AdS/CFT calculations of meson decay rates

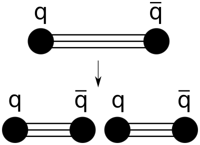
Maciej Matuszewski

Durham University

Supervisors:
Dr Kasper Peeters
Dr Marija Zamaklar

Introduction to Meson Decay: QCD Picture

- Meson decay may be seem as $q\bar{q}$ pair production from a colour field flux tube



- May use Schwinger formula [1] [2] to calculate decay rate Γ for volume V and energy density ϵ:

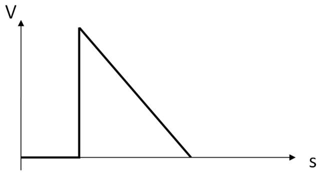
$$\Gamma = 2\mathrm{Im}\,\epsilon$$
$$= -\frac{2}{V}\,\mathrm{Im}\,\ln\int \mathrm{d}X\,\mathrm{e}^{S}$$

[1] Julian Schwinger. On gauge and vacuum polarizaion. *Physical Review*, 82(5):664,June 1951.

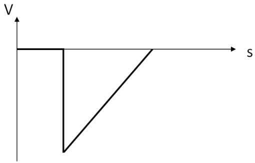
[2] A. Casher, H. Neuberger and S. Nussinov. *Physical Review D*, 20(1):179, July 1979

Introduction to Meson Decay: Instanton Method

- After the new $q\bar{q}$ pair is produced it must gain sufficient energy from the field to come on shell. This may be seen as a tunneling process:



- By Wick rotating the time coordinate we flip the potential:



- The calculation may now be done semi-classically.

Instaton Method: Point Particle Pair Production

- Consider Minkowski point particle action:

$$S_M = \int \mathrm{d}\tau \, \left(\frac{1}{2} \frac{\dot{X}^2}{e} - \frac{1}{2} e m^2 + A_\mu \dot{X}^\mu \right)$$

- Wick rotate to Euclidean spacetime:

$$\tau \to -i\tau$$
$$X^0 \to -iX^0$$
$$A^0 \to -iA^0$$

- Set periodic boundary condition:

$$X^\mu(\tau + 1) = X^\mu(\tau)$$

- Find Euclidean action:

$$S_E = \int_0^1 \mathrm{d}\tau \, \left(\frac{\dot{X}^2}{4T} + m^2 T - iA_\mu \dot{X}^\mu \right)$$

Instaton Method: Point Particle Pair Production

- Recall $A_\mu = -\frac{1}{2} F_{\mu\nu} X^\nu$
- Find Euler-Lagrange equation for T and eliminate it from action:

$$T = \frac{\sqrt{\dot{X}^2}}{2m} \qquad \Rightarrow \qquad S_E = \int_0^1 \mathrm{d}\tau \, \left(m\sqrt{\dot{X}^2} + \frac{i}{2} F_{\mu\nu} X^\nu \dot{X}^\mu \right)$$

- Find Euler-Lagrange equation for X_μ:

$$iF_{\nu\mu}\dot{X}^\nu = \frac{2\ddot{X}_\mu}{\sqrt{\dot{X}^2}} - \frac{2\dot{X}_\mu}{\left(\dot{X}^2\right)^{\frac{3}{2}}} \dot{X}^\nu \ddot{X}_\nu + iF_{\mu\nu}\dot{X}^\nu$$

Instaton Method: Point Particle Pair Production

- Choose constant field:

$$F_{01} = -F_{10} = -iE$$

- Problem admits the solution:

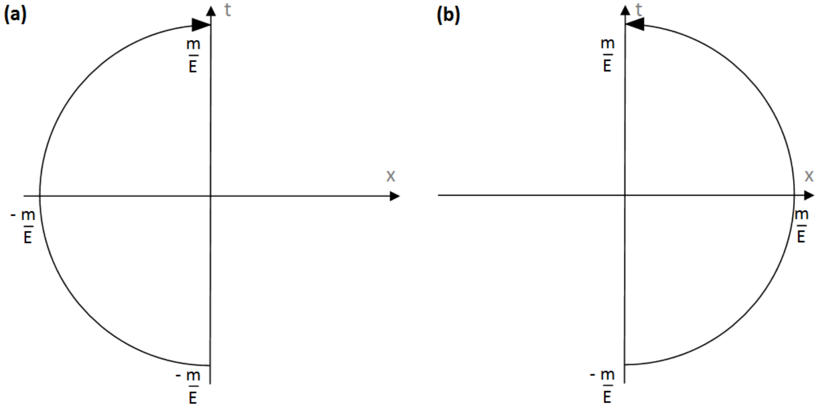
$$X_\mu = R \begin{pmatrix} \cos(2\pi n\tau) \\ sin(2\pi n\tau) \\ 0 \\ 0 \end{pmatrix}$$

- Subsitute X_μ into action and extremise value of R
- Action evaluates to expected result [3]:

$$S_E = \frac{\pi m^2}{E} n$$

[3] Gordon W Semenoff and Konstantin Zarembo. Holographic schwinger effect, September 2011. arXiv:1109.2920 [hep-th].

