Higgs-Assisted Q-balls from Pseudo-Nambu-Goldstone Bosons

Olivier Lennon

with Fady Bishara, George Johnson, and John March-Russell
JHEP **11**, 179 (2017) [arXiv:1708.04620]

UNIVERSITY OF OXFORD

Science & Technology Facilities Council

Part I
What is a Q-ball?

What Is a Q-ball?

- Example of a non-topological soliton that can appear in field theories with scalars and global symmetries
- Coherent configuration of scalars that can be thought of as a bound state with total charge Q
- Spherically symmetric and spatially extended
- Lowest energy state for a collection of charged scalars: kept stable by energy-conservation and a conserved Noether charge

Worked Example: U(1) Scalar Theory

- Worked Example: theory of a complex scalar field with U(1) symmetry

$$\mathcal{L} = \partial^\mu \phi^* \partial_\mu \phi - U(\phi)$$

- Find Q-ball solution by minimising energy for a fixed, non-zero charge

- To do this, we introduce a Lagrange multiplier:

$$\mathcal{E}_\omega = E + \omega \left[Q - i \int \mathrm{d}^3 x \, \left(\dot{\phi}^* \phi - \phi^* \dot{\phi} \right) \right]$$

Worked Example: Q-ball Solutions

- Q-ball solution of the form

$$\phi(x,t) = e^{i\omega t}\phi(x)$$

- Q-ball is absolutely stable if $\omega_0 \leq \omega < m$

- Spatial part minimises

$$S = \int \mathrm{d}^3x \left(\vec{\nabla}\phi \cdot \vec{\nabla}\phi + U(\phi) - \omega^2\phi^2 \right)$$

- This has the form of a "bounce action": well-studied, with spherically symmetric solutions

- Two analytic limits for spatial profile exist: thin-wall (large charge); thick-wall (small charge)

Worked Example: Q-ball Solutions

- Q-ball solution of the form

$$\phi(x,t) = e^{i\omega t}\phi(x)$$

- Q-ball is absolutely stable if $\omega_0 \leq \omega < m$

- Spatial part minimises

$$S = \int \mathrm{d}^3 x \left(\vec{\nabla}\phi \cdot \vec{\nabla}\phi + U(\phi) - \omega^2 \phi^2 \right)$$

- This has the form of a "bounce action": well-studied, with spherically symmetric solutions

- Two analytic limits for spatial profile exist: thin-wall (large charge); thick-wall (small charge)

Motivation

- TeV(or above)-scale strongly-coupled dynamics are *à la mode* (neutral naturalness, ADM theories,…) so good to explore their structure
- Q-balls formed from composite objects
- Q-balls that contain the Standard Model Higgs
- Includes a possible candidate for dark matter
- Standard Model Higgs offers up possible explanation for formation method: "long-range force" to bring together heavy hidden sector states

Outline

- The structure of the hidden sector
- Q-balls from the hidden sector
- Summary and future work

Part II
The Structure of the Hidden Sector

Hidden Sector: Strong Dynamics

- We consider a hidden sector with strong dynamics ($SU(3)'_C$ gauge theory, quarks, confinement, chiral symmetry breaking, asymptotic freedom)
- Gauged $SU(2)'_W$ and introduce hidden leptons for anomaly cancellation; leave $U(1)'_{EM}$ ungauged
- Interested in light stable hidden “pions” (scalars) that result from chiral symmetry breaking
- Will consider two light quarks and arbitrary number of heavy quarks: SU(2) chiral Lagrangian
- This theory possesses a global U(1) symmetry in pion sector, but no Q-balls as it is

Hidden Sector: The Scalar Sector

- Consider an additional complex scalar doublet whose VEV gives mass to hidden sector quarks

- Include portal interaction with the Standard Model Higgs: allows mass eigenstate mixing between both scalars

$$V(H,S) = -\mu_h^2 H^\dagger H + \lambda_h (H^\dagger H)^2 - \mu_s^2 S^\dagger S + \lambda_s (S^\dagger S)^2 + \lambda_p (H^\dagger H)(S^\dagger S)$$

Barger et al. 2009

Hidden Sector: Higgs Coupling to Pions

- The (leading order) chiral Lagrangian for the hidden sector including couplings to new scalar:

Voloshin and Zakharov 1980
Voloshin 1986
Chivukula et al 1989

$$\mathcal{L} = \left(1 + \frac{4n_h}{3\beta_0}\frac{s'}{v_s}\right)\frac{f^2}{4}\mathrm{tr}\left(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right) + \left(1 + \left[1 + \frac{2n_h}{\beta_0}\right]\frac{s'}{v_s}\right)\frac{B_0 f^2}{2}\mathrm{tr}\left(M(\Sigma + \Sigma^\dagger - 2)\right)$$

