YTF Conference 2018
Centre for Particle Theory Durham

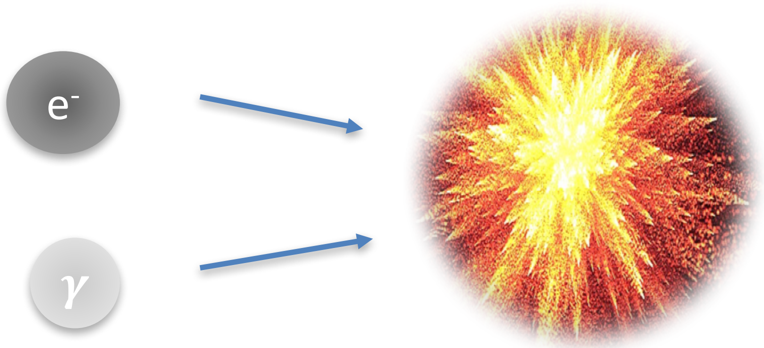
SPIN MODELS AND MONOPOLE CROSS SECTIONS

with thanks to Prof. N. Mavromatos [1], Dr. A. Santra [2]
[1] King's College London
[2] Instituto de Fisica Corpuscular, Valencia

Stephanie Baines 10th Jan 2018

SPIN MODELS OF MONOPOLES

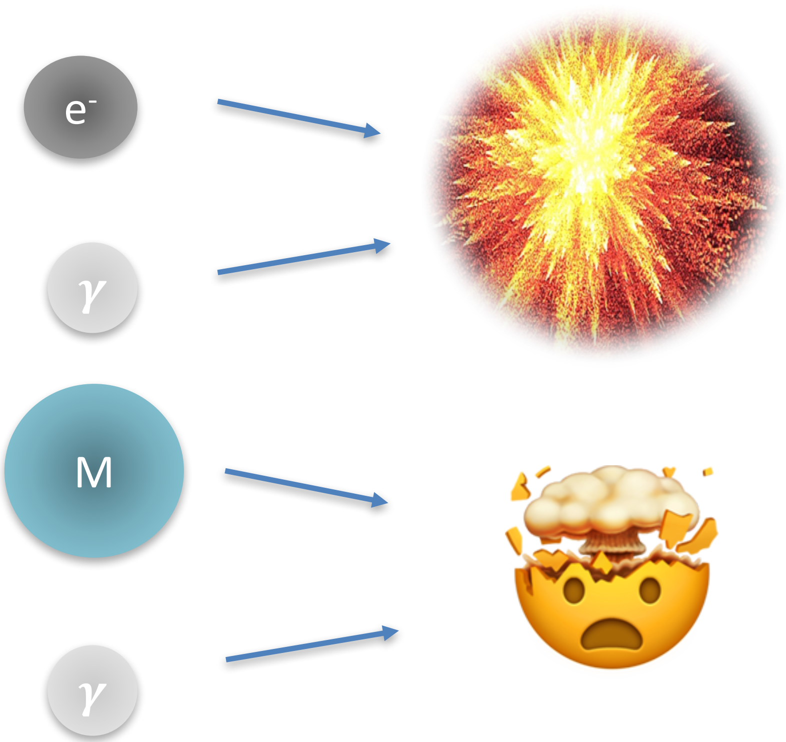
Quantum Electrodynamics



Theory of charged matter fields in U(1) gauge invariant theory.

SPIN MODELS OF MONOPOLES

Quantum Electrodynamics and the Monopole



SPIN MODELS OF MONOPOLES

e^-

M

electric charge: -1

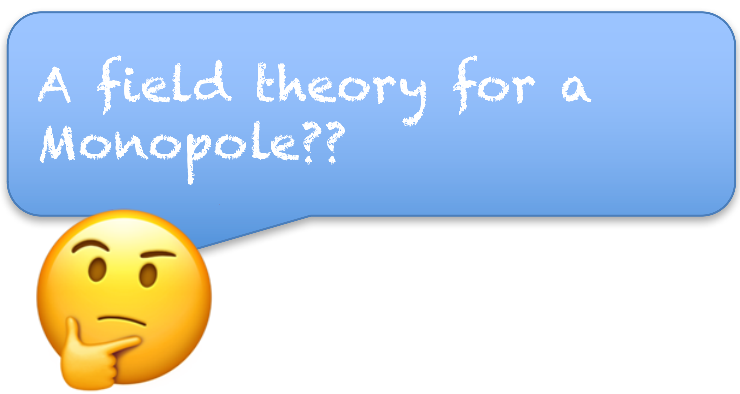
mass: 0.5 MeV

spin: s = -½

magnetic charge: +1

mass: O(TeV)?

spin: s = ?



MoEDAL

SPIN MODELS OF MONOPOLES

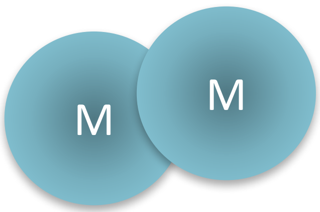
SPIN 0: Scalar Quantum Electrodynamics

SPIN 1 : Proca Field Theory

SPIN 1/2 : Dirac Quantum Electrodynamics

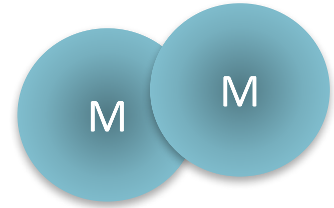
Each model can be derived using a $\beta = \frac{|p|}{E}$ dependent coupling. This only amounts to multiplying the cross sections by a scale factor. We will leave this aside in this discussion.

SPIN 0 MONOPOLES



SPIN 0 : Scalar QED

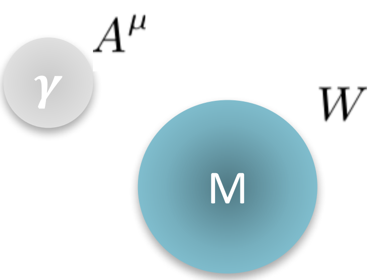
SPIN 0 MONOPOLE LAGRANGIAN



"Scalar Quantum Electrodynamics is described by the U(1) Gauge invariant Lagrangian of a Spin 0 matter field"

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (\partial_\mu - igA_\mu)W^*(\partial_\mu + igA_\mu)W + M^2W^*W$$

GAUGE FIELD
KINETIC TERM

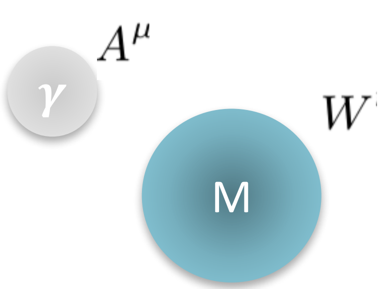


SPIN 0 MONOPOLE LAGRANGIAN

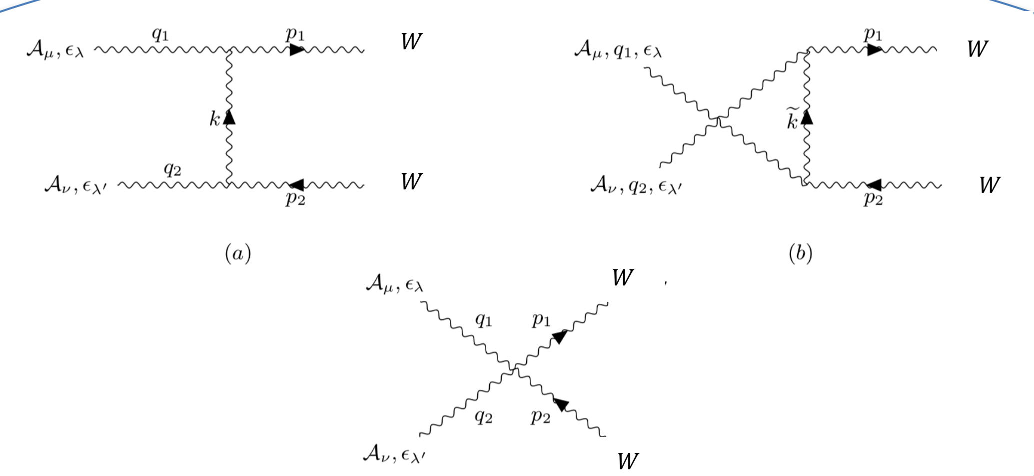
"Scalar Quantum Electrodynamics is described by the U(1) Gauge invariant Lagrangian of a Spin 0 matter field"

