UV conformal window for asymptotically safe gauge-Yukawa theories

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What is Asymptotic Safety?

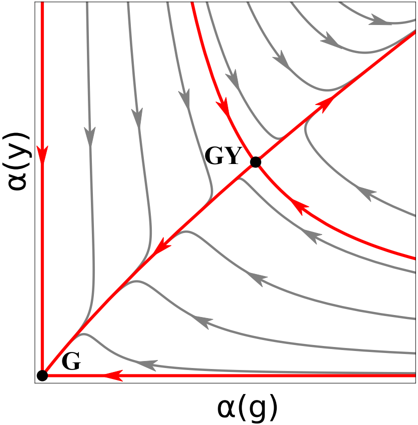
- Asymptotic Safety: theory exhibits UV fixed point
- theory parameters stay finite for high energies
- theory well defined and predictive in the high energy limit - UV completion
- interesting for model building
- no Landau poles (as in SM)

History

- Asymptotic Freedom: free fixed point $\alpha_i^* = 0$ ('73 Wilczek, Gross, Politzer)
- more general: interacting fixed point $\alpha_i^* \neq 0$
- extension of theory space with respect to Asymptotic Freedom
- originally introduced in quantum gravity ('79 Weinberg)
- well studied in various approximations

Asymptotic Safety in 4D Particle Physics

- exact prove of existence for interacting UV fixed point in 4D renormalizable gauge theory (D. Litim, F. Sannino: arXiv:1406.2337)
- Yukawa coupling is a necessary feature (A. Bond, D. Litim: arXiv:1608.00519)
- extended towards semi-simple gauge group (Bond, Litim, arXiv:1707.04217) and supersymmetry (Bond, Litim, arXiv:1709.06953)



Question: How can Asymptotic Safety be guaranteed within this model?

Gauge-Yukawa Model

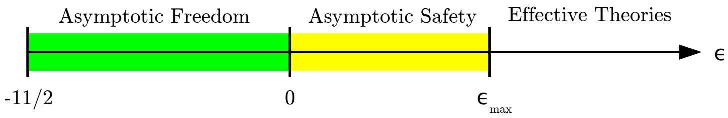
$SU(N_C)$ gauge group

N_F fundamental Fermions: Q_L, Q_R
$N_F \times N_F$ complex meson-like scalar gauge singlets: H

$$
\begin{aligned}
\mathcal{L} = &- \frac{1}{2}\mathrm{Tr}\, F^{\mu\nu} F_{\mu\nu} + \mathrm{Tr}\left(\overline{Q}\, i \not{D}\, Q\right) + \mathrm{Tr}\,(\partial_\mu H^\dagger\, \partial^\mu H) \\
&- \left(y\, \mathrm{Tr}\left(\overline{Q}_L\, H\, Q_R\right) + \mathrm{h.c.}\right) - u\, \mathrm{Tr}\,(H^\dagger H\, H^\dagger H) - v\, (\mathrm{Tr}\, H^\dagger H)^2
\end{aligned}
$$

Veneziano Limit

- mechanism to ensure perturbative control over fixed point
- set $N_F \to \infty$ and $N_C \to \infty$, but ratio fixed
- rescale:
  $\alpha_g = \frac{N_c g^2}{(4\pi)^2} \qquad \alpha_y = \frac{N_c y^2}{(4\pi)^2} \qquad \alpha_u = \frac{N_F u}{(4\pi)^2} \qquad \alpha_v = \frac{N_F^2 v}{(4\pi)^2}$
- new small, tunable parameter $\epsilon = \frac{N_F}{N_C} - \frac{11}{2}$
- conformal window:



- $\alpha^* = \epsilon\lambda_{\rm LO} + \epsilon^2\lambda_{\rm NLO} + \epsilon^3\lambda_{\rm N^2LO} + ...$
- $\lambda_{\rm LO}$: 2-loop β_g, 1-loop $\beta_{y,u,v}$ (D. Litim, F. Sannino: arXiv:1406.2337)
- $\lambda_{\rm NLO}$: 3-loop β_g, 2-loop $\beta_{y,u,v}$ (A. Bond, D. Litim, G. Medina, TS: arXiv:1710.07615)

