Quantum Gravity from Conformal Field Theory

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General Idea

AdS/CFT correspondence

$\mathcal{N} = 4$ SYM gauge group $SU(N)$	$\Longleftrightarrow$	Supergravity on $AdS_5 \times S^5$
4pt correlation functions	$\Longleftrightarrow$	AdS amplitudes (Witten diagrams)

- Interested in loop corrections to supergravity in AdS_5

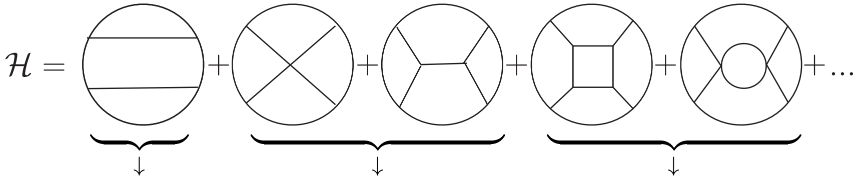
strong coupling ($\lambda \to \infty$) $\Longleftrightarrow$ weak coupling $\to \frac{1}{N^2}$ expansion

General Idea

Consider simplest case:

$\frac{1}{2}$-BPS single-trace operator $\Longleftrightarrow$ graviton multiplet

$$\mathcal{O}_2(x) = y^i y^j \mathrm{Tr}[\varphi_i(x)\varphi_j(x)] \in [0,2,0] \text{ rep of } SU(4)$$



N^0 — free field theory ✓

$\frac{1}{N^2}$ — classical supergravity ✓

$\frac{1}{N^4}$ — quantum corrections to SUGRA

Operator Product Expansion

In $\mathcal{N} = 4$ SYM, consider the correlator

$$\mathcal{H} = \langle \mathcal{O}_2(x_1)\mathcal{O}_2(x_2)\mathcal{O}_2(x_3)\mathcal{O}_2(x_4)\rangle = \frac{1}{g_{12}^2 g_{34}^2}\sum_i \mathcal{H}_{R_i}(u,v),$$

where $R_i \in [0,2,0] \times [0,2,0]$ and $\mathcal{H}_{R_i}(u,v)$ **crossing symmetric**.

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Consequences of **superconformal symmetry**:

$$\sum_i \mathcal{H}_{R_i}(u,v) = (\text{protected}) + \mathcal{I} \cdot \mathcal{F}(u,v),$$

with $\mathcal{I} = (x-y)(x-\bar{y})(\bar{x}-y)(\bar{x}-\bar{y})$.

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Focus on **interacting part** (only $[0, 0, 0]$ rep contributes):

$$\mathcal{F}(u, v) = \mathcal{F}^{(0)} + a \cdot \mathcal{F}^{(1)} + a^2 \cdot \mathcal{F}^{(2)} + \ldots$$

$$a = \frac{1}{N^2 - 1}$$

Operator Product Expansion

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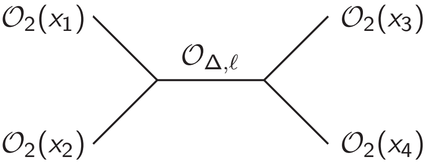
Strategy: use OPE



- Exchanged $\mathcal{O}_{\Delta,\ell}$ can be unprotected (e.g. double-trace, ...)

Operator Product Expansion

Which operators contribute?

Remember: $\mathcal{N} = 4$ SYM at $\lambda \to \infty \iff$ supergravity limit

$\Rightarrow$ long single-trace operators ('string states') decouple

$\Rightarrow$ remaining spectrum: double-trace operators (up to $O(a^2)$)

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Analyse CFT data in large N expansion

$$\Delta = \Delta^{(0)} + a \cdot \eta^{(1)} + a^2 \cdot \eta^{(2)} + \dots$$

$$C_{\Delta,\ell} = C^{(0)}_{\Delta,\ell} + a \cdot C^{(1)}_{\Delta,\ell} + \dots$$

Conformal blocks

Expand contributions from long operators into conformal blocks:

$$\mathcal{F}(u,v) = \sum_{t,\ell \geq 0}^{\infty} C_{t,\ell}^2 \cdot \mathcal{B}_{t,\ell}(u,v)$$

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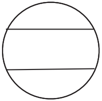
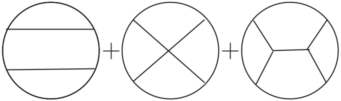
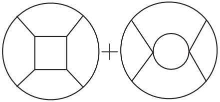
- are Eigenfunctions of the quadratic conformal Casimir
- capture the contribution of a primary operator and all its descendants
- in the OPE-limit $u \to 0$ they behave like

$$\mathcal{B}_{t,\ell}(u,v) \to u^t\,(\ldots)\,, \qquad \text{with twist } t = \Delta - \ell$$

Conformal blocks and large N expansion

Combining conformal block expansion with large N expansion:

- $O(1)$:  ✓
- $O(a)$:  ✓
- $O(a^2)$:  $\longleftarrow$

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$$\mathcal{F}^{(0)}(u,v) = \sum_{t,\ell} (C^{(0)}_{t,\ell})^2 \cdot \mathcal{B}_{t,\ell}(u,v) \qquad \checkmark$$

- $O(a)$:

$$\mathcal{F}^{(1)}(u,v)|_{log(u)} = \sum_{t,\ell} (C^{(0)}_{t,\ell})^2 \cdot \eta^{(1)}_{t,\ell} \cdot \mathcal{B}_{t,\ell}(u,v) \qquad \checkmark$$

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- $O(a^2)$:

$$\mathcal{F}^{(2)}(u,v)|_{log^2(u)} = \frac{1}{2} \sum_{t,\ell} (C^{(0)}_{t,\ell})^2 \cdot (\eta^{(1)}_{t,\ell})^2 \cdot \mathcal{B}_{t,\ell}(u,v)$$

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$\Rightarrow$ double-discontinuity is determined by tree level data!

