Color Superconductivity

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- Color Superconductivity in Dense Matter?
- A warm-up: the NJL Model
- Overview of QC_2D:
  - Thermodynamic observables & deconfinement
  - Hadron spectrum
  - Gluodynamics

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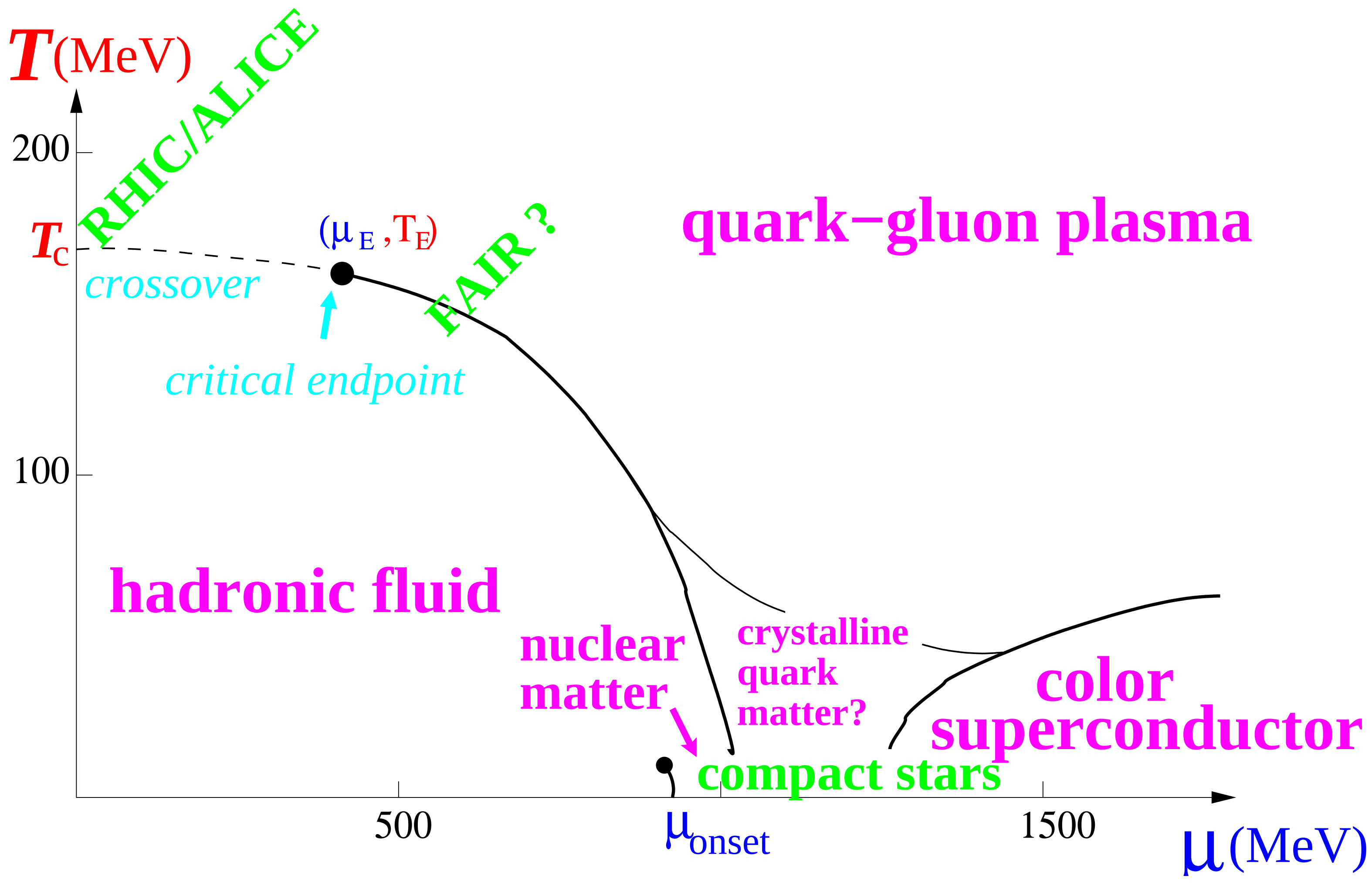
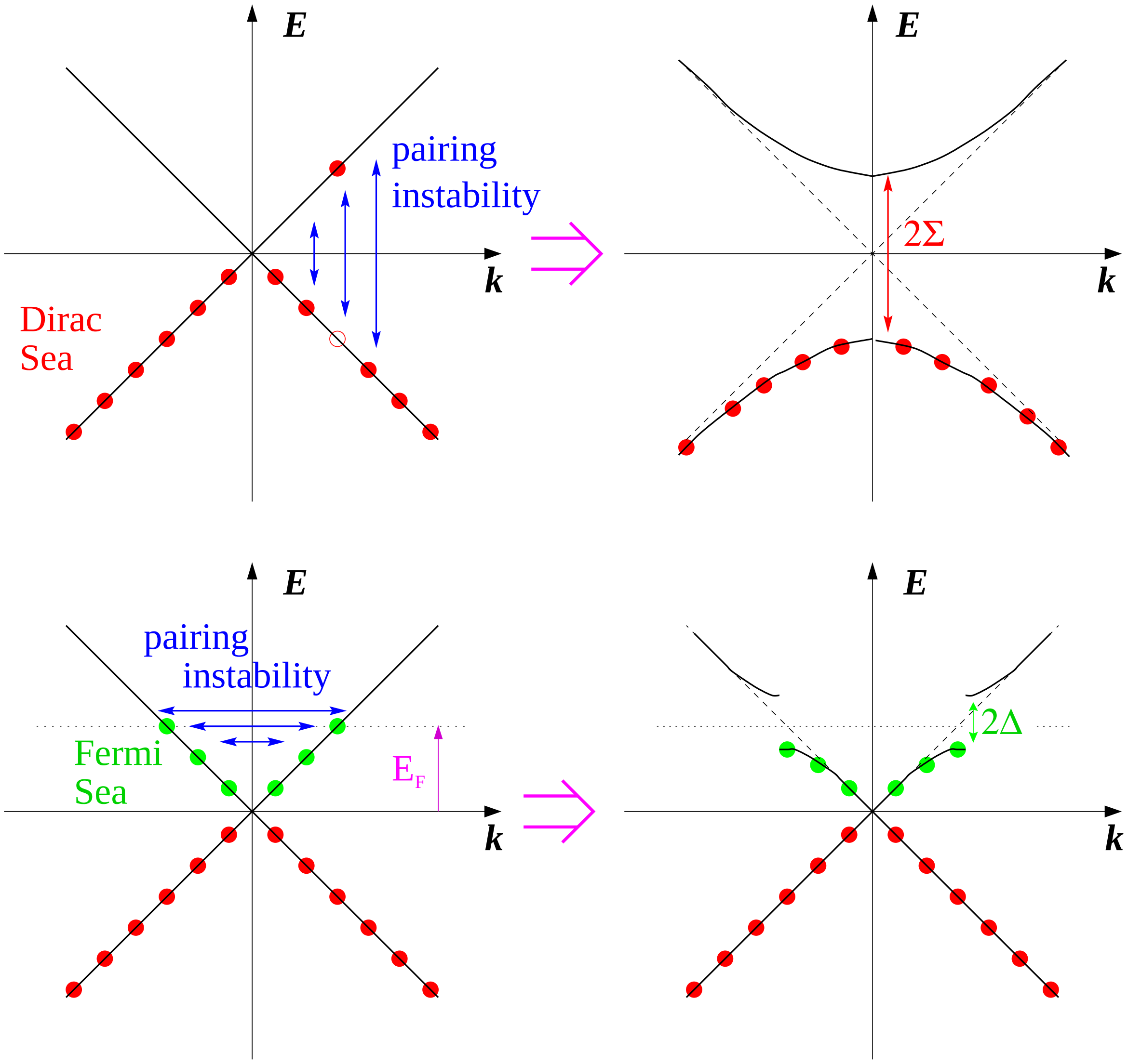
IPPP Durham, 25th October 2007

The QCD Phase Diagram



χSB vs. Cooper Pairing



Color Superconductivity

In the asymptotic limit $\mu \to \infty$, $g(\mu) \to 0$, the ground state of QCD is the *color-flavor locked* (CFL) state characterised by a BCS instability, [D. Bailin and A. Love, Phys.Rep. 107(1984)325]
ie. diquark pairs at the Fermi surface condense via

$$\langle q_i^{\alpha}(p) C\gamma_5 q_j^{\beta}(-p)\rangle \sim \varepsilon^{A\alpha\beta}\varepsilon_{Aij} \times \text{const.}$$

SU(3)$_c\otimes$SU(3)$_L\otimes$SU(3)$_R$ $\otimes$U(1)$_B$ $\otimes$U(1)$_Q$ $\to$SU(3)$_\Delta\otimes$U(1)$_{\tilde{Q}}$
Ground state is simultaneously *superconducting* (8 gapped gluons with mass $O(g\mu)$), *superfluid* (1 exact Goldstone), *chirally broken* (8 pseudo-Goldstones), and *transparent* (quasiparticles with $\tilde{Q} \neq 0$ gapped).