Instanton Method: Point Particle in Static Gauge



AdS/CFT Correspondence

- 't Hooft suggested that that for large N_C QCD is equivalent to theory of free strings [4]
- Seems to violate Weinberg-Witten theorem–which forbids the existance of gravitons in QCD [5]
- Maldacena suggested that a QFT in D dimensions corresponds to string theory in $D+1$ dimensions. [6]
- Good correspondence between $\mathcal{N}=4$ super Yang-Mills theory and a string theory in the $AdS_5 \times S^5$

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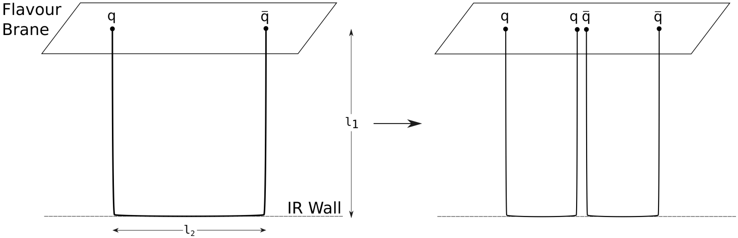
[4] Gerard 't Hooft. A planar diagram theory for strong interactions. *Nuclear Physics B*, 73:461, 1974.

[5] Steven Weinberg & Edward Witten. Limits on massless particles. *Nuclear Physics B*, 96(1-2):59, 1980.

[6] Juan Maldacena. The large-N limit of superconformal field theories and supergravity.*International Journal of Theoretical Physics*, 38(4):1113, 1999.

Meson Decay in the Holographic Picture

- We will eventually want to work in AdS spacetime with a string of the following profile

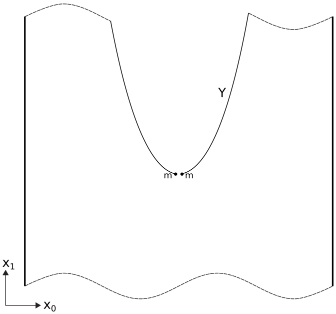


$l_1 \propto$ constituent quark mass $l_2 \propto$ colour flux tube energy

- Will build up to this using simpler examples
- Will work semi-classically using istanton method with a Wick rotated time coordinate

Flat Spacetime: Setting up the Problems

- Consider string with massive endpoints in Eulidean spacetime:



- The Lagrangian for the string with massive endpoints is:

$$\begin{aligned} S_E =& \int_{\tau_1}^{\tau_2} \mathrm{d}\tau \int_0^{\pi} \mathrm{d}\sigma \, \mathcal{L}_{bulk} + \int_{\tau_1}^{\tau_2} \mathrm{d}\tau \, \mathcal{L}_{end} \\ =& \gamma \int_{\tau_1}^{\tau_2} \mathrm{d}\tau \int_0^{\pi} \mathrm{d}\sigma \sqrt{-\left(\dot{X} \cdot X'\right)^2 + \dot{X}^2 X'^2} \\ & + m \int_{\tau_1}^{\tau_2} \mathrm{d}\tau \left(\sqrt{\dot{X}^2(\tau, \sigma = 0)} + \sqrt{\dot{X}^2(\tau, \sigma = \pi)} \right) \end{aligned}$$

Flat Spacetime: Equations of Motion

- Find variation of action:

$$
\begin{aligned}
0 = \delta S = & \int_{\tau_1}^{\tau_2} \mathrm{d}\tau \int_0^{\pi} \mathrm{d}\sigma \left(\frac{\partial}{\partial \tau} \left(\frac{\partial \mathcal{L}_{bulk}}{\partial \dot{X}_\mu} \right) + \frac{\partial}{\partial \sigma} \left(\frac{\partial \mathcal{L}_{bulk}}{\partial X'_\mu} \right) \right) \delta X(\tau, \sigma) \\
& + \int_{\tau_1}^{\tau_2} \mathrm{d}\tau \left(\frac{\partial}{\partial \tau} \left(\frac{\partial \mathcal{L}_{end,\,\sigma=0}}{\partial \dot{X}_\mu} \right) + \frac{\partial \mathcal{L}_{bulk}}{\partial X'{}_\mu} \right) \Bigg|_{\sigma=0} \delta X_\mu(\tau, \sigma = 0) \\
& + \int_{\tau_1}^{\tau_2} \mathrm{d}\tau \left(\frac{\partial}{\partial \tau} \left(\frac{\partial \mathcal{L}_{end,\,\sigma=\pi}}{\partial \dot{X}_\mu} \right) - \frac{\partial \mathcal{L}_{bulk}}{\partial X'{}_\mu} \right) \Bigg|_{\sigma=\pi} \delta X_\mu(\tau, \sigma = \pi)
\end{aligned}
$$

