Flavour Physics Introduction

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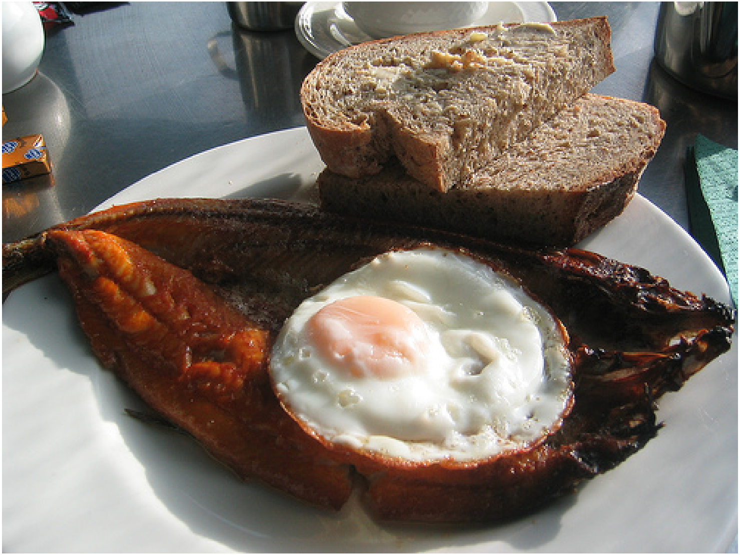
Flavour Physics?



Who ordered that?

Carl Anderson

- Start 1936: Discovery of the muon [Anderson und Neddermeyer]. (Same charge as electron, different mass)
- End 2000: Discovery of ν_τ by the DONUT Collaboration

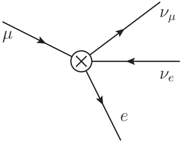
1. Generation	ν_e	e	u	d
2. Generation	ν_μ	μ	c	s
3. Generation	ν_τ	τ	t	b
Electric charge	0	-1	$\frac{2}{3}$	$-\frac{1}{3}$
Colour charge	1	1	3	3

Content

- charged currents
  - Muon decay
  - (Semi-)leptonic decays
- Standard model flavour sector
- Rare decays
- An explicit matching calculation

Fermi Theory

Enrico Fermi



- Effective Theory for $\mu \to e\bar{\nu}\nu$:

$$\mathcal{L}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} \left(\bar{\nu}_\mu \gamma^\lambda P_L \mu\right) \left(\bar{e}\gamma_\lambda P_L \nu_e\right)$$

(Originally without $P_L = (1-\gamma_5)/2$, i.e. left-chirality projector)

- Insertion of operator yields

$$\mathcal{M}_{fi} = -\frac{4G_F}{\sqrt{2}} \left[\bar{u}(\nu_\mu)\gamma^\lambda P_L u(\mu)\right] \left[\bar{u}(e)\gamma_\lambda P_L v(\nu_e)\right]$$

Compare with decay rate Γ respectively lifetime $\tau = \hbar/\Gamma$:

$|\mathcal{M}_{fi}|^2$

Let us determine G_F and compute:

$$|\mathcal{M}_{fi}^{abcd}|^2 = 8G_F^2 \left[\bar{u}^b(\nu_\mu)\gamma^\lambda P_L u^a(\mu)\right]\left[\bar{u}^a(\mu)\gamma^\sigma P_L u^b(\nu_\mu)\right] \left[\bar{u}^c(e)\gamma_\lambda P_L v^d(\nu_e)\right]\left[\bar{v}^d(\nu_e)\gamma_\sigma P_L u^c(e)\right]$$

Average over μ polarisation and sum over final state polarisations:

$$\frac{1}{2}\sum_a\sum_b\sum_c\sum_d \left|\mathcal{M}_{fi}^{abcd}\right|^2$$

$$\sum_a \bar{u}^a(\mu)_i\bar{u}^a(\mu)_j \to (\not{p}+m_\mu)_{ij} \qquad \sum_b u^b(\nu_\mu)_i\bar{u}^b(\nu_\mu)_i \to (\not{q}_2)_{ij}$$

$$\sum_c \bar{u}^c(e)_i\bar{u}^c(e)_j \to (\not{k}+m_e)_{ij} \qquad \sum_d u^d(\nu_e)_i\bar{u}^d(\nu_e)_i \to (\not{q}_1)_{ij}$$

Gives: $|\mathcal{M}_{fi}|^2 = G_F^2 Tr[\not{q}_2\gamma^\mu\not{p}\gamma^\nu(1-\gamma^5)]Tr[\not{q}_1\gamma_\nu\not{k}\gamma_\nu(1-\gamma^5)]$

Decay rate

In μ rest frame: $d\Gamma = \frac{1}{2m_\mu}|\mathcal{M}_{fi}|^2 d\Phi^{(3)}$.

After trace evaluation: $|\mathcal{M}_{fi}|^2 = 64G_F^2(pq_1)(kq_2)$

$$d\Gamma = \frac{32G_F^2}{m_\mu}\frac{(2\pi)^4}{(2\pi)^9}\frac{d^3k}{2E_{\vec{k}}}\frac{d^3q_1}{2E_1}\frac{d^3q_2}{2E_2}\delta^{(4)}(p-k-q_1-q_2)p_\mu q_1^\mu k_\nu q_2^\nu$$

Integration over q_1 and q_2 and neglecting $m_e \ll m_\mu$ gives:

$$\frac{d\Gamma}{dE} = \frac{G_F^2}{12\pi^3}E^2(3m_\mu^2 - 4m_\mu E) \quad \to \quad \Gamma = \frac{G_F^2 m_\mu^2}{192\pi^3}$$

And $\Gamma^{\text{exp}} = 2.99598 \cdot 10^{-19}\text{GeV}^{-2}$ yields:
$G_F = 1.16410^{-5}GeV^{-2}$

Adding QED and m_e Corrections:
$G_F = 1.16637(1)10^{-5}\text{GeV}^{-2}$.

Universality

Apart from elektro-weak corrections are the charged-current interactions of leptons and quarks similar.

E.g. for the interaction of an up-quark

$$\mathcal{L}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}}\left(\bar{\nu}_\mu \gamma^\lambda P_L \mu\right)\left(\bar{e}\gamma_\lambda P_L \nu_e\right) - \frac{4G_F}{\sqrt{2}}\left(\bar{\nu}_\ell \gamma_\lambda P_L \ell\right)\left(\bar{d}' \gamma^\lambda P_L u\right)$$

where d' is the weak partner of the up-quark.

which is related via the unitary CKM Matrix to the mass eigenstates:

$$\bar{d}' = \bar{d}V_{ud}^* + \bar{s}V_{us}^* + \bar{b}V_{ub}^*$$

$\pi^+ \to l^+ \nu$

Adopting the Fermi theory to π^+ ($\pi^+ \leftrightarrow \bar{d}u$):

$$\mathcal{L}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{ud}^* \left(\bar{\nu}_\ell \gamma_\lambda P_L \ell\right) \left(\bar{d}\gamma^\lambda P_L u\right)$$

At leading order we have

$$\mathcal{M}_{fi} = -\frac{4G_F}{\sqrt{2}} V_{ud}^* \langle \ell^+ \nu_\ell | \left[\bar{\nu}_\ell \gamma_\lambda P_L \ell\right] \left[\bar{d}\gamma^\lambda P_L u\right] |\pi^+\rangle^{\text{QCD}}$$

Insert $|0\rangle\langle 0|$ since leptons are colour singlets.

$$\mathcal{M}_{fi} = -\frac{4G_F}{\sqrt{2}} V_{ud}^* \langle \ell^+ \nu_\ell | \left[\bar{\nu}_\ell \gamma_\lambda P_L \ell\right] |0\rangle\langle 0| \left[\bar{d}\gamma^\lambda P_L u\right] |\pi^+\rangle^{\text{QCD}}$$

Matrixelements for $\pi^+ \to l^+\nu$

The Matrixelement for the leptons is:

$$\langle \ell^+ \nu_\ell | \left[\bar{\nu}_\ell \gamma_\lambda P_L \ell \right] |0\rangle = \bar{u}(\nu_\ell)\gamma_\lambda P_L v(\ell)$$

The hadronic $\langle 0| \left[\bar{d}\gamma^\lambda P_L u \right] |\pi^+\rangle$ most be proportional to a Lorentz vector.
The only relevant is the π^+-4-momentum $p^\lambda_{\pi^+}$

$$\langle 0| \left[\bar{d}\gamma^\lambda P_L u \right] (0)|\pi^+\rangle = -\tfrac{1}{2} f_\pi \, p^\lambda_{\pi^+}$$

QCD Effects are absorbed into the decay constant f_π.
f_π can e.g. be calculated with Lattice QCD.
π^+ is pseudo-scalar we have:

$$\langle 0| \left[\bar{d}\gamma^\lambda \gamma_5 u \right] |\pi^+\rangle = f_\pi p^\lambda_{\pi^+}$$

Partial Decay Rate

$|\mathcal{M}_{fi}|^2$ and phase-space integration gives:

$$\Gamma(\pi^+ \to \ell^+ \nu) = \frac{G_F^2}{8\pi} f_\pi^2 |V_{ud}|^2 m_\ell^2 m_\pi \left(1 - m_\ell^2/m_\pi^2\right)^2$$

In particular we have

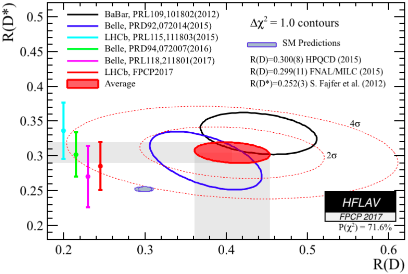
$$\frac{\Gamma(\pi^+ \to e^+ \nu)}{\Gamma(\pi^+ \to \mu^+ \nu)} = \frac{m_e^2}{m_\mu^2} \left(\frac{1 - m_e^2/m_\pi^2}{1 - m_\mu^2/m_\pi^2}\right)^2$$

After adding QED-Corrections one finds agreement with PDG

$$\frac{\Gamma(\pi^+ \to e^+ \nu(\gamma))}{\Gamma(\pi^+ \to \mu^+ \nu(\gamma))} = 1.230(4) \cdot 10^{-4}$$

$b \to c\tau\nu$ Anomaly

B-factories & LHCb show $B \to D^{(*)}\tau\nu$ decay rates larger than expected



$$R(D^{(*)}) = \frac{\mathrm{BR}(B \to D^{(*)}\tau\nu)}{\mathrm{BR}(B \to D^{(*)}\ell\nu)}$$

- 4.1 σ
- SM tree-level
- Experimental uncertainty dominant

Branching Fractions

Branching Fractions of Charged and Neutral Currents have quite different sizes

- charged current
  $\mathcal{B}(K^+ \to \mu^+\nu) = \Gamma(K^+ \to \mu^+\nu)/\Gamma_{\text{tot}} = 64\%$
- neutral current $\mathcal{B}(K_L \to \gamma\gamma) \simeq 5 \cdot 10^{-5}$
- neutral current $\mathcal{B}(K_L \to \mu^+\mu^-) \simeq 7 \cdot 10^{-9}$
- neutral current $\mathcal{B}(B_s \to \mu^+\mu^-) \simeq 3 \cdot 10^{-9}$

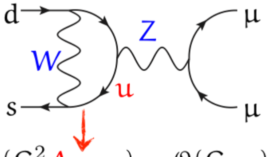
Why are the branching fractions so different?

