# Latest results in meson mixing and lifetimes

Matthew Kirk

YTF 10

based on **1711.02100 (MK, Rauh, Lenz)**
**1712.06572 (Di Luzio, MK, Lenz)**

# Experimental status (~2017)

- $B_s$ Mixing
  - $\Delta M_s$ is extremely well measured (0.1% uncertainty)
  - $\Delta \Gamma_s$ known with sub 10% uncertainty
- B lifetime ratios
  - $\tau(B_s)/\tau(B_d)$ known with < 0.25% uncertainty
- D Mixing
  - First > 5 sigma measurement from LHCb in 2012
  - $O(10\%)$ accuracy
- D lifetime ratios
  - $\tau(D^+)/\tau(D^0)$ known with <1% uncertainty

# Lattice status (~2017)

- Lattice can determine non-perturbative parameters
- We are interested in overlap between meson/anti-meson states

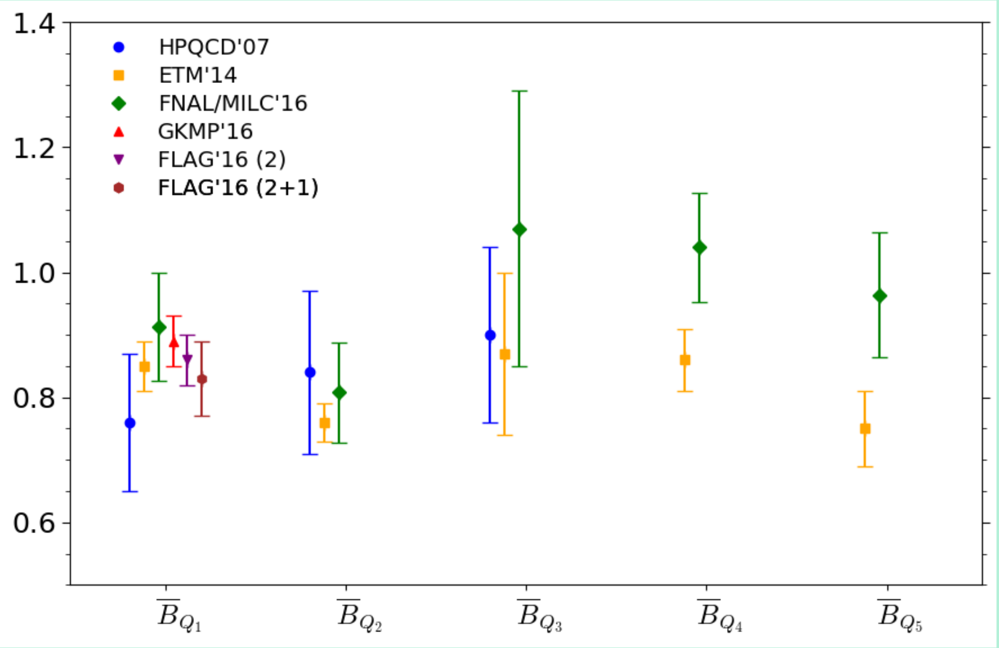
$$\langle \text{meson} | \text{four quark operator} | \text{anti-meson} \rangle \sim B$$

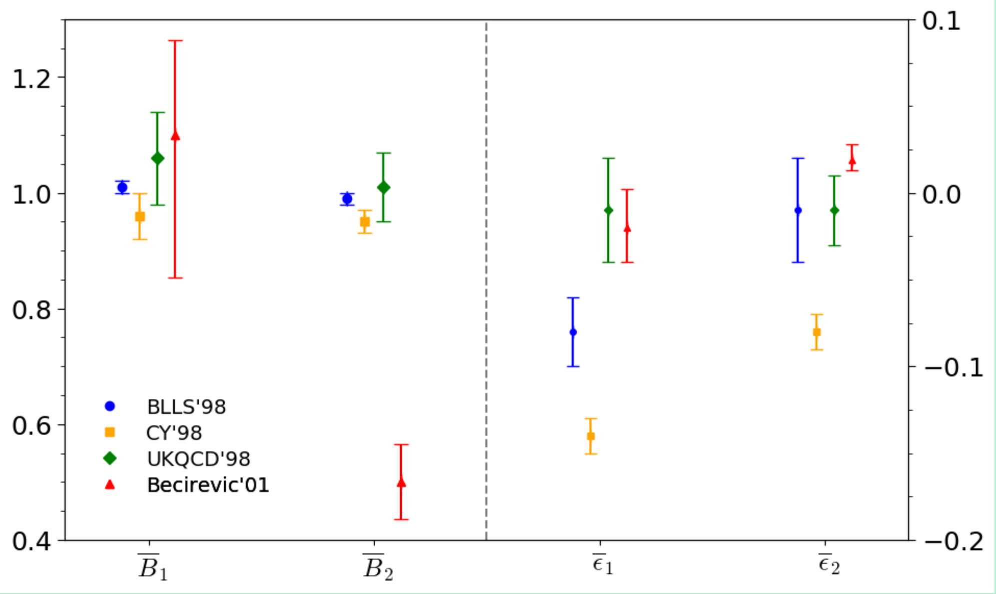
or

$$\langle \text{meson} | \text{four quark operator} | \text{meson} \rangle \sim B/\epsilon$$