Flat Spacetime: Equations of Motion

- Find bulk equation of motion ($0 < \sigma < \pi$):

$$\frac{\partial}{\partial \tau}\left(\frac{\partial \mathcal{L}_{bulk}}{\partial \dot{X}_\mu}\right) + \frac{\partial}{\partial \sigma}\left(\frac{\partial \mathcal{L}_{bulk}}{\partial X'_\mu}\right) = 0$$

- Find boundary equations of motions:

$$\left(\frac{\partial}{\partial \tau}\left(\frac{\partial \mathcal{L}_{end,\,\sigma=0}}{\partial \dot{X}_\mu}\right) + \frac{\partial \mathcal{L}_{bulk}}{\partial X'_\mu}\right)\Bigg|_{\sigma=0} \delta X_\mu(\tau, \sigma = 0) = 0$$

$$\left(\frac{\partial}{\partial \tau}\left(\frac{\partial \mathcal{L}_{end,\,\sigma=\pi}}{\partial \dot{X}_\mu}\right) - \frac{\partial \mathcal{L}_{bulk}}{\partial X'_\mu}\right)\Bigg|_{\sigma=\pi} \delta X_\mu(\tau, \sigma = \pi) = 0$$

Flat Spacetime: General Solution

- Based on the work of Bardeen et al[7], we may choose the solution

$$X_0 = \tau - \tau_0$$
$$X_1 = b\,\frac{\sigma}{\pi}\left(\sqrt{-(\tau-\tau_0)^2 + k^2} + x_0\right)$$

where b and x_0 are constants and $k = \frac{m}{\gamma}$.

- We wish to test that this satisfies the equations of motion. We will write $X_1 = x$ for clarity.

[7] W.A. Bardeen Itzahk Bars, and Ali Teimouri. Study of the logitudinal kink modes of the string. *Physical Review D*, 13(8):2364-2382, 1976

Flat Spacetime: Checking Bulk Equation of Motion

- We find

$$\frac{\partial \mathcal{L}_{bulk}}{\partial \dot{X}_\mu} = \gamma \frac{-(\dot{X} \cdot X')X'^\mu + \dot{X}^\mu X'^2}{\sqrt{-(\dot{X} \cdot X')^2 + \dot{X}^2 X'^2}} \qquad \frac{\partial \mathcal{L}_{bulk}}{\partial X'_\mu} = \gamma \frac{-(\dot{X} \cdot X')\dot{X}^\mu + X'^\mu \dot{X}^2}{\sqrt{-(\dot{X} \cdot X')^2 + \dot{X}^2 X'^2}}$$

$$= +\gamma \frac{-\dot{x}x' X'^\mu + x'^2 \dot{X}^\mu}{\sqrt{-\dot{x}^2 x'^2 + (1+\dot{x}^2)x'^2}} \qquad = +\gamma \frac{-\dot{x}x' \dot{X}^\mu + (1+\dot{x})^2 X'^\mu}{\sqrt{-\dot{x}^2 x'^2 + (1+\dot{x}^2)x'^2}}$$

$$= +\gamma \left(x' \dot{X}^\mu - \dot{x} X'^\mu \right) \qquad = +\gamma \left(-\dot{x}\dot{X}^\mu + \left(\frac{1+\dot{x}^2}{x'} \right) X'^\mu \right)$$

- For $\mu = 0$ we find that

$$\frac{\partial}{\partial \tau} \left(\frac{\partial \mathcal{L}_{bulk}}{\partial \dot{X}_\mu} \right) + \frac{\partial}{\partial \sigma} \left(\frac{\partial \mathcal{L}_{bulk}}{\partial X'_\mu} \right) = \gamma \left(\frac{\partial x'}{\partial \tau} - \frac{\partial \dot{x}}{\partial \sigma} \right) = 0$$

- For $\mu = 1$ we find that

$$\frac{\partial}{\partial \tau} \left(\frac{\partial \mathcal{L}_{bulk}}{\partial \dot{X}_\mu} \right) + \frac{\partial}{\partial \sigma} \left(\frac{\partial \mathcal{L}_{bulk}}{\partial X'_\mu} \right) = \gamma \left(\frac{\partial}{\partial \tau} (x'\dot{x} - \dot{x}x') + \frac{\partial}{\partial \sigma} \left(-\dot{x}^2 + x' \frac{1+\dot{x}^2}{x'} \right) \right)$$

$$= 0$$

Flat Spacetime: Checking Boundary Conditions

- We find that the Dirichlet boundary condition at $\sigma = 0$ is trivially satisfied.
- For the Neumann boundary condition at $\sigma = \pi$ we find

$$\frac{\partial}{\partial \tau}\left(\frac{\partial \mathcal{L}_{end,\sigma=\pi}}{\partial \dot{X}_\mu}\right) - \frac{\partial \mathcal{L}_{bulk}}{\partial X'_\mu} =$$

$$m\frac{\partial}{\partial \tau}\left(\frac{\dot{X}^\mu}{\sqrt{\dot{X}^2}}\right) - \gamma\left(-\dot{x}\dot{X}^\mu + \left(\frac{1+\dot{x}^2}{x'}\right)X'^\mu\right) = 0$$

where

$$\dot{x} = -b\,\frac{\sigma}{\pi}\frac{(\tau-\tau_0)}{\sqrt{-(\tau-\tau_0)^2+k^2}}$$

$$\ddot{x} = -b\,\frac{\sigma}{\pi}\frac{k^2}{(-(\tau-\tau_0)^2+k^2)^{\frac{3}{2}}}$$

