Imperial College London

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Incoherent Mapping of the Gravitational Wave Background

Using 2+ earth-based detectors

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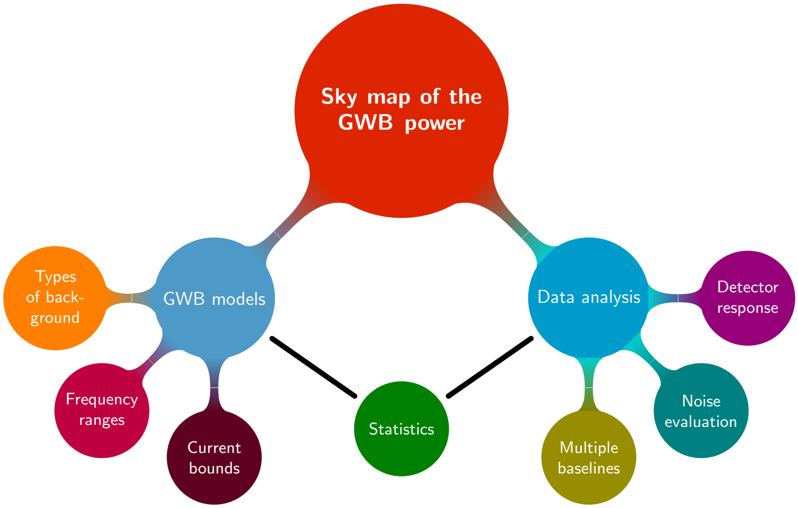
Sky map of the GWB power
GWB models
Types of back-ground
Frequency ranges
Current bounds
Statistics
Data analysis
Detector response
Noise evaluation
Multiple baselines

Introduction: what is the GWB?

Energy spectrum of the Gravitational Wave Background [Allen and Romano, 1999]:

$$\Omega_{GW}(f) = \frac{1}{\rho_c} \frac{d\rho_{GW}}{d \ln f} \tag{1}$$

Astrophysical bgd

overlap of unresolved signals:

- Compact binary inspirals
- Asymmetric SN explosions
- Gravitational captures
- ..

Cosmological bgd

sourced by:

- Inflation
- Phase transitions
- Cosmic strings
- ..

Focus: **stationary background** of multiple overlapping sources.

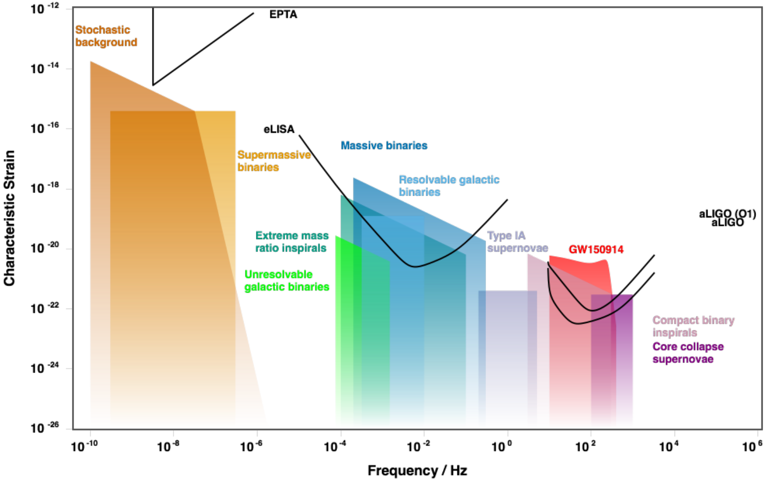

Figure 1: Current constraints on the GWB and models, from [Moore et al., 2014]

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An incoherent approach

In general, the background will be an **incoherent superposition** of GWs, with a **characteristic (anisotropic) power on the sky**, so map **Stokes' Param.s:**

I	**Q**	**U**	**V**
total power	$+/\times$	R/L	R/L

Frequency band will be detector dependent $\Rightarrow$ so will the probed background origin.

$\Rightarrow$ **Incoherent approach**: integrate signal over detector frequency range.

Objectives and motivation

The goal is to **map gravitational wave anisotropies on the sky**, using the detectors we have.

astrophysical bgd:	investigate large scale structure, intrinsic alignment, other polarisation properties
cosmological bgd:	highly model and frequency dependent, not in band...

