_\phi)}{H^2 M_{Pl}^2}$	$\frac{m\kappa}{H M_{Pl}^2}$	0	0
galileon cosmology	[57]	$-\frac{3}{2}\alpha_M^3 H^2 r_c^2 e^{2\phi/M}$	$\frac{\alpha_K}{6} - \alpha_M$	$\frac{-2\dot{\phi}}{HM}$	0
BDK	[26]	$\frac{\dot{\phi}^2 K_{,\dot{\phi}\dot{\phi}} e^{-\kappa}}{H^2 M^2}$	$-\alpha_M$	$\frac{\dot{\kappa}}{H}$	0
metric $f(R)$	[3, 72]	0	$-\alpha_M$	$\frac{B\dot{H}}{H^2}$	0
MSG/Palatini $f(R)$	[73, 74]	$-\frac{3}{2}\alpha_M^2$	$-\alpha_M$	$\frac{2\dot{\phi}}{H}$	0
f (Gauss-Bonnet)	[52, 75, 76]	0	$\frac{-2H\dot{\xi}}{M^2+H\dot{\xi}}$	$\frac{\dot{H}\dot{\xi}+H\ddot{\xi}}{H(M^2+H\dot{\xi})}$	$\frac{\ddot{\xi}-H\dot{\xi}}{M^2+H\dot{\xi}}$



3. DATA — CURRENT & FUTURE

THE ALPHA PARAMETERS

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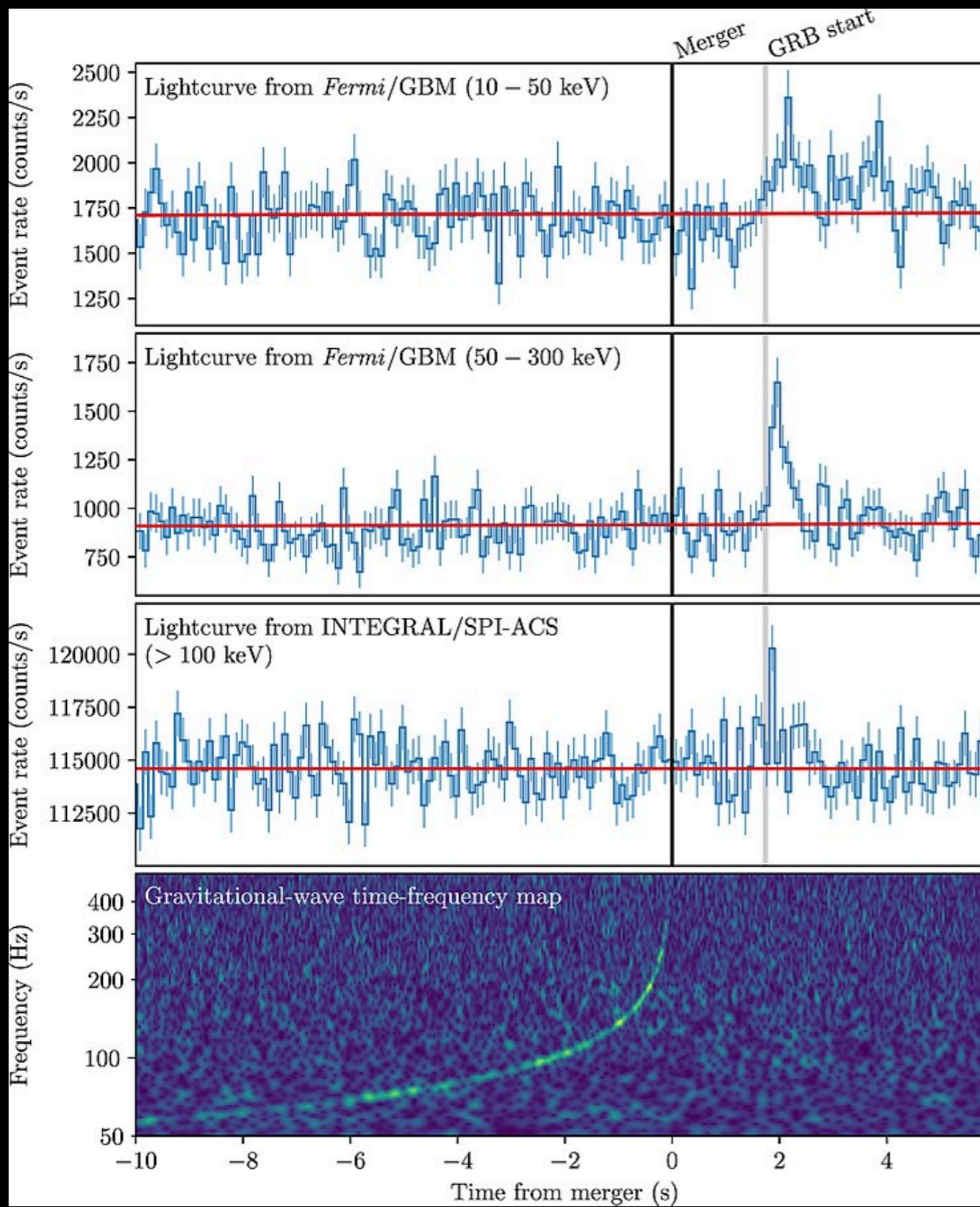
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GW170817 & GRB 170817A