# Lattice status (~2017)

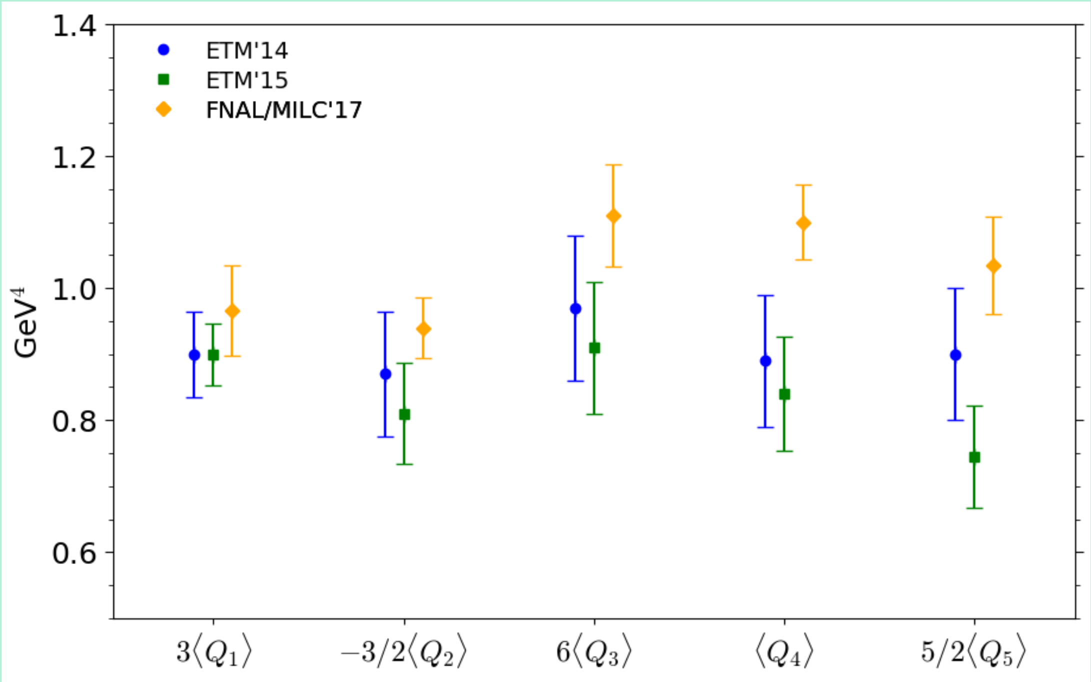
- $B_s$ Mixing
  - Selection of lattice results, all in agreement
- B Lifetimes
  - only old ('98 / '01) lattice results



# Lattice status (~2017)

- D mixing
  - a handful of lattice results




- D lifetimes

# Theory status (~2015)

- B Mixing – $\Delta M_s = 18.3 \pm 2.7\,\text{ps}^{-1}$

  $\Delta \Gamma_s = 0.088 \pm 0.020\,\text{ps}^{-1}$

- B Lifetimes – $\tau(B_s)/\tau(B_d) = 1.0005 \pm 0.0011$

  – $\tau(B^+)/\tau(B_d) = 1.04^{+0.05}_{-0.02}$

- D mixing – 

- D lifetimes – $\tau(D^+)/\tau(D^0) = 2.2 \pm 1.7$

# What has happened since?

- New lattice result from Fermilab-MILC included in FLAG average
  - $f_{B_s}\sqrt{B}$ : $270 \pm 16$ MeV → $274 \pm 8$ MeV
- HQET sum rule calculation
  - Independent determination of non-perturbative matrix elements for all dimension-6 operators
- $V_{cb}$ discrepancy between inclusive / exclusive is perhaps starting to be resolved?
  - (**1703.08170**, **1707.09509**, **1708.07134**, talk by **Stefan Schacht** at **LHCb Implications**)

# What has happened since?

- New lattice result from Fermilab-MILC included in FLAG average
  - $f_{B_s}\sqrt{B}$ : $270 \pm 16$ MeV → $274 \pm 8$ MeV
- HQET sum rule calculation
  - Independent determination of non-perturbative matrix elements for all dimension-6 operators
- $V_{cb}$ discrepancy between inclusive / exclusive is perhaps starting to be resolved?
  - (1703.08170, 1707.09509, 1708.07134, talk by Stefan Schacht at LHCb Implications)

# HQET sum rules

- Made possible by 3-loop calculations done in 2008 by Grozin, Lee (0812.4522)