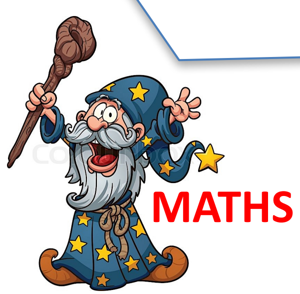
$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (\partial_\mu - igA_\mu)W^*(\partial_\mu + igA_\mu)W + M^2W^*W$$

MONOPOLE KINETIC TERM and MASS TERM with the MONOPOLE INTERACTION TERM



PHOTON FUSION AT TREE LEVEL



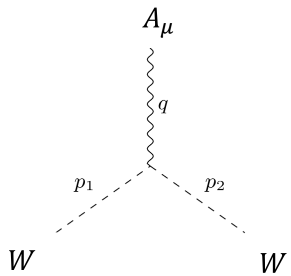


PHOTON FUSION: $\gamma\gamma \rightarrow MM$

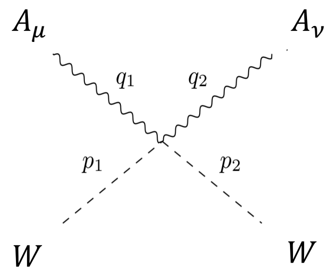
$$\frac{d\sigma_{\gamma\gamma\rightarrow M\bar{M}}}{d(cos\theta)}$$

$$\sigma_{\gamma\gamma\rightarrow M\bar{M}}$$

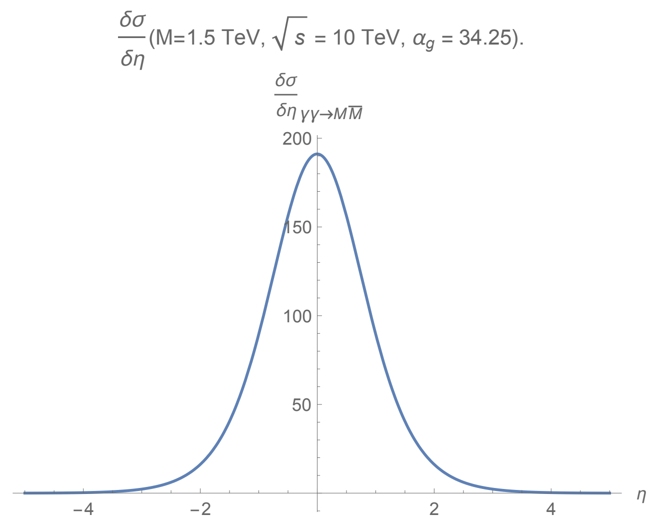
SPIN 0 MONOPOLE CROSS SECTION

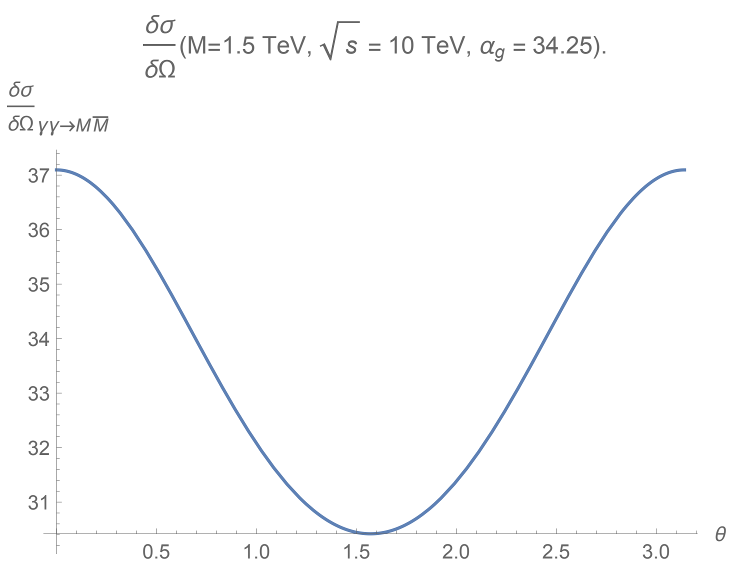


$\propto -ig(\beta)(p_1 + p_2)_\mu$

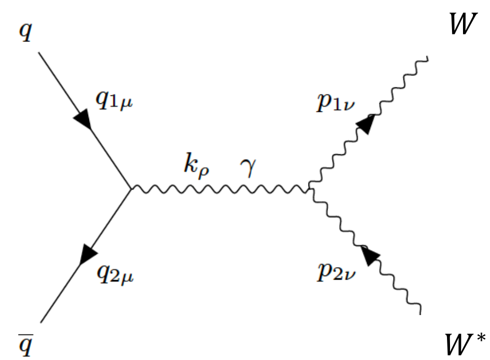


$\propto 2ig(\beta)^2 g_{\mu\nu}$





DRELL-YAN AT TREE LEVEL

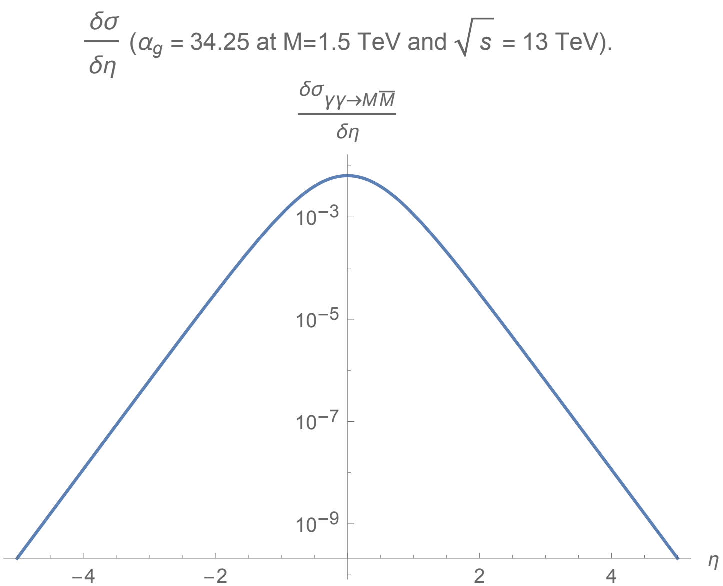


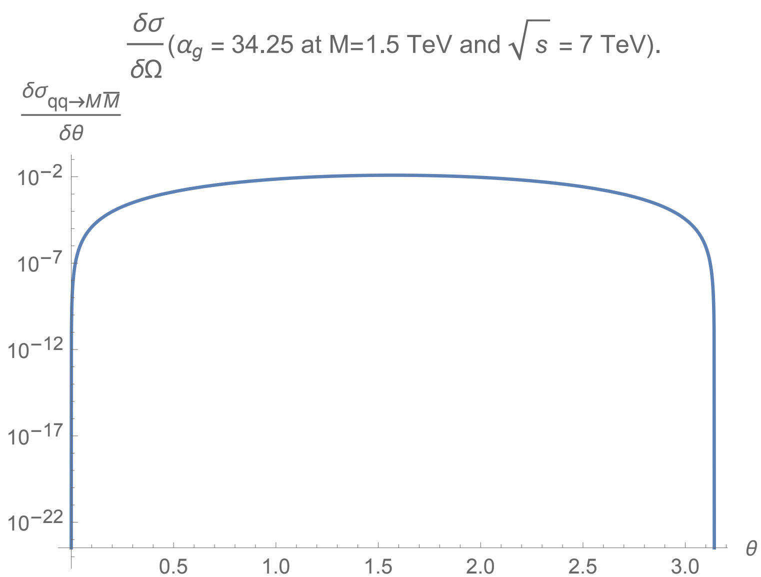
Drell-Yan: qq->MM

$$\frac{d\sigma_{\gamma\gamma\rightarrow M\bar{M}}}{d(cos\theta)}$$

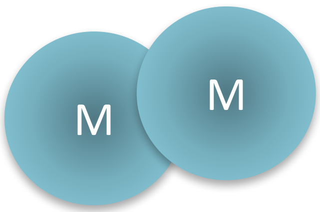
$$\sigma_{\gamma\gamma\rightarrow M\bar{M}}$$

DRELL-YAN SPIN 0 CROSS SECTION





THE SPIN 1 QUESTION



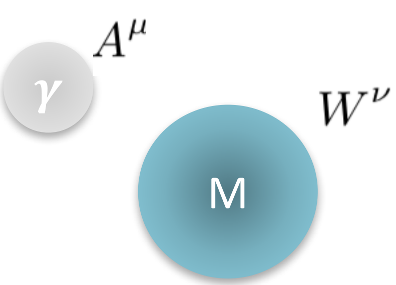
SPIN 1 : Proca Field Theory

WHAT IS A PROCA FIELD?