Before the discovery of charm quark:

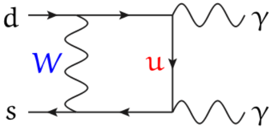
- why are the two Branching ratios

$$\frac{\mathcal{B}(K_L \to \mu^+\mu^-)}{\mathcal{B}(K_L \to \gamma\gamma)} \simeq \frac{6.84(11)\cdot 10^{-9}}{5.47(4)\cdot 10^{-4}}$$ so different in size?

- K_L pseudo-scalar $\Rightarrow$ no 1 γ coupling to $\mu^+\mu^-$:



$\mathcal{O}(G_F^2 \Lambda_{\mathrm{Fermi}}) = \mathcal{O}(G_F \alpha)$

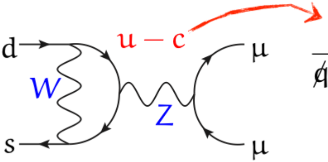


$\mathcal{O}(G_F \alpha)$

- naively they would be off similar size

GIMnastics

- Introduce charm quark to suppress $K_L \to \mu^+\mu^-$
- Charm quark and up quark cancel above $E \geqslant m_c$:



$$\frac{1}{\not{q} - m_u} \to \frac{1}{\not{q} - m_u} - \frac{1}{\not{q} - m_c}$$

Cutoff is then m_c^2

- The rate is hence $\propto \mathcal{O}(G_F^2 m_c^2)$
- Predict $m_c \sim 1.5$GeV [Gaillard, Lee'74]

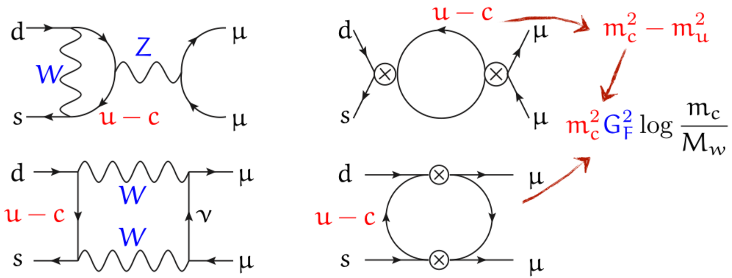
$$\frac{\Delta M_K}{M_K} \simeq \frac{G_F^2 f_K^2}{6\pi^2} |V_{cs} V_{cd}|^2 m_c^2 = 7 \cdot 10^{-15}$$

- similarly $m_t \sim M_W$ from

$$\frac{\Delta M_B}{M_B} \simeq \frac{G_F^2 f_B^2}{6\pi^2} |V_{tb} V_{td}|^2 M_W^2 S(\frac{m_t^2}{M_W^2}) = 6 \cdot 10^{-14}$$

Effective Coefficients Highly Tuned

- In practice semi-leptonic and four-quark operators contribute
- Their effective coefficients have to be highly tuned to be consistent with data



- The constraints in the effective theory is a remnant of the structure of the full theory

The Standard Model Lagrangian

Standard Model: $SU(3) \times SU(2) \times U(1)$, one Higgs $\phi = \begin{pmatrix} 0 \\ (v + h^0)/\sqrt{2} \end{pmatrix}$ and 15 Fermion fields:

$$Q_i(3,2)_{1/6},\ u_i(3,1)_{2/3},\ d_i(3,1)_{-1/3},\ L_i(1,2)_{-1/2},\ e_i(1,1)_{-1}$$

comprises a highly symmetric kinetic term
$(\mathcal{L} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{\text{Higgs}})$

$$\mathcal{L}_{\text{kin}} = \sum_{\substack{\psi=Q,u,d,L,e \\ i=1,2,3}} \overline{\psi}_i \not{D} \psi_i + \frac{1}{4} \sum_{F=B,W,G} F^a_{\mu\nu} F^{a,\mu\nu}$$

with a $U(3)^5$ flavour symmetry. Broken by Yukawa interactions

$$\mathcal{L}_Y = -Y^d_{ij} \overline{Q}_i \phi d_j - Y^u_{ij} \overline{Q}_i \tilde{\phi} u_j - Y^e_{ij} \overline{L}_i \tilde{\phi} e_j + \text{h.c.}$$

CKM Matrix

- In the quark sector $Y^d_{ij}\overline{Q}_i \phi d_j + Y^u_{ij}\overline{Q}_i \tilde{\phi} u_j$ breaks $U(3)^3 = U(3)_Q \times U(3)_u \times U(3)_d \to U(1)_B$.
- This removes $3 \times U(3) - 1 \times U(1)_B$ parameters, i.e. $3(3\text{Re} + 6\text{Im}) - 1\text{Im} = 9\text{Re} + 17\text{Im}$ parameters
- $Y_u \& Y_d$ give $2 \cdot 3 \cdot 3 \cdot (\text{Re} + \text{Im}) - 9\text{Re} + 17\text{Im}$ physical parameters: 6 masses, 3 mixing angles and 1 phase.
- Choosing diagonal $Y^u = \frac{\sqrt{2}}{v}\text{diag}\{m_u, m_c, m_t\}$ we have

$$Y^d = \frac{\sqrt{2}}{v} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}$$

CKM Interaction and Mass Eigenstates

Writing $Q_i = \begin{pmatrix} u_{i_L} \\ d'_{i_L} \end{pmatrix} = \begin{pmatrix} u_{i_L} \\ V_{ij} d_{j_L} \end{pmatrix}$ gives

$$\mathcal{L}_{\text{down mass}} = -\overline{d'}_{i_L} V_{ij} m_{d_j} d_{j_R} = -m_{d_i} \overline{d}_{i_L} d_{i_R} + h.c.$$

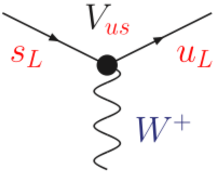
and charged-current flavour violation

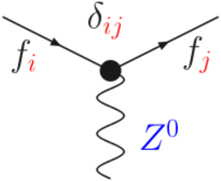
$$\mathcal{L}_{udW^\pm} = -\frac{g}{\sqrt{2}} \overline{u_{i_L}} \gamma^\mu V_{ij} d_{j_L} W^+_\mu + h.c.$$

and no neutral-current flavour violation.

Neutral & Charged Current Interactions

Mass $\neq$ flavour eigenstates





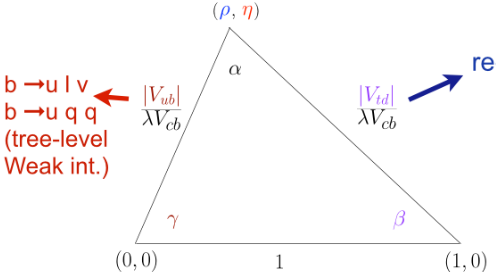
- Only charged currents change flavour ($\propto V_{us}$)
- Neutral currents are flavour diagonal ($i = j$) at tree level

Unitarity Triangle

- 3 CKM angles |Vub|, |Vcb| & |Vus| from (semi)leptonic B & K decays
- CP violation in the standard model area of unitarity triangle

$$\text{Unitarity of } V \Rightarrow \begin{array}{ccccccc} V_{ub}^* V_{ud} & + & V_{cb}^* V_{cd} & + & V_{tb}^* V_{td} & = & 0 \\ A\lambda^3(\rho + i\eta) & - & A\lambda^3 & + & A\lambda^3(1 - \rho - i\eta) & = & 0 \end{array}$$

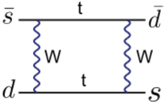
Graphically,

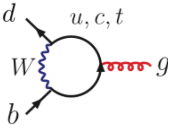


requires top loop

$V_{ub} = |V_{ub}|e^{-i\gamma}$

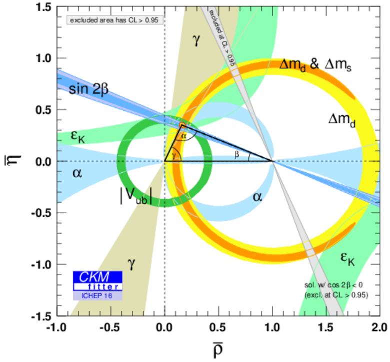
$V_{td} = |V_{td}|e^{-i\beta}$





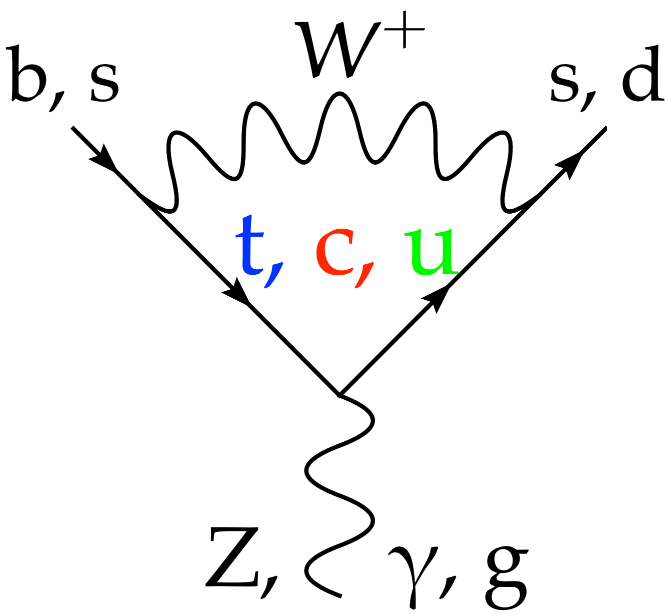
Wolfenstein

- CKM parameters from observables that are less new physics sensitive.
- Can be used to make predictions for new physics sensitive observables.