# HQET sum rules

- Made possible by 3-loop calculations done in 2008 by Grozin, Lee (0812.4522)

$$\begin{aligned}
M_3(\omega_1,\omega_2) = {} & (-2\omega_1)^{3d/2-5}(-2\omega_2)^{3d/2-5}\Gamma^3(d/2-1) \\
& \times \left[ \frac{\Gamma\left(\frac{3}{2}d-4\right)\Gamma^2\left(5-\frac{3}{2}d\right)\Gamma\left(2-\frac{d}{2}\right)}{(d-3)\Gamma(d-2)} \right. \\
& + 2\frac{\Gamma(8-3d)}{d-3}x^{4-3d/2}\,{}_3F_2\left( \left. \begin{matrix} 1, d-2, \frac{3}{2}d-4 \\ \frac{3}{2}d-3, 3d-8 \end{matrix} \right| \frac{1}{x} \right) \\
& + \frac{4\pi\Gamma(6-2d)x^{3d/2-5}}{(3d-10)\Gamma(d-2)\sin(3\pi d)}\,{}_2F_1\left( \left. \begin{matrix} 5-\frac{3}{2}d, 7-2d \\ 6-\frac{3}{2}d \end{matrix} \right| \frac{1}{x} \right) \\
& + 2\frac{\Gamma(8-3d)}{d-3}x^{3d/2-4}\,{}_3F_2\left( \left. \begin{matrix} 1, d-2, \frac{3}{2}d-4 \\ \frac{3}{2}d-3, 3d-8 \end{matrix} \right| x \right) \\
& \left. + \frac{4\pi\Gamma(6-2d)x^{5-3d/2}}{(3d-10)\Gamma(d-2)\sin(3\pi d)}\,{}_2F_1\left( \left. \begin{matrix} 5-\frac{3}{2}d, 7-2d \\ 6-\frac{3}{2}d \end{matrix} \right| x \right) \right].
\end{aligned}$$

# HQET sum rules

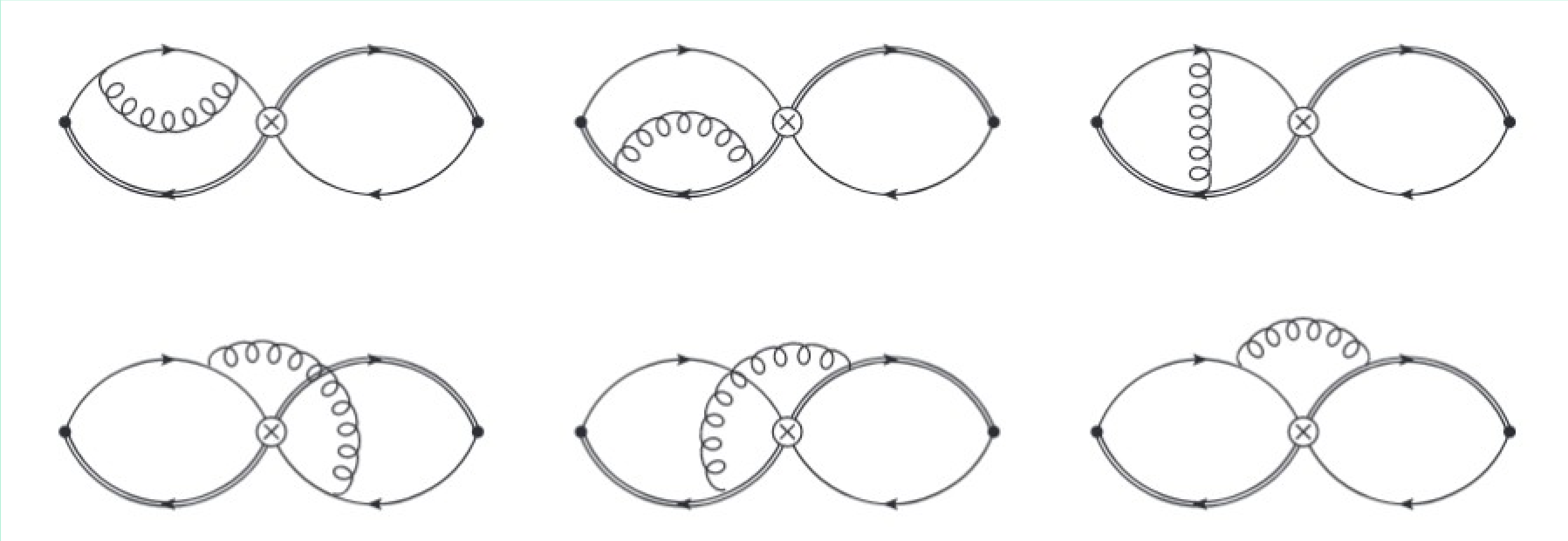
- Made possible by 3-loop calculations done in 2008 by Grozin, Lee (0812.4522)
- First steps made by Grozin, Klein, Mannel, Pivovarov in mid 2016 (1606.06054)
- Late last year, full set of dim-6 operators done by MK, Lenz, Rauh (1711.02100)

