The double copy for scattering of Reissner-Nordström black holes

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Outline

Radiative gravitational scattering

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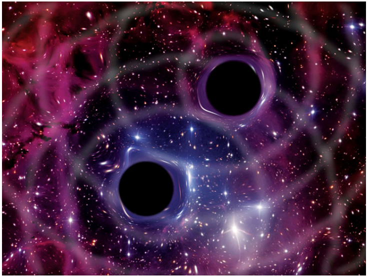

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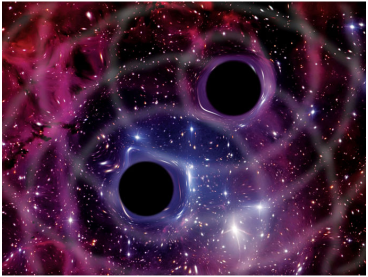

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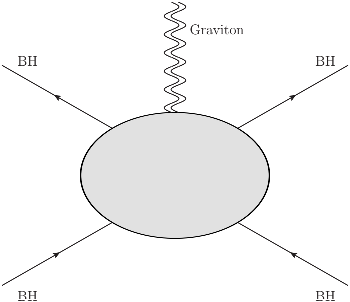

LH image from Nat. Astronomy **1***,0112 (2017).*

The double copy

In Yang-Mills theory, any tree-level m-point amplitude has the form

$$\mathcal{A}_m = g^{m-2} \sum_{i \in \Gamma} \frac{n_i c_i}{\Delta_i}\,, \tag{1}$$

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- n_i are kinematic numerators, which are not unique.

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Gauge freedom makes it possible to always choose numerators satisfying the same Jacobi identities, so

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The *double copy* then says

$$\mathcal{M}_m = i\left(\frac{\kappa}{2}\right)^{m-2} \sum_{i\in\Gamma} \frac{n_i \tilde{n}_i}{\Delta_i} \quad \text{obtained via} \quad g \mapsto \frac{\kappa}{2}, c_i \mapsto n_i \tag{4}$$

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gravity = (YM theory)^2; ?!?!?!?!?!

Little group reps

External particle states are little group irreps, which for $m = 0$ is $SO(d-2)$. If $n = d-2$, then composing two vector reps gives

$$\mathbf{n} \otimes \mathbf{n} = \mathbf{1} \oplus \frac{\mathbf{n(n-1)}}{\mathbf{2}} \oplus \left(\frac{\mathbf{n(n+1)}}{\mathbf{2}} - \mathbf{1}\right) \tag{5}$$

$$\sim \phi \oplus \quad B_{\mu\nu} \quad \oplus \quad h_{\mu\nu}\,, \tag{6}$$

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Upshot

If non-physical gravity DoF are removed then the double copy gives a direct relation between YM theory and GR.

RN black holes

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$$G_{\mu\nu} = \kappa \left(F_{\mu\rho} F^{\rho}_{\nu} - \frac{1}{4} g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} \right) \tag{7}$$

$$\Rightarrow ds^2 = f(r) dt^2 - f(r)^{-1} dr^2 - r^2 d\Omega^2 \,, \tag{8}$$

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EM fields:

$$E_r = \frac{Q}{r^2} \quad \text{and} \quad B_r = \frac{P}{r^2} \,. \tag{9}$$

Hence at large radial distances the black holes appear like point charges.

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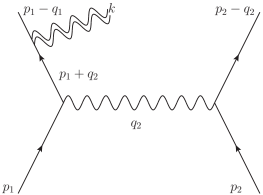
$$\begin{aligned} x^\mu_\alpha(\tau) &= b^\mu_\alpha + v^\mu_\alpha \tau + s^\mu_\alpha(\tau)\,, \\ s^\mu(\tau) &= s^{(1)}_\alpha(\tau) + s^{(2)}_\alpha(\tau) + \dots\,, \\ u^\mu_\alpha(\tau) &= v^\mu_\alpha + u^{\mu(1)}_\alpha + \dots\,, \end{aligned} \tag{11}$$

where $s^\mu(\tau)$ is the EM deflection, $u^\mu_\alpha(\tau) = \frac{d}{d\tau} s^\mu_\alpha(\tau)$. Have BC's $x^\mu_\alpha(\tau \to -\infty) = b^\mu_\alpha + v^\mu_\alpha \tau$, so $u^\mu(\tau \to -\infty) = v^\mu_\alpha$ and $s^\mu(\tau \to -\infty) = 0$.