$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

CKM Factors in Rare decays



Semi-leptonic decays (V_{us}): $\lambda = \mathcal{O}(0.2)$

$$V_{ij} = \mathcal{O}\begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

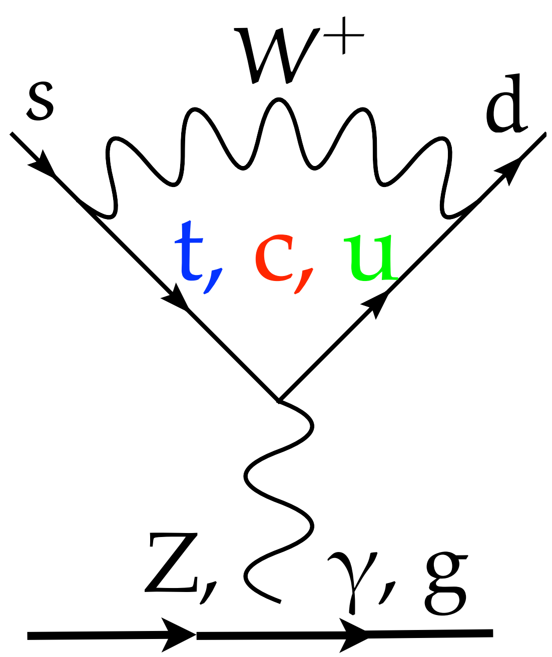
FCNCs which are dominated by top-quark loops:

$b \to s:$	$b \to d:$	$s \to d:$
$\lvert V_{tb}^* V_{ts}\rvert \propto \lambda^2$	$\lvert V_{tb}^* V_{td}\rvert \propto \lambda^3$	$\lvert V_{ts}^* V_{td}\rvert \propto \lambda^5$

Kaon Physics has background from light states:

$$\mathrm{Re} V_{us}^* V_{ud} = -\mathrm{Re} V_{cs}^* V_{cd} = \mathcal{O}(\lambda^1)$$

CKM Factors in Kaon physics



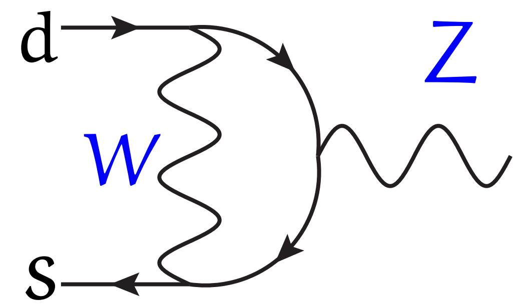
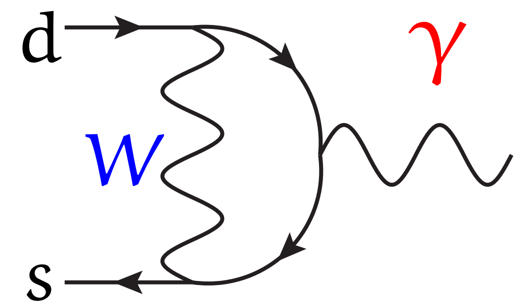
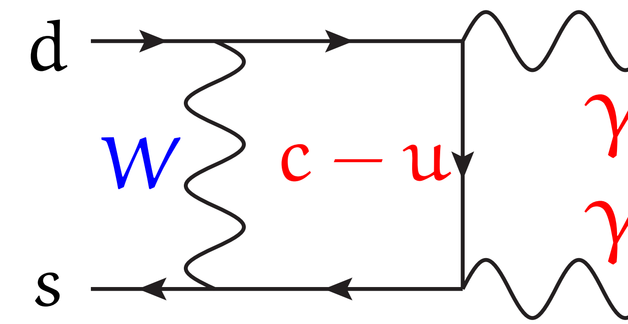
Kaon observables $\propto V_{ts}^* V_{td} \rightarrow$
suppressed in SM
sensitive to flavour violating NP

Using the GIM mechanism,
we can eliminate either $V_{cs}^* V_{cd}$ or
$V_{us}^* V_{ud} \rightarrow - V_{cs}^* V_{cd} - V_{ts}^* V_{td}$

Z-Penguin and Boxes (high virtuality):
power expansion in: $A_c - A_u \propto 0 + O(m_c^2/M_W^2)$

γ/g-Penguin (momentum expansion + e.o.m.):
power expansion in: $A_c - A_u \propto O(\mathrm{Log}(m_c^2/m_u^2))$

Rare Kaon Decays

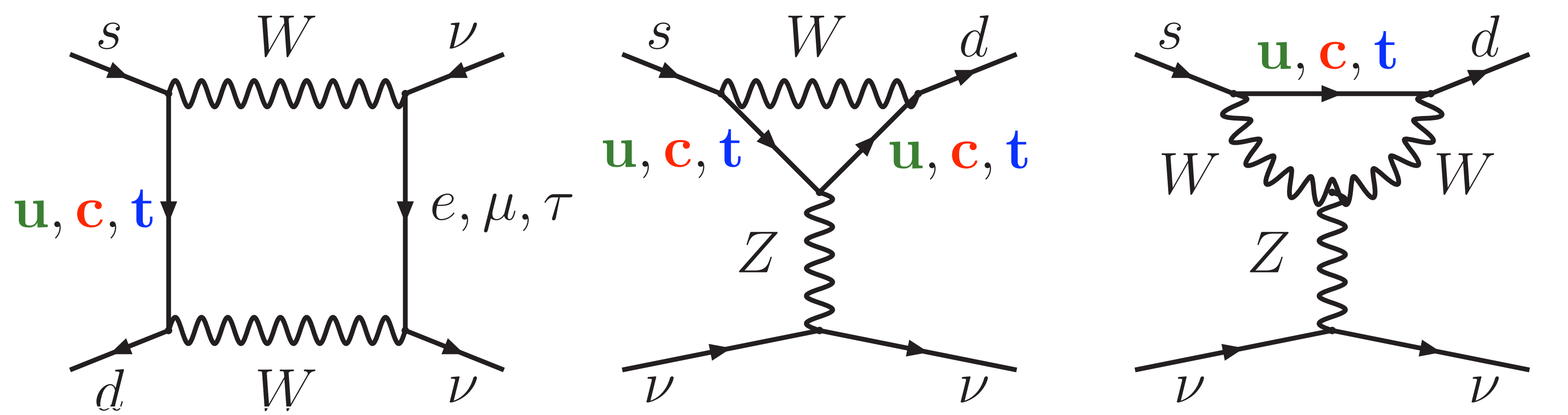
			
$K_L \to \mu^+\mu^-$	SD	–	α_e LD
$K \to \pi\nu\bar{\nu}$	SD	–	–
~~$K_S \to \pi l^+ l^-$~~	~~–~~	~~LD~~	~~–~~
$K_L \to \pi l^+ l^-$	SD	SD $+\epsilon_K$ LD	α_e LD

CP violating

most NNLO QCD known

χPT, Large N [1603.09721 + Ref]
Now: Accessible to Lattice
Talk by Portelli

$K \to \pi \bar{\upsilon} \upsilon$



$$x_i = \frac{m_i^2}{M_W^2}$$

$$\sum_i V_{is}^* V_{id} F(x_i) = V_{ts}^* V_{td}(F(x_t) - F(x_u)) + V_{cs}^* V_{cd}(F(x_c) - F(x_u))$$

Top (SD),

Charm (Renormalisation Group Improved) &

Light Quarks (Non-Perturbative)

$$\lambda^5 \frac{m_t^2}{M_W^2}$$

$$\lambda \frac{m_c^2}{M_W^2} \ln \frac{M_W}{m_c}$$

$$\lambda \frac{\Lambda_{QCD}^2}{M_W^2}$$

$$Q_\nu = (\bar{s}_L \gamma_\mu d_L)(\bar{\nu}_L \gamma^\mu \nu_L)$$

Matrix element from K_{l3} decays (Isospin symmetry: $K^+ \to \pi^0 e^+ \upsilon$)

[Mescia, Smith]

Rare B Decays

B decays do not show the CKM suppression of K decays

2 photon pollution is much smaller in $b \to s\, l^+ l^-$ decays

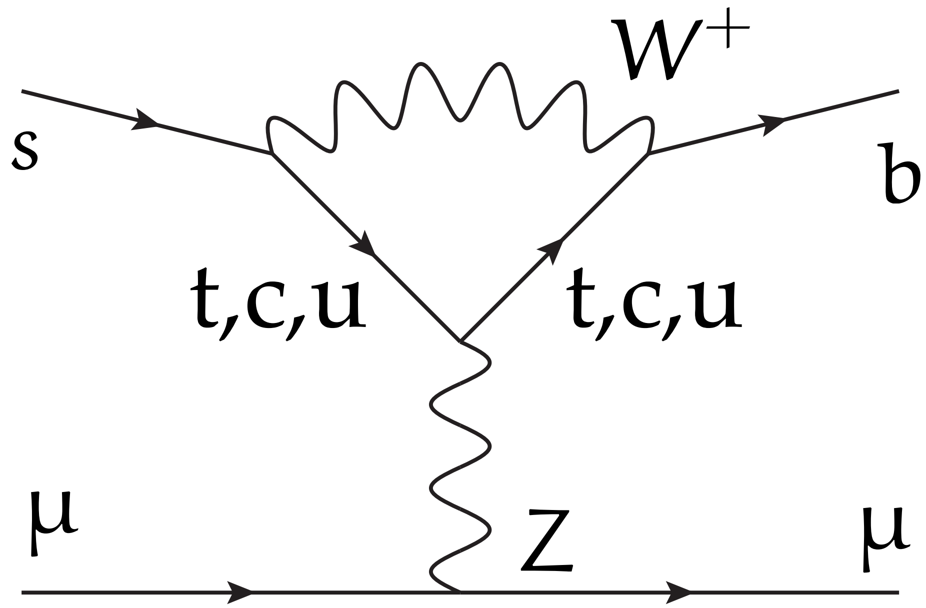
We can test helicity suppressed modes and more operators

$$Q_7 = (\bar{b}_L \sigma_{\mu\nu} s_L) F^{\mu\nu}, \quad Q_V = (\bar{b}_L \gamma_\mu s_L)(\bar{l} \gamma_\mu l), \quad Q_A = (\bar{b}_L \gamma_\mu s_L)(\bar{l} \gamma_\mu \gamma_5 l)$$

$=Q_{10}$ $=Q_9$

E.g. $B_{(s)} \to l^+ l^-$, $B \to K^{(*)} l^+ l^-$, $B \to X_s \gamma$, ...