# HQET sum rules

- Do all dim 6 operators for mixing AND lifetimes
- How?
  - 3 loop diagrams (with 2 external momenta), reduced using FIRE to those known by Grozin, Lee
  - HQET running to scale $m_b$
  - HQET-QCD matching (1-loop) at scale $m_b$

# HQET sum rules

- Do all dim 6 operators for mixing AND lifetimes
- How?
  - 3 loop diagrams (with 2 external momenta), reduced using FIRE to those known by Grozin, Lee



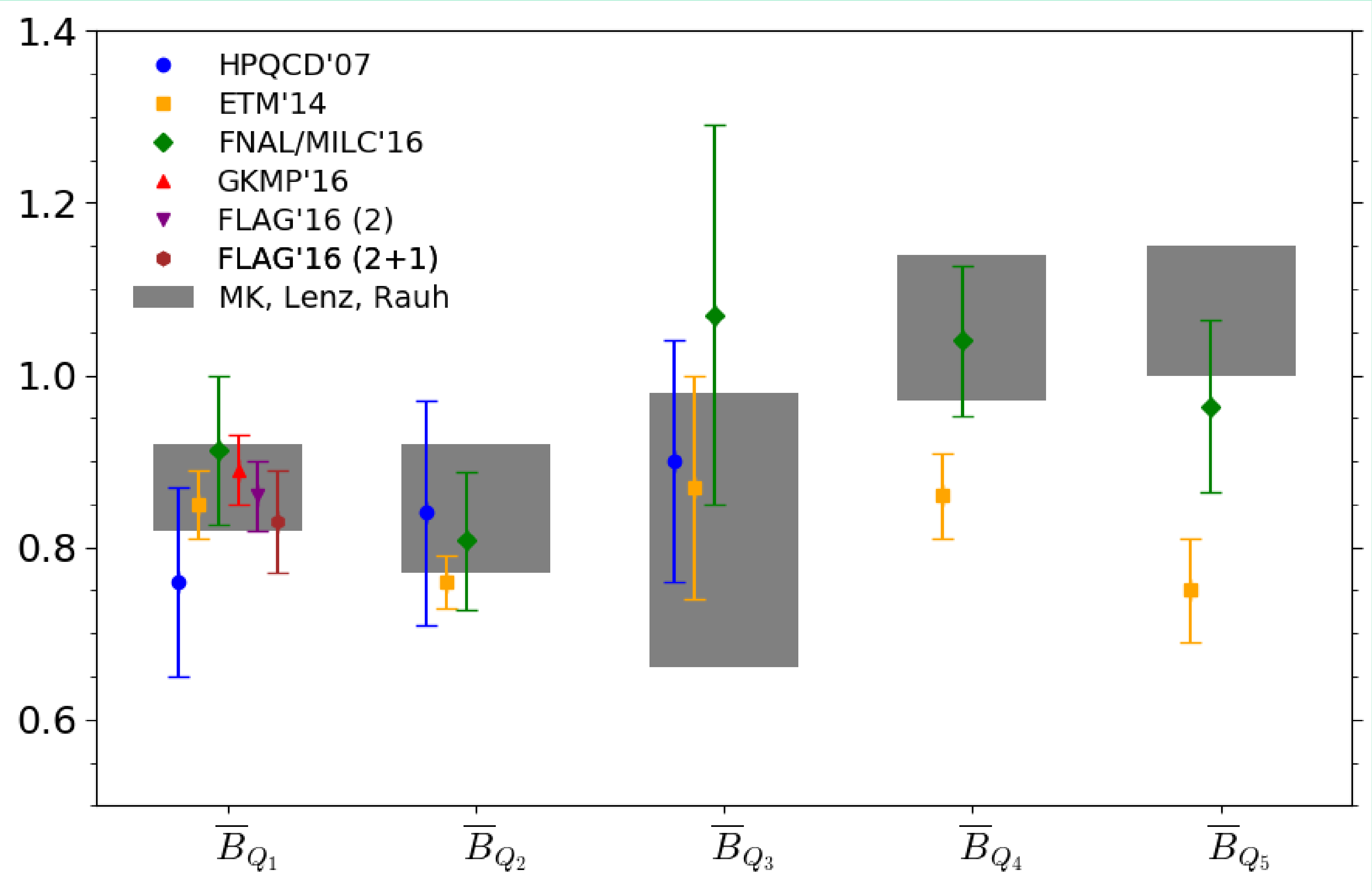
# HQET sum rules

- Do all dim 6 operators for mixing AND lifetimes
- How?
  - 3 loop diagrams (with 2 external momenta), reduced using FIRE to those known by Grozin, Lee
  - HQET running to scale $m_b$
  - HQET-QCD matching (1-loop) at scale $m_b$