[M.G. Alford, K. Rajagopal and F. Wilczek, Nucl.Phys.B537(1999)443]

Asymptotically gap scales as $\Delta \propto \mu \exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right)$

[D.T. Son, Phys.Rev.D59(1999)094019]

At smaller densities such that $\mu/3 \sim k_F \lesssim m_s$, expect pairing between u and d only $\Rightarrow$ "2SC" phase

$$\langle q_i^\alpha(p) C\gamma_5 q_j^\beta(-p)\rangle \sim \varepsilon^{\alpha\beta3}\varepsilon_{ij} \times \text{const.}$$

SU(3)$_c$ $\longrightarrow$ SU(2)$_c$ $\Rightarrow$ 5/8 gluons get gapped
Global SU(2)$_L$$\otimes$SU(2)$_R$$\otimes$U(1)$_B$ unbroken

Another possibility in isopsin asymmetric matter is the so-called "LOFF" phase:

$$\langle u(k_F^u;\uparrow) d(-k_F^d;\downarrow)\rangle \neq 0$$

In the electrically-neutral matter expected in compact stars, $k_F^d - k_F^u = \mu_e \sim 100$MeV $\Rightarrow$ $\langle\psi\psi\rangle$ condensate has $\vec{k} \neq 0$ breaking translational invariance $\Rightarrow$ *crystallisation*

Other ideas:

a 2SC/normal mixed phase (plates? rods?)

or a gapless 2SC where $\langle qq\rangle \neq 0$ but $\Delta = 0$

A Gentle Reminder

QCD condensed matter physics could be very rich!

[M.G. Alford, A. Schmitt, K. Rajagopal & T. Schäfer, arXiv:0709.4635]

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BUT

The most urgent issue of all – whether quark matter exists in a stable form in the cores of compact stars – requires quantitative knowledge of the Equation of State $n_q(\mu), p(\mu), \varepsilon(\mu)$ for all $\mu > \mu_o$

the oldest problem in strong interaction physics?

[L.D. Landau, Phys.Z.Sowjetunion 1(1932)285]

What can we say at smaller densites $\mu \sim O$(1 GeV) where weak coupling methods can't be trusted? Lattice QCD simulations can't help because the Euclidean path integral measure det$M(\mu)$ is not positive definite.

The "Sign Problem"

In condensed matter theory there are two tractable limits:

	Weak Coupling	Strong Coupling
physical d.o.f.'s	weakly interacting fermions	tightly-bound bosons
superfluidity mechanism	BCS condensation	Bose-Einstein Condensation
QFT example	NJL model	QC$_2$D

Both model QFT's can be studied with $\mu \neq 0$ using lattice simulations which evade the Sign Problem.

High-T_c superconducting compounds, cold atoms near a Feshbach resonance, and perhaps QCD, are difficult problems because they belong to neither limit

The NJL Model

SJH, D.N. Walters, PRD69:076011 (2004)

Effective description of soft pions interacting with nucleons/constituent quarks

$$\mathcal{L}_{NJL} = \bar\psi(\partial\!\!\!/ + m + \mu\gamma_0)\psi - \frac{g^2}{2}[(\bar\psi\psi)^2 - (\bar\psi\gamma_5\vec\tau\psi)^2]$$
$$\sim \bar\psi(\partial\!\!\!/ + m + \mu\gamma_0 + \sigma + i\gamma_5\vec\pi.\vec\tau)\psi + \frac{2}{g^2}(\sigma^2 + \vec\pi.\vec\pi)$$

Full global symmetry is $SU(2)_L \otimes SU(2)_R \otimes U(1)_B$

Dynamical χSB for $g^2 > g_c^2 \Rightarrow$ isotriplet Goldstone $\vec\pi$

Scalar isoscalar diquark $\psi^{tr}C\gamma_5 \otimes \tau_2 \otimes A^{color}\psi$ breaks $U(1)_B$

$\Rightarrow$ diquark condensation signals high density ground state *is superfluid*

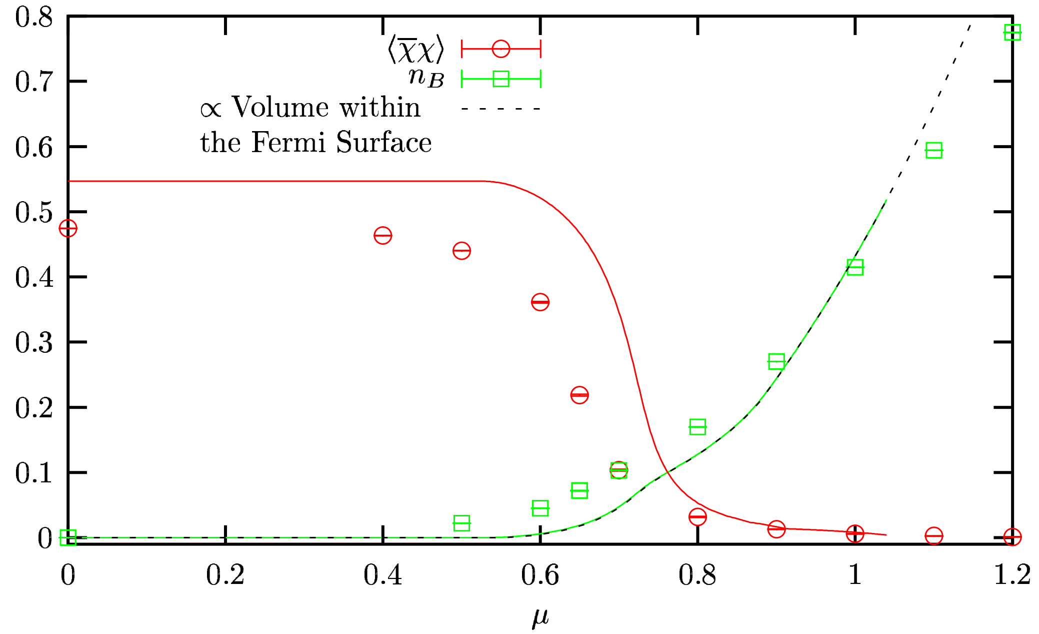
In 3+1d, NJL is non-renormalisable, so an explicit cutoff is required. We follow the large-N_f (Hartree) approach of Klevansky (1992) and match lattice parameters to low energy phenomenology:

Phenomenological Observables fitted	Lattice Parameters extracted
$\Sigma_0 = 400$MeV	$ma = 0.006$
$f_\pi = 93$MeV	$1/g^2 = 0.495$
$m_\pi = 138$MeV	$a^{-1} = 720$MeV

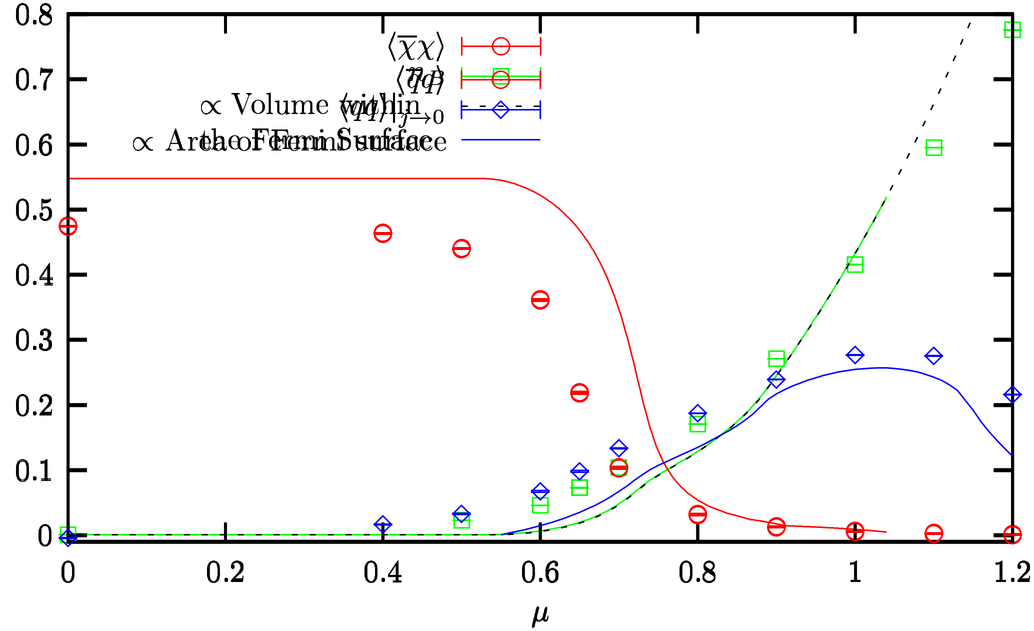
Lattice regularisation preserves SU(2)$_L\otimes$SU(2)$_R\otimes$U(1)$_B$...