"Gauge theory defining the equations of motion of a spin-1 massive boson interacting with a massless gauge boson"

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu A_\nu)(\partial^\nu A^\mu) - \frac{1}{2}\widetilde{W}^\dagger_{\mu\nu}\widetilde{W}^{\mu\nu} - m^2 W^\dagger_\mu g^{\mu\nu} W_\nu + ig\mathcal{A}_\mu W^\dagger_\nu \widetilde{W}^{\mu\nu}$$
$$- ig\widetilde{W}^\dagger_{\mu\nu}\mathcal{A}^\mu W^\nu - g^2 \mathcal{A}_\mu \mathcal{A}^\mu |W|^2 + g^2(\mathcal{A}_\mu W^\mu)(W^\dagger_\nu \mathcal{A}^\nu)$$

GAUGE FIELD KINETIC TERM



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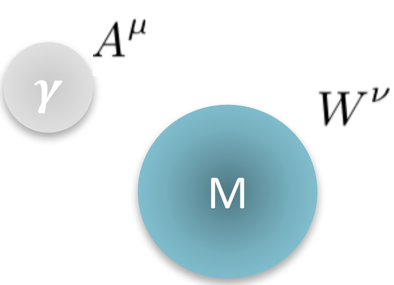
MONOPOLE KINETIC and MASS TERM



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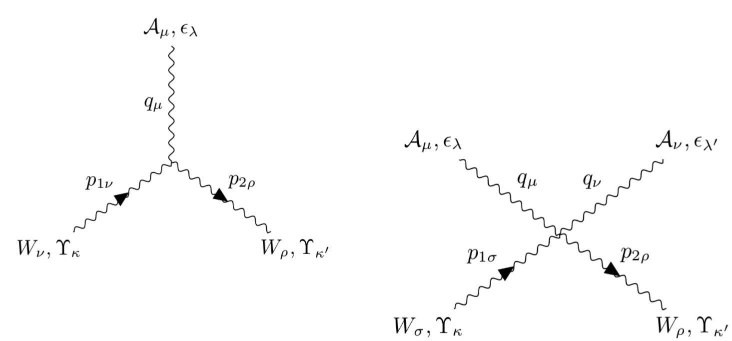
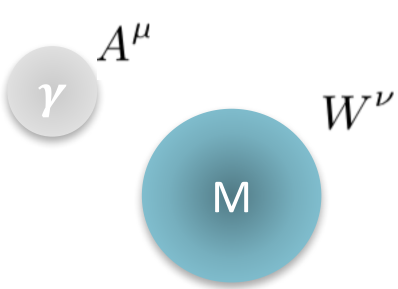
$$- ig\widetilde{W}^\dagger_{\mu\nu}\mathcal{A}^\mu W^\nu - g^2\mathcal{A}_\mu\mathcal{A}^\mu |W|^2 + g^2(\mathcal{A}_\mu W^\mu)(W^\dagger_\nu \mathcal{A}^\nu)$$

MMγ INTERACTIONS





THE LEE-YANG MODEL [1]

A Magnetic Moment Term Renders the Model Completely General

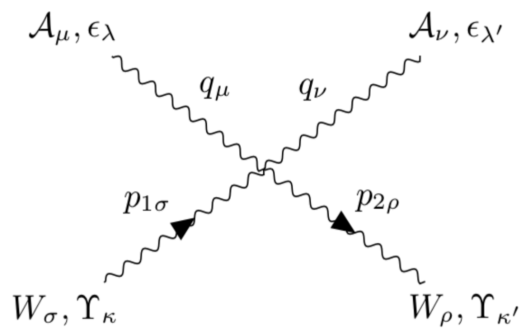
$$\mathcal{L} = -\xi(\partial_\mu W^{\dagger\mu})(\partial_\nu W^\nu) - \frac{1}{2}(\partial_\mu A_\nu)(\partial^\nu A^\mu) - \frac{1}{2}\widetilde{W}^\dagger_{\mu\nu}\widetilde{W}^{\mu\nu} - m^2 W^\dagger_\mu g^{\mu\nu} W_\nu + ig\mathcal{A}_\mu W^\dagger_\nu \widetilde{W}^{\mu\nu}$$
$$- ig\widetilde{W}^\dagger_{\mu\nu}\mathcal{A}^\mu W^\nu - g^2\mathcal{A}_\mu\mathcal{A}^\mu |W|^2 + g^2(\mathcal{A}_\mu W^\mu)(W^\dagger_\nu\mathcal{A}^\nu) - ig\kappa F^{\mu\nu} W^\dagger_\mu W_\nu .$$

GAUGE FIXING TERM
(covariance, renormalizability)

MAGNETIC MOMENT TERM
(Highly divergent, introduces ghosts, extra graphs)

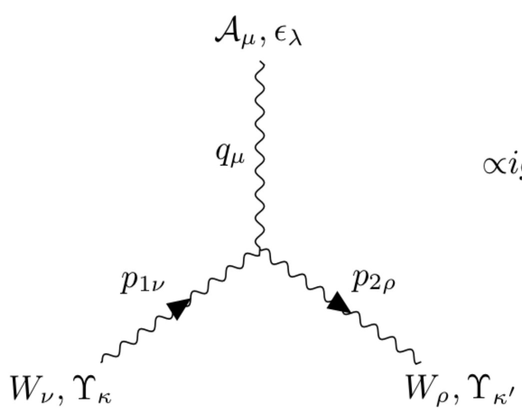
[1] T Lee, C Yang, Phys Rev 128.2 (1962)

THE FEYNMAN RULES



$$\propto -2ig^2(g^{\mu\nu}g^{\sigma\rho}) + ig^2(g^{\mu\sigma}g^{\nu\rho} + g^{\mu\rho}g^{\nu\sigma})$$

Unaffected by κ



$$\propto ig(-g^{\nu\mu}(-\kappa p_2 + \kappa p_1 + p_1)^\rho - g^{\mu\rho}(p_2 + \kappa p_2 - \kappa p_1)^\nu + g^{\rho\nu}(p_1 + p_2)^\mu)$$

κ dependence in the Feynman rule

You would recognize these as SM Feynman rules if $\kappa = 1$

THE LEE-YANG MODEL [2]

Before Weinberg-Salam model was verified experimentally, studies of angular distributions for different κ were used to study $W^{\pm}$ theories.

$$-ig\kappa F_{\mu\nu}W^{\dagger\mu}W^{\nu} \rightarrow -igF_{\mu\nu}W^{\dagger\mu}W^{\nu}$$

$\kappa = 1$

$g_{SM} = \pm 2$

- → Term appears naturally in SM Spontaneous Symmetry Breaking
- → This is the *only Renormalizable Model*
- → This is the *only Unitary Model* (convergence of $\sigma_{s \to \infty}$)
- → Generates Unique Differential Cross Section Distributions

[1] T. D. Lee Phys Rev 128.899 (1973)

[2] Tupper & Samuel Phys Rev D 23.9 (1981)

[3] Mikaelian, Samuel & Sahdev Phys Rev Lett 43.746 (1979)

THE LEE-YANG MODEL [2]

M M

Changing κ Leads to Observable Effects

$$-ig\kappa F_{\mu\nu}W^{\dagger\mu}W^{\nu}$$

- Magnetic moment (to 1st order) $\mu_M = \frac{g}{2M_W}(\kappa + 1)$
  - Gyromagnetic ratio $g_R = \kappa + 1$
  - Quadrupole moment (to 1st order) $Q_E = -\frac{g\kappa}{M_W^2}$
- Distinct angular distributions for different κ