$B_s \to \mu^+ \mu^-$ in the Standard Model



+ Box diagrams

B_s is (pseudo)scalar – no photon penguin

$$Q_A = (\bar{b}_L \gamma_\mu s_L)(\bar{l}\gamma_\mu \gamma_5 l)$$

Dominant operator in the SM

helicity suppression $\left(\propto \frac{m_l^2}{M_B^2}\right)$

$$\propto |V_{tb}^* V_{ts}| \simeq \left|1 - \lambda^2 \left(\tfrac{1}{2} - i\eta - \rho\right)\right| V_{cb}$$

Effective Lagrangian in the SM:

$$\mathcal{L}_{\text{eff}} = G_F^2 M_W^2 \, V_{tb}^* V_{ts} \left(C_A Q_A + C_S Q_S + C_P Q_P\right) + \text{h.c.}$$

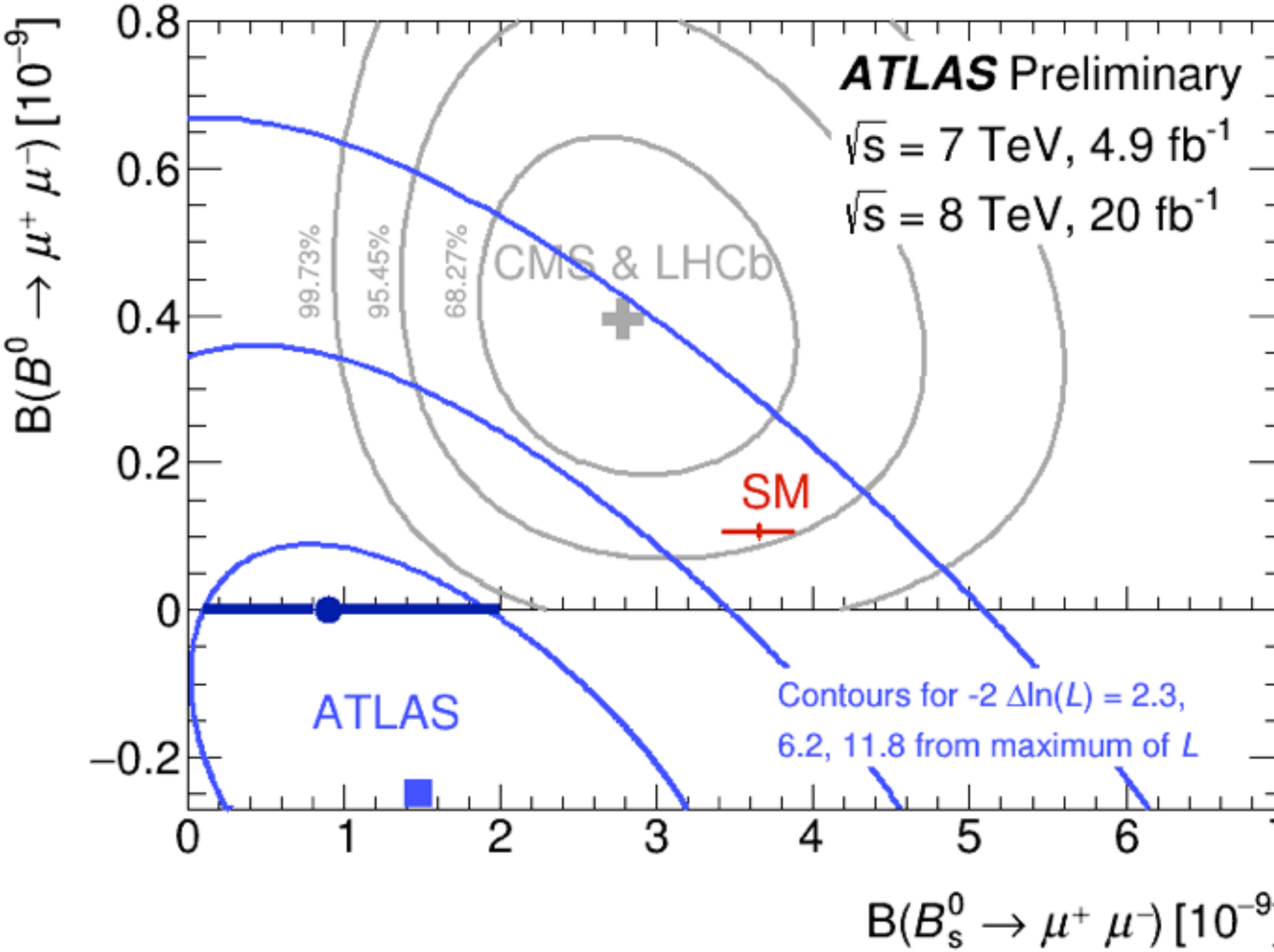
Scalar operators: $Q_S = (\bar{b}_R q_L)(\bar{l}l)$ $\quad Q_P = (\bar{b}_R q_L)(\bar{l}\gamma_5 l)$

Standard Model: C_S & C_P are highly suppressed

Theory Prediction $B_s \to \mu^+\mu^-$

We find for the time integrated BR @ NNLO & EW

[Bobeth MG, Hermann, Misiak, Steinhauser, Stamou `13] $Br_{the} = (3.65 \pm 0.23)\ 10^{-9}$



$B_s \to \mu^+ \mu^-$ and New Physics

Contribution of Q_S and Q_P are not helicity suppressed

Potentially large coefficients C_S and C_P in 2HDM

Yet, only if contribution to ΔM_s is suppressed,
i.e. type 2 Higgs potential, $\lambda_5 \ll 1$ and type 3 Yukawas

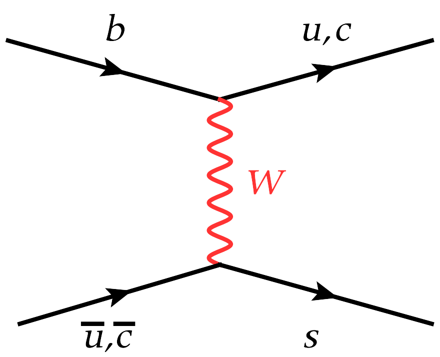
which is the MSSM at $\tan\beta \gg 1$, with the Branching Ratio

$$BR \propto (\tan\beta)^6 \; M_A^{-4}$$

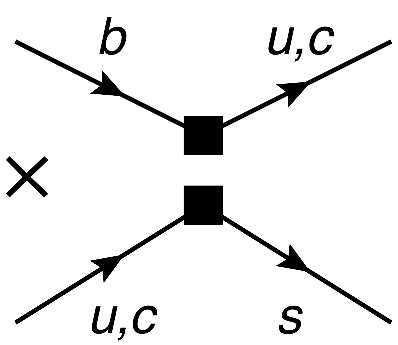
Non-zero $\Delta\Gamma_s$ allows for another untagged observable beyond the BR via an effective lifetime measurement.
[Bruyn, Fleischer, Knegjens et.al. `12]

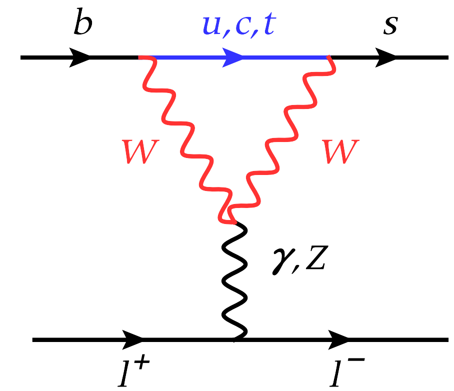
Status of $\mathcal{L}_{\text{eff}}$ for $b \to s\, l^+ l^-$

SM Wilson coefficients: Matching at $\mu \approx M_W$

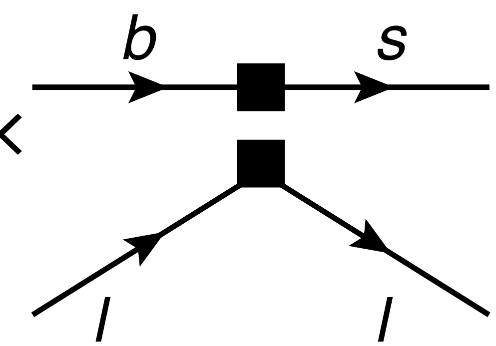


$\to C_2 \times$



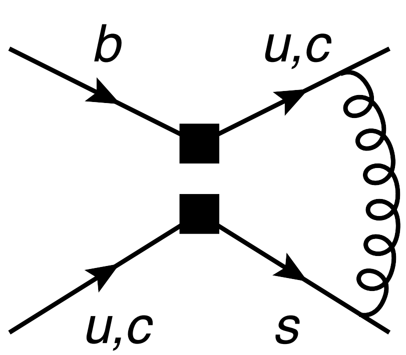


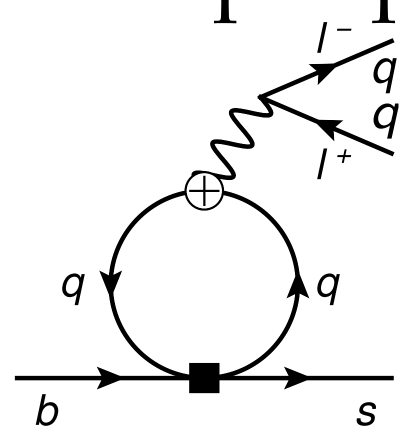
$\to C_{A,V} \times$ $(C_{10,9})$



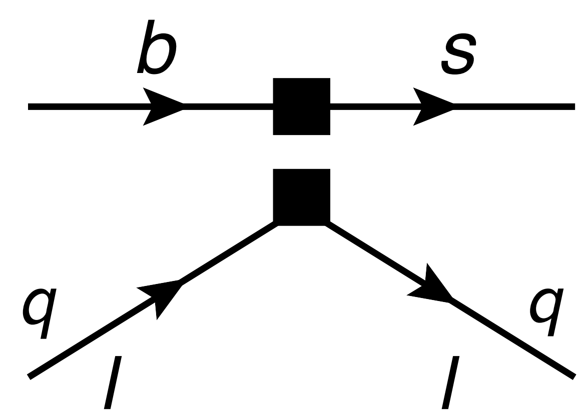
Known at two-loops in QCD for NNLL [Bobeth, Misiak, Urban, `99]

