Scherk Schwartz twists in GUT models

Dumitru Dan Smaranda

Supervisor: Dr. David J. Miller

University of Glasgow

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Outline

Why GUTs?

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Grand Unified Theories (**GUTs**) can address these problems!

GUTs 101

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Under the Standard model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ they transform as:

$$u_R^\dagger : (\mathbf{3}, 1)_{\frac{2}{3}} \qquad d_R^\dagger : (\mathbf{3}, 1)_{-\frac{1}{3}} \qquad e_R^\dagger : (1, 1)_{-1}$$

$$\bar{q}_R = (\bar{d}_R^\dagger, \bar{u}_R^\dagger) : (\bar{\mathbf{3}}, \mathbf{2})_{-\frac{1}{6}} \qquad \bar{\ell}_R = (\bar{e}_R^\dagger, \bar{\nu}_R^\dagger) : (1, \mathbf{2})_{\frac{1}{2}}$$

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$$\psi = \bar{\mathbf{5}} = \begin{pmatrix} \bar{d}^r & \bar{d}^b & \bar{d}^g & e & -\nu \end{pmatrix}$$

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Similarly the $SU(2)$ **Higgs** doublet sits in another $\bar{5}$:

$$\mathcal{H} = (\phi^r, \phi^b, \phi^g, \underbrace{\boxed{\varphi^-, \varphi^0}}_{H})$$

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But we need to keep the $W^{\pm}, Z^0, H$ **light**. This is known as the **doublet triplet splitting problem**(2-3).

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The idea is simple we'll write a theory in $D = d + 4$ dimensions with $4D$ Minkowski space-time $\times$ the d dimmensional compact manifold:

$$\mathscr{M}_4 \times \mathcal{C} : \qquad S_D = \int d^D z \, \mathcal{L}[\phi(z^\mu)]$$

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Integrating out the extra coordinates gives us a 4D effective Lagrangian:

$$\mathcal{L}_4 = \int d^d y \, \mathcal{L}_D[\phi(x^\mu, y^m)]$$

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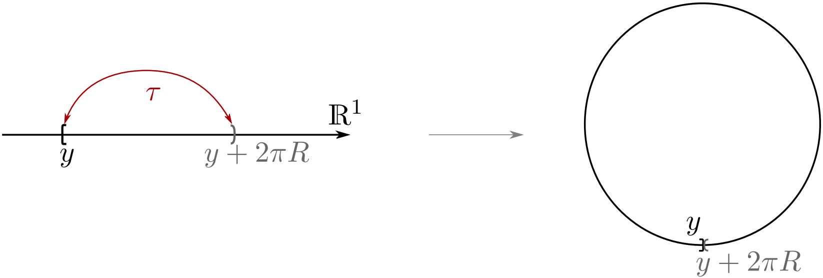

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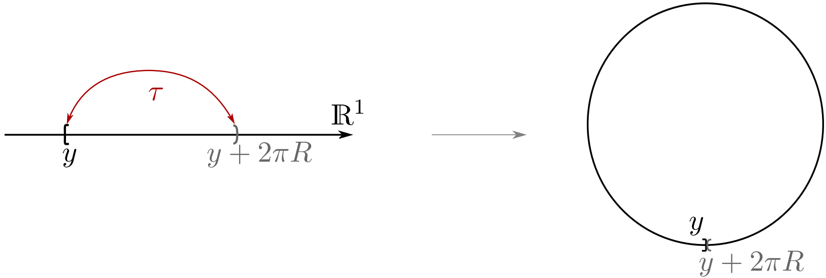

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Chirality Problem

This isn't enough though since we can't get chiral fermions.

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The ladder is known as a Scherk - Schwartz compactification.