CERN

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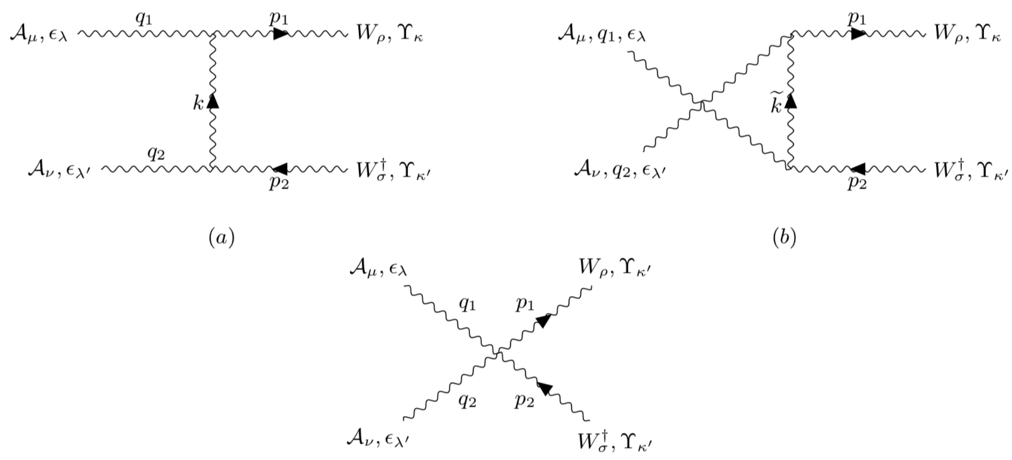
PHOTON FUSION AT TREE LEVEL

M M



WOLFRAM MATHEMATICA

PHOTON FUSION $\gamma\gamma$->MM

$$\frac{d\sigma_{\gamma\gamma\to M\bar{M}}}{d(cos\theta)}$$

$$\sigma_{\gamma\gamma\to M\bar{M}}$$

CERN

MoEDAL

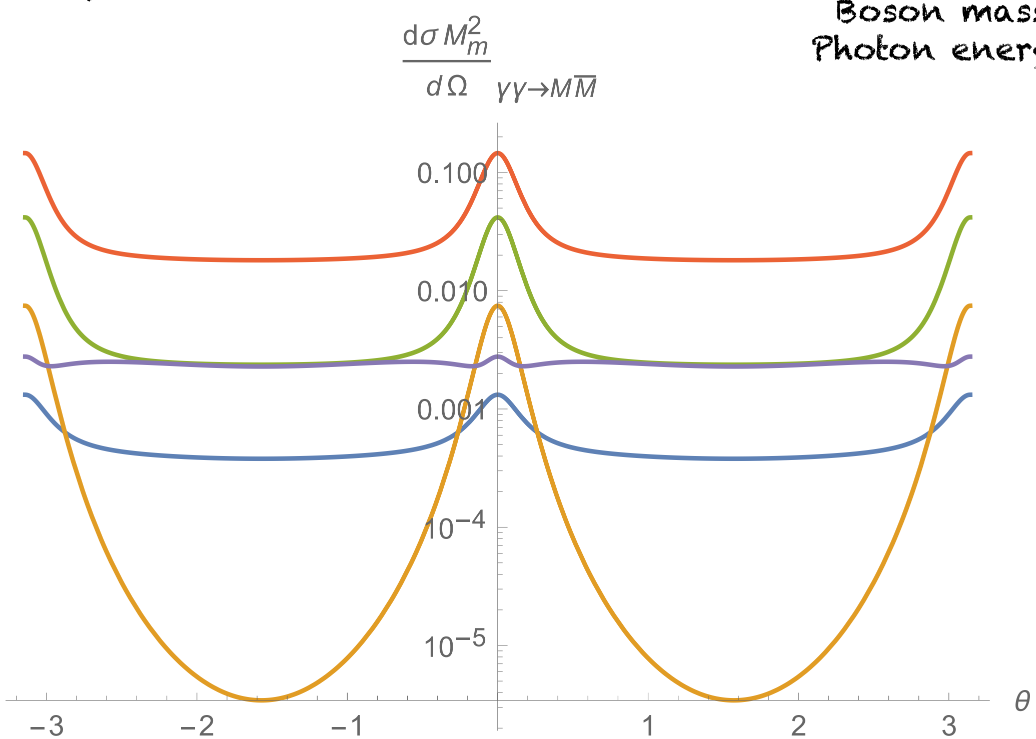
SPIN 1 PRODUCTION CROSS SECTIONS

Differential Cross Section Distributions for difference gyromagnetic ratios $g_R = \kappa + 1$

β independent

Boson mass $M_M = 1.5\,TeV$

Photon energy $E_\gamma = 6 M_M$



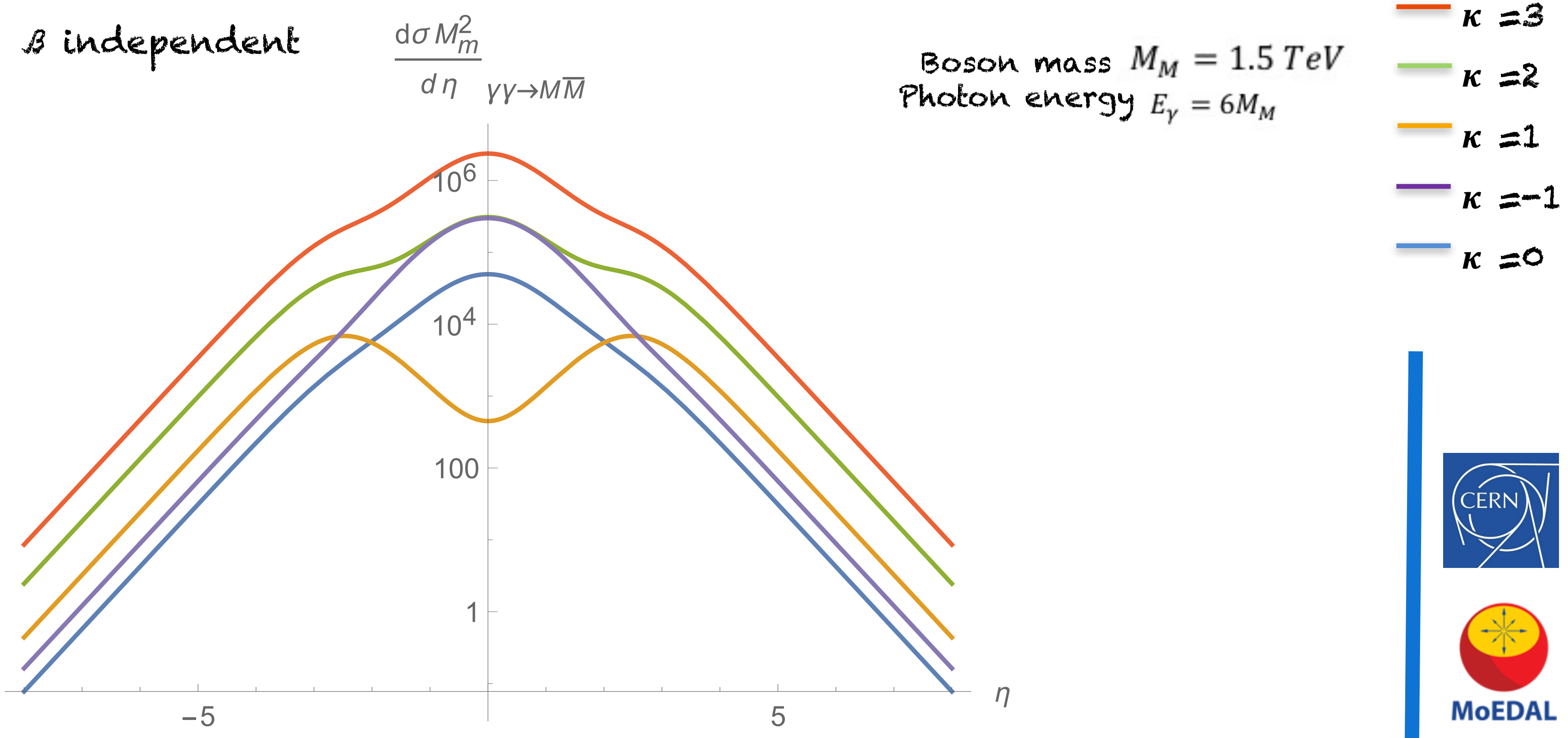
- $\kappa = 3$
- $\kappa = 2$
- $\kappa = 1$
- $\kappa = -1$
- $\kappa = 0$

SPIN 1 PRODUCTION CROSS SECTIONS

Differential Cross Section Distributions for difference gyromagnetic ratios $g_R = \kappa + 1$