Limit for inspiral-dominated GWB [Abbott et al., 2017]:

$$\Omega_{2/3}(\boldsymbol{n}) < 2 - 6 \times 10^{-8}\,\mathrm{sr}^{-1}$$

obtained in a specific, 2-detector frame. The multiple baseline search needs a more general frame and will be more constraining.

What we measure

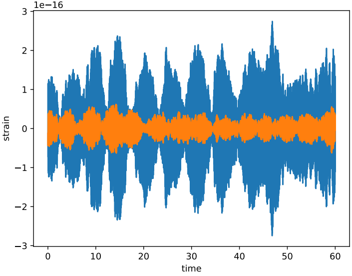

Figure 2: Signal in time

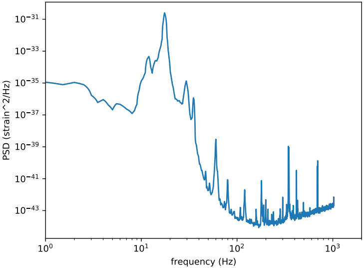

Figure 3: Power Spectrum Density

The measured strain is

$$h = F_{+}(\boldsymbol{n})\, h_{+}(\boldsymbol{n}, f) + F_{\times}(\boldsymbol{n})\, h_{\times}(\boldsymbol{n}, f) \qquad (2)$$

F_{+} and $F_{\times}$ are the **polarisation response functions** of the detector - depend on (lat,lon), pol. response, position of earth w.r.t. pol. basis $\Rightarrow \boldsymbol{n} \equiv \boldsymbol{n}(t)$.

Correlating detecors, creating a beam on the sky

Correlate detectors to eliminate detector noise. Focus on GWB intensity $\boldsymbol{I}$ on the sky

$$I \propto < h_+ \, h'^{\star}_+ > + < h_\times \, h'^{\star}_\times >$$

where $h_+(\boldsymbol{n}, f)$ and $h'_+ \equiv h_+(\boldsymbol{n'}, f')$ from $\neq$ detectors.

The combined response to I of a single baseline (pair of dect.s) is

$$\gamma_{I,\,ab}(\boldsymbol{n}) = F^+_a F^{+\,\star}_b + F^\times_a F^{\times\,\star}_b \tag{3}$$

a, b: dect. labels.

note: $\neq$ pol. modes have $\neq$ **overlap functions**; we use **quaternions** to deal with geometry+polarisation.

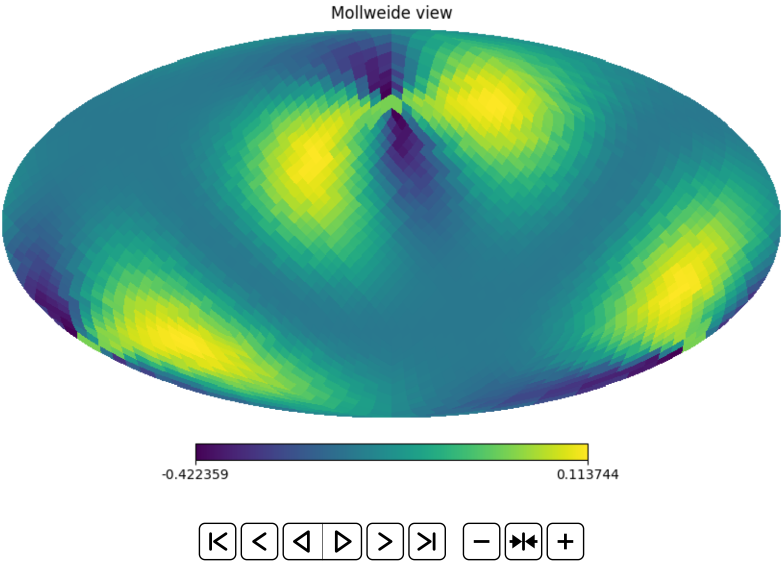

Figure 4: The overlap function γ_I on the sky for the LIGO detector pair.