Renormalisation Group Equation $\to \mu \approx M_W$





$\to C_{1\ldots10} \times$

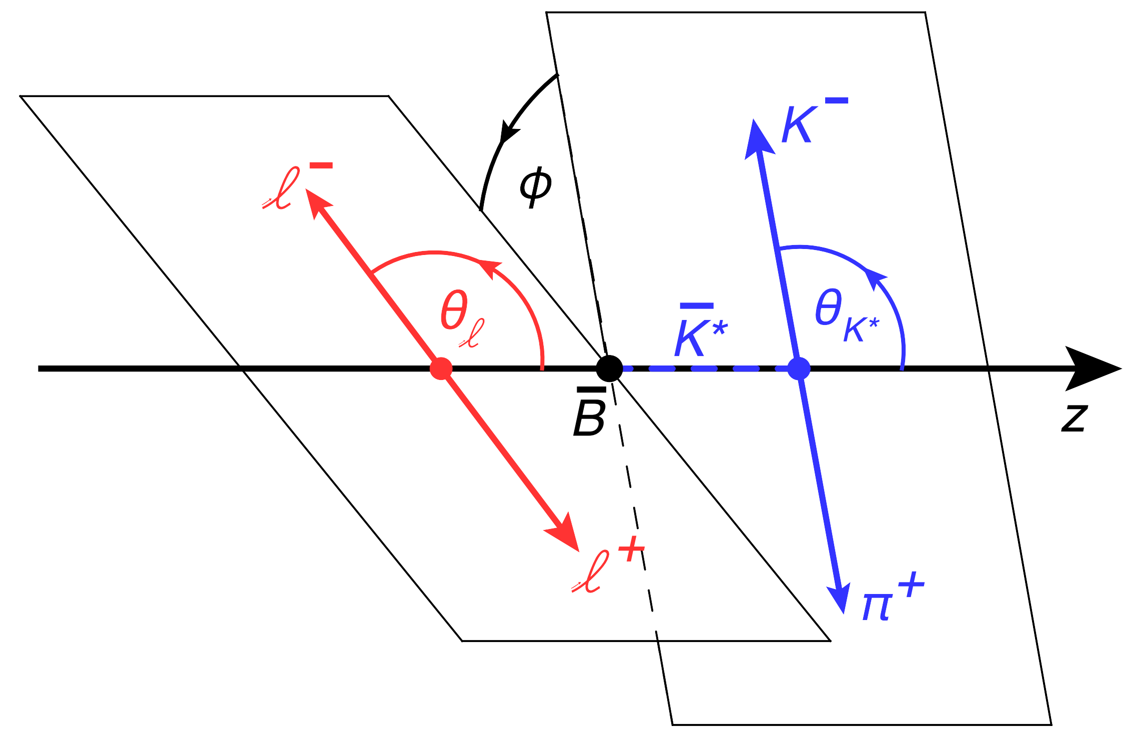


$\mathcal{L}_{\text{eff}}$ @ NNLL in QCD and NLL EW for all but C_9 & C_{10} EW matching

[Gambino Haisch`01; Haisch `05, Bobeth, Gambino, MG, Haisch `04, MG, Haisch `05, Huber et. al. `05]

$B \to K^{(*)}[\to K\pi] + \ell^+\ell^-$

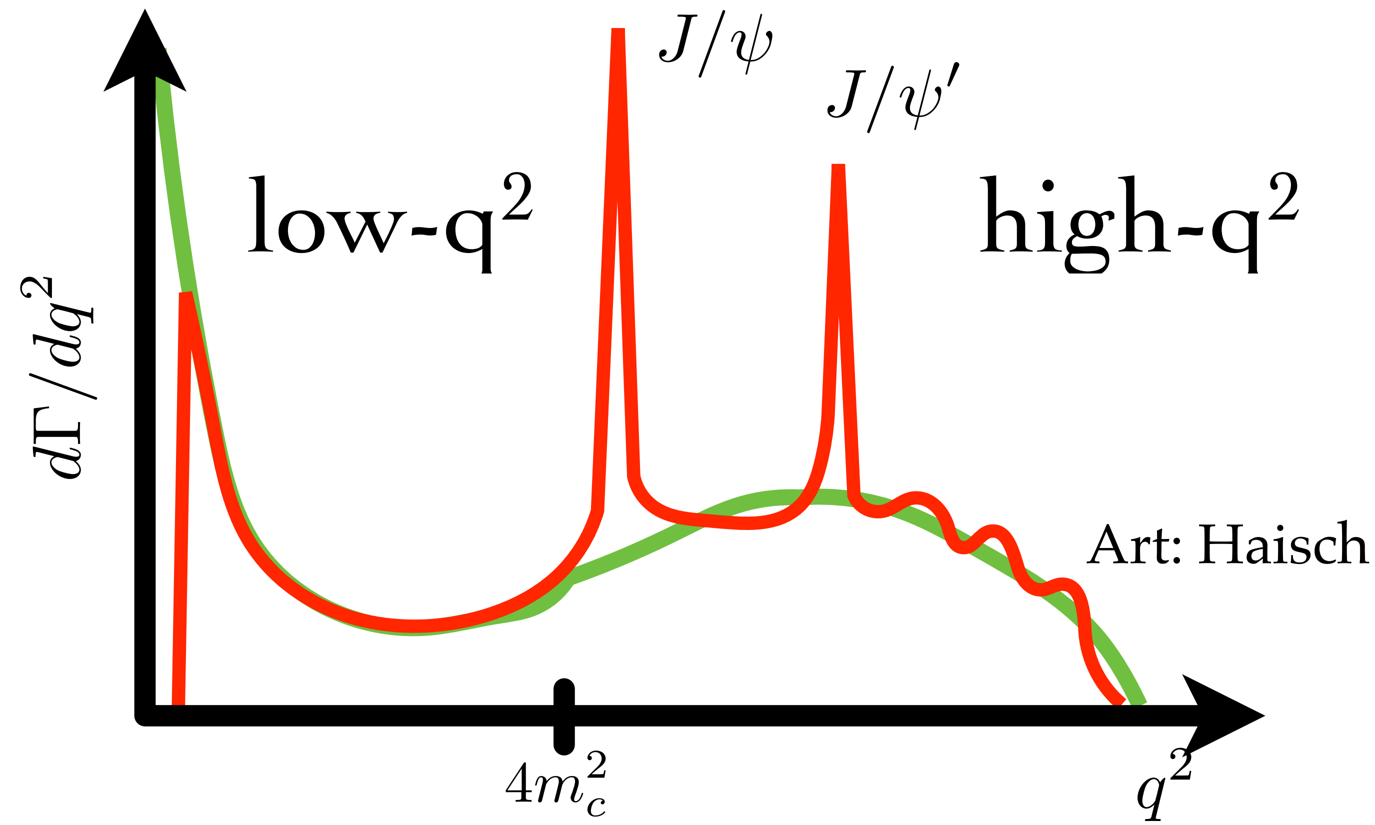
Many angular observables
for
$B \to K^{(*)}[\to K\pi] + \ell^+\ell^-$



$$\frac{32\pi}{9}\frac{d^4\Gamma}{dq^2\, d\cos\theta_\ell\, d\cos\theta_K\, d\phi} = J_{1s}\sin^2\theta_K + J_{1c}\cos^2\theta_K + (J_{2s}\sin^2\theta_K + J_{2c}\cos^2\theta_K)\cos 2\theta_\ell$$
$$+J_3\sin^2\theta_K\sin^2\theta_\ell\cos 2\phi + J_4\sin 2\theta_K\sin 2\theta_\ell\cos\phi + J_5\sin 2\theta_K\sin\theta_\ell\cos\phi$$
$$+(J_{6s}\sin^2\theta_K + J_{6c}\cos^2\theta_K)\cos\theta_\ell + J_7\sin 2\theta_K\sin\theta_\ell\sin\phi$$
$$+J_8\sin 2\theta_K\sin 2\theta_\ell\sin\phi + J_9\sin^2\theta_K\sin^2\theta_\ell\sin 2\phi$$

From these one can construct
observables as e.g. forward backward asymmetry

Exclusive $B \rightarrow K^{(*)} l^+ l^-$ decays



Systematic heavy-quark expansion in Λ_{QCD}/mb (SCET) for $q^2 << m^2(J/\psi)$
[Benke, Feldmann, Seidel `01]

OPE for $q^2 >> m^2(J/\psi)$
[Grinstein et.al. ; Beylich et. al. `11]

Non-perturbative input: Form factors from sum-rules (small q^2) and Lattice QCD (large q^2)