- Mixing of the new scalar and the Higgs leads to chiral Lagrangian where we make the replacement:

$$s' \approx s - \theta h \approx s - \frac{\lambda_p v_h}{2\lambda_s v_s} h$$

Part III
Q-balls from the Hidden Sector

Minimising Energy for Fixed Charge

- To minimise the energy in a sector of fixed charge, we introduce a Lagrange multiplier and minimise the functional

$$\mathcal{E}_\omega = E + \omega \left(Q - i \int \mathrm{d}^3 x \, \left(1 - \varphi \frac{2\kappa}{3} \frac{h}{v_s} \right) \pi^+ \overleftrightarrow{\partial}_t \pi^- \right)$$

- Minimising the time-dependent terms of the cubic expansion yields

$$\pi^\pm(x,t) = e^{\pm i\omega t} \pi^\pm(x)$$
$$\pi^0(x,t) = \pi^0(x)$$
$$h(x,t) = h(x)$$

Analytic Example: Properties

- Can solve analytically if there are no heavy quarks in hidden sector; also make following assumptions for clarity of resulting expressions:

$$\frac{1}{2}\frac{m_h^2}{m_\pi^2}\alpha^2 \ll 1 \quad \text{and} \quad \frac{\lambda v_h v_s}{\theta m_\pi^2}\alpha^2 \ll 1$$

- Mass and radius of Q-ball given by

$$\frac{M_Q}{Q m_\pi} = 1 - \frac{1}{6}\epsilon^2 - \mathcal{O}(\epsilon^4) \quad \text{and} \quad R_Q^{-1} \sim \frac{\epsilon m_\pi}{\sqrt{3}}\left(1 + \frac{1}{2}\epsilon^2 + \mathcal{O}\left(\epsilon^4\right)\right)$$

$$\epsilon \equiv \frac{4}{9\sqrt{3}S_\psi}\frac{Q\theta^2 m_\pi^2}{v_s^2} < \frac{1}{2}$$

Analytic Example: Constraints

- No lower bound on charge for (classical) stability

- Upper bounds on charge:

$$Q \lesssim 76 \left(\frac{v_s}{\theta m_\pi}\right)^2, \quad Q \ll 150 \frac{v_s}{\theta m_\pi \sqrt{\lambda}}, \quad Q \ll 430 \frac{v_s f}{\theta m_\pi^2}$$

- Fermi repulsion constraint:

$$Q \lesssim 0.1 \left(\frac{m_f}{m_\pi}\right)^{3/2}$$

Numerical Analysis

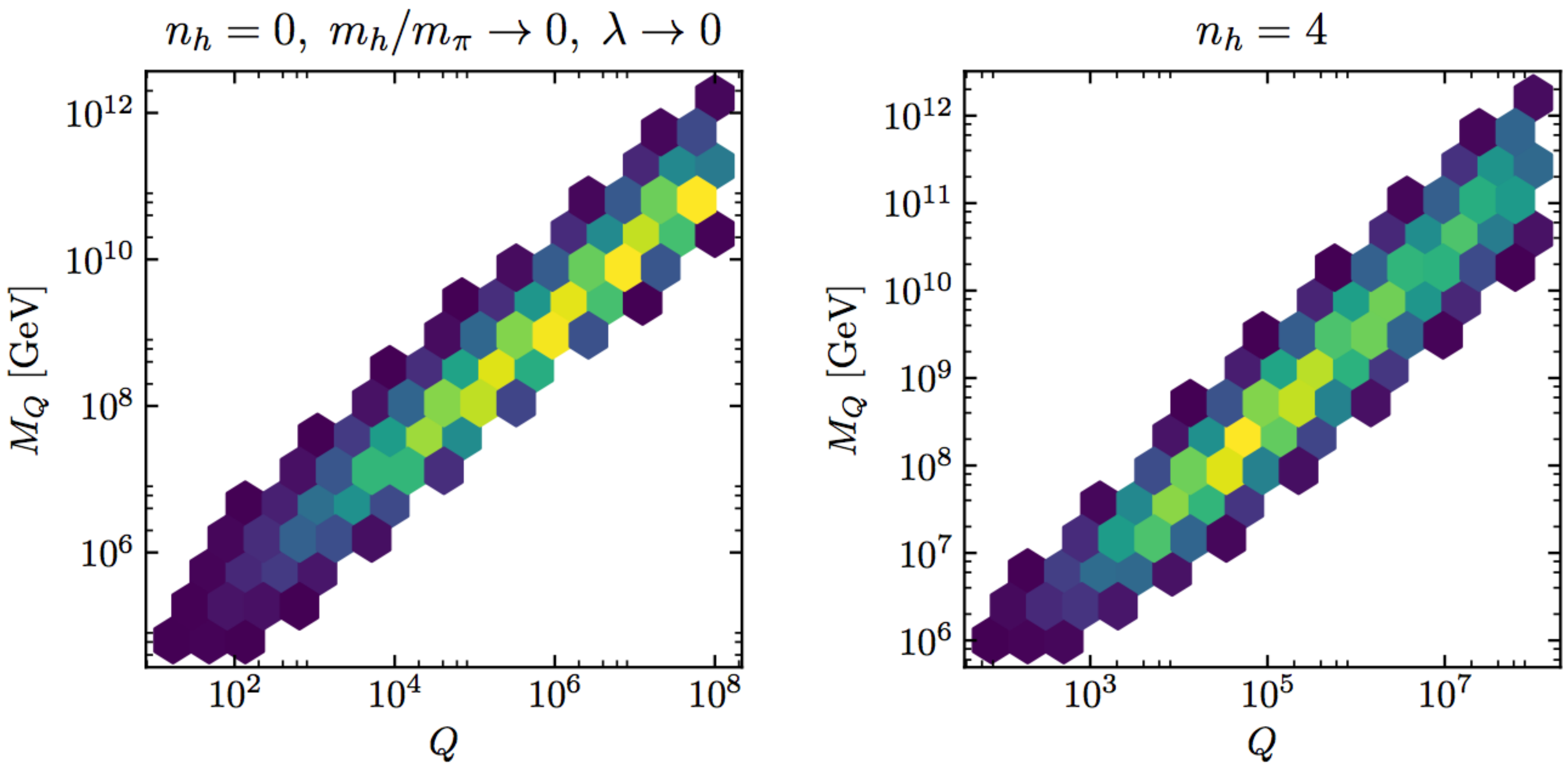
- Scan over parameters of the theory to find Q-ball solutions

