Pion Scattering and Lattice QCD
UKQCD

James Richings
Supervised by: Dr A.Jüttner

University of Southampton

January 14, 2018

Plan for this talk

1. Why is Lattice QCD interesting & useful?
2. Pion Scattering
3. Why is any of this useful to you?
4. Frontiers in Lattice QCD

Why is low energy physics interesting?

- We know the SM is not the full picture: Dark matter, CP-violation ... we are missing something.
- After a many years of searching we have not detected a signal of BSM Physics. Despite a few 3 sigma hints (R.I.P 750 GeV)
- "So its time to get the shovel and start digging."-David Newbold (CMS)

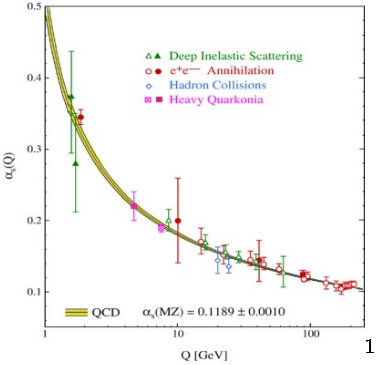
However a number of resonances have started to apear in the hadronic sector.

Where do we have large errors?

- Hadronic (QCD) uncertainties are often the dominant errors in theoretical predictions.

Why is low energy difficult?

- Perturbation theory breaks down when applied to QCD at low energies.
- Due to perturbations in the strong force becoming large at low energy. i.e. coupling increases as energy decreases.

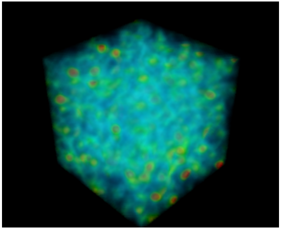


[1]

[1] https://arxiv.org/pdf/hep-ex/0606035.pdf

How does Lattice QCD help?

QCD Lagrangian:



$$\mathcal{L}_{QCD} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a + \sum_f \bar{\psi}_f(i\gamma^\mu D_\mu - m_f)\psi_f$$

Lattice Fermion action:

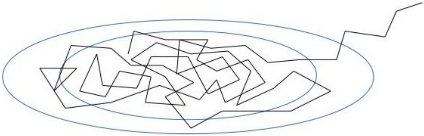
$$S_F[U,\psi,\bar{\psi}] = a^4 \sum_{n,m\in\Lambda}\sum_{f=1}^{N_f} \bar{\psi}^{(f)}(n)D^{(f)}(n|m)\psi^{(f)}(m)$$

How do you solve a path integral without perturbation theory?

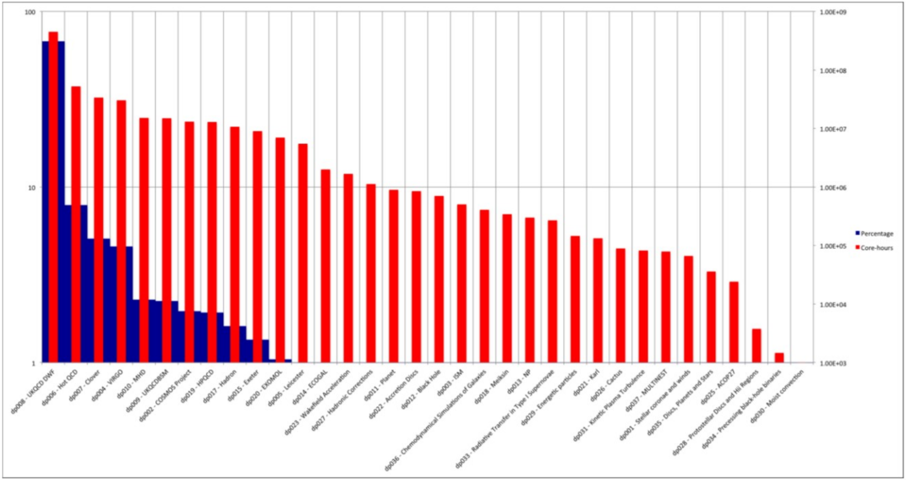
$$<\hat{\mathcal{O}}>= \frac{1}{Z}\int \mathcal{D}[U,\psi,\bar{\psi}]e^{-S_{Lat}[U,\psi,\bar{\psi}]}\mathcal{O}[\psi,\bar{\psi}]$$

- Boltzman factor

$$<\mathcal{O}>= \lim_{N\to\infty}\frac{1}{N}\sum_{n=1}^{N}\mathcal{O}_n$$



Trade Off: Computing time



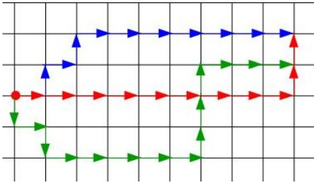
[2]

[2] DiRAC-3 Technical Case

Trade Off: Computationally Expensive

Discretised path integrals can be evaluated by means of Hybrid Monte Carlo methods.