β independent

Boson mass $M_M = 1.5\ TeV$
Photon energy $E_\gamma = 6M_M$



SPIN 1 PRODUCTION CROSS SECTIONS

The behaviour of the differential cross section changes with $\boldsymbol{\kappa}$

The differential cross section diverges unless κ=1

Standard Model Case

$$\frac{d\sigma_{\gamma\gamma\to M\bar{M}}}{d(cos\theta)} \propto E_\gamma^2 \quad \text{for all } \kappa \neq 1$$

$$\frac{d\sigma_{\gamma\gamma\to M\bar{M}}}{d(cos\theta)} \propto \frac{1}{E_\gamma^2} \quad \text{for } \kappa = 1$$

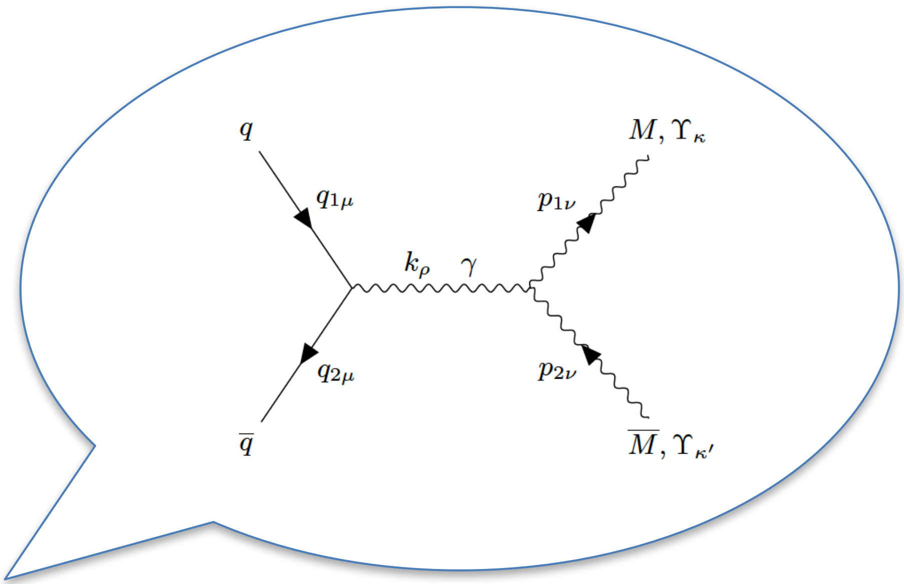
The behaviour of the total cross section at high $s=4E^2_\gamma$.

$$\sigma_{DY\ \gamma\gamma\to M\bar{M}(s\to\infty)} \propto E_\gamma^2$$

This is not a problem in an Effective Field Theory. Unitary can be resorted in an overarching theory at some higher energy scale.

DRELL-YAN TREE LEVEL

M M



WOLFRAM MATHEMATICA

$$\frac{d\sigma_{\gamma\gamma\to M\bar{M}}}{d(\cos\theta)}$$

$$\sigma_{\gamma\gamma\to M\bar{M}}$$

DRELL-YAN qq->MM

CERN

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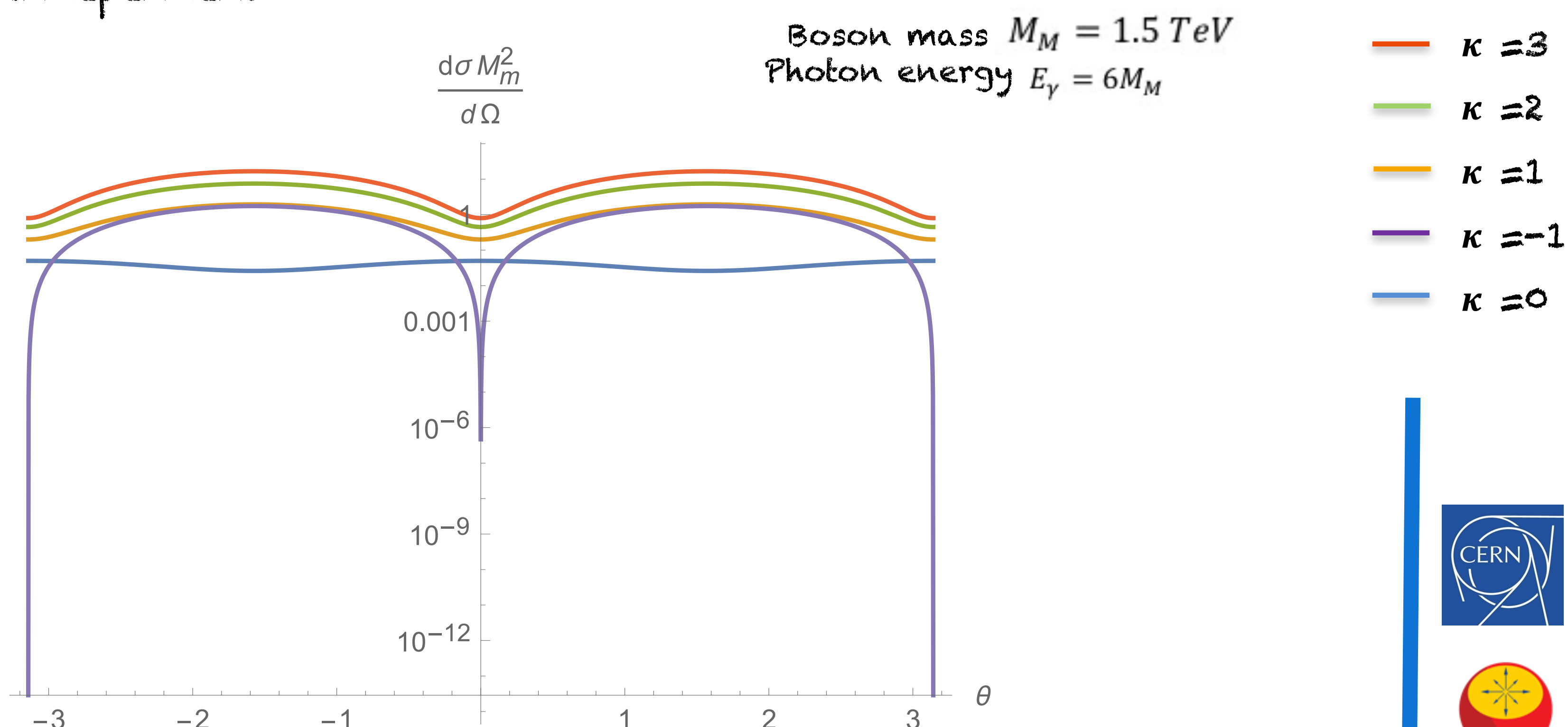
SPIN 1 PRODUCTION CROSS SECTIONS

Differential Cross Section Distributions for difference gyromagnetic ratios $g_R = \kappa + 1$