Scattering

To leading order the 5 possible diagrams are:



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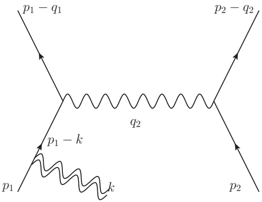
$+ 1 \leftrightarrow 2$



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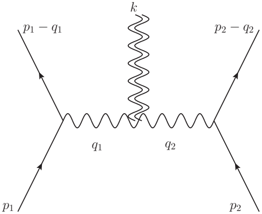

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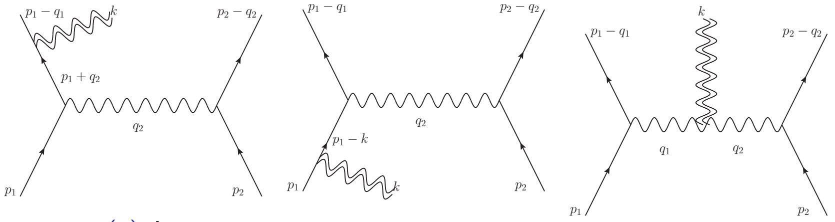

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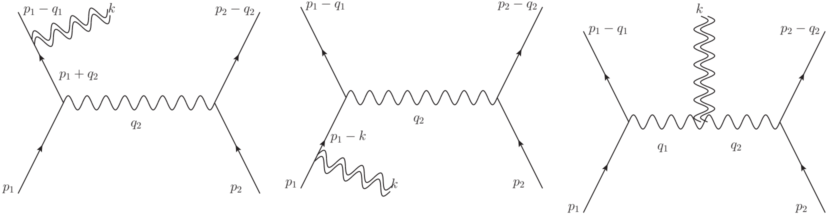

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As outgoing radiation is on-shell $k^2 = 0$ and $k \cdot \epsilon = 0$, inside the amplitude $k^\mu = 0$.

Scattering

In all, leading order radiation field can be written

$$k^2 h^{\mu\nu}_{(1)}(k) = \frac{e^2\kappa}{2}\int đ^d q_1 đ^d q_2 \hat{\delta}^d(q_1\cdot v_1)\hat{\delta}^d(q_2\cdot v_2)\hat{\delta}^d(k-q_1-q_2) \quad (13)$$

$$\frac{e^{i(q_1\cdot b_1+q_2\cdot b_2)}}{q_1^2\, q_2^2}\left[\frac{Q_{12}^\mu P_{12}^\nu + Q_{12}^\nu P_{12}^\mu}{q_1^2 q_2^2} + (v_1\cdot v_2)\left(\frac{Q_{12}^\mu Q_{12}^\nu}{q_1^2 q_2^2} - \frac{P_{12}^\mu P_{12}^\nu}{(k\cdot v_1)^2(k\cdot v_2)^2}\right)\right]$$

where

$$P_{12}^\mu = k\cdot v_1 v_2^\mu - k\cdot v_2 v_1^\mu\,, \quad (14)$$

$$Q_{12}^\mu = (q_1-q_2)^\mu - \frac{q_1^2}{k\cdot v_1}v_1^\mu + \frac{q_2^2}{k\cdot v_2}v_2^\mu\,. \quad (15)$$

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Physically use a set of diagrams with vector bosons only, and a set with both vector and scalar bosons.

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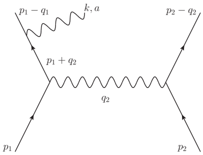

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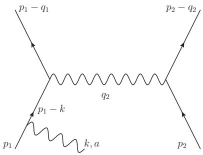

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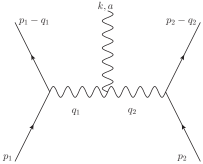

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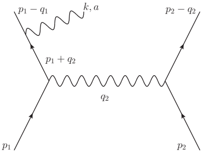

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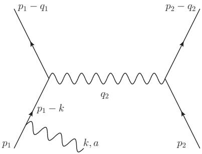

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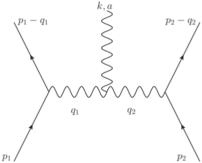

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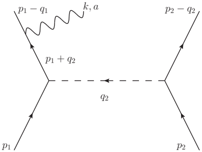

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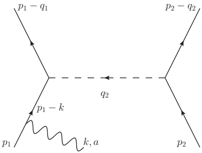

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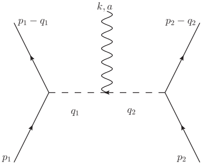

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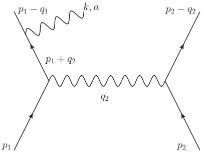

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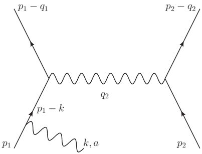

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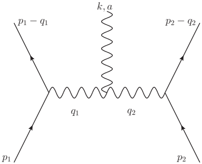

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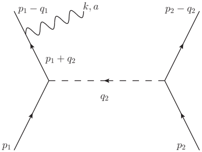

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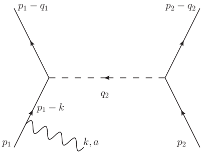

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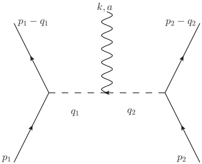