See talk by Straub & Jäger

Anomalies

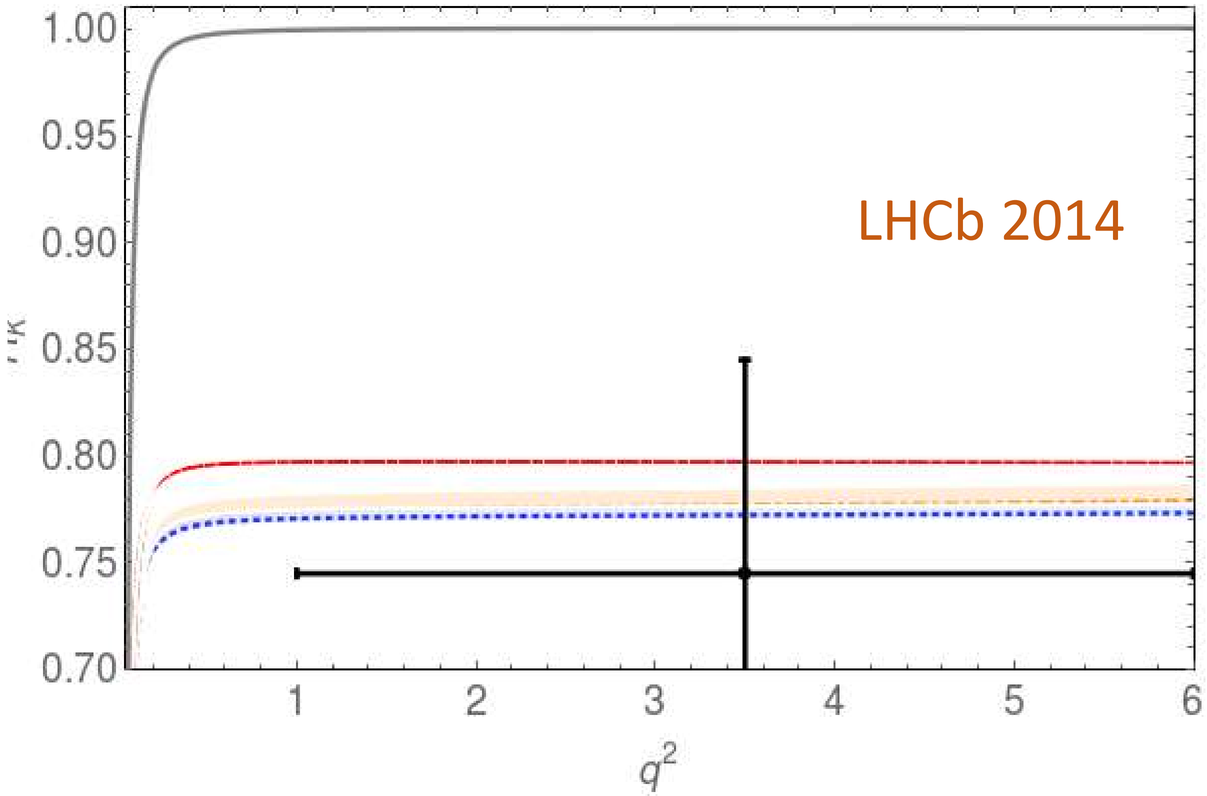
Various Anomalies, in low Branching ratios in angular observables such as P'$_5$.

Plenty of discussion of SM uncertainty, which determines the significance.

Yet theory uncertainty small ir

$R_{K(*)} = \Gamma_{\mu(*)}/\Gamma_{e(*)} = 1.00(1)$

Bordone et.al. [1605.07633]



$$R_{K^{(*)}}[a,b] = \frac{\int_a^b \frac{d\Gamma}{dq^2}(B \to K^{(*)}\mu^+\mu^-)dq^2}{\int_a^b \frac{d\Gamma}{dq^2}(B \to K^{(*)}e^+e^-)dq^2}$$

Anomalies

Observable	Anomaly	σ
$\mathrm{BR}(B_{d/s} \to (K^{(*)}/\Phi)\mu^+\mu^-)$	Below @ small q^2	1-2 ?
$P_5'(B \to (K^{(*)})\mu^+\mu^-)$	Below @ some q^2	2-3 ?
$\frac{\mathrm{BR}(B \to D^{(*)}\tau\nu)}{\mathrm{BR}(B \to D^{(*)}\ell\nu)}$	Enhanced w.r.t SM	4.1
$\frac{\mathrm{BR}(B \to K^{(*)}\mu^+\mu^-)}{\mathrm{BR}(B \to K^{(*)}e^+e^-)}$	Below SM	3.7
$\frac{\varepsilon'}{\varepsilon}$, direct $K \to \pi\pi$ CP violation	Below SM	2.9
$(g-2)_\mu$	Below SM	3.9

Higher Order Corrections

- In effective theory calculations we can systematically add higher corrections.
- These will involve extra light particles such as Gluons and light quarks.
- But how can we expand $\frac{1}{(l+k)^2 - 0^2}$ in external momenta k if the masses are light or even zero?
- This will generate infrared divergences?

The power of effective theories is that IR divergences cancel out in

$$\mathcal{A}_{full} = \mathcal{A}_{eff}$$

Matching of the Weak Hamiltonian

We will match $b\bar{d} \to c\bar{u}$ using the standard model

$$\mathcal{L}_{\text{full}} = \mathcal{L}_{\text{SM}}(G^a_\mu, u, d, s, c, b, t, W, Z)$$

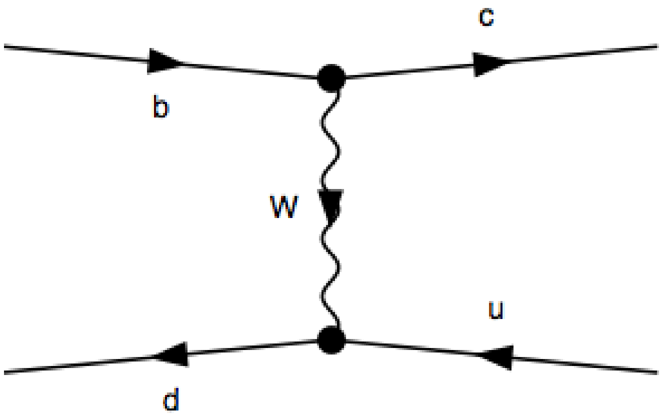
and the Effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \mathcal{L}(G^a_\mu, u, d, s, c, b) - \frac{4}{\sqrt{2}} G_F V_{cb} V^*_{ud} \sum_{i=1}^{2} C_i Q_i$$

$$\begin{aligned} Q_1 &= (\bar{c} T^a \gamma_\mu P_L b)(\bar{d} T^a \gamma^\mu P_L u) \\ Q_2 &= (\bar{c} \gamma_\mu P_L b)(\bar{d} \gamma^\mu P_L u)\,, \end{aligned}$$

T^a are the SU(3) colour matrices and $P_L = (1 - \gamma_5)/2$.

Diagram



Tree Level Calculation

In the calculation of the truncated $b\bar{d} \to c\bar{u}$ Green's function, the following structures will appear:

$$
\begin{aligned}
S_1 &= (\bar{c}T^a\gamma_\mu b_L)(\bar{d}T^a\gamma^\mu u_L) \\
S_2 &= (\bar{c}\gamma_\mu b_L)(\bar{d}\gamma^\mu u_L)
\end{aligned}
$$

In the full theory we find at tree ($G_F = \sqrt{2}e^2/(8M_W^2 s_W^2)$):

$$
\mathcal{A}_{full,0}^{b\bar{d}\to c\bar{u}} = \frac{-e^2}{2s_W^2}\frac{V_{cb}V_{ud}^*}{M_W^2 - k^2}S_2 \simeq \frac{-4}{\sqrt{2}}G_F V_{cb}V_{ud}^* S_2
$$

$$
\mathcal{A}_{eff,0}^{b\bar{d}\to c\bar{u}} = -\frac{4}{\sqrt{2}}G_F V_{cb}V_{ud}^* \sum C_i\langle Q_i\rangle_0 = \frac{-4}{\sqrt{2}}G_F V_{cb}V_{ud}^* \sum_{i=1}^{2} C_i S_i
$$

Tree Level Matching

The tree-level matching equation

$$\mathcal{A}^{b\bar{d}\to c\bar{u}}_{full,0} = \mathcal{A}^{b\bar{d}\to c\bar{u}}_{eff,0}$$

results in

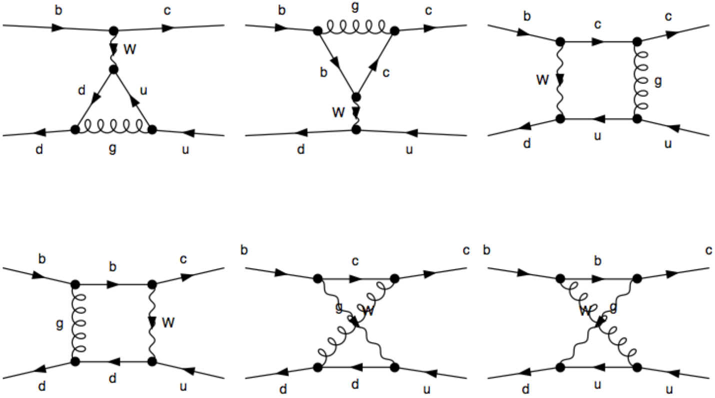
$$\frac{-e^2}{2s_W^2}\frac{V_{cb}V_{ud}^*}{M_W^2 - k^2}S_2 = \frac{-4}{\sqrt{2}}G_F V_{cb}V_{ud}^* \sum_{i=1}^{2}(C_1 S_1 + C_2 S_2)$$

which implies for the leading order coefficients:

$$C_1^{(0)} = 0 \quad and \quad C_2^{(0)} = 1$$

One-Loop Matching

We will now perform the one-loop matching assuming a common mass m for all light quarks (u,d,s,c,b). This mass will regularise the IR divergences.



Full Theory: Diagram 1 + 2

Defining $\mathcal{N} = \frac{\alpha_s}{4\pi}\frac{-4}{\sqrt{2}}G_F V_{cb} V_{ud}^*$ diagram 1 and 2 give

$$\mathcal{A}^{b\bar{d}\to c\bar{u}}_{full,1+2} = \mathcal{N}C_F\left(\frac{2}{\epsilon}S_2 - S_2(3 + 2\log\frac{m^2}{\mu^2}) + 2S_Y\right)$$

where $C_F = (N_C^2 - 1)/(2N_C)$ and

$$S_Y = (\bar{c}T^a\gamma_\mu b_L)(\bar{d}T^a\gamma^\mu u_R) + RL$$

- The light quark mass m regularises the IR divergences.
- There is still a divergent result. What is missing?

Full Theory: Counterterm

The W-quark interaction receives a counterterm contribution from $Z_\psi = 1 + \alpha_s/(4\pi) Z_\psi^{(1)}$, where $Z_\psi^{(1)} = -(C_F/\epsilon)$

$$\mathcal{A}_{full,ct.}^{b\bar{d}\to c\bar{u}} = -2Z_\psi^{(1)} \mathcal{N} S_2 = -\mathcal{N} C_F \frac{2}{\epsilon} S_2$$

so that

$$\mathcal{A}_{full,1+2}^{b\bar{d}\to c\bar{u}} + \mathcal{A}_{full,ct.}^{b\bar{d}\to c\bar{u}} = \mathcal{N} C_F \left(-S_2 (3 + 2\log \frac{m^2}{\mu^2}) + 2S_Y \right)$$

is finite, but still dependent on the light quark mass m.