# HQET sum rules – results

# HQET sum rules – results

B mixing



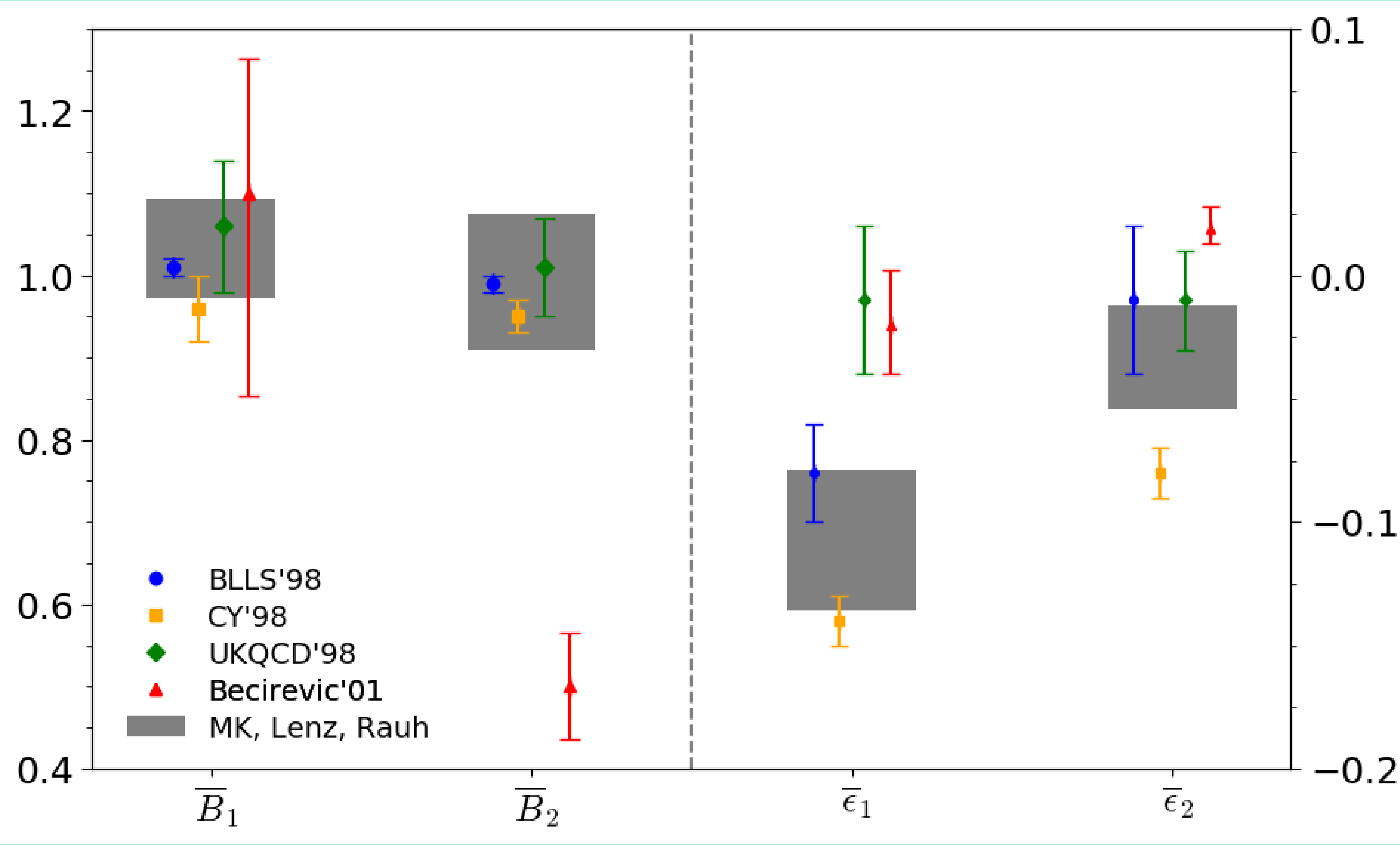
# Effect on observables

- $\Delta M_s = 18.1 \pm 1.9\,\mathrm{ps}^{-1}$
- $\Delta \Gamma_s = 0.079 \pm 0.023\,\mathrm{ps}^{-1}$
- $a_{sl}^s = 2.0 \pm 0.3 \times 10^{-5}$

- Gives errors that are comparable ( $\pm 15\,\%$ ) with lattice data → lattice not the only game in town