Beta functions

$$\beta_g^{(1)} = \frac{4}{3}\epsilon\,\alpha_g^2$$
$$\beta_g^{(2)} = \left(25 + \tfrac{26}{3}\epsilon\right)\alpha_g^3 - 2\left(\tfrac{11}{2} + \epsilon\right)^2 \alpha_y\,\alpha_g^2$$
$$\beta_g^{(3)} = \left(\tfrac{701}{6} + \tfrac{53}{3}\epsilon - \tfrac{112}{27}\epsilon^2\right)\alpha_g^4 - \tfrac{27}{8}(11+2\epsilon)^2\alpha_g^3\alpha_y + \tfrac{1}{4}(11+2\epsilon)^2(20+3\epsilon)\alpha_y^2\alpha_g^2$$

$$\beta_y^{(1)} = (13+2\epsilon)\,\alpha_y^2 - 6\,\alpha_y\,\alpha_g$$
$$\beta_y^{(2)} = \tfrac{20\epsilon-93}{6}\alpha_g^2\,\alpha_y + (49+8\epsilon)\alpha_g\alpha_y^2 - 4\left[(11+2\epsilon)\alpha_y - \alpha_u\right]\alpha_u\alpha_y - \left(\tfrac{385}{8} + \tfrac{23}{2}\epsilon + \tfrac{\epsilon^2}{2}\right)\alpha_y^3$$

$$\beta_u^{(1)} = -(11+2\epsilon)\,\alpha_y^2 + 4\alpha_u(\alpha_y + 2\alpha_u)$$
$$\beta_u^{(2)} = \alpha_u\alpha_y\left[10\alpha_g - 16\alpha_u - 3(11+2\epsilon)\alpha_y\right] + (11+2\epsilon)\left[(11+2\epsilon)\alpha_y - 2\alpha_g\right]\alpha_y^2 - 24\alpha_u^3$$

$$\beta_v^{(1)} = 12\alpha_u^2 + 4\alpha_v\left(\alpha_v + 4\alpha_u + \alpha_y\right)$$
$$\beta_v^{(2)} = 8\alpha_v\alpha_y\left[\tfrac{5}{4}\alpha_g - 4\alpha_u - \alpha_v - (\tfrac{33}{8} + \tfrac{3}{4}\epsilon)\alpha_y\right] + (11+2\epsilon)\left[(11+2\epsilon)\alpha_y + 4\alpha_u\right]\alpha_y^2 - 8\alpha_u^2\left[12\alpha_u + 5\alpha_v + 3\alpha_y\right]$$

- extended by scalar two-loop equations (blue)

Fixed point

$$\alpha_g^* = 0.4561\epsilon + 0.7808\epsilon^2 + O\left(\epsilon^3\right)$$
$$\alpha_y^* = 0.2105\epsilon + 0.5082\epsilon^2 + O\left(\epsilon^3\right)$$
$$\alpha_u^* = 0.1998\epsilon + 0.4403\epsilon^2 + O\left(\epsilon^3\right)$$
$$\alpha_v^* = -0.1373\epsilon - 0.6318\epsilon^2 + O\left(\epsilon^3\right)$$

- same sign contributions
- no bound on ϵ from unphysical regime of complex coupling
- strong coupling bound from $|\alpha^*| > 0$: $\epsilon_{max} = 0.8767$
- vacuum stability unclear

Vacuum stability

- Condition for vacuum stability (D. Litim, M. Mojaza, F. Sannino: arXiv:1501.03061)

$$\alpha_u^* + \alpha_v^* > 0$$

- Next-to-leading order:

$$\alpha_u^* + \alpha_v^* = 0.0625\epsilon - 0.1915\epsilon^2$$

- upper bound: $\epsilon_{max} = 0.326$

Critcial exponents

- critical exponent ϑ: eigenvalues of $\frac{\partial \beta_i}{\partial \alpha_j}$ at the fixed point
- describe the flow around a fixed point

$$\begin{array}{lcc} \vartheta < 0 & \text{relevant,} & \text{UV attractive} \\ \vartheta > 0 & \text{irrelevant,} & \text{UV repulsive} \\ \vartheta = 0 & \text{marginal} & \end{array}$$