β independent

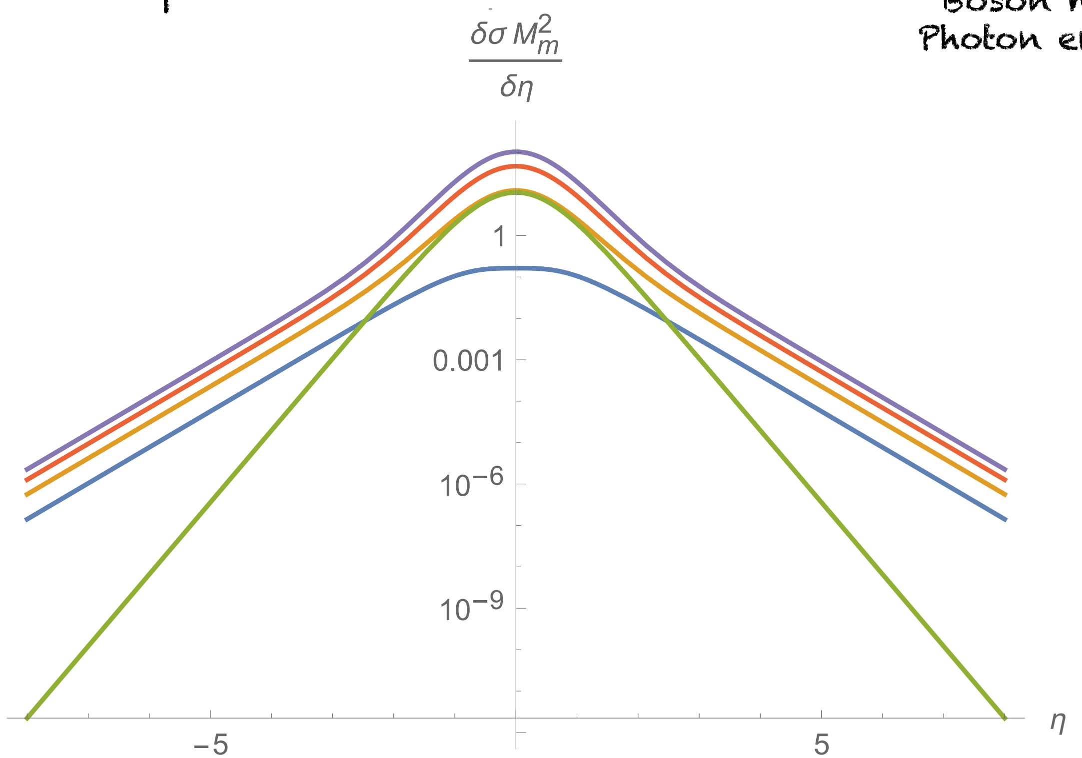


SPIN 1 PRODUCTION CROSS SECTIONS

Differential Cross Section Distributions for difference gyromagnetic ratios $g_R = \kappa + 1$

ß independent

Boson mass $M_M = 1.5\,TeV$
Photon energy $E_\gamma = 6M_M$



- $\kappa = 3$
- $\kappa = 2$
- $\kappa = 1$
- $\kappa = -1$
- $\kappa = 0$

SPIN 1 PRODUCTION CROSS SECTIONS

The behaviour of the cross section changes with $\boldsymbol{\kappa}$

The differential cross section goes as

$$\frac{d\sigma_{q\bar{q}\to M\bar{M}(s\to\infty)}}{d(\cos\theta)} \propto \frac{1}{s} \quad \text{if } \kappa = 0$$

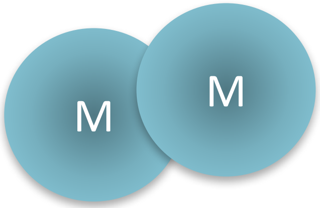
$$\propto s \quad \text{otherwise}$$

The cross section diverges unless $\kappa = 0$

$$\sigma_{DY\ (s\to\infty)} \to \frac{5\pi\alpha_e\alpha_g}{36M^4} s(\kappa^2 + (4\kappa^2 + 12\kappa + 10)\frac{M^2}{s})$$

This process is made regular in the SM though addition of the other SM production modes.

SPIN ½ PRODUCTION CROSS SECTIONS



Spin 1/2: Dirac QED

SPIN ½ LAGRANGIAN

"Dirac Quantum Electrodynamics is described by the U(1) Gauge invariant Lagrangian of a Spin ½ matter field"

$$\mathcal{L}_{Dirac} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(\gamma^{\mu}\partial_{\mu} - m)\psi + ie\bar{\psi}\gamma^{\mu}A_{\mu}\,\psi$$

GAUGE FIELD KINETIC TERM

SPIN ½ LAGRANGIAN

"Dirac Quantum Electrodynamics is described by the U(1) Gauge invariant Lagrangian of a Spin ½ matter field"

$$\mathcal{L}_{Dirac} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(\gamma^{\mu}\partial_{\mu} - m)\psi + ie\bar{\psi}\gamma^{\mu}A_{\mu}\,\psi$$

MONOPOLE KINETIC TERM
and MASS TERM

SPIN ½ MOMENT TERM

Making the Model Completely General by adding the Magnetic Moment Term

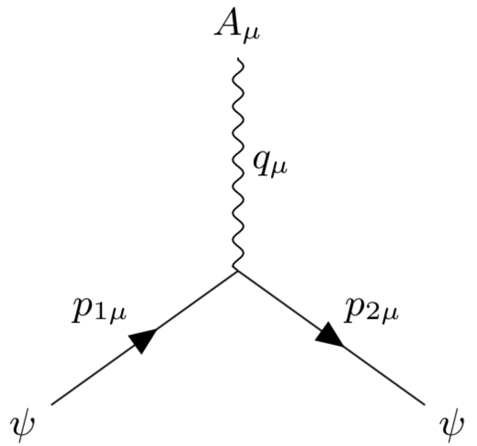
$$\mathcal{L}_{Dirac} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(\gamma^{\mu}\partial_{\mu} - m)\psi$$

$$+ie\bar{\psi}\gamma^{\mu}A_{\mu}\psi - ig\kappa\bar{\psi}F_{\mu\nu}[\gamma^{\mu},\gamma^{\nu}]\psi$$

Interaction Term

The second term is not in SM QED as the electron moment appears through anomalous spin interactions.

SPIN ½ FEYNMAN RULES

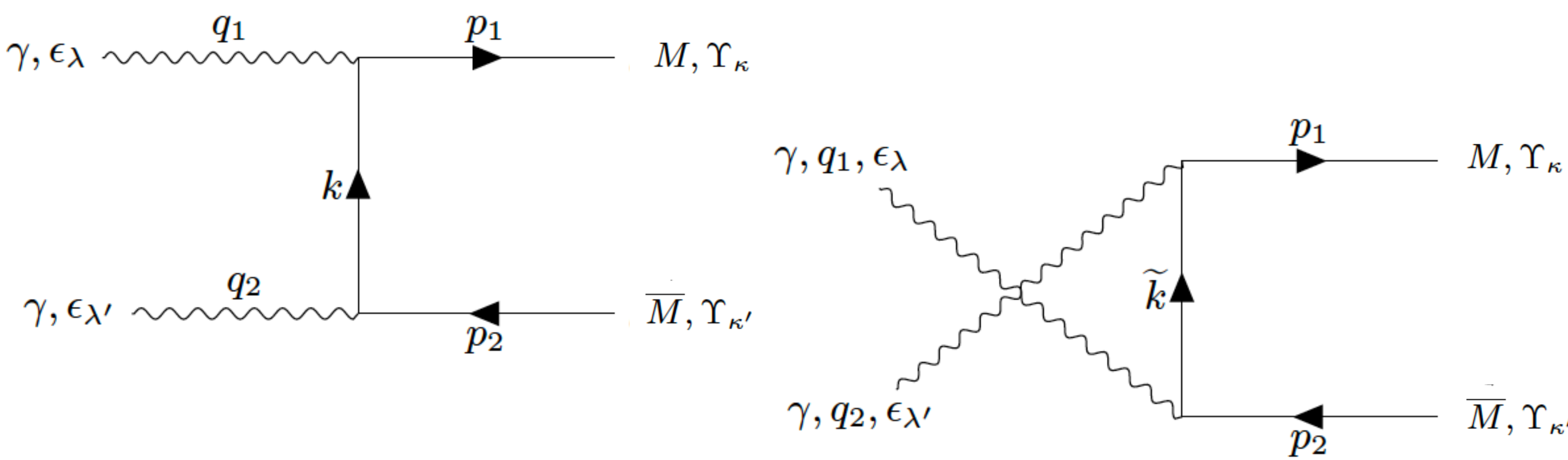


$$\propto -ig(\beta)\gamma^\nu - i\kappa\frac{1}{2}g(\beta)q_\mu[\gamma^\mu,\gamma^\nu]$$