- Theoretically possible.
- Practically hazardous...



The propagator is the sum over paths of link varriables.

Scale of a propagator:

$$4_{spin} \times 3_{colour} = 12_{dof\,/\,site}$$

Lattice sizes vary but $\approx (96_{spacial})^3 \times 192_{time}$.

$$12 \times 96^3 \times 192 = 2038431744_{floats}$$

$$(2038431744_{floats} \times 8_{bytes} \times 2_{complex})^2 = Large$$

Lattice QCD: Current state of the art

What can we do?

- Simulations of QCD with dynamic (sea) u,d,s,c quarks with physical masses.
- Inverse lattice spacing $a^{-1} \leq 4GeV$
- Volume $L \leq 6fm$

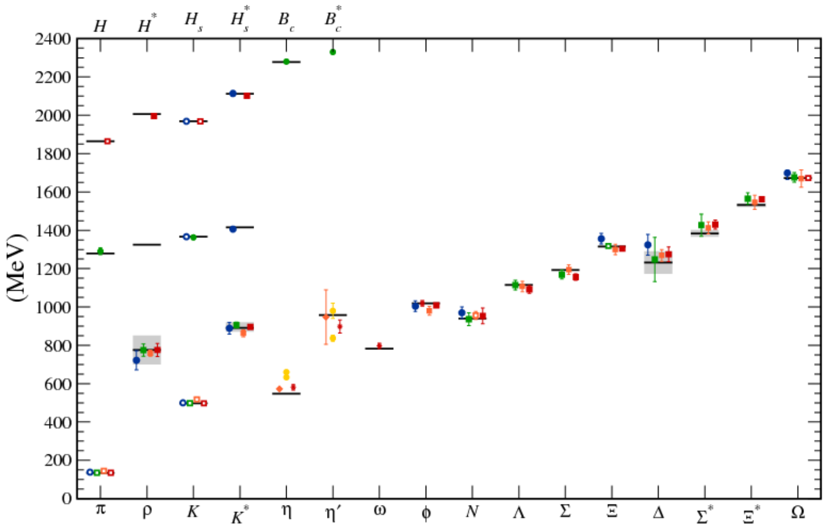
- Standard: Hadron Spectroscopy
- Difficult: Interractions of pairs of hadrons.
- "Under construction": multi channel final states.

Parameter tuning

- Tune light quark mass am_l such that: $\frac{am_\pi}{am_p} = \frac{am_\pi^{PDG}}{am_p^{PDG}}$
- Tune strange quark mass: $\frac{am_\pi}{am_K} = \frac{am_\pi^{PDG}}{am_K^{PDG}}$
- Fix lattice spacing $a = \frac{af_\pi}{f_\pi^{PDG}}$

Once these parameters are "tuned" theory is fully predictive! No more input required.

Lattice QCD and the Hadronic spectrum



[3] [4]

[3]b-flavored meson masses are offset by -4000MeV.

[4]http://inspirehep.net/record/1092992/plots

Other Lattice Effects

Regualisation

IR regulator

- The finite size of the box.

UV regulator

- The inverse lattice spacing.

Discretisation errors

$$\frac{d\psi(x)}{dx} = lim_{a\rightarrow 0}\frac{\psi(x+a)-\psi(x-a)}{2a} + O(a^2)$$

- We often run a number of simulations of the same physics with different lattice spacings.
- Extrapolation techniques can be used to estimate the continuum limit. $a \rightarrow 0$

Finite volume errors

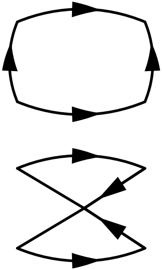
- We have to settle for a reasonable box size and deal with the finite volume errors.