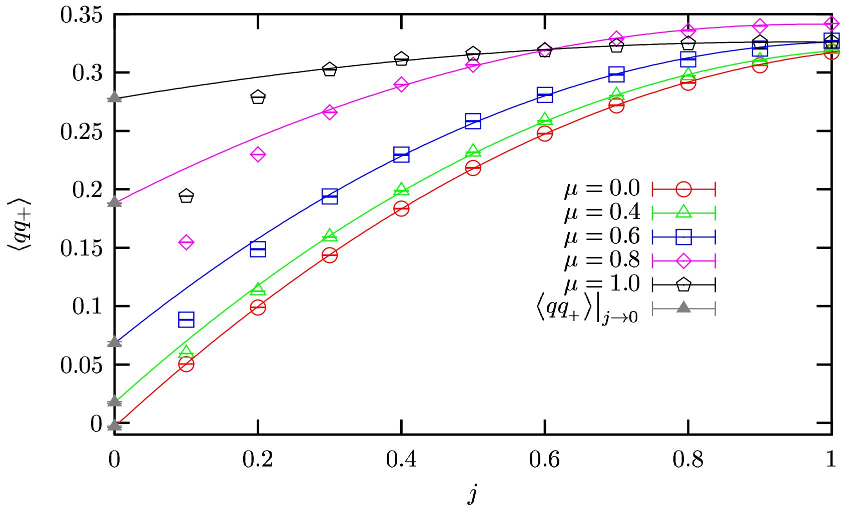
...but phenomenological NJL is barely a field theory

Equation of State and Diquark Condensation



Equation of State and Diquark Condensation





Add source $j[\psi^{tr}\psi + \bar{\psi}\bar{\psi}^{tr}]$

Diquark condensate estimated by taking $j \to 0$

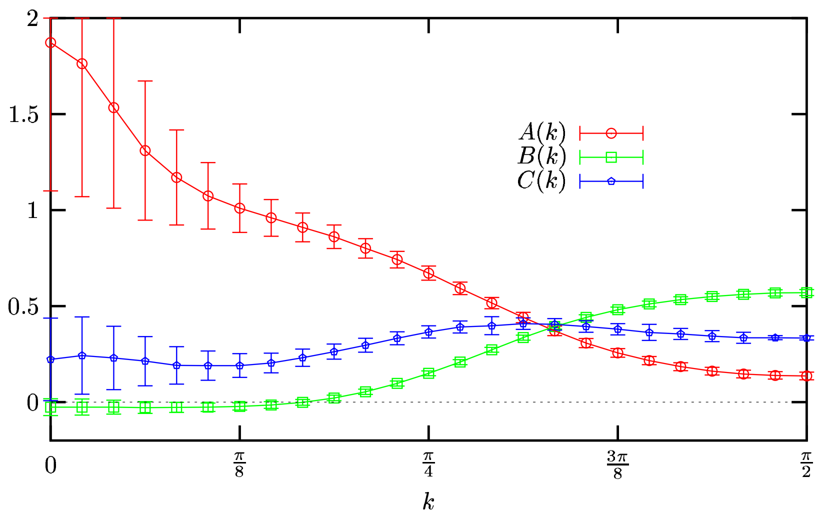
Our fits exclude $j \leq 0.2$

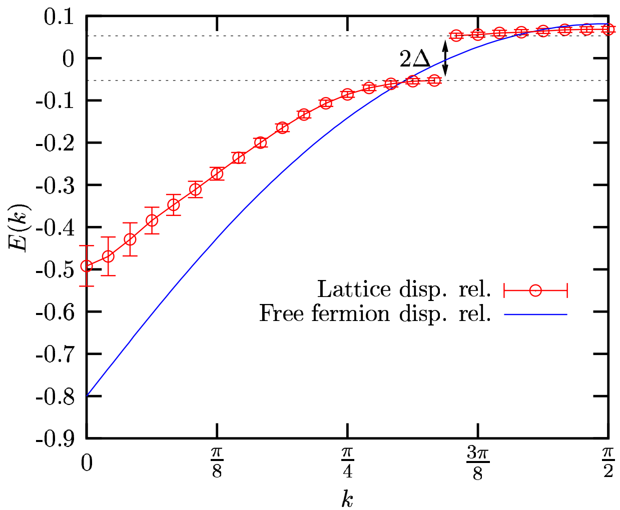
The Superfluid Gap

Quasiparticle propagator:

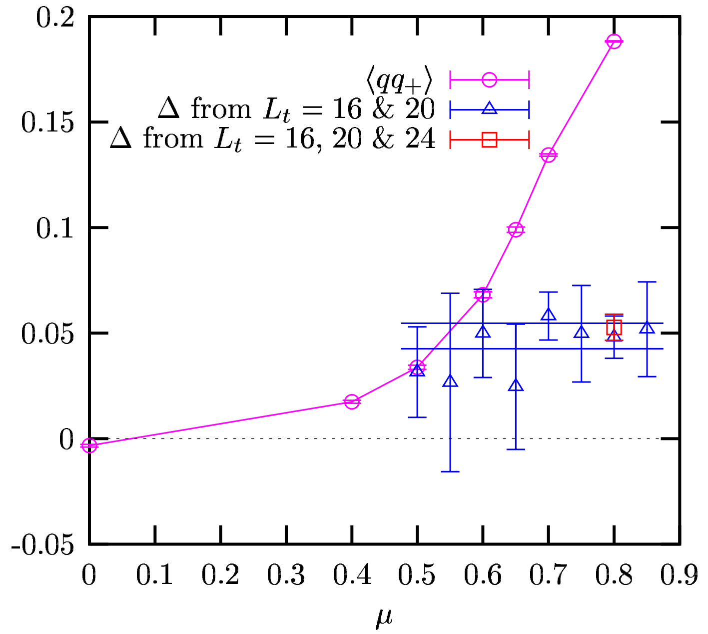
$$\langle \psi_u(0)\bar{\psi}_u(t)\rangle = Ae^{-Et} + Be^{-E(L_t-t)}$$
$$\langle \psi_u(0)\psi_d(t)\rangle = C(e^{-Et} - e^{-E(L_t-t)})$$

Results from $96 \times 12^2 \times L_t$, $\mu a = 0.8$ extrapolated to $L_t \to \infty$ (ie. $T \to 0$) then $j \to 0$





The gap at the Fermi surface signals superfluidity



- Near transition, $\Delta \sim$ const, $\langle\psi\psi\rangle \sim \Delta\mu^2$
- $\Delta/\Sigma_0 \simeq 0.15 \Rightarrow \Delta \simeq 60$MeV
in agreement with self-consistent approaches
- $\Delta/T_c = 1.764$ (BCS) $\Rightarrow L_{tc} \sim 35$
explains why $j \to 0$ limit is problematic
- Currently studying $\mu_I = (\mu_u - \mu_d) \neq 0$,
which "re"introduces a sign problem!

Two Color QCD - the large-N_c^{-1} limit

- $\tau_2 \not{D}(\mu)\tau_2 = \not{D}^*(\mu) \Rightarrow$ det$(\not{D} + m)$ is real
- Spectrum contains degenerate $q\bar{q}$ mesons and qq baryons so that chiral group at $\mu = 0$ is enlarged from SU$(N_f)_L \otimes$SU$(N_f)_R \otimes$U(1)$_B$ to SU$(2N_f) \longrightarrow$ Sp$(2N_f)$

Can analyse response to $\mu \neq 0$ using chiral effective theory:

$$\mathcal{L}_{\chi PT} = \frac{f_\pi^2}{2}\mathrm{ReTr}\Big[\partial_\nu \Sigma \partial_\nu \Sigma^\dagger - 2m_\pi^2 \begin{pmatrix} & 1 \\ -1 & \end{pmatrix} \Sigma + 4\mu \begin{pmatrix} 1 & \\ & -1 \end{pmatrix} \Sigma^\dagger \partial_t \Sigma$$
$$- 2\mu^2 \left\{ \Sigma \begin{pmatrix} 1 & \\ & -1 \end{pmatrix} \Sigma^\dagger \begin{pmatrix} 1 & \\ & -1 \end{pmatrix} + \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \right\} \Big]$$