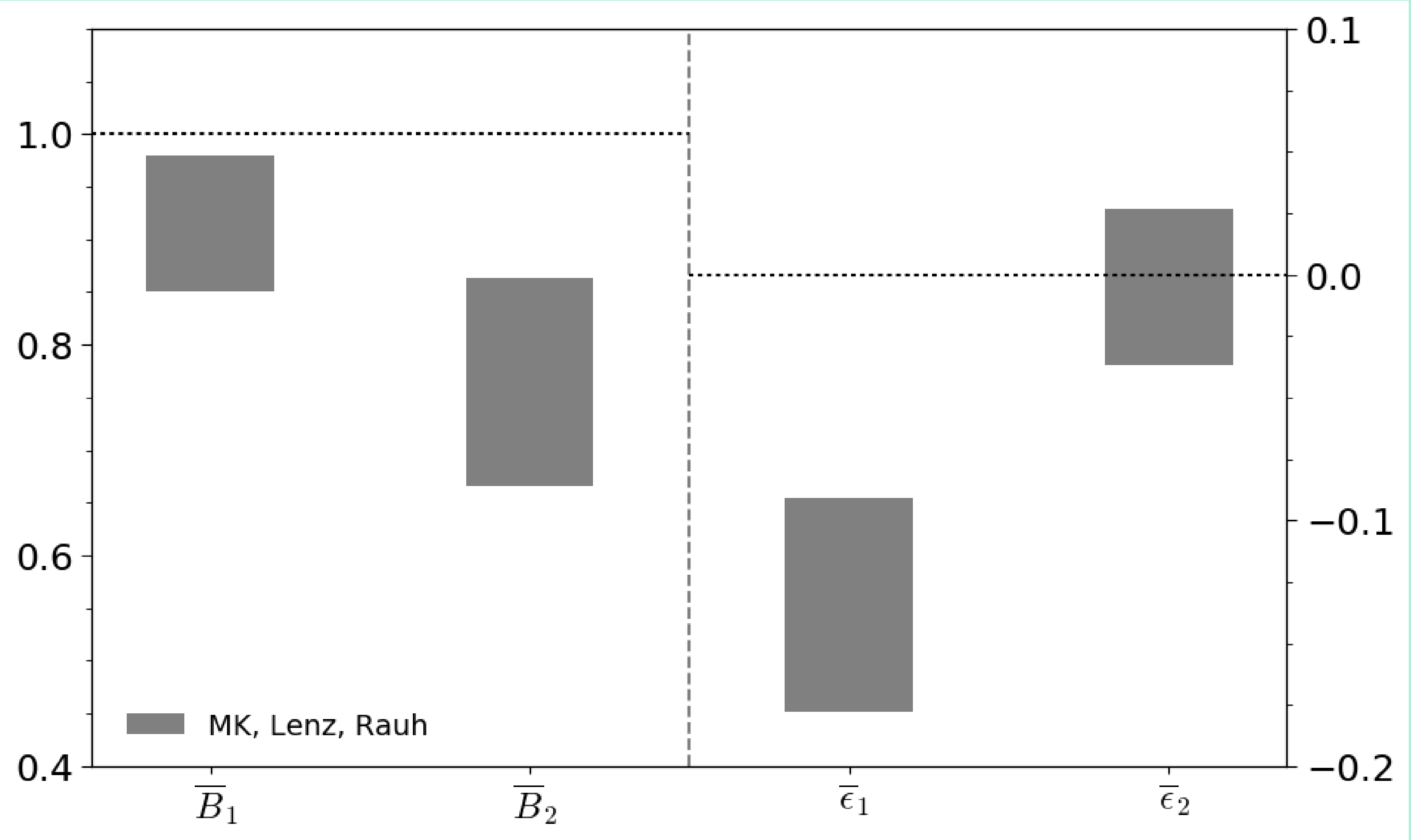
# HQET sum rules – results

B lifetimes



# HQET sum rules – results

D lifetimes



# Effect on observables

- $\tau(B_s)/\tau(B_d) = 0.9994 \pm 0.0025$
- $\tau(B^+)/\tau(B_d) = 1.082^{+0.022}_{-0.026}$
- $\tau(D^+)/\tau(D^0) = 2.7^{+0.7}_{-0.8}$

- For lifetimes, lattice hasn't yet arrived → sum rules the only game in town

# Effects on NP models

- Non-perturbative parameters very important
- Constraints from B mixing depend sensitively on values

| Source | $f_{B_s}\sqrt{\hat{B}}$ | $\Delta M_s^{\text{SM}}$ |
|---|---|---|
| HPQCD14 [116] | $(247 \pm 12)$ MeV | $(16.2 \pm 1.7)\,\text{ps}^{-1}$ |
| HQET-SR [71] | $(261 \pm 8)$ MeV | $(18.1 \pm 1.1)\,\text{ps}^{-1}$ |
| ETMC13 [117] | $(262 \pm 10)$ MeV | $(18.3 \pm 1.5)\,\text{ps}^{-1}$ |
| HPQCD09 [118] = FLAG13 [119] | $(266 \pm 18)$ MeV | $(18.9 \pm 2.6)\,\text{ps}^{-1}$ |
| **FLAG17** [65] | $\mathbf{(274 \pm 8)}$ **MeV** | $\mathbf{(20.01 \pm 1.25)\,ps^{-1}}$ |
| Fermilab16 [67] | $(274.6 \pm 4)$ MeV | $(20.1 \pm 0.7)\,\text{ps}^{-1}$ |
| HPQCD06 [120] | $(281 \pm 20)$ MeV | $(21.0 \pm 3.0)\,\text{ps}^{-1}$ |
| RBC/UKQCD14 [121] | $(290 \pm 20)$ MeV | $(22.4 \pm 3.4)\,\text{ps}^{-1}$ |
| Fermilab11 [122] | $(291 \pm 18)$ MeV | $(22.6 \pm 2.8)\,\text{ps}^{-1}$ |