Flat Spacetime: Checking Boundary Conditions

- Looking at $\mu = 0$ we find

$$\frac{\partial}{\partial\tau}\left(\frac{\partial\mathcal{L}_{end,\sigma=\pi}}{\partial\dot{X}_0}\right) - \frac{\partial\mathcal{L}_{bulk}}{\partial X'_0}\bigg|_{\sigma=\pi} = \frac{\partial}{\partial\tau}\left(\frac{m}{\sqrt{1+\dot{x}^2}}\right) + \gamma\dot{x}$$

$$= m\frac{\partial}{\partial\tau}\left(\sqrt{\frac{-(\tau-\tau_0)^2+k^2}{(b^2-1)(\tau-\tau_0)^2+k^2}}\right) - \frac{\gamma b(\tau-\tau_0)}{\sqrt{-(\tau-\tau_0)^2+k^2}}$$

- We note that this only satisfies the required condition when $b=-1$, in which case we find

$$\frac{\partial}{\partial\tau}\left(\frac{\partial\mathcal{L}_{end,\sigma=\pi}}{\partial\dot{X}_0}\right) - \frac{\partial\mathcal{L}_{bulk}}{\partial X'_0}\bigg|_{\sigma=\pi} = -m\frac{(\tau-\tau_0)}{k^2}\sqrt{\frac{k^2}{-(\tau-\tau_0)^2+k^2}} + \frac{\gamma(\tau-\tau_0)}{\sqrt{-(\tau-\tau_0)^2+k^2}} = 0$$

Flat Spacetime: Checking Boundary Conditions

- Similarly for $\mu = 1$ we find

$$\frac{\partial}{\partial\tau}\left(\frac{\partial\mathcal{L}_{end,\sigma=\pi}}{\partial\dot{X}_1}\right) - \frac{\partial\mathcal{L}_{bulk}}{\partial X'_1}\bigg|_{\sigma=\pi} = m\frac{\partial}{\partial\tau}\left(\frac{\dot{x}}{\sqrt{1+\dot{x}^2}}\right) - \gamma$$

$$= m\frac{\partial}{\partial\tau}\left(\frac{(\tau-\tau_0)}{\sqrt{-(\tau-\tau_0)^2+k^2}}\sqrt{\frac{-(\tau-\tau_0)^2+k^2}{k^2}}\right) - \gamma = 0$$

- We note that the $\sigma = \pi$ endpoint satisfies

$$(\tau-\tau_0)^2 + (x-x_0)^2 = \left(\frac{m}{\gamma}\right)^2 .$$

Flat Spacetime: Full Solution

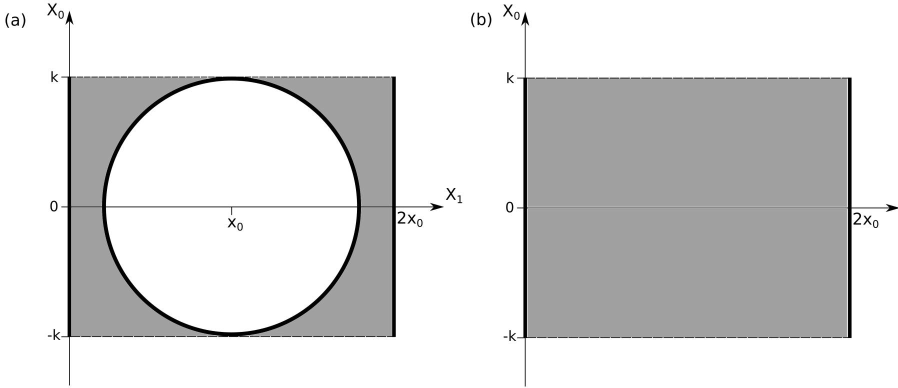
- For the solution, we find:

$$X_{L0} = X_{R0} = \tau$$
$$X_{L1} = x_L = -\frac{\sigma}{\pi}\left(\sqrt{-\tau^2 + k^2} - x_0\right)$$
$$X_{R1} = x_R = \left(1 - \frac{\sigma}{\pi}\right)\left(\sqrt{-\tau^2 + k^2} + x_0\right) + 2x_0\frac{\sigma}{\pi}$$