(c) $\tilde{D}_C$

$$c_A = \tilde{c}_A = i(T_1^a \cdot T_1^b)T_2^b \quad c_B = \tilde{c}_B = i(T_1^b \cdot T_1^a)T_2^b \quad c_C = \tilde{c}_A = f^{abc}T_1^b T_2^c$$

$$\Rightarrow c_A - c_B = c_C \quad \text{and} \quad \tilde{c}_A - \tilde{c}_B = \tilde{c}_C \,.$$

Double copy

Can make both sets of numerators satisfy colour-kinematics duality. Then vector diagram amplitude:

$$\mathcal{A}_{\text{vec}} = g^3 \left(\frac{n_A c_A}{(2p_1 \cdot k - q_1^2 + q_2^2) q_2^2} + \frac{n_B c_B}{(-2p_1 \cdot k)\, q_2^2} + \frac{n_C c_C}{q_1^2 q_2^2} + \frac{n_D c_D}{(2p_2 \cdot k - q_2^2 + q_1^2) q_1^2} + \frac{n_E c_E}{(-2p_2 \cdot k)\, q_1^2} \right), \tag{18}$$

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However could also antisymmetrise, representing emission of an external axion with an antisymmetric polarisation tensor. Hence let

$$\mathcal{M}^{\mu\nu}_{\mathrm{ax}} = \mathcal{M}^{[\mu\nu]}_{\mathrm{RN}}\,. \tag{23}$$

Mass expansion

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$$p_1^\mu = m_1 v_1^\mu \quad \text{and} \quad p_2^\mu = m_2 v_2^\mu \tag{24}$$

and Taylor expand. The classical limit then given by dominant initial terms, which are

$$\mathcal{M}_{\mathrm{RN}}^{\mu\nu} = e^2 \kappa \frac{4 m_1^2 m_2^2}{(k \cdot v_1)^2 (k \cdot v_2)^2 q_1^2 q_2^2} \left[2(k \cdot v_2)^2 q_1^2 (q_1^2 - q_2^2) v_1 \cdot v_2 v_1^\mu v_1^\nu + \ldots \right]. \tag{25}$$

Mass expansion

Desired classical limit corresponds to the black hole masses being large. Introduce mass by

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Recall

$$P_{12}^\mu = k \cdot v_1 v_2^\mu - k \cdot v_2 v_1^\mu \,, \tag{26}$$

$$Q_{12}^\mu = (q_1 - q_2)^\mu - \frac{q_1^2}{k \cdot v_1} v_1^\mu + \frac{q_2^2}{k \cdot v_2} v_2^\mu \,. \tag{27}$$

Results

With these quantities, and using $k^\mu = 0$, get

$$\mathcal{M}^{\mu\nu}_{\rm RN} = e^2\kappa\, 8m_1^2m_2^2\Bigg[\frac{Q_{12}^\mu P_{12}^\nu + Q_{12}^\nu P_{12}^\mu}{q_1^2 q_2^2} + (v_1\cdot v_2)\Bigg(\frac{Q_{12}^\mu Q_{12}^\nu}{q_1^2q_2^2} - \frac{P_{12}^\mu P_{12}^\nu}{(k\cdot v_1)^2(k\cdot v_2)^2}\Bigg)\Bigg]. \tag{28}$$

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Classically:

$$k^2 h^{\mu\nu}_{(1)}(k) = \frac{e^2\kappa}{2} \int đ^d q_1 đ^d q_2 \hat{\delta}^d(q_1 \cdot v_1) \hat{\delta}^d(q_2 \cdot v_2) \hat{\delta}^d(k - q_1 - q_2)$$
$$\frac{e^{i(q_1 \cdot b_1 + q_2 \cdot b_2)}}{q_1^2\, q_2^2} \left[\frac{Q_{12}^\mu P_{12}^\nu + Q_{12}^\nu P_{12}^\mu}{q_1^2 q_2^2} + (v_1 \cdot v_2) \left(\frac{Q_{12}^\mu Q_{12}^\nu}{q_1^2 q_2^2} - \frac{P_{12}^\mu P_{12}^\nu}{(k \cdot v_1)^2 (k \cdot v_2)^2} \right) \right]$$

Results

Call couplings and part in brackets $\mathcal{T}^{\mu\nu}$; then

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Do same steps for antisymmetric $\mathcal{M}^{\mu\nu}_{\mathrm{ax}} = \mathcal{M}^{[\mu\nu]}_{\mathrm{RN}}$, giving the axion emission as

$$\mathcal{M}^{\mu\nu}_{\mathrm{ax}} = \frac{8 m_1^2 m_2^2}{q_1^2 q_2^2} \left(Q^{\nu}_{12} P^{\mu}_{12} - Q^{\mu}_{12} P^{\nu}_{12} \right) \,. \tag{30}$$

This has no classical perturbative counterpart, however external axions have recently been studied for spinning particles in [6].

1 Introduction

2 Reissner-Nordström black hole scattering

3 Double copy using charged scalar diagrams

4 Summary

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References

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