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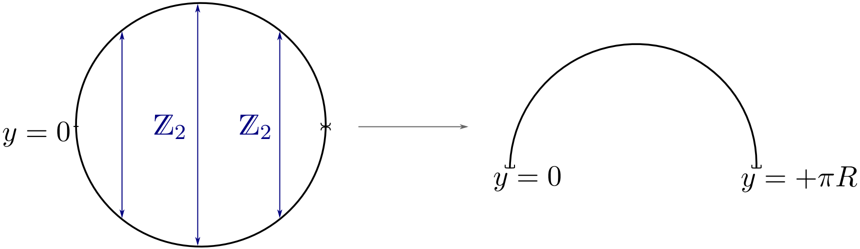

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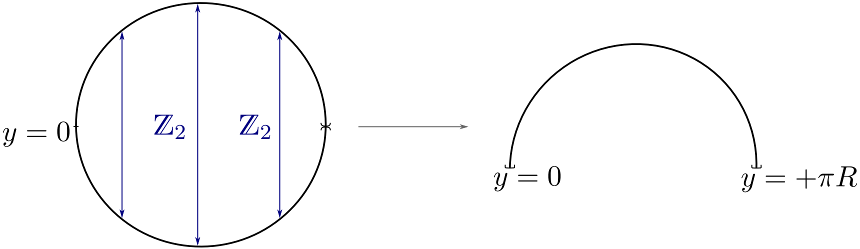

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The resulting manifold is no longer a smooth manifold but now has singularities called fixed points, and is called an **orbifold**.

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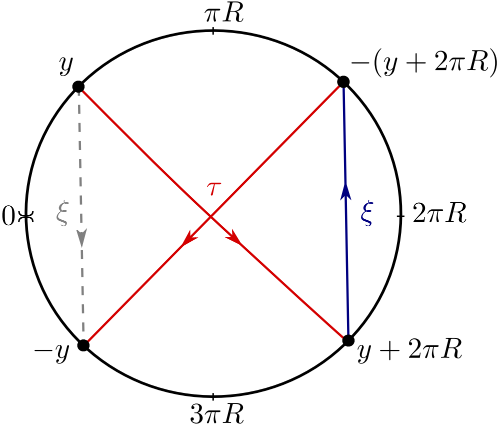

Figure: Geometrical flow of $\tau\xi\tau(y) \equiv \xi(y)$

Consistency - 2

The consistency conditions boil down to:

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For example, say we have a theory with a global $SU(2)$ field symmetry. Therefore we could have a SS compactification with:

$$Z = \sigma^3 \qquad T = \exp\big(2\pi i \alpha \sigma^2\big)$$

Kaluza Klein modes

The Z choice will give a ± 1 eigenvalue on the respective fields, and since we have a compact dimmension, we have only certain modes allowed (**Kaluza Klein** modes):

$$\tilde{\phi}_{+} = \phi^{(0)} + \sqrt{2} \sum_{n=1}^{\infty} \cos \frac{ny}{R} \phi_{+}^{(n)}$$

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The $+$ fields have '0 modes', and the $-$ ones don't.
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- Gives '0 modes' to our light GUT particles, therefore with masses $\mathcal{O}(\text{TeV})$
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Bonus

Since GUTs require SUSY, SS also gives us soft SUSY breaking masses, via $T = \exp\left(2\pi i \alpha \sigma^2\right)$.

Terminology

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We have a choice if we put our fields either in the bulk or the branes, which changes what fields get soft SUSY masses.