- For the background, we find

$$X_{B0} = \tau$$
$$X_{B1} = 2x_0\frac{\sigma}{\pi}$$

Flat Spacetime: Full Solution



Flat Spacetime: Full Solution

$$
\begin{aligned}
S_{sol} = &+ \gamma \int_{-\frac{m}{\gamma}}^{\frac{m}{\gamma}} \mathrm{d}\tau \int_0^\pi \mathrm{d}\sigma \sqrt{-\left(\dot{X}_L \cdot X_L'\right)^2 + \dot{X}_L^2 X_L'^2} \\
&+ m \int_{-\frac{m}{\gamma}}^{\frac{m}{\gamma}} \mathrm{d}\tau \left(\sqrt{\dot{X}_L^2(\tau,\sigma=0)} + \sqrt{\dot{X}_L^2(\tau,\sigma=\pi)}\right) \\
&+ \gamma \int_{-\frac{m}{\gamma}}^{\frac{m}{\gamma}} \mathrm{d}\tau \int_0^\pi \mathrm{d}\sigma \sqrt{-\left(\dot{X}_R \cdot X_R'\right)^2 + \dot{X}_R^2 X_R'^2} \\
&+ m \int_{-\frac{m}{\gamma}}^{\frac{m}{\gamma}} \mathrm{d}\tau \left(\sqrt{\dot{X}_R^2(\tau,\sigma=0)} + \sqrt{\dot{X}_R^2(\tau,\sigma=\pi)}\right) \\
&+ \gamma \int_{-\frac{m}{\gamma}}^{\frac{m}{\gamma}} \mathrm{d}\tau \int_0^\pi \mathrm{d}\sigma \, {x_L}' + m \int_{-\frac{m}{\gamma}}^{\frac{m}{\gamma}} \mathrm{d}\tau \left(\sqrt{\dot{X}_L^2(\tau,\sigma=0)} + \sqrt{\dot{X}_L^2(\tau,\sigma=\pi)}\right) \\
&+ \gamma \int_{-\frac{m}{\gamma}}^{\frac{m}{\gamma}} \mathrm{d}\tau \int_0^\pi \mathrm{d}\sigma \, {x_R}' + m \int_{-\frac{m}{\gamma}}^{\frac{m}{\gamma}} \mathrm{d}\tau \left(\sqrt{\dot{X}_R^2(\tau,\sigma=0)} + \sqrt{\dot{X}_R^2(\tau,\sigma=\pi)}\right) \\
= &+ \gamma \int_{-\frac{m}{\gamma}}^{\frac{m}{\gamma}} \mathrm{d}\tau \left(x_L(\tau,\sigma=\pi) - x_L(\tau,\sigma=0)\right) + m \int_{-\frac{m}{\gamma}}^{\frac{m}{\gamma}} \mathrm{d}\tau \left(1 + \sqrt{1 + \dot{x}_L^2(\tau,\sigma=\pi)}\right) \\
&+ \gamma \int_{-\frac{m}{\gamma}}^{\frac{m}{\gamma}} \mathrm{d}\tau \left(x_R(\tau,\sigma=\pi) - x_R(\tau,\sigma=0)\right) + m \int_{-\frac{m}{\gamma}}^{\frac{m}{\gamma}} \mathrm{d}\tau \left(\sqrt{1 + \dot{x}_R^2(\tau,\sigma=0)} + 1\right) \\
= &+ \int_{-\frac{m}{\gamma}}^{\frac{m}{\gamma}} \mathrm{d}\tau \left(m\sqrt{1 + \dot{x}_L^2(\tau,\sigma=\pi)} + \gamma x_L(\tau,\sigma=\pi)\right) \\
&+ \int_{-\frac{m}{\gamma}}^{\frac{m}{\gamma}} \mathrm{d}\tau \left(m\sqrt{1 + \dot{x}_R^2(\tau,\sigma=0)} - \gamma x_R(\tau,\sigma=0)\right) + \int_{-\frac{m}{\gamma}}^{\frac{m}{\gamma}} \mathrm{d}\tau \left(2m + 2x_0\right).
\end{aligned}
$$

Flat Spacetime: Full Solution Action

$$
\begin{aligned}
S_{back} =& \gamma \int_{-\frac{m}{\gamma}}^{\frac{m}{\gamma}} \mathrm{d}\tau \int_0^{\pi} \mathrm{d}\sigma \sqrt{-\left(\dot{X}\cdot X'\right)^2 + \dot{X}^2 X'^2} \\
& + m \int_{-\frac{m}{\gamma}}^{\frac{m}{\gamma}} \mathrm{d}\tau \left(\sqrt{\dot{X}^2(\tau,\sigma=0)} + \sqrt{\dot{X}^2(\tau,\sigma=\pi)}\right) \\
& + \gamma \int_{-\frac{m}{\gamma}}^{\frac{m}{\gamma}} \mathrm{d}\tau \int_0^{\pi} \mathrm{d}\sigma \, \frac{2_0}{\pi} \; - 2m \int_{-\frac{m}{\gamma}}^{\frac{m}{\gamma}} \mathrm{d}\tau \\
=& + \int_{-\frac{m}{\gamma}}^{\frac{m}{\gamma}} \mathrm{d}\tau \left(2m + 2x_0\right).
\end{aligned}
$$