Noise-domination

The correlated signal $< H_a \, H_b'^{\star} > \equiv d_t(\boldsymbol{b}, f)$ is[1]

$$d_t(\boldsymbol{b}, f) = \delta_{\boldsymbol{n},\boldsymbol{n}'} \, \delta_{f,f'} \int_{S^2} d\boldsymbol{m} \, \gamma_{I,\,ab}(\boldsymbol{m}) \, I(f,\, \boldsymbol{m}) \, e^{2\pi i f \boldsymbol{m}\cdot\boldsymbol{b}} \tag{4}$$

this is noise-dominated:

$$d_t(\boldsymbol{b}, f) = s_t(\boldsymbol{b}, f) + n_t(f)\,. \tag{5}$$

$\Rightarrow$ **The PSD of the noise is just the PSD of the data:**

$$< n_{t,\,a}(f) \, n^{\star}_{t,\,b}(f') > = \delta_{tt'} \delta_{ff'} N_{t,\,t'}(f, f')\,, \tag{6}$$

$$N_{t,\,t'}(f, f') = P_{t,\,a}(f) P_{t',\,b}(f')\,, \qquad P_t(f) = < d_t(f) >^2\,.$$

[1] if all anisotropies are 0.

Making the map: spherical harmonics decomposition

$$d_t(\boldsymbol{b}, f) = \int_{S^2} d\boldsymbol{m}\, I(\boldsymbol{m})\, \gamma_I(\boldsymbol{m})\, E(f)\, e^{2\pi i f\, \boldsymbol{b}\cdot\boldsymbol{m}} \tag{7}$$

is the correlated data as a function of frequency f and the **baseline** $\boldsymbol{b}(t)$; decompose it:

$$\begin{aligned} d_t(\boldsymbol{b}, f) &= \sum_{lm} d^t_{lm}(f)\, Y^\star_{lm}(\hat{b})\,, \\ d^t_{lm} &= 4\pi i^l F_l(b) \sum_{LM,L'M'} a^I_{LM} \gamma^I_{L'M'} K_{LM,L'M',lm}\,. \end{aligned} \tag{8}$$

$K_{LM,L'M',lm}$: coupling kernel. Summing over (lm), $(L'M')$ we get

$$d_t(\boldsymbol{b}, f) = \sum_{LM} a^I_{LM} M^I_{LM}(\boldsymbol{b}, f)\,. \tag{9}$$

Making the map: maximum likelihood map

Extract $\boldsymbol{a}^{I}_{LM}$ by minimizing the χ^2 of the map given the data [Thrane et al, 2009]; defining the **dirty map** z_{LM}:

$$z_{LM} := \sum_{T,f} \frac{M^{I\star}_{LM}(\hat{b})}{N(f)} d^T(f,\hat{b})\,; \tag{10}$$

$$z_{LM} = \sum_{\tilde{L}\tilde{M}} A_{\tilde{L}\tilde{M}LM} a^{I}_{\tilde{L}\tilde{M}}\,, \quad A_{\tilde{L}\tilde{M}LM} = \sum_{T,f} \frac{M^{I}_{\tilde{L}\tilde{M}}(\hat{b}) M^{I\star}_{LM}(\hat{b})}{N(f)}\,, \tag{11}$$

and the map solution which maximizes likelyhood is

$$a^{I}_{\tilde{L}\tilde{M}} = \sum_{LM} \left(A_{\tilde{L}\tilde{M}LM}\right)^{-1} z_{LM}\,. \tag{12}$$

We call $a^{I}_{\tilde{L}\tilde{M}}$ the **clean map** and A the **beam-pattern matrix**.

Making the map: what happens in the code

Track time-coincident blocks of data in the dect.s and divide into equal segments, which are FFTd to get d_f; then:

$$z_{LM} = \sum_{T \in \text{ block}} \sum_{b \in \text{base}} \sum_{f \in F} M_{LM\, f_i}\, \tilde{d}_{f_i}\,, \qquad \tilde{d}_{f_i} = \delta_{ij} N^{-1}_{f_i f_j}\, d_{f_j}\,, \quad (13)$$

$$f_i, f_j \in \tilde{T} = F = \{f_1, f_2, f_3, ..., f_N\}\,.$$

Build $A_{\tilde{L}\tilde{M}LM}$ and invert it to get the clean map $a_{\tilde{L}\tilde{M}}$

$$A_{\tilde{L}\tilde{M}LM} = \sum_{T \in \text{ block}} \sum_{b \in \text{base}} \sum_{f \in F} M^{T}_{\tilde{L}\tilde{M},\, f}\, N^{-1}_{ff}\, M^{T}_{LM\,,f}\,, \qquad (14)$$

$$a_{\tilde{L}\tilde{M}} = A^{-1}_{\tilde{L}\tilde{M}\, LM} z_{LM}\,,$$

which gets **accumulated over time**.