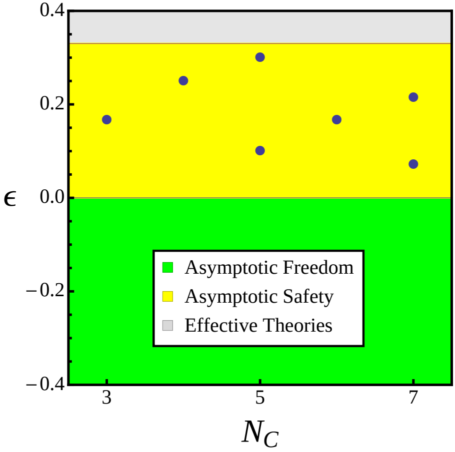
- change in sign of ϑ may indicate a fixed point merger
- next-to-leading order:

$$\vartheta_1 = -0.6082\epsilon^2 + 0.7067\epsilon^3 + 3.322\epsilon^4$$
$$\vartheta_2 = 2.737\epsilon + 6.676\epsilon^2$$
$$\vartheta_3 = 2.941\epsilon + 1.041\epsilon^2$$
$$\vartheta_4 = 4.039\epsilon + 9.107\epsilon^2$$

- $\vartheta_{2,3,4}$ UV repulsive
- bound on ϑ_1 to be UV attractive: $\epsilon_{max} = 0.335$

Conformal window - constraints from fixed points

- blue dots indicate the first integers for N_C, N_F to lie within Asymptotic Safety region
- upper bound from vacuum stability $\epsilon_{max} = 0.326$
- but: constraints from beta functions are tighter



Constraints from beta functions

- estimate constraints on the conformal window from beta functions directly
- ϵ - dependence due to $\alpha \sim \epsilon$

$$\beta^{(n)}_{y,u,v} \sim (1 \ldots \epsilon^n)\alpha^{1+n} \sim O\left(\epsilon^{1+n}\right)$$
$$\beta^{(1)}_g = \frac{4}{3}\epsilon\,\alpha_g^2 \sim O\left(\epsilon^3\right)$$
$$\beta^{(n>1)}_g \sim (1 \ldots \epsilon^n)\alpha^{1+n} \sim O\left(\epsilon^{1+n}\right)$$

- modify beta functions to cut out ϵ dependence which contributes to subleading terms
- subleading terms shift $\beta_{g,u,v}$ upwards, β_y downwards

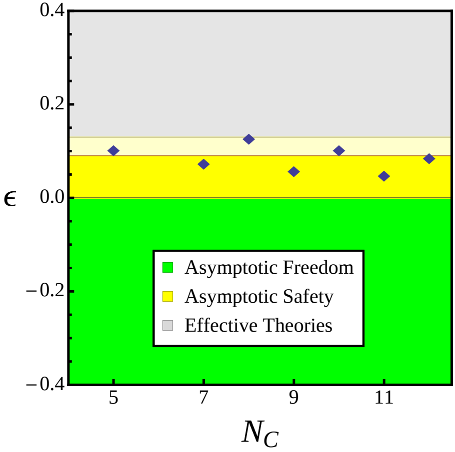
Constraints from beta functions - systematic analysis

- constraints for full beta functions ($\epsilon_{\text{subl.}}$) and only leading terms (ϵ_{strict})
- for different loop orders, constraints arise due to:
  a) strong coupling regime $\alpha > 1$
  b) fixed point mergers
  c) vacuum instability

Couplings	Orders in perturbation theory								
β_{gauge}	2	2	2	2	2	3	3	3	3
β_{Yukawas}	1	1	1	2	2	1	1	2	2
β_{quartics}	0	1	2	1	2	1	2	1	2
ϵ_{strict}	2.192^a	2.192^a	0.135^c	16.16^a	0.222^c	0.029^b	0.029^b	0.145^b	0.095^c
$\epsilon_{\text{subl.}}$	1.048^a	1.048^a	0.116^c	3.112^b	0.208^c	0.027^b	0.027^b	0.117^b	0.087^c

Conformal window - constraints from beta functions

- constraints tighter $\epsilon_{max} \approx 0.09...0.13$
- light yellow band denotes contraints from 321 case (merger)
- darker yellow band is constraint from 322 case (vacuum stability)
- blue dots indicate the first integers for N_C, N_F to lie within Asymptotic Safety region



Conclusions

- beta functions at NLO have been calculated and utilized to determine the system consistently to order ϵ^2
- UV conformal window studied at NLO, with vacuum stability giving the tightest constraints
- the conformal window is situated within the perturbative regime! - convenient for model building