$$
\begin{aligned}
S =& S_{sol} - S_{back} \\
=& \int_{-\frac{m}{\gamma}}^{\frac{m}{\gamma}} \mathrm{d}\tau \left(m\sqrt{1+\dot{x}_L^2(\tau,\sigma=\pi)} + \gamma x_L(\tau,\sigma=\pi)\right) \\
& + \int_{-\frac{m}{\gamma}}^{\frac{m}{\gamma}} \mathrm{d}\tau \left(m\sqrt{1+\dot{x}_R^2(\tau,\sigma=0)} - \gamma x_R(\tau,\sigma=0)\right) = \frac{\pi m^2}{\gamma},
\end{aligned}
$$

Flat Spacetime: Full Solution

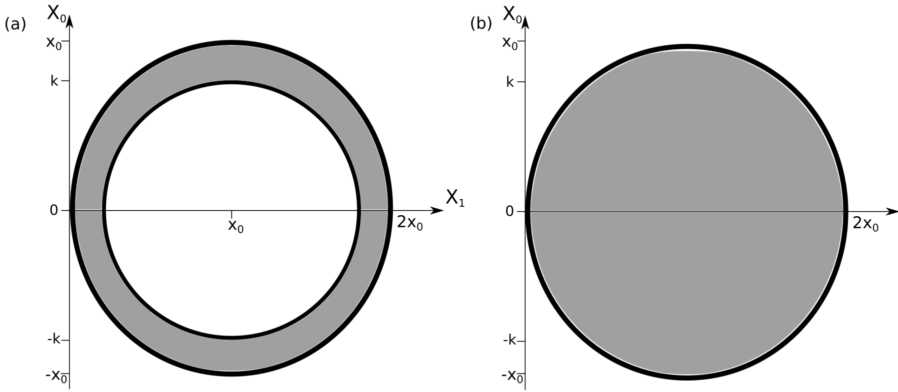
- May have the outer endpoints move in any shape

$$
\begin{aligned}
X_{L0} &= X_{R0} = \tau \\
X_{L1} &= x_L = -\frac{\sigma}{\pi}\left(\sqrt{-\tau^2+k^2} - x_0\right) + \left(1-\frac{\sigma}{\pi}\right)\left(\sqrt{-\tau^2+{x_0}^2}\right) \\
X_{R1} &= x_R = \left(1-\frac{\sigma}{\pi}\right)\left(\sqrt{-\tau^2+k^2} + x_0\right) + \frac{\sigma}{\pi}\left(\sqrt{-\tau^2+{x_0}^2} + x_0\right)
\end{aligned}
$$

- We find the same solution $S = \frac{\pi m^2}{\gamma}$
- This makes sense as, looking at the problem geometrically, we find

$$
\begin{aligned}
S =& m(\text{Circumeference of circle}) - \gamma(\text{Area of circle}) \\
=& m(2\pi k) - \gamma(\pi k^2) \\
=& 2\frac{\pi m^2}{\gamma} - \frac{\pi m^2}{\gamma} = \frac{\pi m^2}{\gamma}.
\end{aligned}
$$

Flat Spacetime: Full Solution



Flat Spacetime: Full Solution

- Find where $k = \frac{m}{\gamma}$ comes from
- For generic outer path, we find

$$
\begin{aligned}
S_{sol} =& \gamma \int_{-T}^{T} \mathrm{d}\tau \left(\tilde{x}_L(\tau, \sigma = \pi) - \tilde{x}_L(\tau, \sigma = 0) \right) \\
& + m \int_{-T}^{T} \mathrm{d}\tau \left(\sqrt{1 + \dot{\tilde{x}}_L{}^2(\tau, \sigma = 0)} + \sqrt{1 + \dot{\tilde{x}}_L{}^2(\tau, \sigma = \pi)} \right) \\
& + \gamma \int_{-T}^{T} \mathrm{d}\tau \left(\tilde{x}_R(\tau, \sigma = \pi) - \tilde{x}_R(\tau, \sigma = 0) \right) \\
& + m \int_{-T}^{T} \mathrm{d}\tau \left(\sqrt{1 + \dot{\tilde{x}}_R{}^2(\tau, \sigma = 0)} + \sqrt{1 + \dot{\tilde{x}}_R{}^2(\tau, \sigma = \pi)} \right) \\
=& - \gamma \pi k^2 + 2 m \pi \gamma + C
\end{aligned}
$$

- Extremising this gives the required condition.