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What's in a map

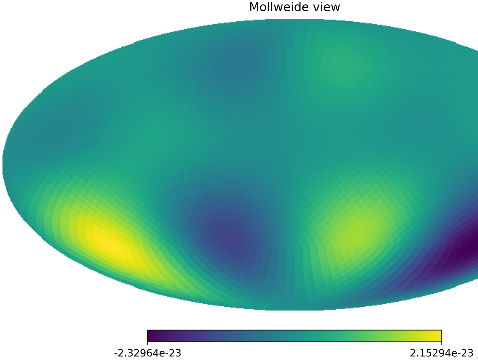

Figure 5: The output of the analysis of ~ 2 days of LIGO S6 (2009-2010) open data. Unexciting as $\ell_{max} = 4$.

We are still interpreting this... and inputting simulated data.

Multiple baselines

The beam-pattern matrix for 1 baseline isn't easily invertible, we've been relying on SVD techniques.

We need multiple baselines to break the singularity of the beam-pattern matrix, and scan the LM space more efficiently.

The code allows an input from any number of detectors on earth - one just needs to simulate data from extra detectors → *coming soon!*

Input maps

We simulate input maps by treating the signal and the noise separately;

input noise: take PSD of existing data and simulate gaussian noise around it

input signal: create input h_+, $h_\times$ signals and feed them into the detector [Cornish, 2001]:

$$H_a(f) = \int_{S^2} d\boldsymbol{n} \sum_{P=+,\times} h_P(f, \boldsymbol{n})\, F_a^P(f, \boldsymbol{n}) \exp(2\pi i f\, \boldsymbol{n} \cdot \boldsymbol{x_a}) \quad (15)$$

Comparing input/output maps is key to testing the mapper.

Next steps

In bullet points:

- Test the code and interpret spherical harmonic response of the setup
- Produce model-based input maps to input
- Include polarisation reconstruction
- Get better data (e.g. Advanced LIGO and Virgo runs)
- Generalize to other detector types: LISA

LISA is expected to probe extensively the frequency range for the AGWB... good news!

Thanks for listening!

Making the map: maximum likelihood map

The goal is to extract the a^l_{LM}. We do this by minimizing the χ^2 of the map given the data [Thrane et al, 2009]:

$$\chi^2(a^l_{LM}) = -\frac{1}{2}\sum_{t,f}\frac{(d_t - < d_t >)(d_t - < d_t >)^\star}{N(f)} \tag{16}$$

$$\text{where} \quad d_t(f) = s^\star_{a,t}(f)\, s_{b,t}(f)\,, \qquad < d_t(\boldsymbol{b}, f) > \equiv d_t(\boldsymbol{b}, f).$$

$$\text{from (9)}: \quad \frac{\partial < d_t(\boldsymbol{b}, f) >}{\partial\, a_{LM}} = M^l_{LM}(\hat{b})\,;$$

then we carry out the calculations...

Making the map: maximum likelihood map

Defining the **dirty map** z_{LM}:

$$z_{LM} := \sum_{T,f} \frac{M^{I\star}_{LM}(\hat{b})}{N(f)} d^T(f, \hat{b}) \, ; \tag{17}$$

$$z_{LM} = \sum_{\tilde{L}\tilde{M}} A_{\tilde{L}\tilde{M}LM} a^I_{\tilde{L}\tilde{M}} \, , \quad A_{\tilde{L}\tilde{M}LM} = \sum_{T,f} \frac{M^I_{\tilde{L}\tilde{M}}(\hat{b}) M^{I\star}_{LM}(\hat{b})}{N(f)} \, , \tag{18}$$

and the map solution which maximizes likelyhood is

$$a^I_{\tilde{L}\tilde{M}} = \sum_{LM} \left(A_{\tilde{L}\tilde{M}LM}\right)^{-1} z_{LM} \, . \tag{19}$$

We call $a^I_{\tilde{L}\tilde{M}}$ the **clean map** and A the **beam-pattern matrix**.