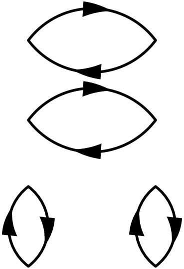
Pion Scattering: $\pi\pi \to \pi\pi$

- Two possible states $I = 0, 2$

$$|00> = \frac{1}{\sqrt{3}}\left(|\pi^+\pi^- > -|\pi^0\pi^0 > +|\pi^-\pi^+ > \right)$$
$$|22> = |\pi^+\pi^+ >$$

- We have to be careful to constrain our simulation so that we stay below the inelastic limit.





Pion Scattering: The Correlator

Correlator of a pion created, propagating and then annihilated:

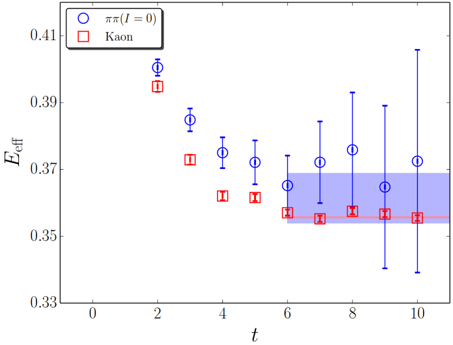
$$< 0|\mathcal{O}_\pi(t)\mathcal{O}_\pi(0)^\dagger|0> = \frac{1}{Z}\int \mathcal{D}[U,\psi,\bar{\psi}]\mathcal{O}_\pi(t)\mathcal{O}_\pi(0)^\dagger e^{-S[U,\psi,\bar{\psi}]}$$

$$\begin{aligned} < 0|\mathcal{O}_\pi(t)\mathcal{O}_\pi(0)^\dagger|0> &= \sum_n < 0|\mathcal{O}_\pi(t)|n >< n|\mathcal{O}_\pi(0)^\dagger|0> \\ &= \sum_n |< 0|\mathcal{O}_\pi(0)|n >|^2 e^{-E_n t} \\ &= |A_\pi|^2 e^{-E_\pi t}(1 + \mathcal{O}(e^{-\Delta E t})) \end{aligned}$$

What is the form of the pion correlator that we want to fit to our data?

$$C_{\pi\pi} =< \mathcal{O}_{\pi\pi}(t)\mathcal{O}_{\pi\pi}(0)^\dagger >= |A_{\pi\pi}|^2\left(e^{-E_{\pi\pi}t} + e^{-E_{\pi\pi}(T-t)}\right) + K$$

$$E_{\pi\pi} = ln\left(\frac{C_{\pi\pi}(t+2) - C_{\pi\pi}(t+1)}{C_{\pi\pi}(t+1) - C_{\pi\pi}(t)}\right)$$



[5]

Result: $E_{\pi\pi}^{I=0} \approx 0.36$ in lattice units, $aE_{\pi\pi}^{I=0} = 498MeV$

[5] https://arxiv.org/pdf/1505.07863.pdf

Pion Scattering: Phase shift

Luscher quantisation condition:

$$n\pi - \delta(k) = \phi(q)$$

We calculate the pion momentum:

$$k = \sqrt{\frac{E_{\pi\pi}^2}{4} - m_\pi^2}, \qquad q = \frac{Lk}{2\pi}$$

Therfore we have an equation for the phase shift:

$$\delta(k) = -\arctan\left(\frac{-q\pi^{3/2}}{Z_{00}(1;q^2)}\right)$$

The result is for the S wave scattering is:

$$\delta_0(k)_{lat} = 23.8^o(4.9^o)$$ [6]

$$\delta_0(k)_{ch.sym/exp} \approx 30 - 35^o$$ [7]

[6] https://arxiv.org/pdf/1505.07863.pdf

[7] https://arxiv.org/pdf/hep-ph/0103088.pdf

CP Violation and Kaon Decays

The experimental measure of CP violation:

$$Re\left(\frac{\epsilon'}{\epsilon}\right) = Re\left(\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\epsilon}\left[\frac{ImA_2}{ReA_2} - \frac{ImA_0}{ReA_0}\right]\right)$$

This is dependent on the pion scattering phases δ_0 we have just evaluated.