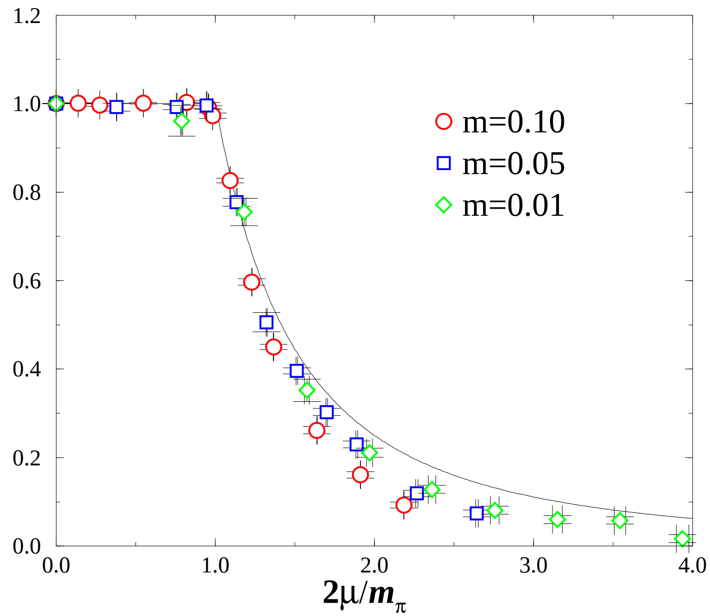
The μ-dependent terms in $\mathcal{L}_{\chi PT}$ are completely determined by the global symmetries

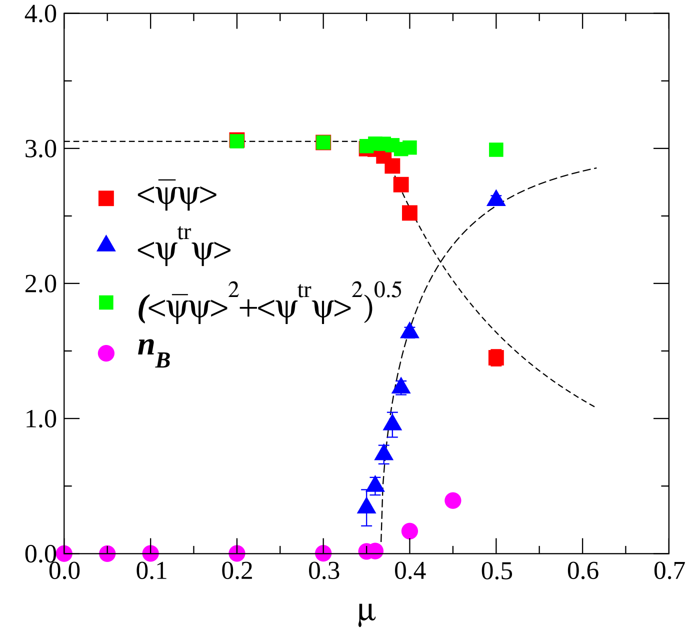
Quantitatively, for $\mu \gtrsim \mu_o$ χPT predicts ($\mu_o \equiv \frac{1}{2}m_\pi$)

$$\frac{\langle\bar\psi\psi\rangle}{\langle\bar\psi\psi\rangle_0} = \left(\frac{\mu_o}{\mu}\right)^2; \quad n_q = 8N_f f_\pi^2 \mu \left(1 - \frac{\mu_o^4}{\mu^4}\right); \quad \frac{\langle qq\rangle}{\langle\bar\psi\psi\rangle_0} = \sqrt{1 - \left(\frac{\mu_o}{\mu}\right)^4}$$

[Kogut, Stephanov, Toublan, Verbaarschot & Zhitnitsky, Nucl.Phys.B582(2000)477]

confirmed by QC_2D simulations with staggered fermions





[SJH, I. Montvay, S.E. Morrison, M. Oevers, L. Scorzato J.I. Skullerud, Eur.Phys.J.C17(2000)285, *ibid* C22(2001)451]

Thermodynamics at $T = 0$ from χPT

quark number density $n_{\chi PT} = 8N_f f_\pi^2 \mu \left(1 - \frac{\mu_o^4}{\mu^4}\right)$ [KSTVZ]

pressure $p_{\chi PT} = -\frac{\Omega}{V} = \int_{\mu_o}^{\mu} n_q d\mu = 4N_f f_\pi^2 \left(\mu^2 + \frac{\mu_o^4}{\mu^2} - 2\mu_o^2\right)$

energy density $\varepsilon_{\chi PT} = -p + \mu n_q = 4N_f f_\pi^2 \left(\mu^2 - 3\frac{\mu_o^4}{\mu^2} + 2\mu_o^2\right)$

interaction measure (aka conformal anomaly)

$$\delta_{\chi PT} = \Theta_{\mu\mu} = \varepsilon - 3p = 8N_f f_\pi^2 \left(-\mu^2 - 3\frac{\mu_o^4}{\mu^2} + 4\mu_o^2\right)$$

NB $\delta_{\chi PT} < 0$ for $\mu > \sqrt{3}\mu_o$

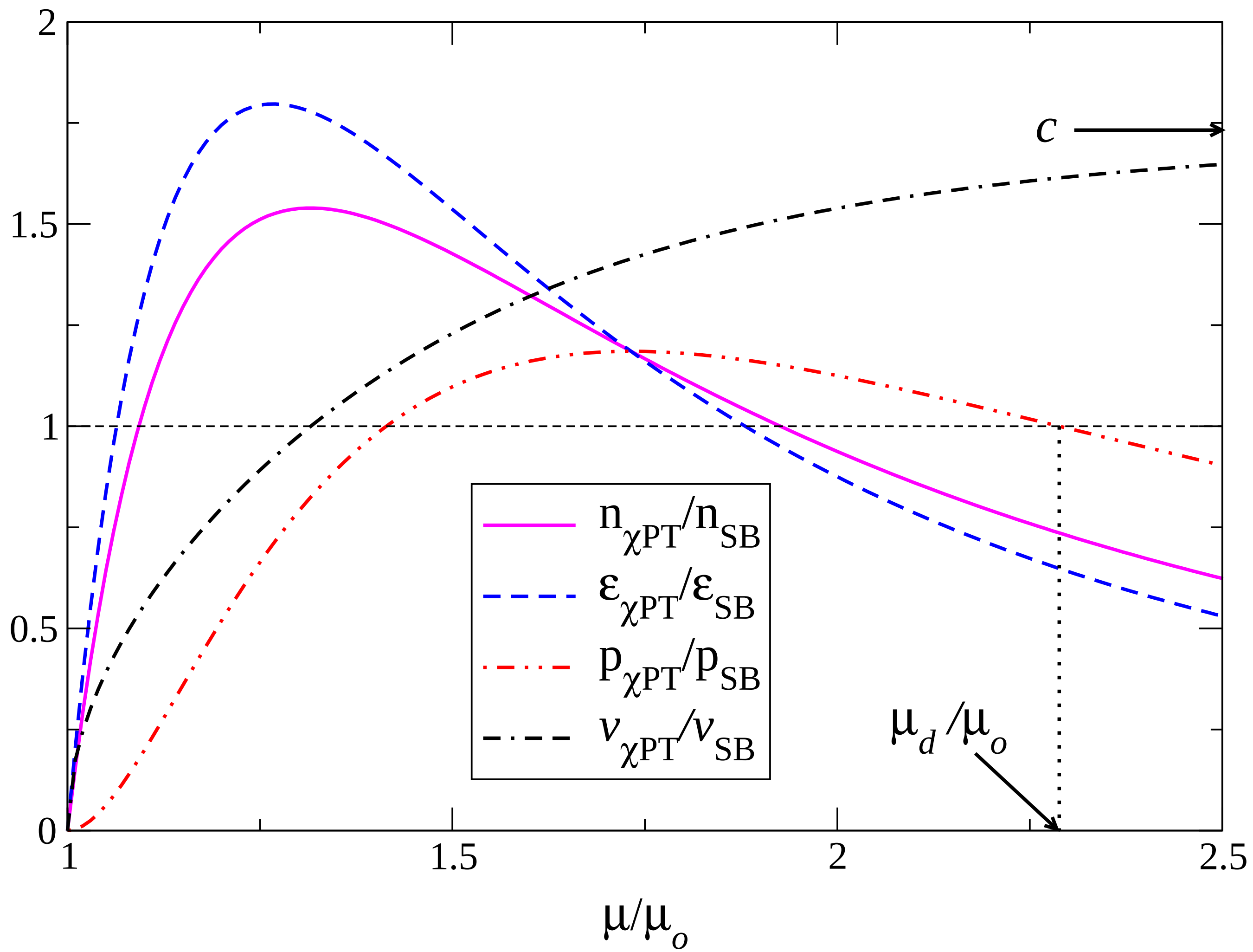
speed of sound $v_{\chi PT} = \sqrt{\frac{\partial p}{\partial \varepsilon}} = \left(\frac{1 - \frac{\mu_o^4}{\mu^4}}{1 + 3\frac{\mu_o^4}{\mu^4}}\right)^{\frac{1}{2}}$

This is to be contrasted with our other paradigm for cold dense matter, namely a degenerate system of weakly interacting (deconfined) quarks populating a Fermi sphere up to some maximum momentum $k_F \approx E_F = \mu$

$$\Rightarrow \quad n_{SB} = \frac{N_f N_c}{3\pi^2}\mu^3; \quad \varepsilon_{SB} = 3p_{SB} = \frac{N_f N_c}{4\pi^2}\mu^4;$$
$$\delta_{SB} = 0; \quad v_{SB} = \frac{1}{\sqrt{3}}$$

Superfluidity arises from condensation of diquark Cooper pairs from within a layer of thickness Δ centred on the Fermi surface:

$$\Rightarrow \quad \langle qq \rangle \propto \Delta \mu^2$$



By equating free energies, we naively predict a first order deconfining transition from BEC to quark matter;
eg. for $f_\pi^2 = N_c/6\pi^2$, $\mu_d \approx 2.3\mu_o$.