Sakstein & Jain 1710.05893
 Ezquiaga & Zuma. 1710.05901

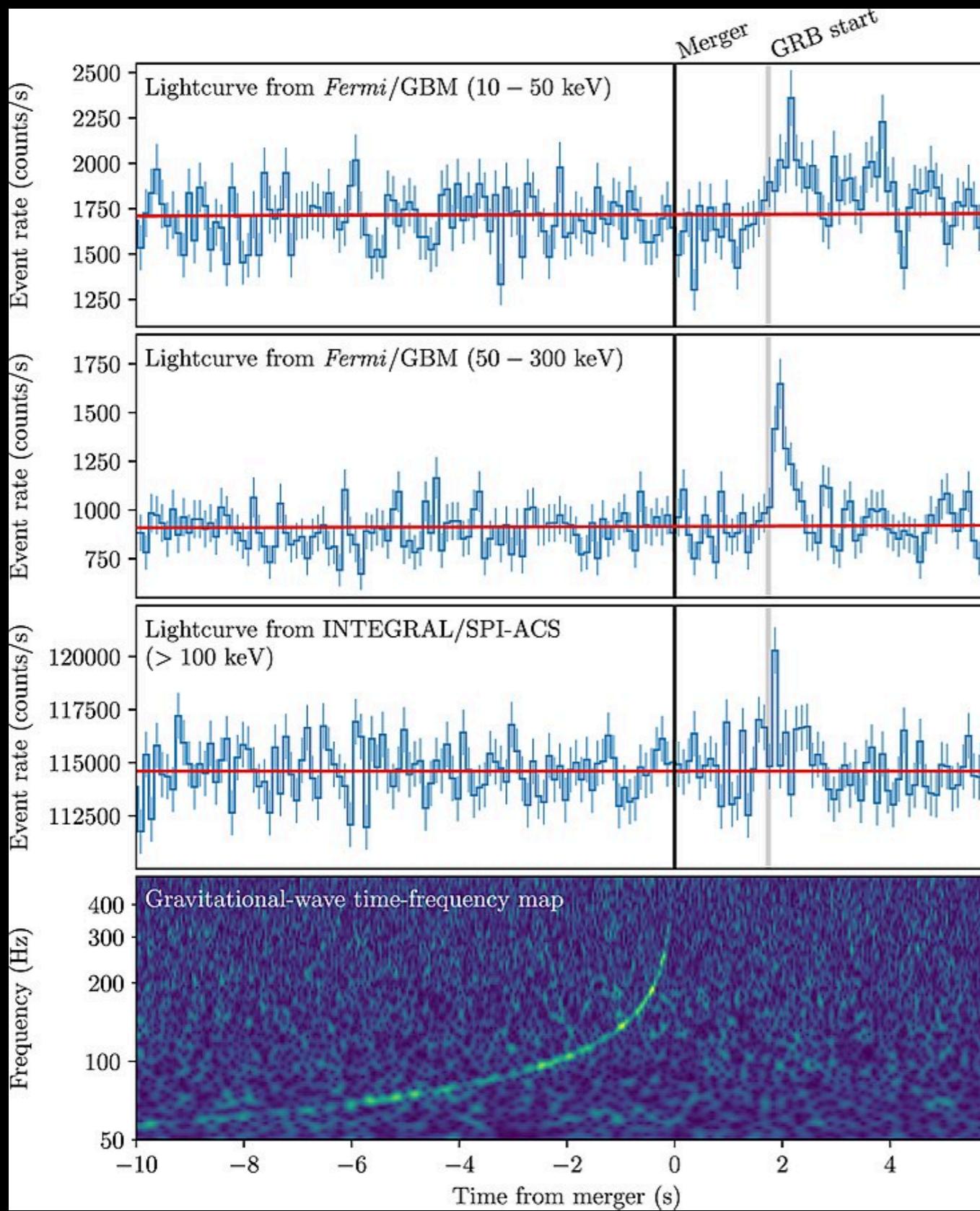
TB++ 1710.06394