$$Re(\epsilon'/\epsilon)_{Lat} = 1.38(5.15)(4.59) \times 10^{-4}$$ [8]

$$Re(\epsilon'/\epsilon)_{Exp} = 16.6(2.3) \times 10^{-4}$$

Improve by increasing statistics:

$$\frac{1}{\sqrt{N}} = \frac{1}{\sqrt{216}} = \frac{1}{14.7} = 0.068$$
$$\frac{1}{\sqrt{N}} = \frac{1}{\sqrt{1000}} = \frac{1}{31.6} = 0.032$$

[8] https://arxiv.org/pdf/1505.07863.pdf

Frontiers in Lattice QCD

- QED effects
  - QED not currently included due to finite volume effects. Photons iteract over long range.
  - Large volumes not currently available, so use effective theory to subtract finite volume effect.
  - Analytically compute the difference between the finite volume and infinite volume self energies.
  - QED in meason decay: Assume photons are soft and do not resolve the hadronic structure.
- Isospin breaking effects
  - Currently assusme $d = u = l$ and have equal mass.

Conclusion

- Lattice QCD is a method for making predictions of the SM in a low energy regime.
- Introducing previously challenging effects and using increased statistics we can drive down errors
- This means we can tightly constrain the Standard Model and hopefully break it in the hadronic sector.

Thank you

Conclusion

- Lattice QCD is a method for making predictions of the SM in a low energy regime.
- Introducing previously challenging effects and using increased statistics we can drive down errors
- This means we can tightly constrain the Standard Model and hopefully break it in the hadronic sector.

Thank you

Mesons on the Lattice

Now we have a way of calculating the correlators we need operators:

$$\mathcal{O}_M(n) = \bar{\psi}^{(f_1)}(n)\Gamma\psi(n)^{(f_2)}$$

Operator product of a meason propagating from a point n to m:

(supressed intergral and indices)

$$\begin{aligned} < \mathcal{O}_T(n)\bar{\mathcal{O}}_T(m) > &= < \bar{d}(n)\Gamma u(n)\bar{u}(m)\Gamma d(m) > \\ &= \Gamma\Gamma < \bar{d}(n)u(n)\bar{u}(m)d(m) > \\ &= \Gamma\Gamma < u(n)\bar{u}(m) >< d(m)\bar{d}(n) > \\ &= \Gamma\Gamma D_u^{-1}(n|m)D_d^{-1}(n|m) \\ &= tr[\Gamma D_u^{-1}(n|m)\Gamma D_d^{-1}(n|m)] \end{aligned}$$

Final form of a hadron correlator:

$$< \mathcal{O}_\pi(n)\bar{\mathcal{O}}_\pi(m) >= \frac{1}{Z}\int \mathcal{D}e^{-S_{Lat}} det[D_u]det[D_d]tr[\Gamma D_u^{-1}(n|m)\Gamma D_d^{-1}(n|m)]$$

form of the Zeta function

The form of the zeta function is:

$$Z_{00}(s; q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\vec{n} \in \mathbb{Z}^3} (\vec{n}^2 - q^2)^{-s}$$

Phase shift of Pion Scattering

Luscher quantisation condition:

$$n\pi - \delta(k) = \phi(q)$$

we relate k and q in the following way:

$$q = \frac{Lk}{2\pi}$$

We calculate the pion momentum:

$$k = \sqrt{\frac{E_{\pi\pi}^2}{4} - m_\pi^2}$$

but we dont calculate the mass we have E_π. To calculate the mass we can use the relation:

$$m_\pi = \sqrt{E_\pi^2 - 3\left(\frac{\pi}{L}\right)^2}$$

Analysis: Effective Mass

$$E_{\pi\pi} = ln\Big(\frac{C_{\pi\pi}(t)}{C_{\pi\pi}(t+1)}\Big) \tag{1}$$

$$E_{\pi\pi} = ln\Big(\frac{C_{\pi\pi}(t+2) - C_{\pi\pi}(t+1)}{C_{\pi\pi}(t+1) - C_{\pi\pi}(t)}\Big) \tag{2}$$

Pion Scattering: Analysis Schematic

1. Raw data from simulation output.
2. Form of the data is a $C \times T$ matrix (Configurations, Timeslices)
3. Perform Jackknife or Bootstrap Resampling.
4. Calculate the covarriance matrix.
5. Fit functional form of the correlator to data.
6. The result is one value for each of the fitted free parameters this is done for each configuration.
7. Average to get central value and take standard deviation to get the error.