Sakai-Sugimoto Holographic Background

- We wish to consider double loops in the background[8]

$$\mathrm{d}s^2 = \frac{R^{\frac{3}{2}}}{z^{\frac{3}{2}}}\left(\mathrm{d}x_0{}^2 + \mathrm{d}\rho^2 + \rho^2\mathrm{d}\theta^2 + \rho^2\sin\theta\mathrm{d}\phi^2 + f(z)\mathrm{d}x_4{}^2\right) + \frac{1}{z^{\frac{5}{2}}R^{\frac{3}{2}}}f^{-1}(z)\mathrm{d}z^2$$

  where

$$z = \frac{1}{r} \qquad f(z) = 1 - \frac{z^3}{z_t{}^3}$$

- Use parametrisation

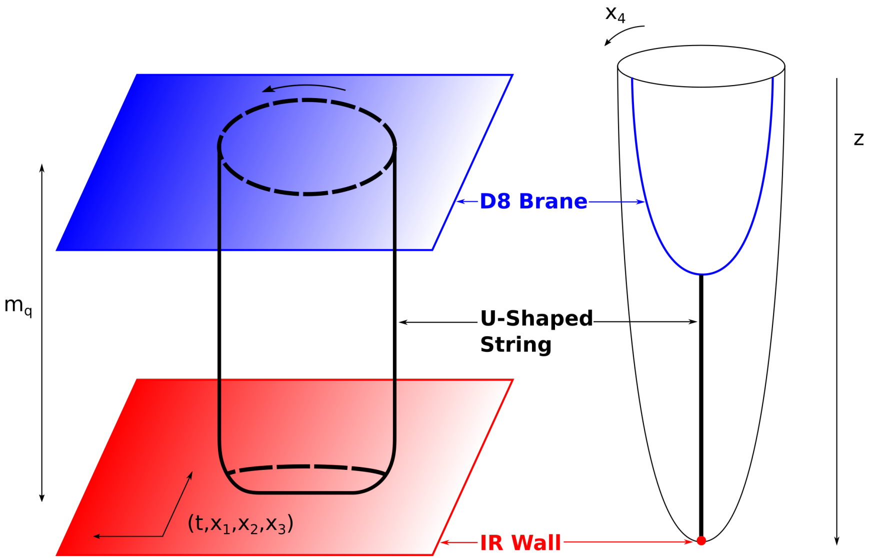
$$z = \sigma \qquad \rho = \rho(z)$$
$$\theta = \frac{\pi}{2} \qquad \phi = \tau$$

  to find the Lagrangian

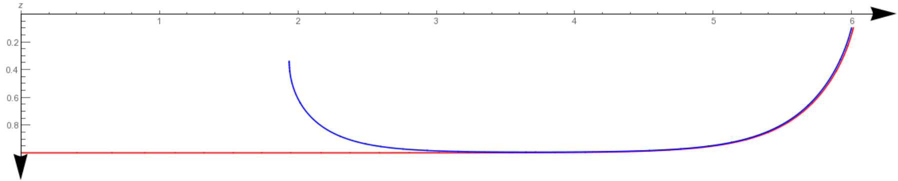
$$\mathcal{L} = \sqrt{-(\dot{X}\cdot X')^2 + (X')^2(\dot{X})^2}$$
$$= \frac{\rho}{z^{\frac{3}{2}}}\sqrt{R^3(\rho')^2 + \frac{1}{1-\frac{z^3}{z_t{}^3}}\frac{1}{z}}$$

[8] T. Sakai and S. Sugimoto. Low energy hadron physics in holographic QCD. *Prog. Theor. Phys*, 113(2005):843. arXiv:hep-th/0412141.

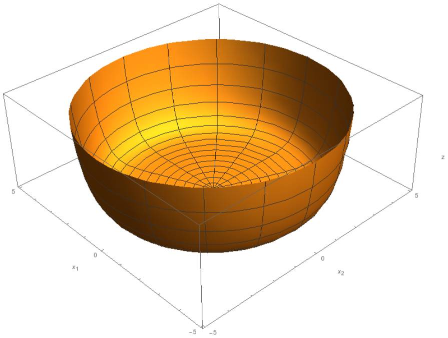
Sakai-Sugimoto Holographic Background

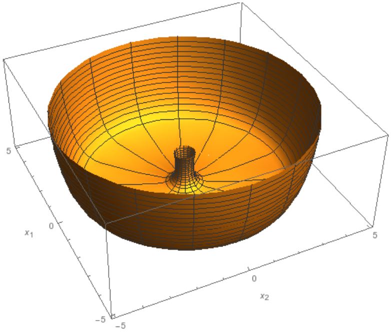


Sakai-Sugimoto Holographic Background



Sakai-Sugimoto Holographic Background





Sakai-Sugimoto Holographic Background

- We apply the Dirichlet boundary condition for the outer leg and the z direction of the inner leg. We apply the the Neumann boundary condition for the inner leg ρ direction.

$$\frac{\partial \mathcal{L}}{\partial X'^{\mu}} = -X'^{\mu}\sqrt{\frac{(\dot{X})^2}{(X')^2}} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \rho'} = R^{\frac{3}{2}}\rho\left(R^3(\rho')^2 + \frac{1}{1-\frac{z^3}{{z_t}^3}}\right)^{-\frac{1}{2}}\rho'$$

