s and c quark mass determination from lattice QCD

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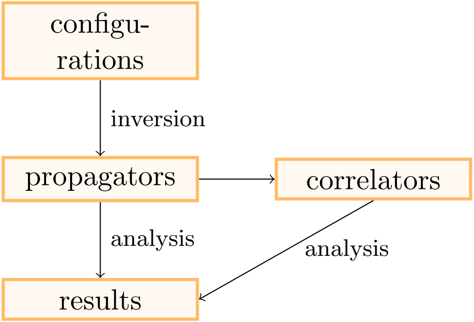
Introduction and Motivation

- Quark masses are difficult to determine precisely due to confinement
- Precise determination desirable for some precision Higgs calculations
- PDG strange and charm masses:
  - $\overline{m}_s(2 \text{ GeV}) = 96 \pm^{8}_{4} \text{ MeV}$
  - $\overline{m}_c(\overline{m}_c) = 1.28 \pm 0.03 \text{ GeV}$
- Lattice results typically considerably more precise

Lattice Gauge Configurations

- Spacetime is discretised
  - Allows numerical calculations
  - Regulates divergences
- Gauge configuration is a group of SU(3) matrices; one for each link between lattice sites
- Gauge configurations chosen by MC to follow e^{-S} distribution
- Correlation functions are evaluated configuration by configuration and then averaged



Propagators

- Grassmann variables in path integral can be performed analytically
  - $\int \mathcal{D}\bar{\psi}\mathcal{D}\psi\bar{\psi}\psi e^{-\bar{\psi}M\psi} = M^{-1}\mathrm{det}(M)$
  - $\mathrm{det}(M)$ included in gauge field generation
- The propagator is the inverse of the Dirac matrix $M \equiv \not{D} - m$
- M is a very large matrix that is inverted numerically
- Correlation functions are constructed from the propagators
- Calculation time diverges as $am \to 0$

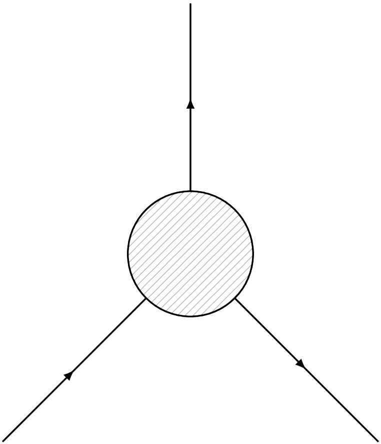
https://www.hpc.cam.ac.uk/images/darwin.jpg/view

Tuning Quark Masses

- Quark masses are input into simulations and tuned to produce correct meson mass
  - Input as am
- am_s and am_c are tuned on each lattice to give correct η_s and η_c meson masses
- Lattice QCD is a regularisation and the bare input masses therefore need to be renormalised
- Renormalisation is done nonperturbatively in a momentum subtraction scheme and then perturbatively matched to $\overline{\text{MS}}$ in the continuum

RI-SMOM: Setup

- To renormalise a current, consider the vertex function of (G_Γ) the relevant operator ($\bar{\psi}\Gamma\psi$)
- Conditions are placed on the amputated vertex functions in the limit of zero quark mass
- Kinematic setup is chosen to satisfy $p_1^2 = p_2^2 = q^2$
- Momentum space Green's functions are constructed from Fourier transformed propagators



RI-SMOM: Conditions

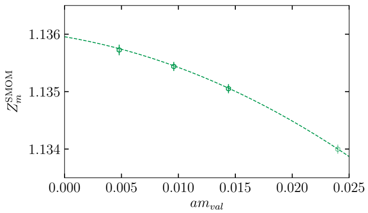
- Propagator condition gives $Z_q = 1$ in the free theory
- Other conditions satisfy Ward identities
- $\Lambda_\Gamma = S^{-1}(p_2) G_\Gamma S^{-1}(p_1)$

$$\frac{1}{12p^2}\mathrm{Tr}[S^{-1}(p)\not{p}] = -Z_q$$
$$\frac{1}{12}\frac{Z_S}{Z_q}\mathrm{Tr}[\Lambda_S(p_1, p_2)] = 1$$
$$Z_m = \frac{1}{Z_S}$$

- Only propagator and scalar operator are required for mass renormalisation

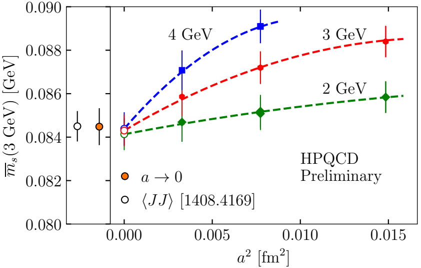
$\overline{\text{MS}}$ Matching

- To obtain masses in the $\overline{\text{MS}}$ scheme a matching calculation must be performed in the continuum
- Done perturbatively
- Perturbation theory does not include condensate effects that are present in the lattice calculation
- Chiral condensates are removed by $am_{\text{val}} \to 0$
- Other condensate is allowed for in continuum extrapolation



Continuum Extrapolation

- Calculate m_s and m_c on three different lattice spacings: $\sim \{0.12, 0.09, 0.06\}$ fm
- Extrapolate these results to $a = 0$
- Take account of $(a\mu)^2$, $(a\mu)^4$, $(a\Lambda)^2$ and $(a\Lambda)^4$ discretisation effects
  - HISQ action has disc. errors removed to this order
- Allow for condensate term



Future Work

- Largest error from bare mass tuning
- May reduce extrapolation error by using smaller lattice spacing
  - Will also allow for higher μ points
- Look into Z_V; relevant for $g-2$ calculation