Model example

The action of the isometries will look like:

$$Z = \pm(\sigma^3)_R \otimes (\sigma^3)_H \otimes (+,+,+,+,+)$$
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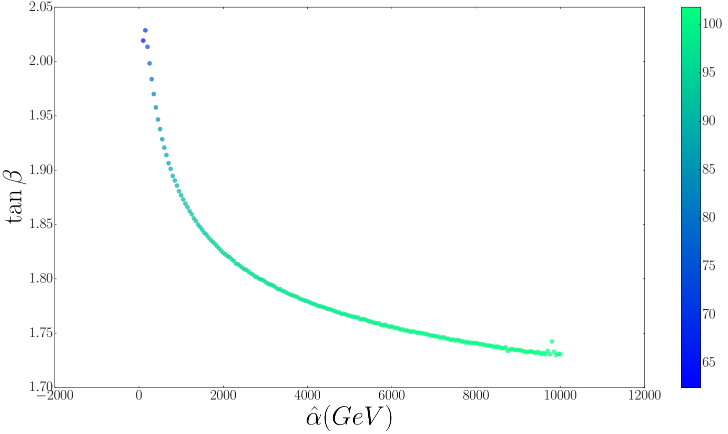
Which depending on the matter placement gives soft SUSY breaking masses something like:

$$m_{1/2} = \hat{\alpha} \equiv \alpha/R$$
$$m^2_{h_u,h_d} = \hat{\alpha}^2 \qquad m^2_{\tilde{q},\tilde{u},\tilde{d},\tilde{l},\tilde{e}} = 0 \qquad A_0 = -\hat{\alpha}$$
$$\mu = \hat{\gamma} \equiv \gamma/R \qquad \mu B = -2\hat{\alpha}\hat{\gamma}$$

5D $SU(5)$ GUT

The most basic GUT which results in a low energy **MSSM**:



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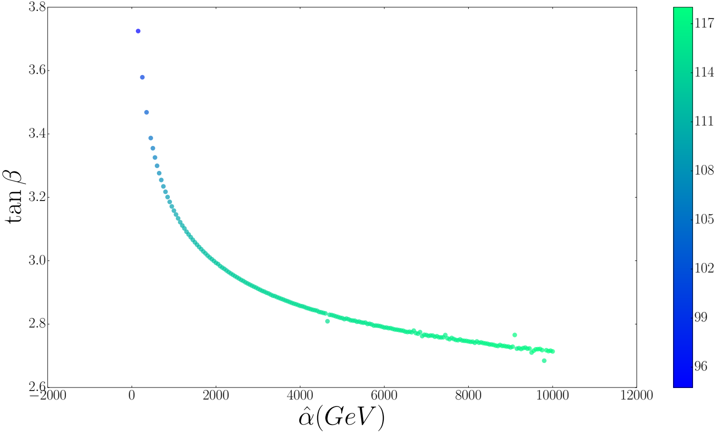

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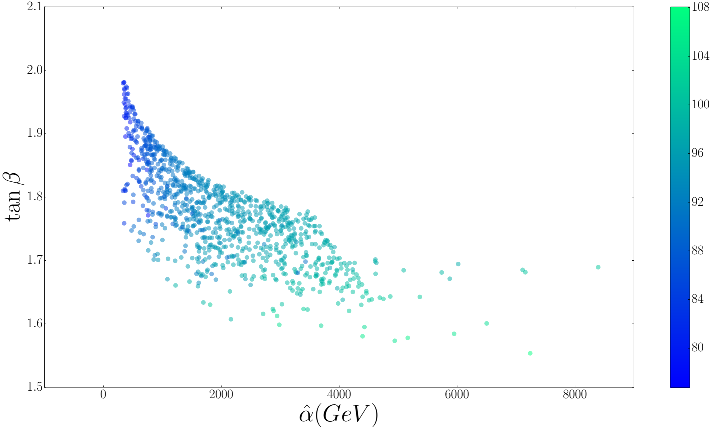