Problem: double-trace operator mixing

Problem: double-trace operators are degenerate!

$$\mathcal{O}_{t,\ell} \longrightarrow \mathcal{O}^{i}_{t,\ell}$$

At twist t we have $\mathbf{\frac{t}{2} - 1}$ operators:

$$\mathcal{O}^{i}_{t,\ell} = \left\{ (\mathcal{O}_2 \Box^{\frac{t}{2}-2} \partial^\ell \mathcal{O}_2),\ (\mathcal{O}_3 \Box^{\frac{t}{2}-3} \partial^\ell \mathcal{O}_3), \ldots,\ (\mathcal{O}_{\frac{t}{2}} \Box^{0} \partial^\ell \mathcal{O}_{\frac{t}{2}}) \right\}$$

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Not discussed here:

- Operator mixing can be resolved using data from higher charge correlators
- Consider $\langle \mathcal{O}_p \mathcal{O}_p \mathcal{O}_q \mathcal{O}_q \rangle$ instead of just $\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \rangle$
- Closed expressions for $C^{(0)}_{t,\ell,i}$ and $\eta_{t,\ell,i}$ found

Results: loop predictions

Predict one-loop $log^2(u)$ term at $O(a^2)$:

$$\mathcal{F}^{(2)}(u,v)|_{log^2(u)} = \frac{1}{2}\sum_{t,\ell}(C^{(0)}_{t,\ell})^2 \cdot (\eta^{(1)}_{t,\ell})^2 \cdot \mathcal{B}_{t,\ell}(u,v)$$

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this resums to:

$$\mathcal{F}^{(2)}|_{log^2(u)} = \frac{1}{uv}\Big[p(u,v)\frac{\mathrm{Li}_1(x)^2 - \mathrm{Li}_1(\bar{x})^2}{x-\bar{x}} + 2\Big[p(u,v) + p\Big(\frac{1}{v},\frac{u}{v}\Big)\Big]\frac{\mathrm{Li}_2(x)-\mathrm{Li}_2(\bar{x})}{x-\bar{x}}$$
$$+ q(u,v)(\mathrm{Li}_1(x)+\mathrm{Li}_1(\bar{x})) + r(u,v)\frac{\mathrm{Li}_1(x)-\mathrm{Li}_1(\bar{x})}{x-\bar{x}} + s(u,v)\Big]$$

with

$$p(u,v) = 24uv\partial_x^2\partial_{\bar{x}}^2\left[\frac{u^2v^2(1-u-v)[(1-u-v)^4 + 20uv(1-u-v)^2 + 30u^2v^2]}{(x-\bar{x})^{10}}\right]$$

Results: loop predictions

Promote double-discontinuity of $\mathcal{F}^{(2)}$ to full crossing-symmetric amplitude:

$$\mathcal{F}^{(2)}(u,v) = \frac{1}{uv}\left[f(u,v) + \frac{1}{u} f\left(\frac{1}{u},\frac{v}{u}\right) + \frac{1}{v} f\left(\frac{1}{v},\frac{u}{v}\right)\right].$$

$$f(u,v) = \frac{uv}{x-\bar{x}}\partial_x^2\partial_{\bar{x}}^2\left[(x-\bar{x})\left(g^{(4)}+g^{(3)}+g^{(2)}+g^{(1)}+g^{(0)}\right)\right]$$

$$\begin{aligned}
g^{(4)}(u,v) &= P_-^{(4)}(u,v)\Phi^{(2)}(u,v)\\
g^{(3)}(u,v) &= P_+^{(3)}(u,v)\Psi(u,v) + P_-^{(3)}(u,v)\log(uv)\Phi^{(1)}(u,v)\\
g^{(2)}(u,v) &= P_+^{(2)}(u,v)\log u \log v + P_-^{(2)}(u,v)\Phi^{(1)}(u,v)\\
g^{(1)}(u,v) &= P_+^{(1)}(u,v)\log(uv)\\
g^{(0)}(u,v) &= P_+^{(0)}(u,v)
\end{aligned}$$

Only ladder integrals $\Phi^{(1)}$ and $\Phi^{(2)}$ appearing!

Results: new CFT data

From our result, we can extract new CFT data at order $O(\frac{1}{N^4})$:

- At $t = 4$, we find for the second correction to the dimension:

$$\Delta_{t=4,\ell} = \Delta^{(0)}_{4,\ell} + a \cdot \eta^{(1)}_{4,\ell} + a^2 \cdot \eta^{(2)}_{4,\ell} + \ldots,$$

$$\eta^{(2)}_{4,\ell} = \frac{1344(\ell-7)(\ell+14)}{(\ell-1)(\ell+1)^2(\ell+6)^2(\ell+8)} - \frac{2304(2\ell+7)}{(\ell+1)^3(\ell+6)^3}$$

- For $t \geq 6$, mixing with *triple*-trace operators obstructs the extraction of more data.

Summary and Outlook

- We bootstrapped the supergravity loop correction to $\langle \mathcal{O}_2\mathcal{O}_2\mathcal{O}_2\mathcal{O}_2 \rangle$ from OPE consistency and crossing symmetry
- We relied on being able to resolve double-trace mixing from classical data ('solving' the theory)
- Result is structurally simpler than one could have hoped for
- Can extract new subsubleading CFT data at $O(a^2)$

Outlook:

- Mellin representation of our result - simpler form?
- Consider loop corrections to higher charge correlators ($\langle \mathcal{O}_2\mathcal{O}_2\mathcal{O}_3\mathcal{O}_3 \rangle$ already done)
- Bootstrapping higher loops?