$$c_T^2 = 1 + \alpha_T$$

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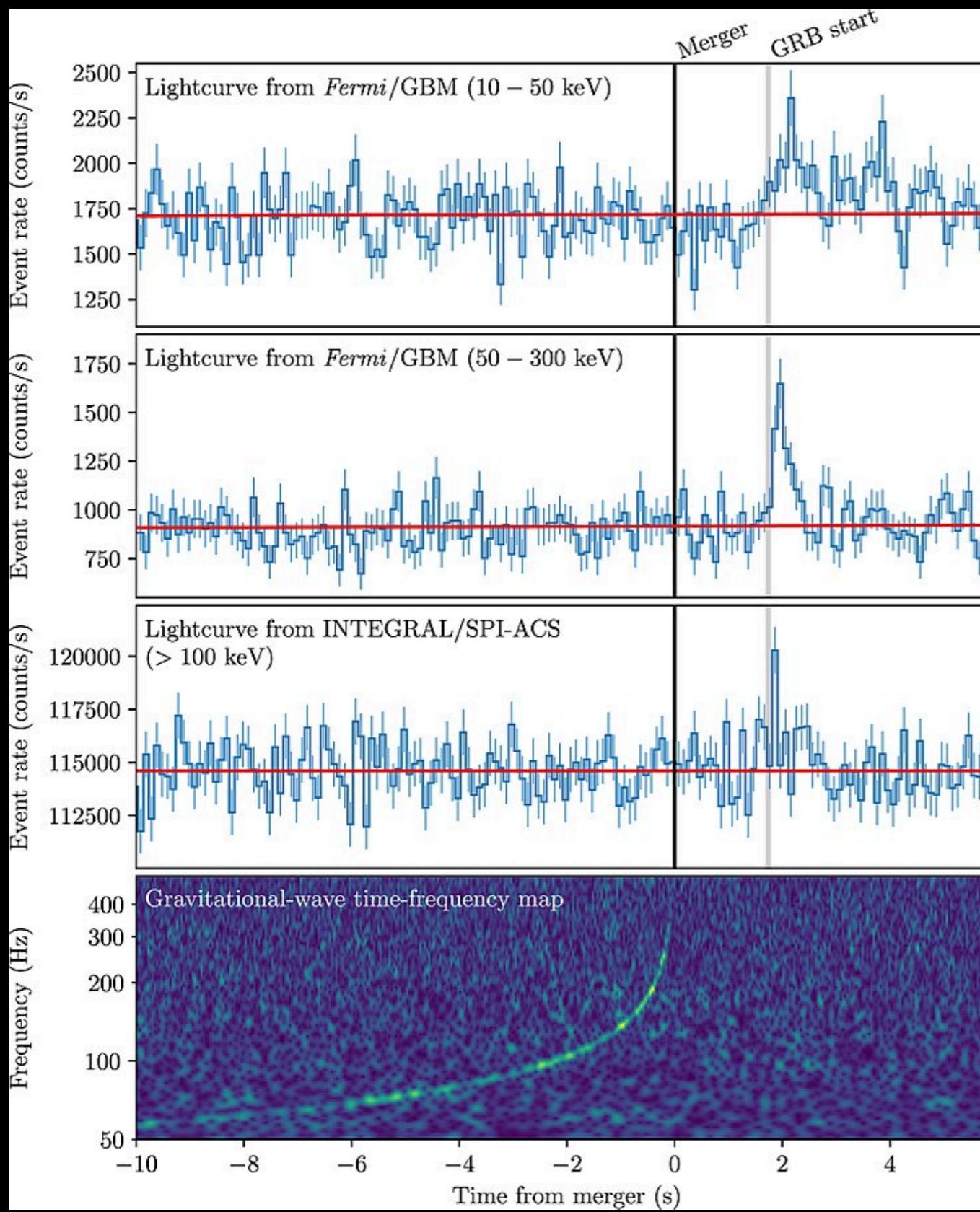