κ dependence in the Feynman rule

You would recognize these as SM Feynman rules if $\kappa = 0$

PHOTON FUSION AT TREE LEVEL



PHOTON FUSION $\gamma\gamma$->MM

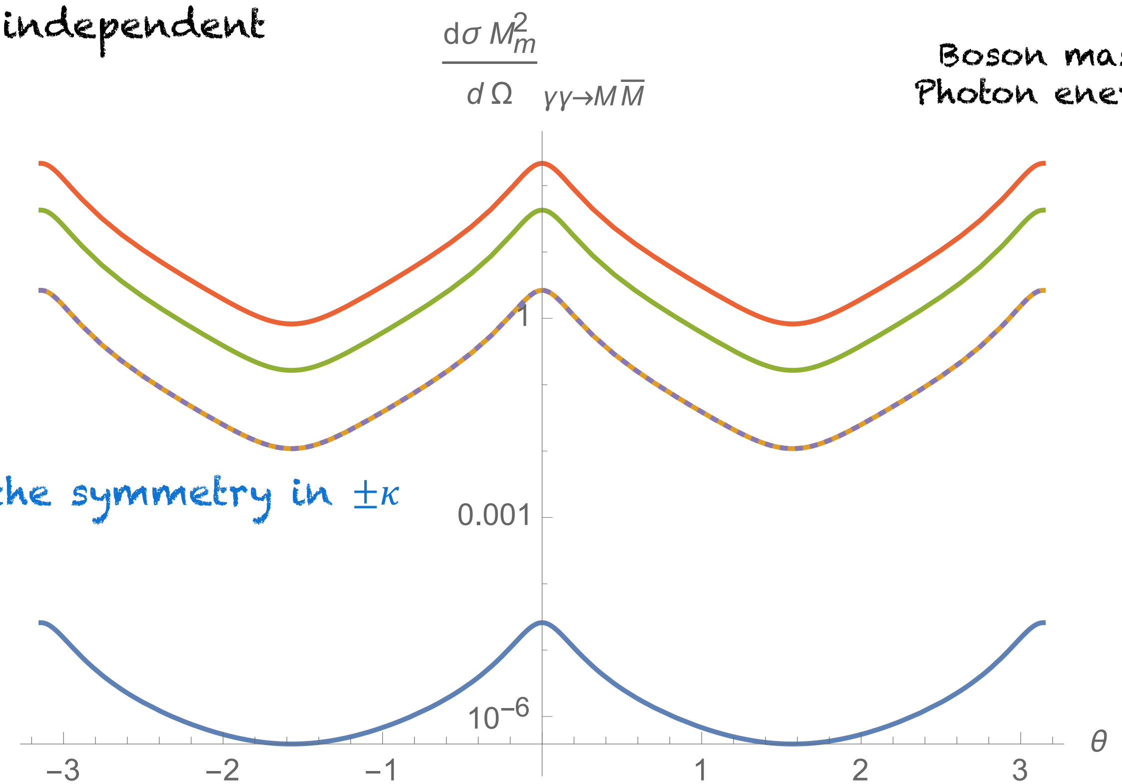
Enough cartoons for one presentation...

SPIN ½ PRODUCTION CROSS SECTIONS

Differential Cross Section Distributions for difference gyromagnetic ratios $g_R = \kappa + 1$

β independent

Boson mass $M_M = 1.5\ TeV$
Photon energy $E_\gamma = 6M_M$



- $\kappa = 3$
- $\kappa = 2$
- $\kappa = 1$
- $\kappa = -1$
- $\kappa = 0$

Note the symmetry in $\pm\kappa$

$\kappa \neq 0$ ensures a non-trivial production for all angles

SPIN ½ PRODUCTION CROSS SECTIONS

Differential Cross Section Distributions for
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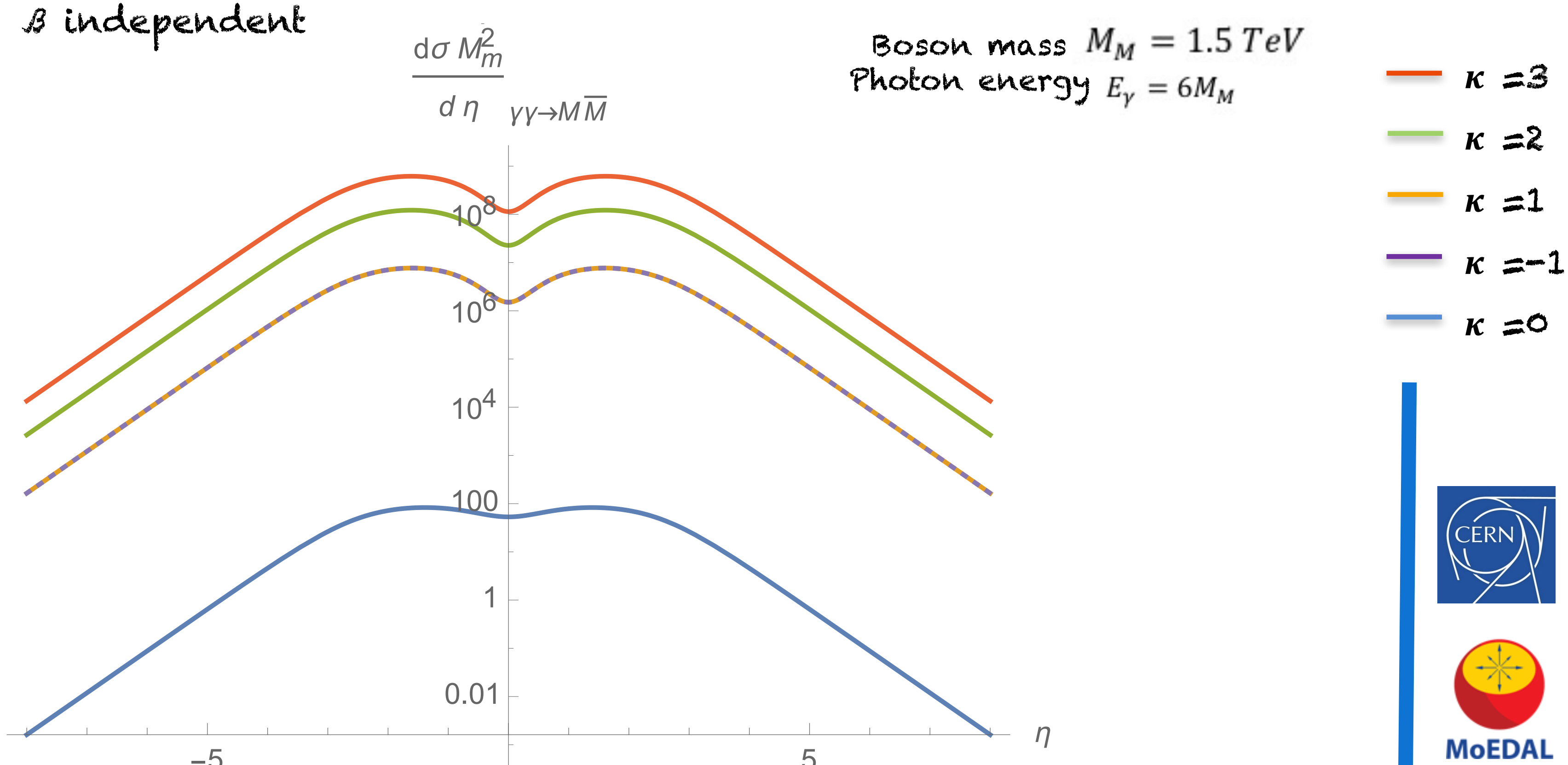
β independent



SPIN ½ PRODUCTION CROSS SECTIONS

The behaviour of the cross section stays quite similar for all κ, but the distribution scales with κ

The differential cross section diverges as

$$\frac{d\sigma_{\gamma\gamma\to M\bar{M}(s\to\infty)}}{d(cos\theta)} \propto E_\gamma^2$$

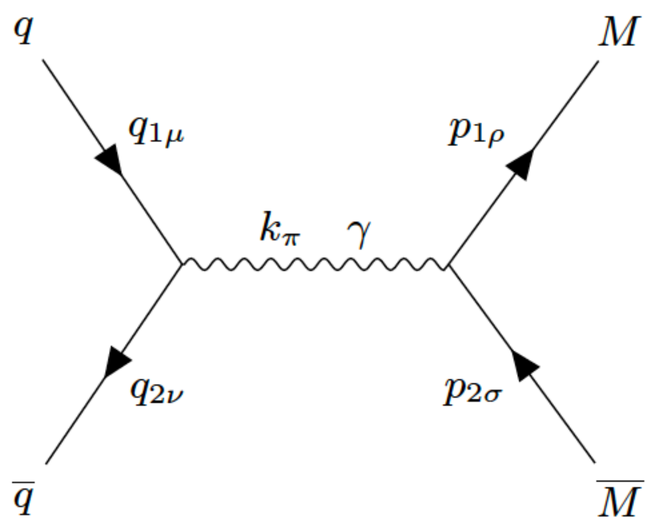
unless κ=0 is the Standard Model case for Dirac QED.