- From this we read off the condition

$$\frac{\mathrm{d}\rho}{\mathrm{d}z} = 0$$

Sakai-Sugimoto Holographic Background

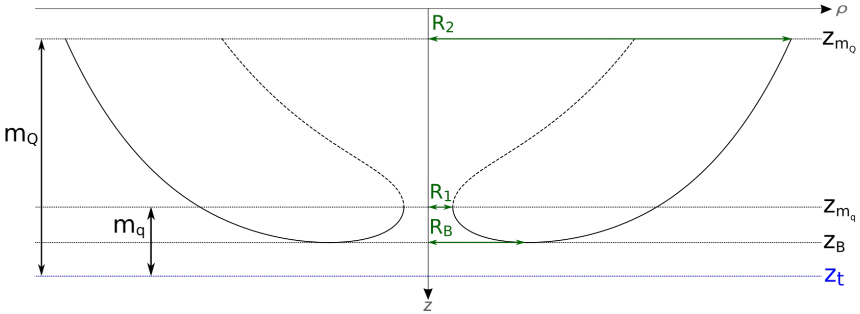
- We find the mass to be

$$m_q = \int_{\sigma^*}^{\sigma_{max}} \sqrt{g_{tt} g_{zz}} \mathrm{d}\sigma = \int_{z^*}^{z_{max}} \frac{1}{z^2} \sqrt{\frac{1}{1 - \frac{z^3}{{z_t}^3}}} \mathrm{d}z$$

- We find the tension to be

$$T = \sqrt{g_{tt} g_{xx}} = \frac{1}{z^{\frac{3}{2}}}$$

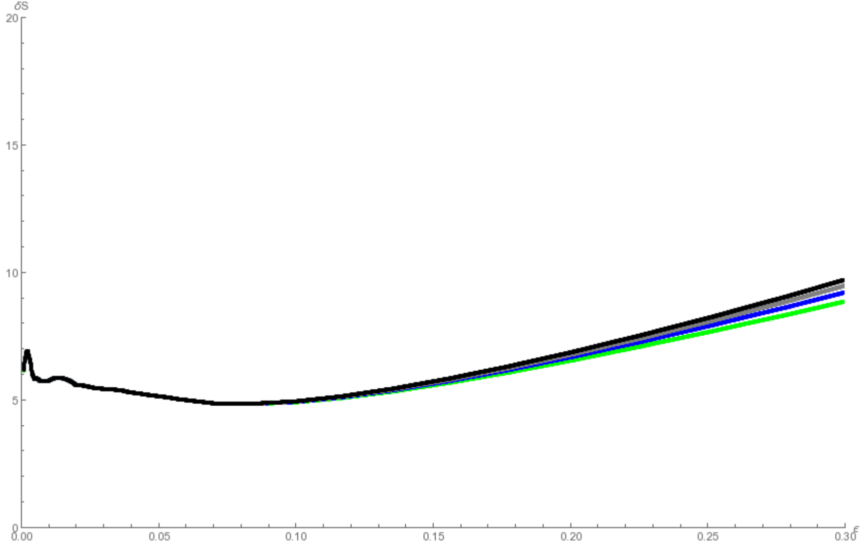
Sakai-Sugimoto Holographic Background



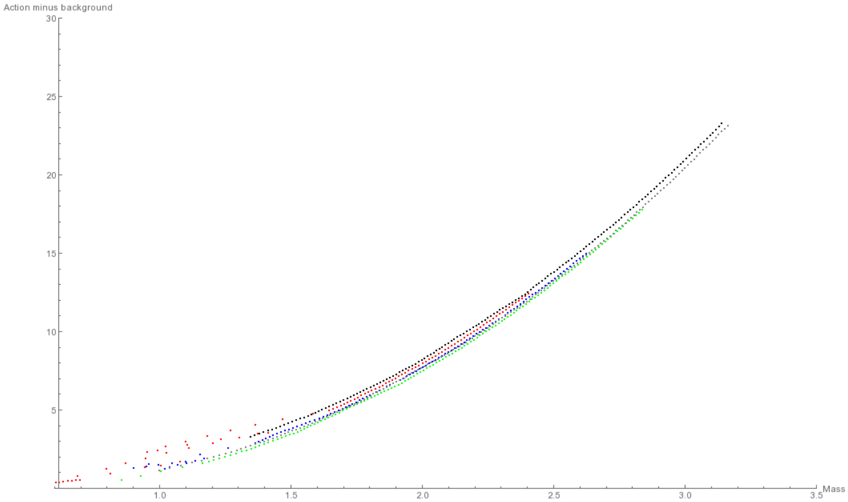
Sakai-Sugimoto Holographic Background

- Bottom of the loop is the point (z_B, R_B).
- If $\epsilon = z_t - z_B$ we want to work in a low ϵ regime as this eliminates the effect of the outer quarks.
- We expect to find the relationship $S \sim \frac{m^2}{T}$

Sakai-Sugimoto Holographic Background



Sakai-Sugimoto Holographic Background



Sakai-Sugimoto Holographic Background