Parameter	Range	Distribution
Q	$[1, 10^8]$	log-uniform
v_s [TeV]	$[1, 10]$	log-uniform
m_π [TeV]	$[0.5, 2v_s]$	log-uniform
θ	$[10^{-4}, 0.1]$	log-uniform
λ	$[10^{-6}, 10^{-1}]$	log-uniform

- Results are largely independent of number of heavy quarks: only enters in small modification

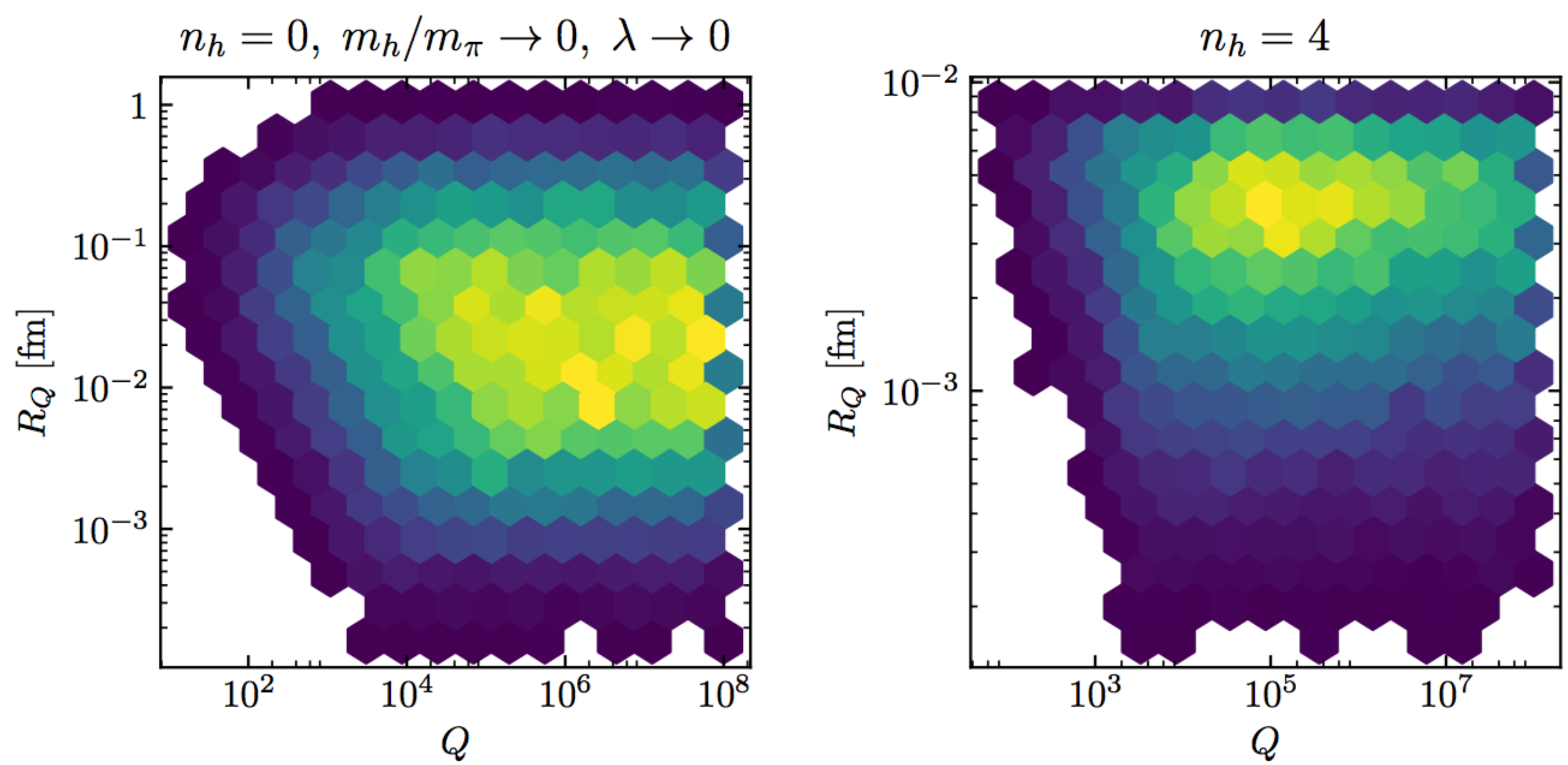
- Results are more dependent on re-introducing Higgs mass and Higgs cubic self-coupling