Simulation Details ($N_f = 2$ Wilson flavors)

Initial runs used a $8^3 \times 16$ lattice with parameters $\beta = 1.7$, $\kappa = 0.1780$ (Wilson gauge action)

$\Rightarrow a = 0.220$ fm, $m_\pi a = 0.79(1)$, $m_\pi / m_\rho = 0.779(4)$

Now have preliminary data from a matched $12^3 \times 24$ lattice with $\beta = 1.9$, $\kappa = 0.1680$

$\Rightarrow a = 0.15$ fm, $m_\pi a = 0.68(1)$, $m_\pi / m_\rho = 0.80(1)$

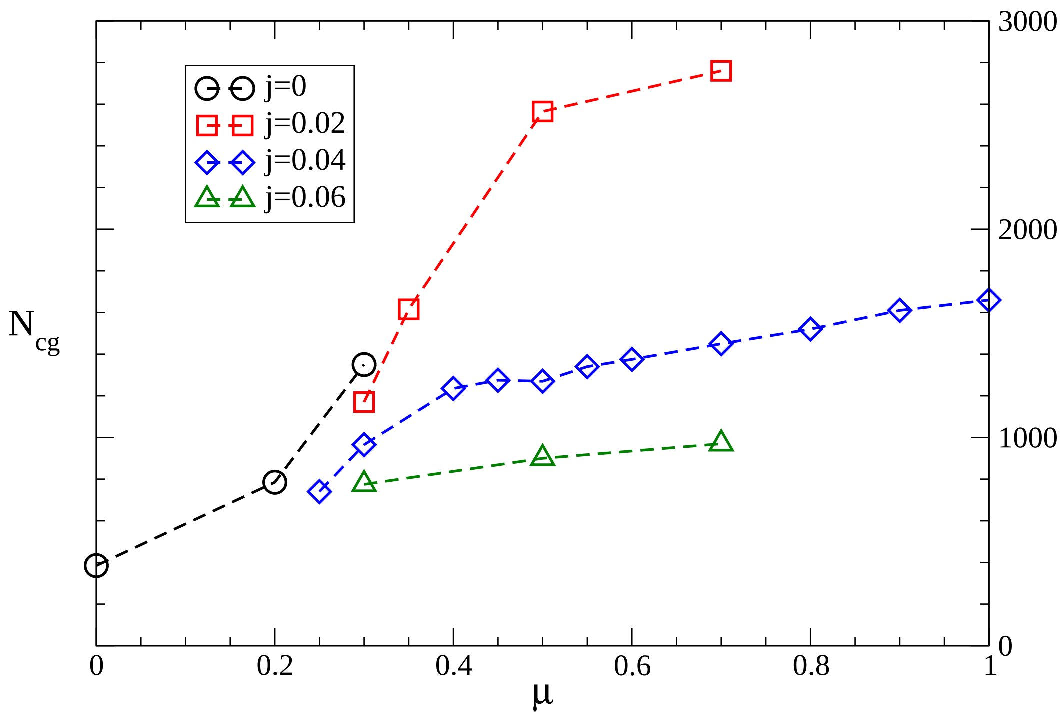
$\Rightarrow T \approx 60$MeV in both cases

To counter IR fluctuations and to maintain ergodicity, we introduce a diquark source $j\kappa(-\bar{\psi}_1 C \gamma_5 \tau_2 \bar{\psi}_2^{tr} + \psi_2^{tr} C \gamma_5 \tau_2 \psi_1)$

So far have accumulated roughly 300 trajectories of mean length 0.5 on $8^3 \times 16$ and 100 trajectories on $12^3 \times 24$

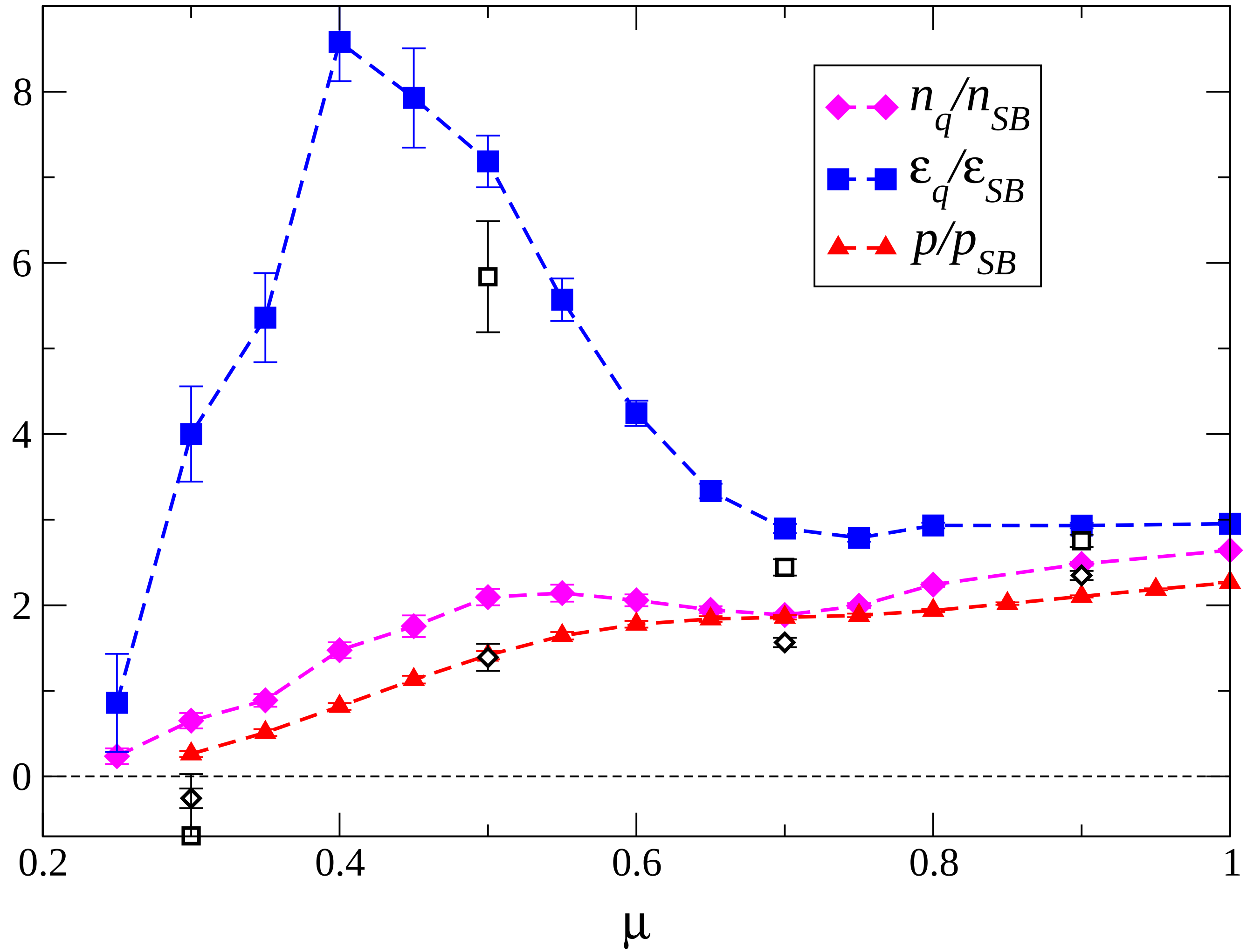
SJH, Kim, Skullerud, Eur.Phys. J. C48 (2006)193