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$$\Delta t \simeq 1.7 \text{ s}$$

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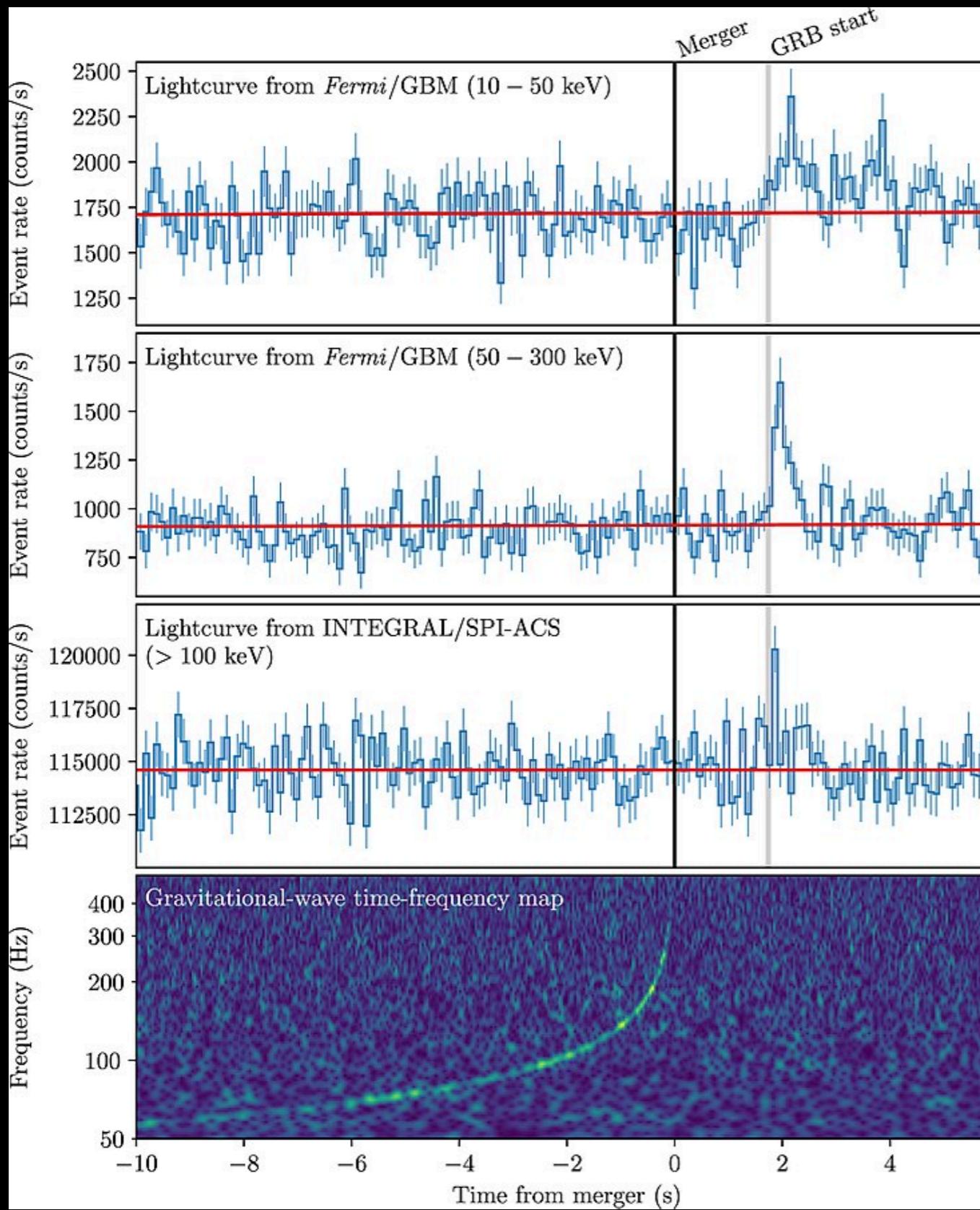
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$$\text{!! c. f. } |\alpha_M|, |\alpha_B| \lesssim \mathcal{O}(1)$$