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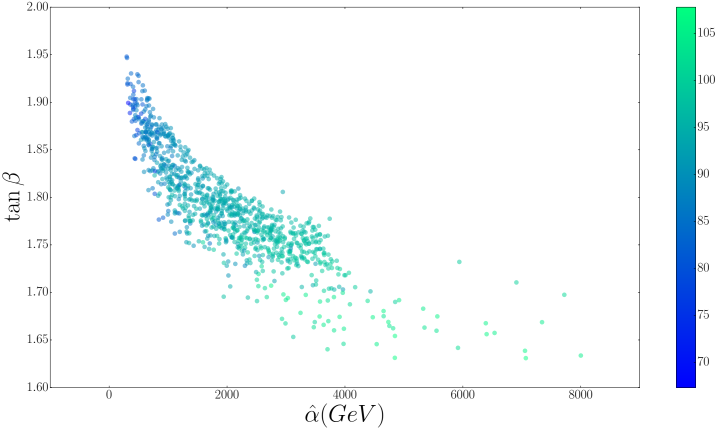

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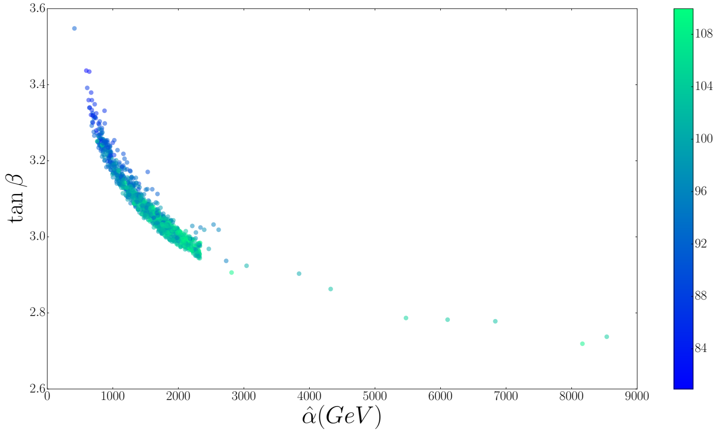

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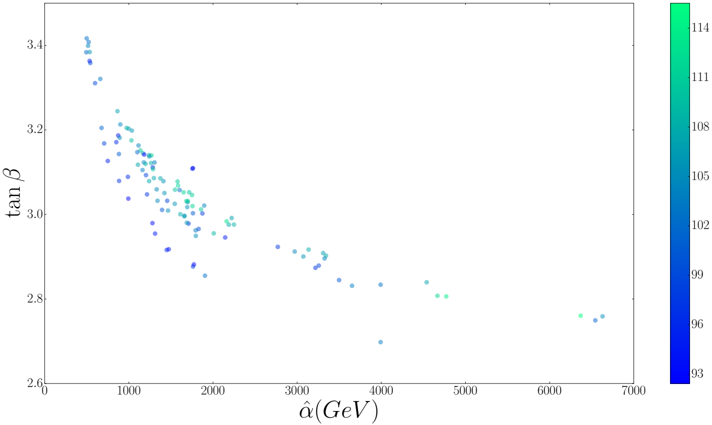

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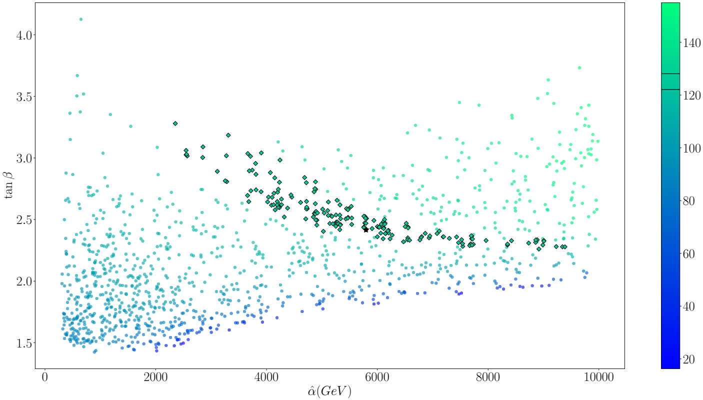
5D $SU(5) \times U(1)_\chi \times U(1)_\psi$ GUT

The most basic GUT which results in a low energy **modified UMSSM**:



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Thank you for your attention!