Computer effort



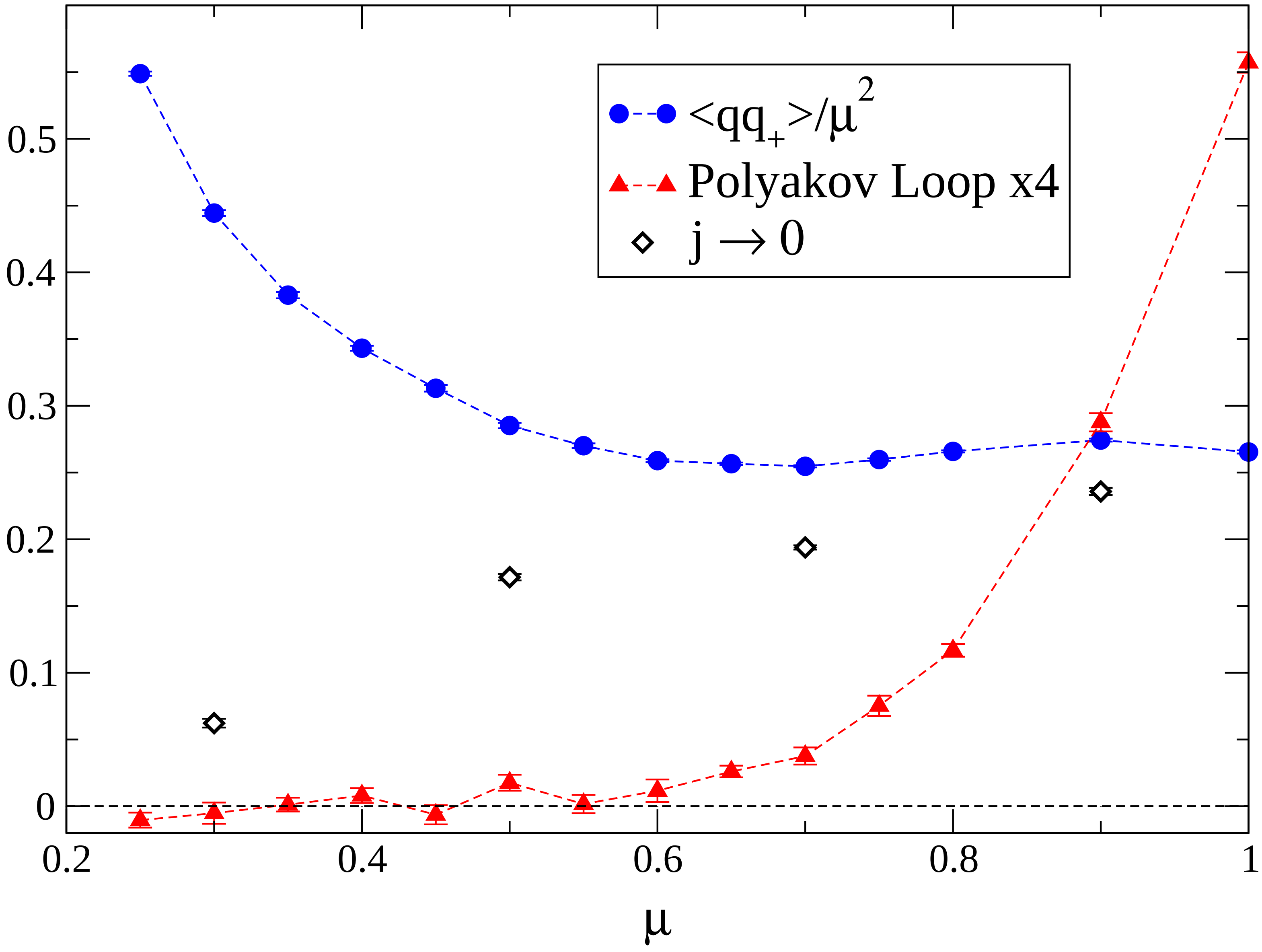
The number of `congrad` iterations required for convergence during HMC guidance rises as μ increases $\Leftrightarrow$ accumulation of small eigenvalues of M.

Equation of State ($8^3 \times 16$)



Open symbols denote $j \to 0$ extrapolation

Evidence for Deconfinement at $\mu a \simeq 0.65$

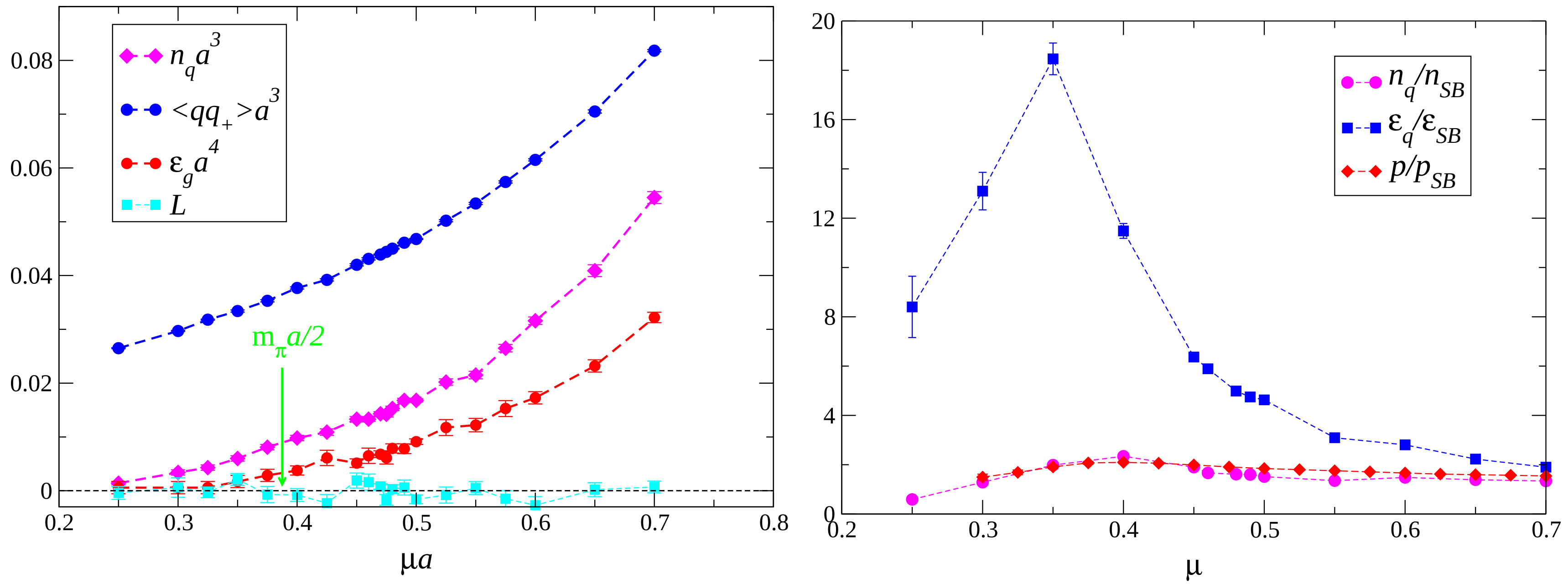


We conclude there is a transition from confined bosonic "nuclear matter" to deconfined fermionic "quark matter" at $\mu_d \approx 0.65$. Both phases are superfluid, but for $\mu > \mu_d$ the scaling is that expected of a degenerate system.

In condensed matter parlance we are observing a BEC/BCS crossover.

What is the nature of the transition between these two régimes?

Towards the continuum limit. . .



$12^3 \times 24$ results at $\beta = 1.9$ $\kappa = 0.168$ $ja = 0.04$

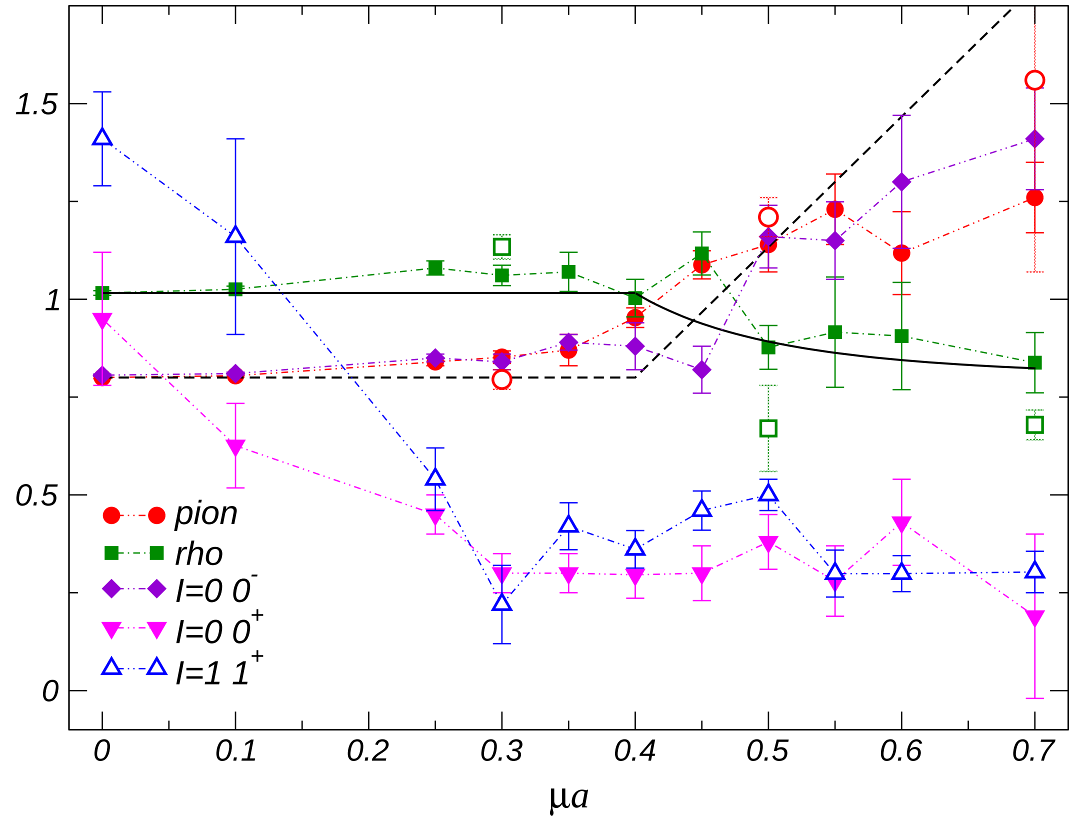
Identify onset transition at $\mu a \approx 0.32$ and (very tentatively) a deconfining transition at $\mu a \approx 0.5$

i.e. with $\mu_q \approx 670\,\text{MeV}$, $n_q \approx 5\,\text{fm}^{-3}$, $\Delta\varepsilon_g \lesssim 2\,\text{GeVfm}^{-3}$