The total cross section also diverges unless κ=0

$$\sigma_{\gamma\gamma\to M\bar{M}(s\to\infty)} \to E_\gamma^2$$

DRELL-YAN AT TREE LEVEL

DRELL-YAN qq->MM



I assume quark masses are m=0

SPIN ½ PRODUCTION CROSS SECTIONS

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β independent

Boson mass $M_M = 1.5\ TeV$
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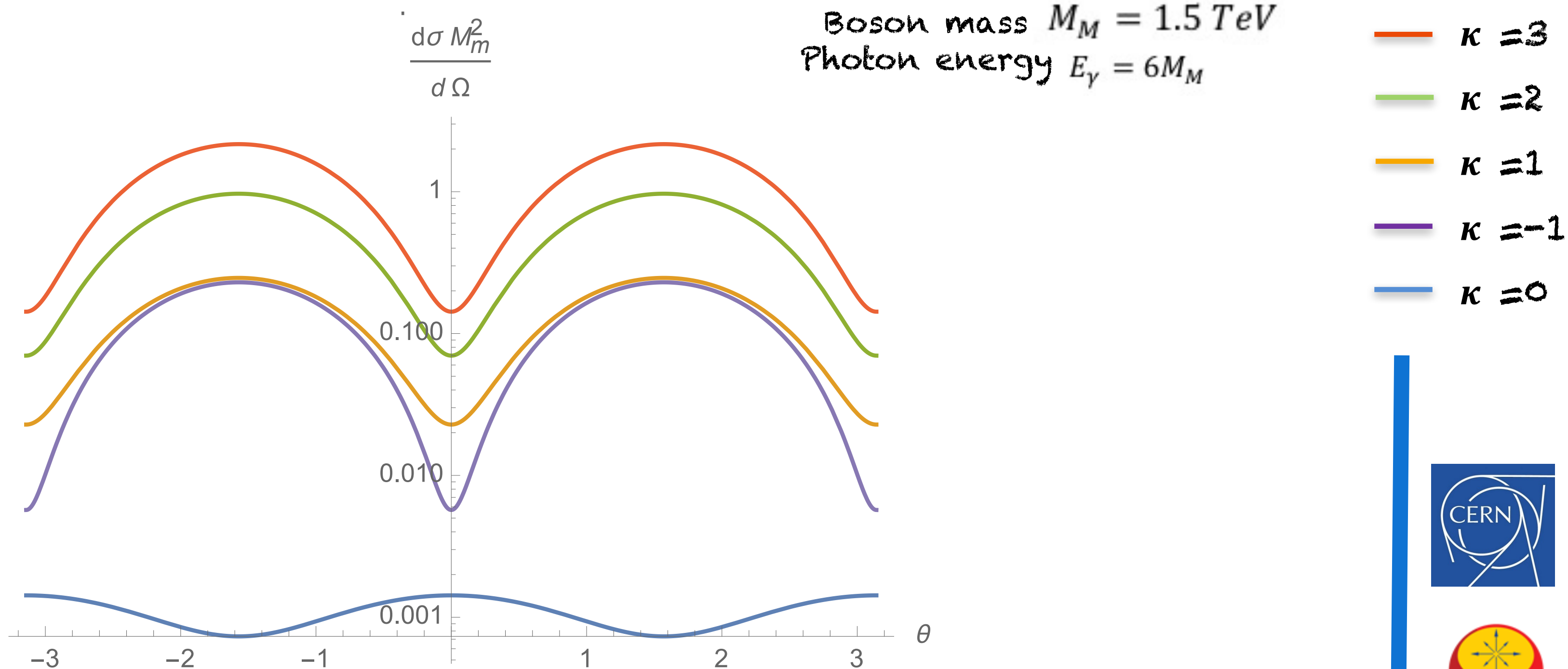

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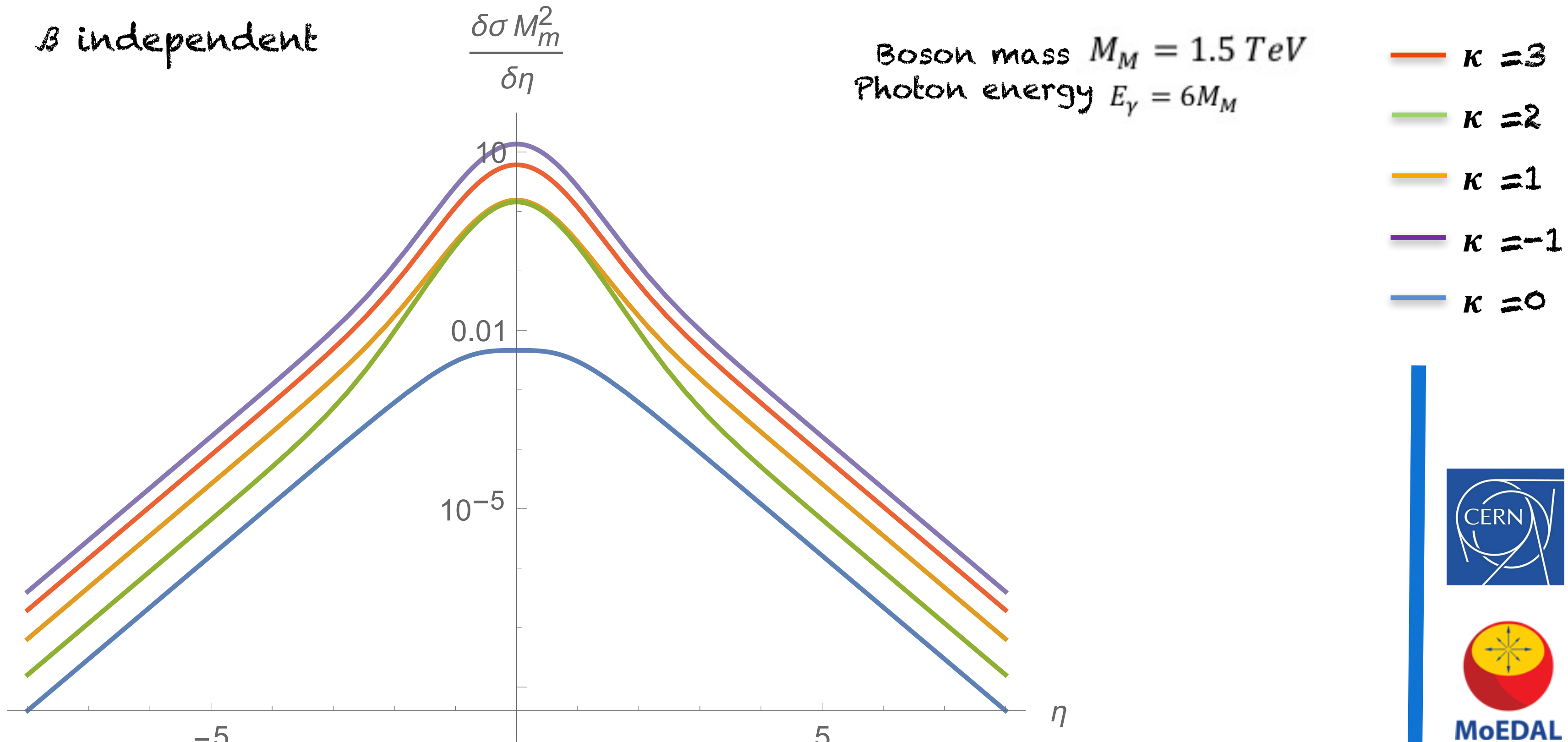

SPIN ½ PRODUCTION CROSS SECTIONS

The behaviour of the cross section changes with κ

$$\frac{d\sigma_{q\bar{q}\to M\bar{M}(s\to\infty)}}{d(\cos\theta)}$$ converges for all κ

The total cross section converges for all κ

$$\sigma_{DY\ (s\to\infty)} \to \frac{5\pi\alpha_e\alpha_g}{18M^2}\beta^2$$

$$\beta^2 = (1 - \frac{4M^2}{s})$$

CONCLUSION

MONOPOLE PRODUCTION

- The cross section distributions have been evaluated for monopole production in the spin 0, 1 and ½, for photon fusion and Drell-Yan processes
- The spin 1 and ½ have been treated more generally for arbitrary magnetic moments

CONCLUSION

The addition of a moment term in the Lagrangian of spin 1 and spin 1/2 monopoles

- Adds a new phenomenological parameter κ
- Gives different cross section distributions for different κ
- Constrains the theoretical fluidity in the search of a FT or EFT

STAY TUNED TO THE MOEDAL COLLABORATION PUBLICATIONS FOR MORE DETAILS ON THE MODELS

Thank you for listening