# Effects on NP models

- Using the latest FLAG average → much less space for e.g. Z' model
- See **1712.06572** (Di Luzio, MK, Lenz)

## One constraint to kill them all?

Luca Di Luzio, Matthew Kirk, Alexander Lenz

*Institute for Particle Physics Phenomenology, Durham University, DH1 3LE Durham, United Kingdom*
*luca.di-luzio@durham.ac.uk, m.j.kirk@durham.ac.uk, alexander.lenz@durham.ac.uk*

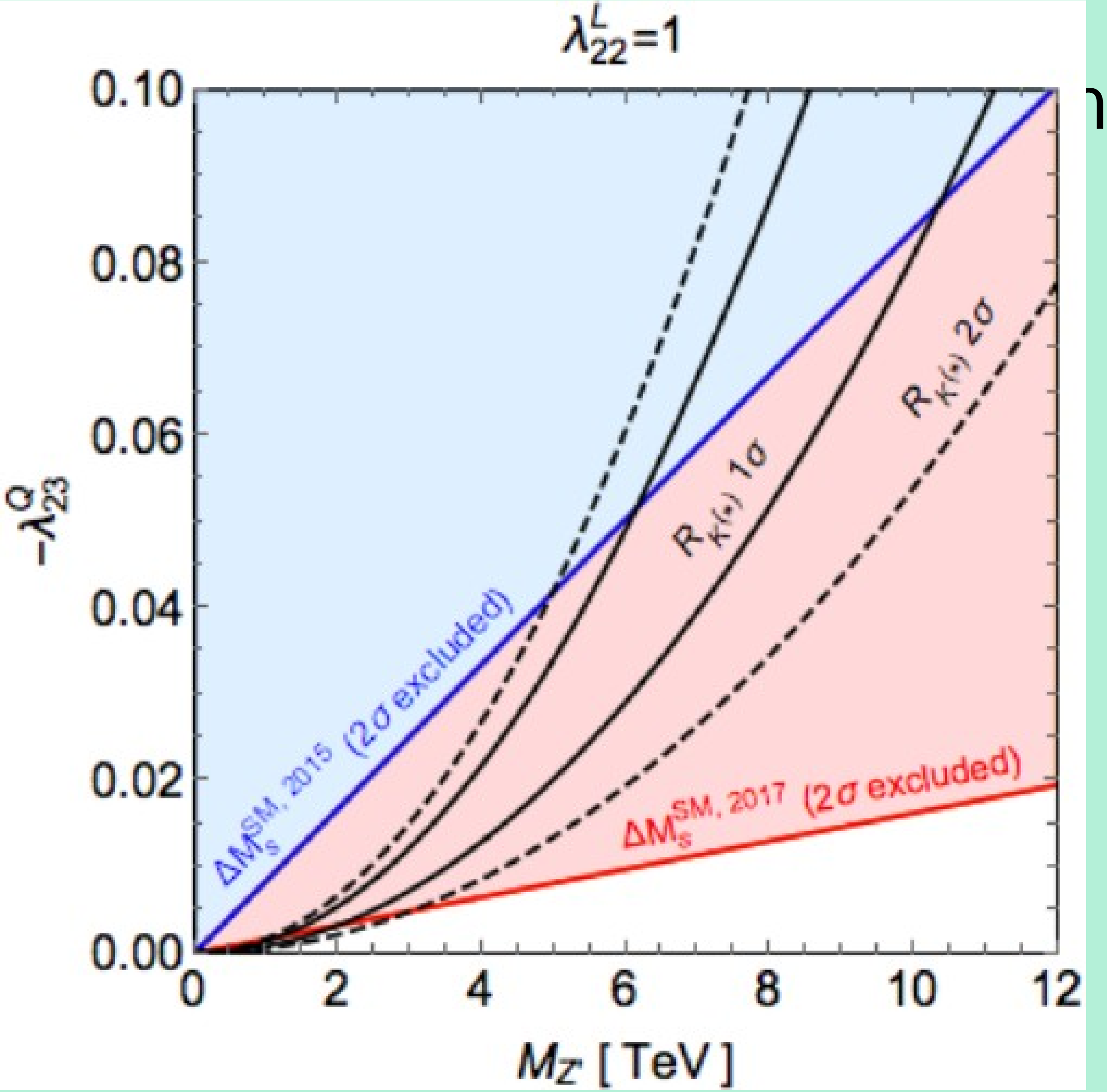
**Abstract**

Many new physics models that explain the intriguing anomalies in the $b$-quark flavour sector are severely constrained by $B_s$-mixing, for which the Standard Model prediction and experiment agreed well until recently. New non-perturbative calculations point, however, in the direction of a small discrepancy in this observable. Using up-to-date inputs to determine $\Delta M_s^{\rm SM}$, we find a severe reduction of the allowed parameter space of $Z'$ and leptoquark models explaining the $B$-anomalies. Remarkably, in the former case the upper bound on the $Z'$ mass approaches dangerously close to the energy scales already probed by the LHC. We finally identify some model building directions in order to alleviate the tension with $B_s$-mixing.