On $N_\tau = 24$ the unrenormalised Polyakov loop L has very poor signal:noise

Mesons on $8^3 \times 16$

SJH, P. Sitch, J.I. Skullerud arXiv:0710.1966



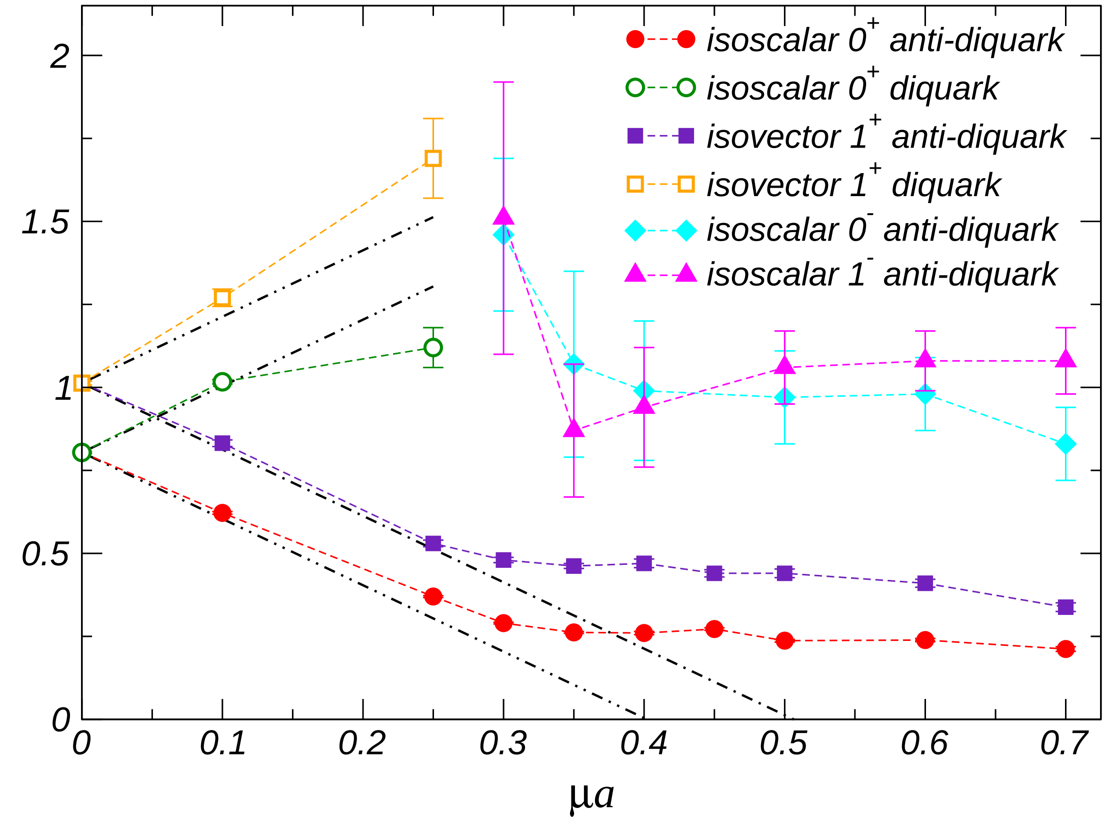
Meson spectrum roughly constant up to onset. Then $m_\pi \approx 2\mu$ in accordance with χPT, while m_ρ decreases once $n_q > 0$, in accordance with effective spin-1 action

[Lenaghan, Sannino & Splittorff PRD65:054002(2002)]

Cf. Hiroshima group

[Muroya, Nakamura & Nonaka PLB551(2003)305]

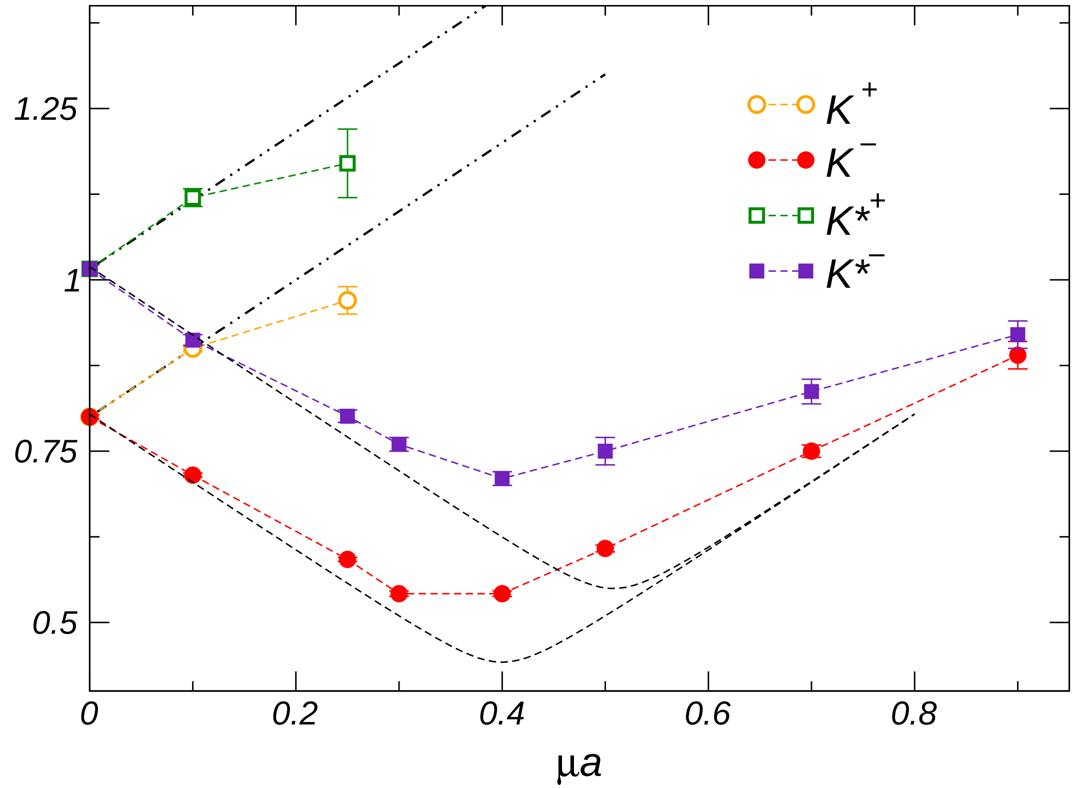
Diquark Spectrum on $8^3 \times 16$



Diquark spectrum modelled by $m_{\pi,\rho} \pm 2\mu$ up to onset, while post-onset:

- Splitting of “Higgs/Goldstone” degeneracy in $I = 0\ 0^+$ channel
- Meson/Baryon degeneracy in $I = 0\ 0^+$ and $I = 1\ 1^+$ channels