Effective Theory: Diagram 1 + 2

The effective Theory results in the same expression:

$$\mathcal{A}^{b\bar{d}\to c\bar{u}}_{eff,1+2}(C_2^{(0)}Q_2) = C_2^{(0)}\mathcal{N}C_F\left(\frac{1}{\epsilon}2S_2 - S_2(3+2\log\frac{m^2}{\mu^2}) + 2S_Y\right)$$

The field renormalisation of the four quark fields in Q_2 gives the same counterterm as before.

$$\begin{aligned}\mathcal{A}^{b\bar{d}\to c\bar{u}}_{eff,1+2}(C_2^{(0)}Q_2) \; + \; 2Z_\psi^{(1)}\mathcal{A}^{b\bar{d}\to c\bar{u}}_{eff,0}(C_2^{(0)}Q_2)\\ = C_2^{(0)}\mathcal{N}C_F\left(-S_2(3+2\log\frac{m^2}{\mu^2}) + 2S_Y\right)\end{aligned}$$

which equals our full theory result for $C_2^{(0)} = 1$.

- These diagrams do not contribute in the matching calculation.

Full Theory: Diagram 3 – 6

The contribution of the remaining diagrams are all finite

$$\mathcal{A}^{b\bar{d}\to c\bar{u}}_{full,3-6} = \mathcal{N}\left((S_3 - 10S_1)\left(1 + \ln\frac{m^2}{M_W^2}\right) - S_X \right)$$

- In this calculation we pick up the effects from the heavy W Boson via the $\ln M_W^2$.
- The $\ln m^2$ term regularises the infrared divergences.

Here we have defined the following structures:

$$\begin{aligned} S_3 &= (\bar{c}T^a\gamma_\mu\gamma_\nu\gamma_\lambda b_L)(\bar{d}T^a\gamma^\mu\gamma^\nu\gamma^\lambda u_L) \\ S_4 &= (\bar{c}\gamma_\mu\gamma_\nu\gamma_\lambda b_L)(\bar{d}\gamma^\mu\gamma^\mu\gamma^\lambda u_L) \\ S_X &= \left[(\bar{c}T^a\gamma_\mu\gamma_\nu b_L)(\bar{d}T^a\gamma^\mu\gamma^\nu u_L) - LR - RL + RR\right] \\ &\quad -8\left[(\bar{c}T^a b_L)(\bar{d}T^a u_R) + RL\right] \end{aligned}$$

Effective Theory: Diagram 3 – 6

The effective field diagrams are divergent

$$\mathcal{A}^{b\bar{d}\to c\bar{u}}_{eff,3-6}(C_2^{(0)}Q_2) = \mathcal{N}\frac{1}{\epsilon}C_2^{(0)}\left(10S_1 - S_3\right) + \mathcal{N}C_2^{(0)}\left(-S_1 - \frac{1}{2}S_3 - S_X + (10S_1 - S_3)\ln\left(\frac{\mu^2}{m^2}\right)\right)$$

But we have to expand our bare Wilson Coefficient

$$C_{02}Q_{02} = C_2^{(0)}Q_2 + \frac{\alpha_s}{(4\pi)}\frac{1}{\epsilon}(C_2 Z_{2i}^{(1,1)}Q_i)$$

$$\mathcal{A}^{b\bar{d}\to c\bar{u}}_{eff}(\sum_i Z_{2i}^{(1,1)}Q_i) = \mathcal{N}\frac{1}{\epsilon}C_2^{(0)}\sum_i Z_{2i}^{(1,1)}S_i$$

This would involve new operators that have tree-level contribution to S_3. Assuming they exist, $\mathcal{A}^{b\bar{d}\to c\bar{u}}_{eff,3-6}(C_2^{(0)}Q_2) + \mathcal{A}^{b\bar{d}\to c\bar{u}}_{eff}(\sum_i Z_{2i}^{(1,1)}Q_i)$ would be finite.

Matching Equation $\mathcal{A}_{full} = \mathcal{A}_{eff}$

Adding the NLO Wilson coefficients

$$\mathcal{A}^{b\bar{d}\to c\bar{u}}_{eff,(1)}(\sum_i C_i^{(1)} Q_i) = \mathcal{N} \sum_i C_i^{(1)} S_i$$

to the matching equation

$$(S_3 - 10S_1)\left(1 + \ln\frac{m^2}{M_W^2}\right) - S_X = \sum_i C_i^{(1)} S_i +$$
$$C_2^{(0)}\left(-S_1 - \frac{1}{2}S_3 - S_X + (10S_1 - S_3)\ln\left(\frac{\mu^2}{m^2}\right)\right)$$

we see that the IR divergence cancel with $C_2^{(0)} = 1$:

$$\frac{3}{2}S_3 - 9S_1 + (S_3 - 10S_1)\ln\frac{\mu^2}{M_W^2} = \sum_i C_i^{(1)} S_i$$

IR Dimensional Regularisation

The calculation simplifies if we use dimensional regularisation for the infrared divergences.

- Set the light quark masses to zero and expand in external momenta.
- Massless diagrams vanish
- Only operator renormalisation contributes from effective theory.
- Wave function renormalisation drops out.

Full theory calculation (by hand) gives:

Wilson Coefficients

- Structures $S_3 = (\bar{c}T^a\gamma_\mu\gamma_\nu\gamma_\lambda b_L)(\bar{d}T^a\gamma^\mu\gamma^\nu\gamma^\lambda u_L)$ and $S_1 = (\bar{c}T^a\gamma_\mu b_L)(\bar{d}T^a\gamma^\mu u_L)$ are not independent for d=4.
- Define $\langle E_1\rangle_{(0)} = S_3 - 16S_1 = S_{E1}$
- Show that $\langle E_1\rangle_{(0)} = 0$ in $d = 4$ by using the relation $\gamma^{\mu\nu\rho} = \eta^{\mu\nu}\gamma^\rho + \eta^{\nu\rho}\gamma^\mu - \eta^{\mu\rho}\gamma^\nu - i\epsilon^{\sigma\mu\nu\rho}\gamma_\sigma\gamma^5$
- We can then rewrite the matching equations:

$$\frac{3}{2}S_3 - 9S_1 + (S_3 - 10S_1)\ln\frac{\mu^2}{M_W^2} =$$

$$(6S_1 + S_{E1})\ln\frac{\mu^2}{M_W^2} + 15S_1 + \frac{3}{2}S_{E1} = \sum_i C_i^{(1)}S_i$$

- Hence $C_1 = 0 + \frac{\alpha_s}{4\pi}(15 + 6\ln\frac{\mu^2}{M_W^2})$ and $C_2 = 1 + \alpha_s \cdot 0$.

Operator Counterterms

In the basis $\{Q_1, Q_2, E_1, E_2\}$ the renormalisation matrix reads:

$$Z_{ij}^{(1,1)} = \begin{pmatrix} -2 & 4/3 & 5/12 & 2/9 \\ 6 & 0 & 1 & 0 \\ 0 & 0 & -7 & -4/3 \\ 0 & 0 & -6 & 0 \end{pmatrix}$$

From $\sum_j Z_{2j}^{(1,1)} \langle Q_j \rangle_{(0)} = 6S_1 + S_{E1} = -10S_1 + S_3$ this gives the contribution that results in a finite matching correction.

Counting Dimensions

The action is dimensionless. Consider e.g. the effective interaction

$$\int d^4x\mathcal{L}_{eff} = -\frac{1}{2}\left((\partial h)^2 - M_H^2 h^2\right) - 1/4 G^a_{\mu\nu}G^{a,\mu\nu} + c_g h G^a_{\mu\nu}G^{a,\mu\nu}$$

where $G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + if^{abc} g G^b_\mu G^c_\nu$.

Object	$(\partial\ h)^2$	∂	h	$(G^a_{\mu\nu})^2$	G_μ	$h\ (G^a_{\mu\nu})^2$	c_g
Dimension D	4	1	1	4	1	5	-1

We see that only the effective operator has mass dimension $D > 4$.

Power Counting

- The Dimension D>4 plays an important role if we count the superficial degree of divergence
- Superficial: Divergence after subdivergencies are subtracted
- In a scalar ϕ^n theory the degree of divergence is $Div = (n-4)V + 4 - N_e$
  - For n=5 we would have a D=5>4 operator ϕ^5
  - One ϕ^5 insertion: Diagrams with $N_e = 5$ external legs become divergent. Renormalise with D=5 operator.
  - Two ϕ^5 insertions: Diagrams with $N_e = 6$ divergent. Renormalise with D=6 operator.
- Dimensional regularisation: Operators with dimension D mix only into dimension D operators.

Renormalisation Group Equations

- Higher dimensional operators are generated via matching.
- In the effective theory these operators have to be renormalised.
- This leads to renormalisation group running and large logarithms.
- We can resum these logarithms using Renormalisation Group Equations (RGE).

Dimensional Regularization

Let us consider only the Lagrangian of colour charged fermion:

$$\mathcal{L}_{QCD} = i\bar{\psi}_i \left(\partial\!\!\!/ \delta_{ij} + g T^a_{ij} G\!\!\!/^a \right) \psi_j - m\bar{\psi}_i\psi_i - 1/4 G^a_{\mu\nu} G^{a,\mu\nu}$$

In Dimensional Regularisation we work in d space-time dimensions. E.g. we have $red g_{\mu\nu} g^{\mu\nu} = d$.

The dimensionless of the action $\int d^d x \mathcal{L}_{QCD}$ implies:

Object	∂	ψ	G_μ	g
Dimension D	1	(d-1)/2	(d-2)/2	ϵ

where $\epsilon = (4-d)/2$ such that $D[\, g\bar{\psi} G\!\!\!/^a \psi \,] = d$. I.e. $(4-d)/2 + (d-1) + (d-1)/2 = d$.

Renormalisation

The *regularisation* renders the divergences finite but regulator dependent. I.e. they will appear as $\frac{1}{\epsilon}$ poles. We absorb these in the so-called bare parameters in the *renormalisation* procedure and write

$$\mathcal{L}^{(0)}_{fermion} = i\bar{\psi}_{0i}\left(\not\partial\delta_{ij} + g_0 T^a_{ij}\not G^a_0\right)\psi_{0j} - m_0\bar{\psi}_{0i}\psi_{0i}$$

in terms of the bare parameters.

$$\psi_0 = Z_\psi^{\frac{1}{2}}\psi, \qquad m_0 = Z_m m,$$
$$G_0 = Z_G^{\frac{1}{2}}, G \qquad g_0 = \mu^\epsilon Z_g g$$

Here the scale μ appears so that g is dimensionless.