Numerical Results



Mass vs Charge

Radius vs Charge



Part IV
Summary

Summary

- Presented an existence "proof" for Q-balls formed from pseudo-Nambu-Goldstone bosons arising from a strongly interacting hidden sector

- These Q-balls are kept stable by the inclusion of the SM Higgs

- Future work: Thin-wall Q-balls? Dark matter formation and phenomenology of the model

Appendices

Q-balls: Thin-Wall

Coleman 1985

- Lower bound of stability condition: $\omega = \omega_0 = \sqrt{\frac{U(\phi_0)}{\phi_0^2}}$

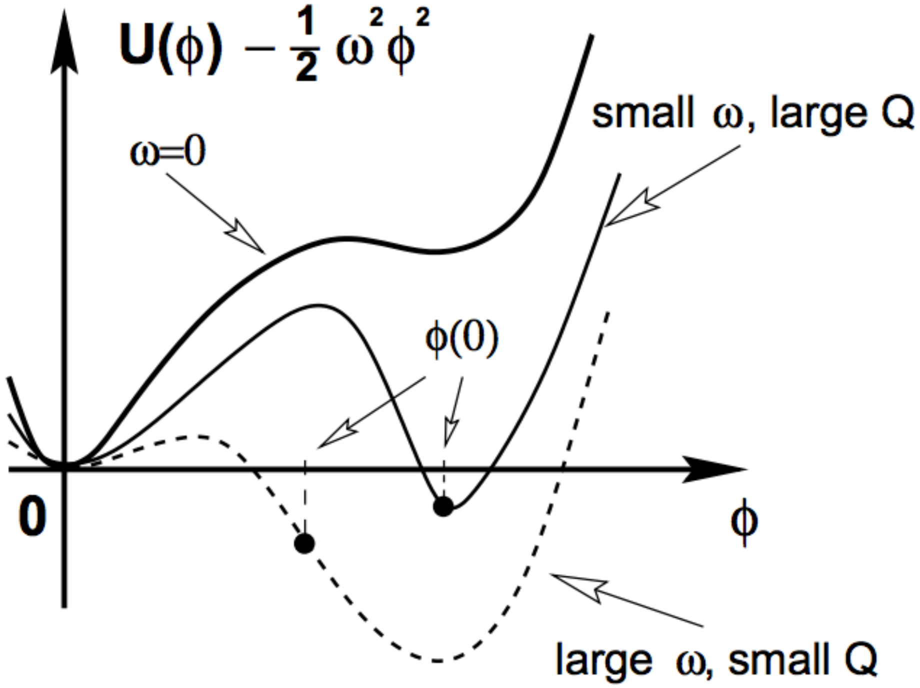
- Large charge/large radius limit, where volume effects dominate the surface effects

- Mass of a thin-wall Q-ball: $m_Q = Q\omega_0$

Q-balls: Thick-Wall

Kusenko 1997

- Upper bound of stability condition: $\omega \to m$
- Small charge/small field limit



- Mass in terms of small parameter:

$$m_Q = mQ\left(1 - \frac{1}{6}\epsilon^2 - \mathcal{O}(\epsilon^4)\right) \qquad 0 < \epsilon < 1/2$$