*Keywords:* New Physics, B-Physics, B-mixing

# Effects on NP models

- Using [obscured by figure] less space [obscured by figure]
- See 17[obscured by figure]



# Effects on NP models

- Good example of why independent determinations necessary
- From different lattice groups AND other methods

# What next?

- Determination of dimension-7 operators

$$R_2 = \frac{1}{m_b^2} \bar{b}_i \overleftarrow{D}_\lambda \gamma_\mu (1-\gamma^5) D^\lambda q_i \; \bar{b}_j \gamma^\mu (1-\gamma^5) q_j$$

  from lattice / sum rules – reduce error in $\Delta\Gamma_s$

- Lattice confirmation of dimension-6

# What next?

- Determination of dimension-7 operators

$$R_2 = \frac{1}{m_b^2} \bar{b}_i \overleftarrow{D}_\lambda \gamma_\mu (1-\gamma^5) D^\lambda q_i \; \bar{b}_j \gamma^\mu (1-\gamma^5) q_j$$

  from lattice / sum rules – reduce error in $\Delta\Gamma_s$
- Lattice confirmation of dimension-6

- HPQCD working on both
- MK, Rauh, Lenz working on dim-7 now

# What next?

- Determination of dimension-7 operators

$$R_2 = \frac{1}{m_b^2} \bar{b}_i \overleftarrow{D}_\lambda \gamma_\mu (1-\gamma^5) D^\lambda q_i \; \bar{b}_j \gamma^\mu (1-\gamma^5) q_j$$

  from lattice / sum rules – reduce error in $\Delta\Gamma_s$

- Lattice confirmation of dimension-6

- Know $\tau(B_s)/\tau(B_d)$ better from experiment – while already doing very well, theory is currently ahead

Thanks!

# Backup

# Mixing operators

$$Q_1 = \bar{b}_i \gamma_\mu (1-\gamma^5) q_i \; \bar{b}_j \gamma^\mu (1-\gamma^5) q_j,$$

$$Q_2 = \bar{b}_i (1-\gamma^5) q_i \; \bar{b}_j (1-\gamma^5) q_j, \qquad Q_3 = \bar{b}_i (1-\gamma^5) q_j \; \bar{b}_j (1-\gamma^5) q_i,$$

$$Q_4 = \bar{b}_i (1-\gamma^5) q_i \; \bar{b}_j (1+\gamma^5) q_j, \qquad Q_5 = \bar{b}_i (1-\gamma^5) q_j \; \bar{b}_j (1+\gamma^5) q_i,$$

# Lifetime operators

$$Q_1^q = \bar{b}\gamma_\mu(1-\gamma^5)q\ \bar{q}\gamma^\mu(1-\gamma^5)b, \qquad T_1^q = \bar{b}\gamma_\mu(1-\gamma^5)T^A q\ \bar{q}\gamma^\mu(1-\gamma^5)T^A b,$$

$$Q_2^q = \bar{b}(1-\gamma^5)q\ \bar{q}(1+\gamma^5)b, \qquad T_2^q = \bar{b}(1-\gamma^5)T^A q\ \bar{q}(1+\gamma^5)T^A b.$$

# Vaccum saturation approximation

$$\langle B_s|(\bar{q}\,\Gamma\,b)(\bar{q}\,\Gamma\,b)|\overline{B_s}\rangle = \sum_{\text{all states}} \langle B_s|(\bar{q}\,\Gamma\,b)|X\rangle\langle X|(\bar{q}\,\Gamma\,b)|\overline{B_s}\rangle$$

$$\approx \langle B_s|(\bar{q}\,\Gamma\,b)|0\rangle\langle 0|(\bar{q}\,\Gamma\,b)|\overline{B_s}\rangle$$

These then look like decay constants for meson to vacuum – extracted from experimental decay width

$$\langle B_s|(\bar{q}\,\Gamma\,b)(\bar{q}\,\Gamma\,b)|\overline{B_s}\rangle = B_\Gamma \langle B_s|(\bar{q}\,\Gamma\,b)|0\rangle\langle 0|(\bar{q}\,\Gamma\,b)|\overline{B_s}\rangle$$

Bag parameter

# HQET sum rules – results

D mixing