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What does this mean for gravity theories?

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Scalar case clearest; full theory is **Horndeski gravity**.

$$S = \int d^4x \sqrt{-g} \sum_{i=2}^5 \mathcal{L}_i + S_M$$

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where $G_i = G_i(\phi, X)$ and $X = -\nabla_\nu \phi \nabla^\nu \phi / 2$.

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Linearised theory maps to alpha parameters:

$$\Rightarrow \alpha_T(t) = \frac{2X}{M_*^2} \left[2\textcolor{red}{G}_{4,X} - 2\textcolor{red}{G}_{5,\phi} - \left(\ddot{\phi} - \dot{\phi}H \right) \textcolor{red}{G}_{5,X} \right]$$

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Vector-tensor theories are a little bruised, but survive.

$$\mathcal{L}_4 \propto R$$

$$\mathcal{L}_5 \propto G_{\mu\nu} \nabla^\mu A^\nu$$

(+ Maxwell term etc.)

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(+ Maxwell term etc.)

For bimetric theories, get a bound on graviton mass:

$$m_g \lesssim 10^{-22} \text{ eV}$$

THE NEW THEORY LANDSCAPE

Quintessence

Horndeski

Quintic Galileons

K-essence

Generalised Proca

Quartic Galileons

Bigravity

Einstein-Aether

TeVeS

Massive Gravity

DHOST

SVT

Brans-Dicke

Horava-Lifschitz

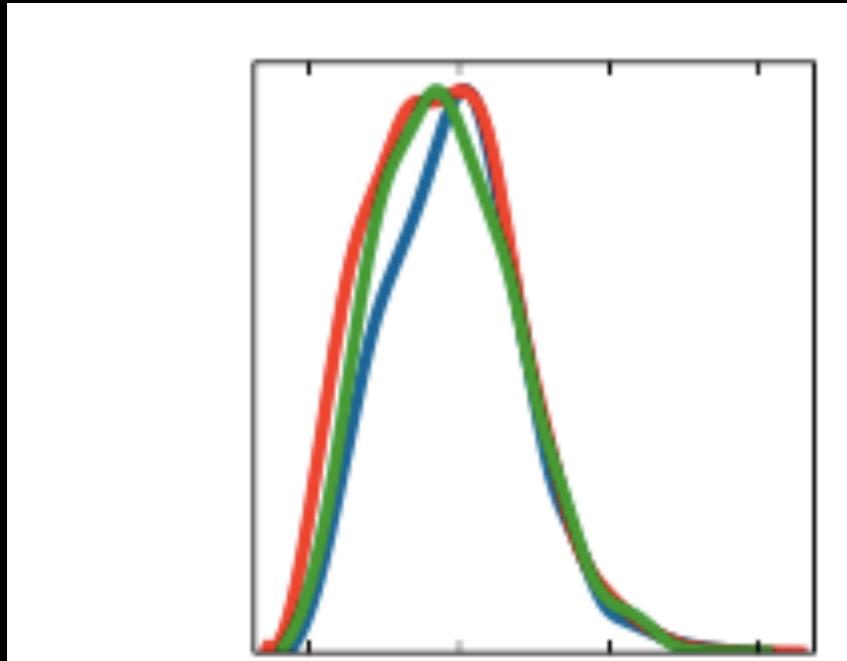
Fab Four

$f(R)$

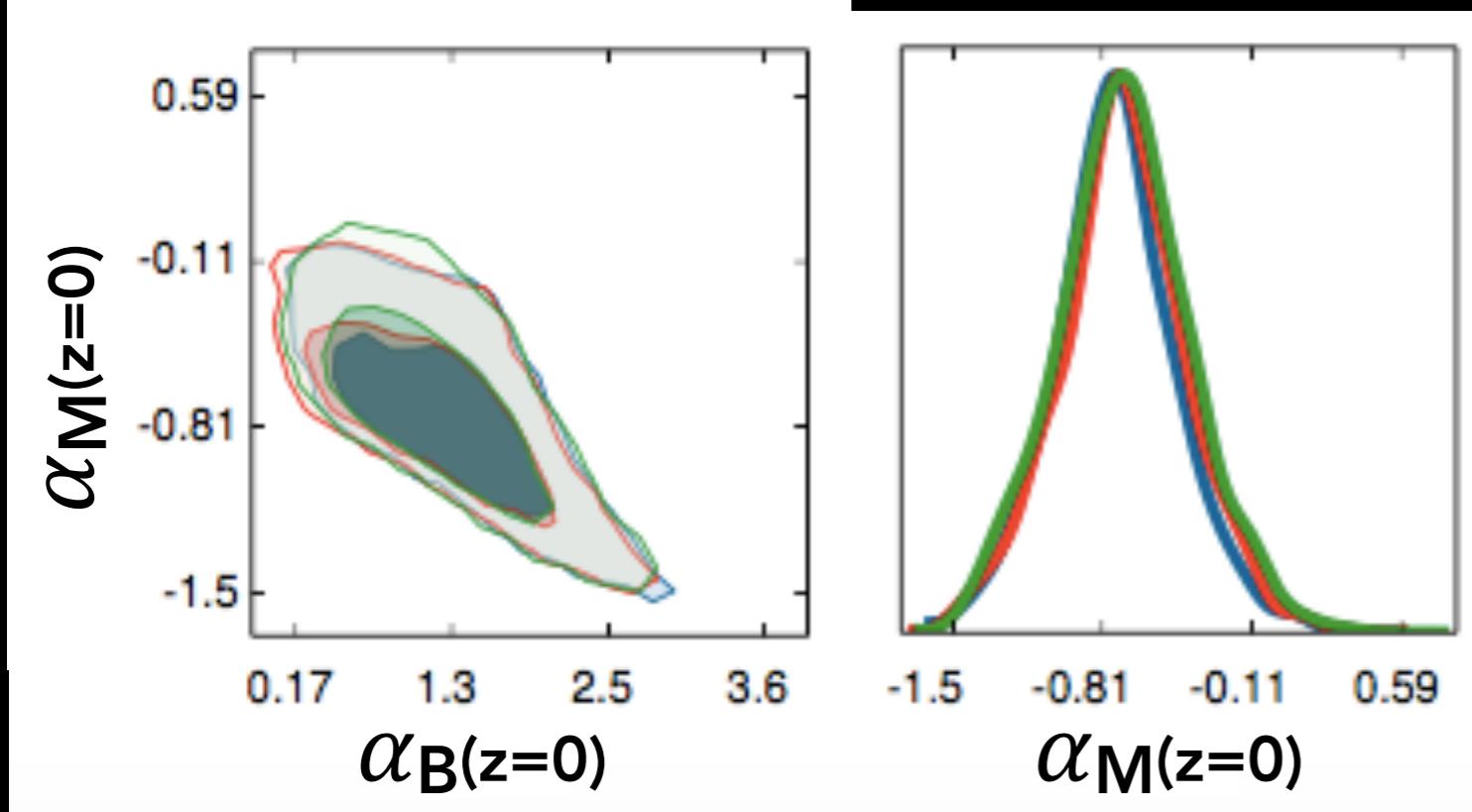
KGB

Cubic Galileon

THE CURRENT STATE OF PLAY

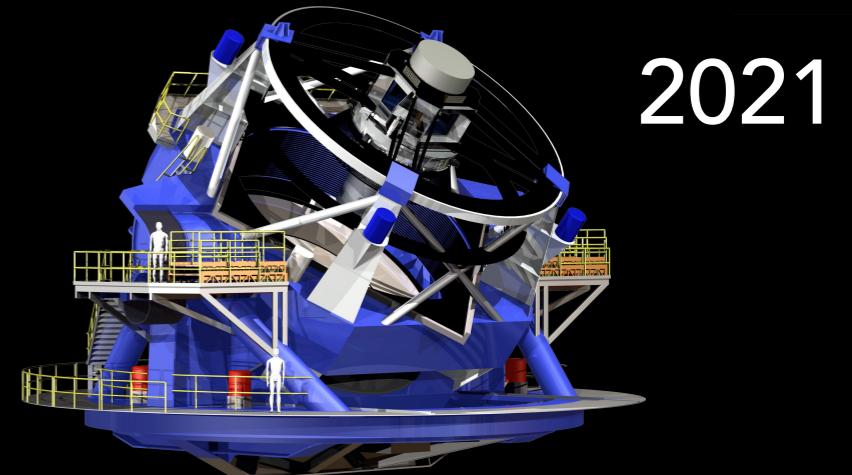
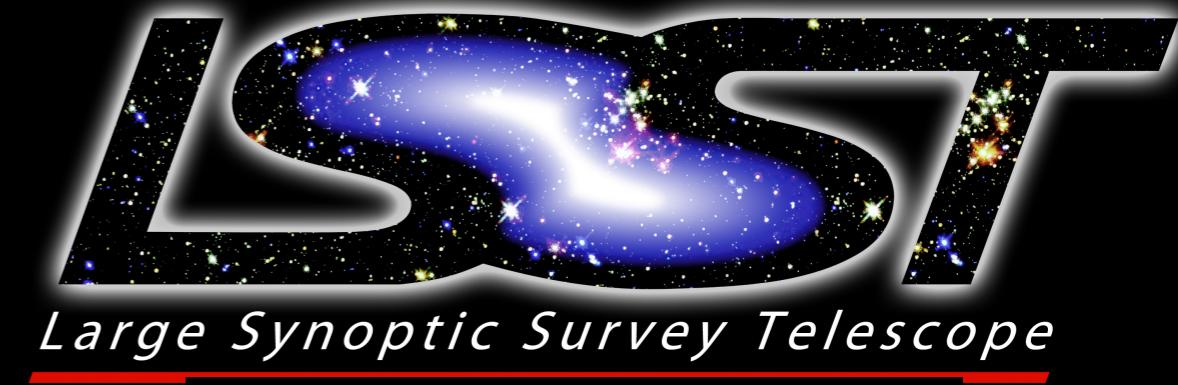
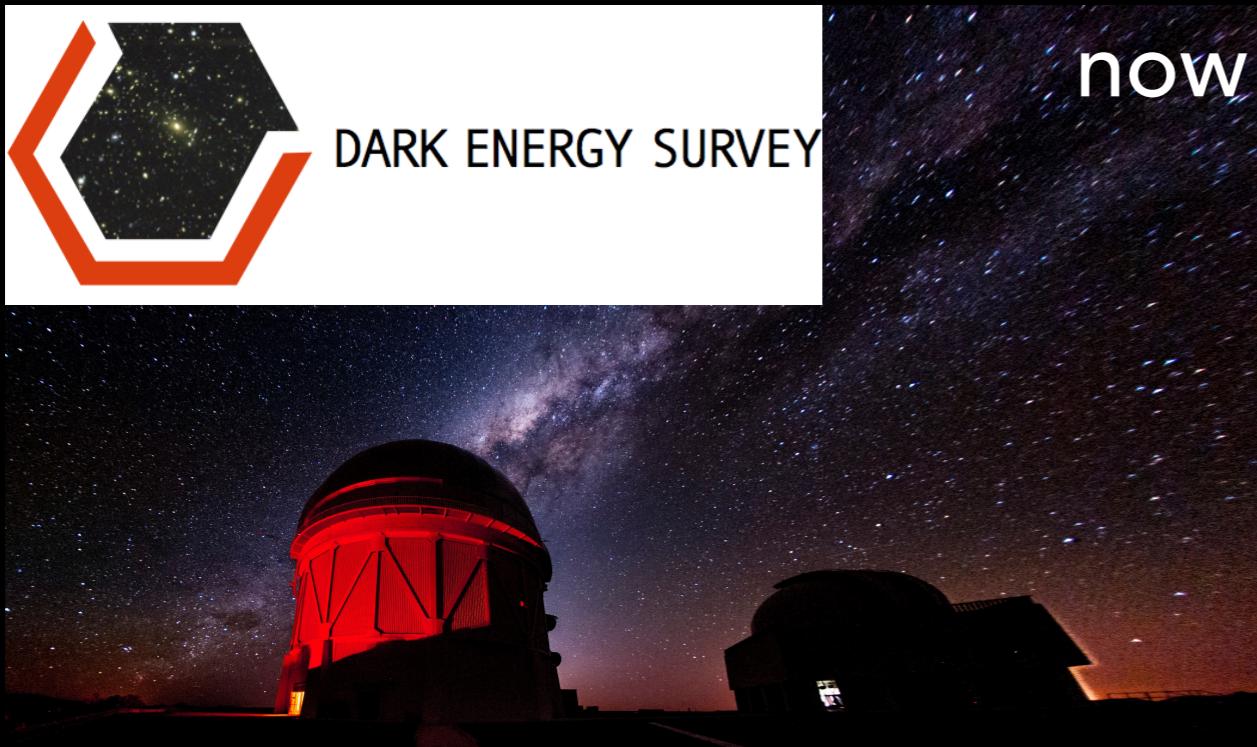


Planck CMB data + galaxy surveys:
BOSS, VIPERS, WiggleZ.



- $c_K = 0$
- $c_K = 1$
- $c_K = 10$

ONGOING & FUTURE EXPERIMENTS



CONCLUSIONS



References:

[1604.01396](#)

[1710.06394](#)

(+1 in prep.)

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CONCLUSIONS

- The landscape of extensions to GR.



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CONCLUSIONS

- The landscape of extensions to GR.
- EFT-style model-independent treatment.



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CONCLUSIONS

- The landscape of extensions to GR.
- EFT-style model-independent treatment.
- $\alpha_T(t)$ strongly constrained by GWs; others in progress.



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