Bare Parameters

- The bare parameters are *scheme* and *scale* independent: They do not depend on the choice of μ and the Z's.
- Yet the relation between bare and renormalised parameters depends on μ and the choice of Z.

Hence we write $g_0 = \mu^{\epsilon} Z_g g(\mu)$ and expand [1]

$$Z_g = 1 + \frac{g^2}{(4\pi)^2}\left(\frac{1}{\epsilon}Z^{(1,1)} + Z^{(1,0)}\right) + \frac{g^4}{(4\pi)^4}\left(\frac{1}{\epsilon^2}Z^{(2,2)} + \frac{1}{\epsilon}Z^{(2,1)} + Z^{(2,0)}\right) + \dots$$

[1] we assume the $\overline{MS}$ choice for μ here

Beta Function

To derive the renormalisation group equations for g we use the scheme invariance of $g_0 = \mu^\epsilon g Z_g$:

$$0 = \mu \frac{d}{d\mu} g_0 = \epsilon \mu^\epsilon g Z_g + \mu^\epsilon (\mu \frac{d}{d\mu} g) Z_g + \mu^\epsilon g \mu \frac{d}{d\mu} Z_g$$

Solving for $\mu \frac{d}{d\mu} g$ gives

$$\beta(g, \epsilon) \equiv \mu \frac{d}{d\mu} g = -\epsilon g - g Z_g^{-1} \mu \frac{d}{d\mu} Z_g \equiv -\epsilon g + \beta(g)$$

where we have

$$\beta(g) = -g Z_g^{-1} \mu \frac{d}{d\mu} Z_g = -g Z_g^{-1} \mu \frac{dg}{d\mu} \frac{d}{dg} Z_g =$$

$$= g Z_g^{-1} (\beta(g) - \epsilon g) \frac{d}{dg} Z_g \overset{\epsilon^0}{=} \frac{2g^3}{(4\pi)^2} Z_g^{(1,1)} + \frac{4g^5}{(4\pi)^4} Z_g^{(2,1)} + \dots$$

Renormalisation of Composite Operators

Consider the effective Lagrangian that comprises a dimension $d \leqslant 4$ term $\mathcal{L}^{(4)}$ and $d = 6$ operators Q_i and a new physics scale Λ:

$$\mathcal{L}_{eff} = \mathcal{L}^{(4)} + \Lambda^{-2} \sum_i C_i Q_i \,.$$

If we only consider one insertion of Q_i we renormalise

$$C_i \to C_{0i} = C_j Z_{ji} = (\hat{Z}^T \vec{C})_i$$

$$Z_{ij} = \delta_{ij} + \frac{g^2}{(4\pi)^2} \left(\frac{1}{\epsilon} Z_{ij}^{(1,1)} + Z_{ij}^{(1,0)} \right) + \frac{g^4}{(4\pi)^4} \left(\frac{1}{\epsilon^2} Z_{ij}^{(2,2)} + \dots \right) + \dots$$

plus the fields and couplings in $Q_i \to Q_{0i} = Z_{Q\psi} Q_i$.

RGE for Wilson Coefficients

Using similar steps as before one finds

$$\mu \frac{d}{d\mu}\vec{C} = \hat{\gamma}^T \vec{C}, \quad where \quad \hat{\gamma} = \beta(g)\hat{Z}\frac{d}{dg}\hat{Z}^{-1}$$

and

$$\begin{aligned} \hat{\gamma}^{(0)} &= 2\hat{Z}^{(1,1)}, \\ \hat{\gamma}^{(1)} &= 4\hat{Z}^{(2,1)} - 2\hat{Z}^{(1,1)}\hat{Z}^{(1,0)} - 2\hat{Z}^{(1,0)}\hat{Z}^{(1,1)} + 2\beta_0\hat{Z}^{(1,0)}. \end{aligned}$$

- In the $\overline{MS}$-scheme only UV divergences are subtracted.
- $\hat{Z}^{(1,0)}$ only present when generated by UV $1/\epsilon$ times ϵ from gamma algebra.

Leading Order Solution

Writing $\vec{C}(\mu) = \hat{U}(\mu, \mu_0)\vec{C}(\mu_0)$ we have to show
$\mu \frac{d}{d\mu}\hat{U}(\mu, \mu_0) = \gamma^T \hat{U}(\mu, \mu_0)$ at LO, where

$$\hat{U}^{(0)}(\mu, \mu_0) = \hat{V}\,\mathrm{diag}(\alpha_s(\mu)^{-a_i})\mathrm{diag}(\alpha_s(\mu_0)^{a_i})\hat{V}^{-1}$$

and

$$\left(\hat{V}^{-1}\hat{\gamma}^{(0)\,T}\hat{V}\right)_{ij} = 2\beta_0 a_i \delta_{ij}$$

Exercise: Show using

$$\mu\frac{d}{d\mu}\mathrm{diag}(\alpha_s(\mu)^{-a_i}) = -2\beta_0\frac{\alpha_s(\mu)^2}{4\pi}\frac{d}{d\alpha_s(\mu)}\mathrm{diag}(\alpha_s(\mu)^{-a_i}) + \ldots$$

Resummation of Logarithms

Defining $\eta = \alpha_s(\mu_0)/\alpha_s(\mu)$

$$\hat{U}^{(0)}(\mu, \mu_0) = \hat{V}\,\mathrm{diag}(\eta^{a_i})\hat{V}^{-1}$$

we can expand the leading order expression using

$$\alpha_s(\mu) = \alpha_s(\mu_0) - \frac{\alpha_s^2(\mu_0)}{4\pi}2\beta_0 \ln(\frac{\mu}{\mu_0}) + \frac{\alpha_s^3(\mu_0)}{(4\pi)^2}4\beta_0^2 \ln(\frac{\mu}{\mu_0})^2 \ldots$$

Example: One Operator where $\hat{\gamma}_0 = \gamma_0$:

$$U^{(0)}(\mu, \mu_0) = 1 + \frac{\alpha_s(\mu_0)}{4\pi}\gamma_0 \ln\frac{\mu}{\mu_0} + \frac{\alpha_s^2(\mu_0)}{(4\pi)^2}(\gamma_0^2/2 - 2\beta_0\gamma_0)\ln^2\frac{\mu}{\mu_0} + \ldots$$

This is a sum of infinite terms

$$U^{(0)}(\mu, \mu_0) = \sum_{n=0}^{\infty} a_n(\alpha_s \ln(\mu/\mu_0))^n$$

RG Improved Perturbation Theory

The evolution term $\hat{U}^{(0)}(\mu, \mu_0)$ resums the large logs:

- E.g. $\ln(M_W/\Lambda_{NP})$ in case of SM effective
- or $\ln(m_b/M_W)$ for B meson decays.

Consider B decays where $\vec{C}(\mu_b) = \hat{U}(\mu_b, \mu_W)\vec{C}(\mu_W)$:

- Expand $\hat{U}^{(0)}(\mu_b, \mu_W)$ around $\hat{U}^{(0)}(\mu_b, M_W)$
  - $\hat{U}^{(0)}(\mu_b, \mu_W) = \hat{U}^{(0)}(\mu_b, M_W)\hat{U}^{(0)}(M_W, \mu_W) = \hat{U}^{(0)}(\mu_b, M_W)(1 + \frac{\alpha_s}{4\pi}\hat{\gamma}_0^T \ln(M_W/\mu_w))$
- The $\frac{\alpha_s}{4\pi}\hat{\gamma}_0^T \ln(M_W/\mu_W)$ is a small log.
- The $\ln(\mu_W)$ will cancel in $\hat{U}(\mu_b, \mu_W)\vec{C}(\mu_W)$ through higher order corrections to $\vec{C}(\mu_W)$.
- This will be $\alpha_s \times$ small log correction to $\vec{C}(\mu_W)$.

Evanescent Operators

- We had to introduce additional operators to perform the calculation in d dimensions.
- Yet there should be only be two physical operators – Q_1 and Q_2 – for our process.
- But we have to introduce additional evanescent operators for each loop order.
- The evanescent operators should not contribute to physical processes.
- But if we consider the one loop matrix element $\langle E_1 \rangle_{(1)} = X_1 \langle Q_1 \rangle_{(0)} + X_2 \langle Q_2 \rangle_{(0)} + \dots$
  - The X_1 are finite that origin from UV divergences.
  - The gamma algebra gives $\propto \epsilon$ if we project onto Q_1 and Q_2 since $E_1 = 0$ if $d = 4$ and is multiplied with $1/\epsilon$ from loop Integral.

Evanescent Renormalisation

Renormalising the evanescent operators we have

$$\langle E_1 \rangle_{(1)} + Z^{(1)}_{E,Q_1} \langle Q_1 \rangle_{(0)} + Z^{(1)}_{E,Q_2} \langle Q_2 \rangle_{(0)} =$$
$$(X_1 + Z^{(1)}_{E,Q_1}) \langle Q_1 \rangle_{(0)} + (X_2 + Z^{(1)}_{E,Q_2}) \langle Q_2 \rangle_{(0)} + \dots .$$

by choosing $Z^{(1)}_{E,Q_i} = -X_1$. → Matrix elements of evanescent are not contributing to physical operators. While the ADM reads

$$\hat{\gamma} = \begin{pmatrix} \hat{\gamma}_{QQ} & \hat{\gamma}_{QE} \\ \hat{0} & \hat{\gamma}_{EE} \end{pmatrix}$$

via the cancellation (locality) at NLO and beyond:

$$\hat{\gamma}^{(1)}_{EQ} = 4\hat{Z}^{(2,1)}_{EQ} - 2\hat{Z}^{(1,1)}_{EE}\hat{Z}^{(1,0)}_{EQ} = 2\hat{Z}^{(1,1)}_{EE}\hat{Z}^{(1,0)}_{EQ} - 2\hat{Z}^{(1,1)}_{EE}\hat{Z}^{(1,0)}_{EQ} = 0 .$$

Only brief introduction

- Many important discoveries have been made with the help of flavour physics
- Several suppression mechanism give unique sensitivities to physics beyond the standard model
- The flavour sector might still provide us with new exciting results
- Calculations provide conceptually interesting aspects of QFT