Kaon Spectrum on $8^3 \times 16$

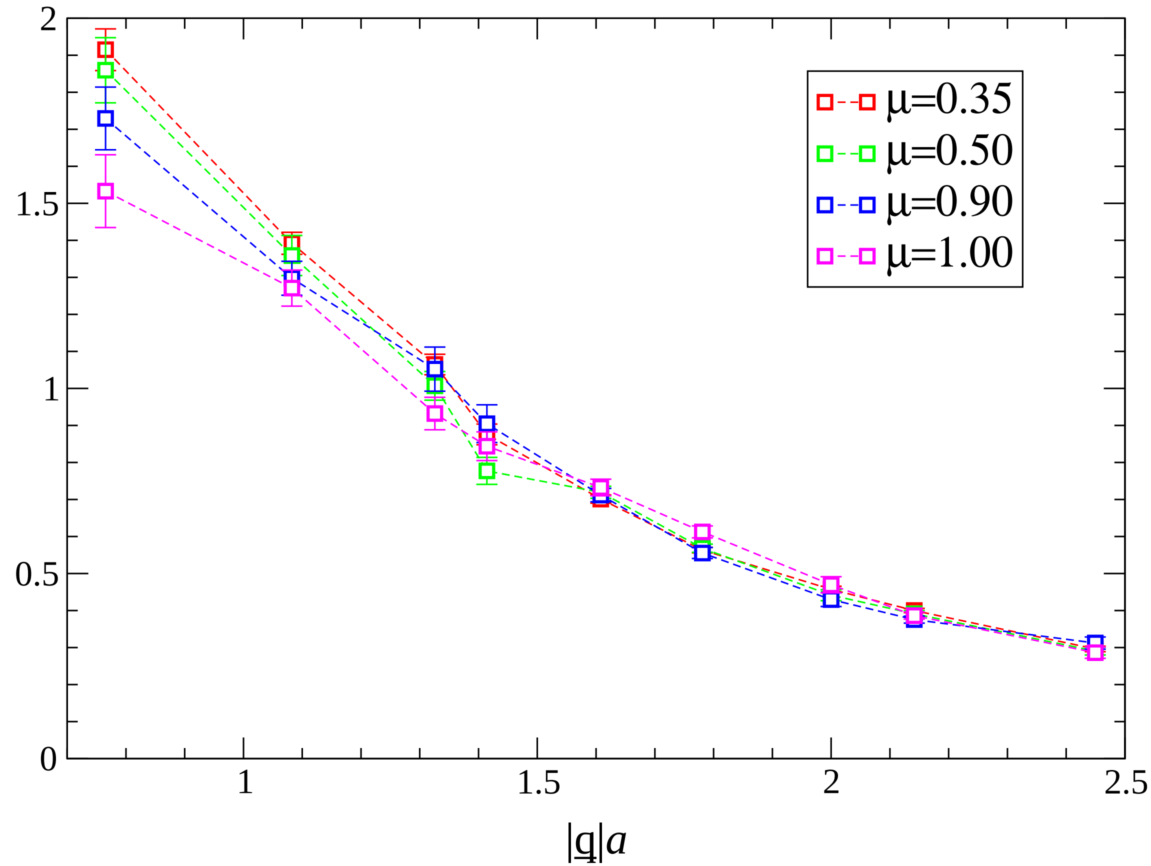


"Kaons" have one quark propagating with $\mu_s \equiv 0$.

Kaon spectrum modelled by $m_{\pi,\rho} \pm \mu$ up to onset, while post-onset $m_{K^-} \gtrsim \mu$, as if it were a weakly-bound state of an s-quark and a u-hole.

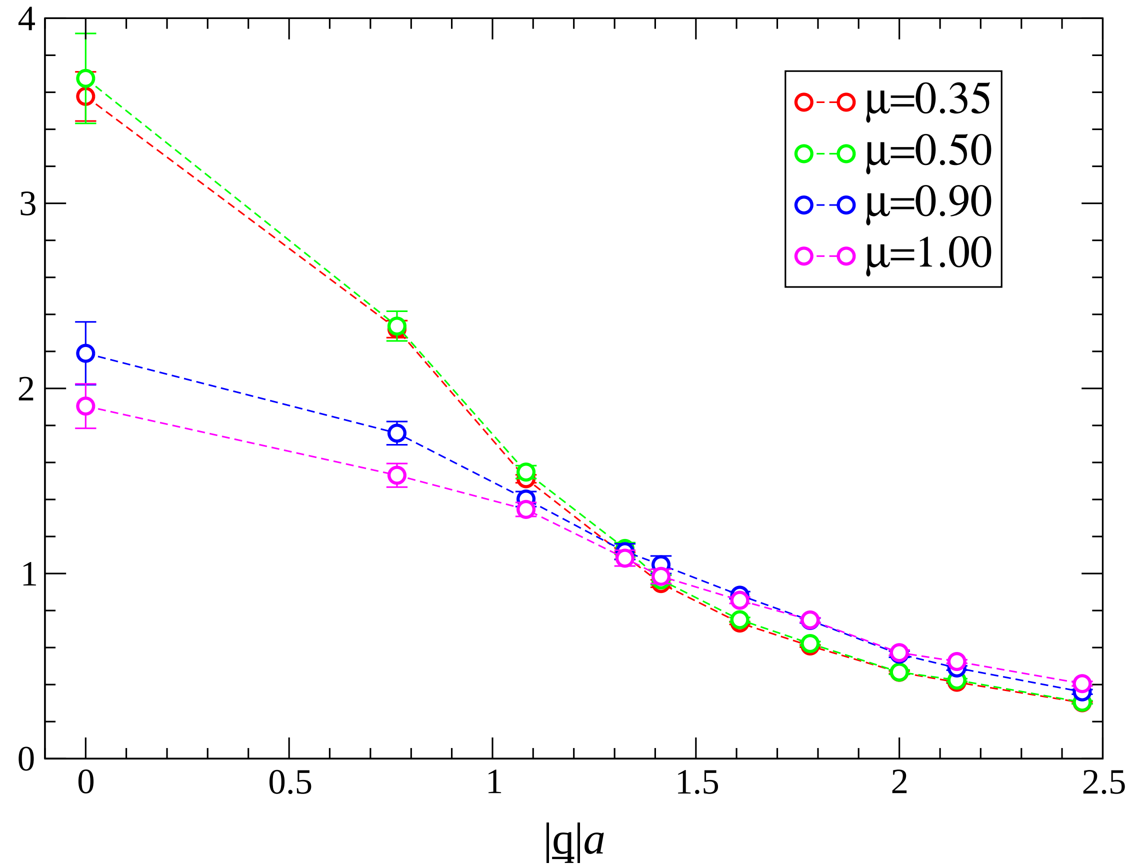
Suggestive of bound kaonic states in nuclear medium?

Electric gluon propagator (Landau gauge)



Plot $D^E(q_0, \vec{q})$ for fixed q_0 as a function of $|\vec{q}|$. The electric gluon in the static limit $q_0 = 0$ shows some evidence of Debye screening as $|\vec{q}| \to 0$ for $\mu \gtrsim 0.9$.

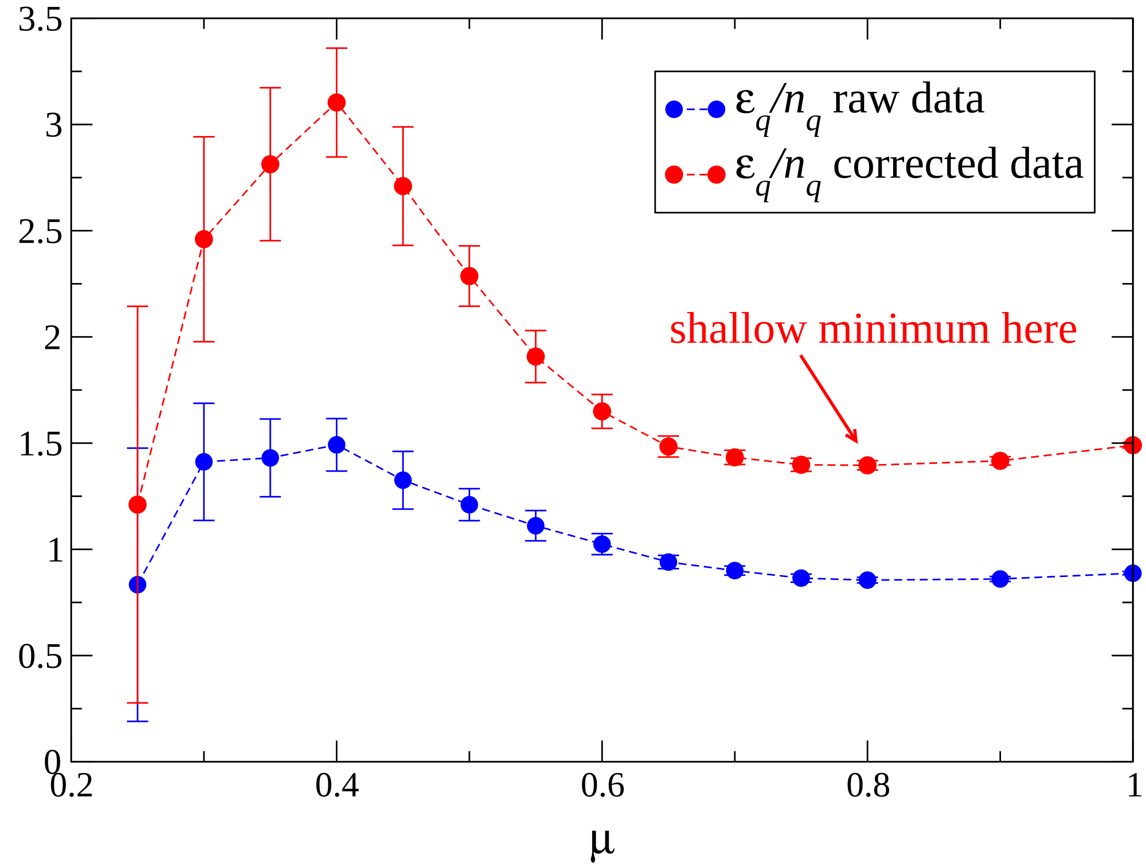
Magnetic gluon



The effect of $\mu \neq 0$ is much more dramatic in the magnetic sector.

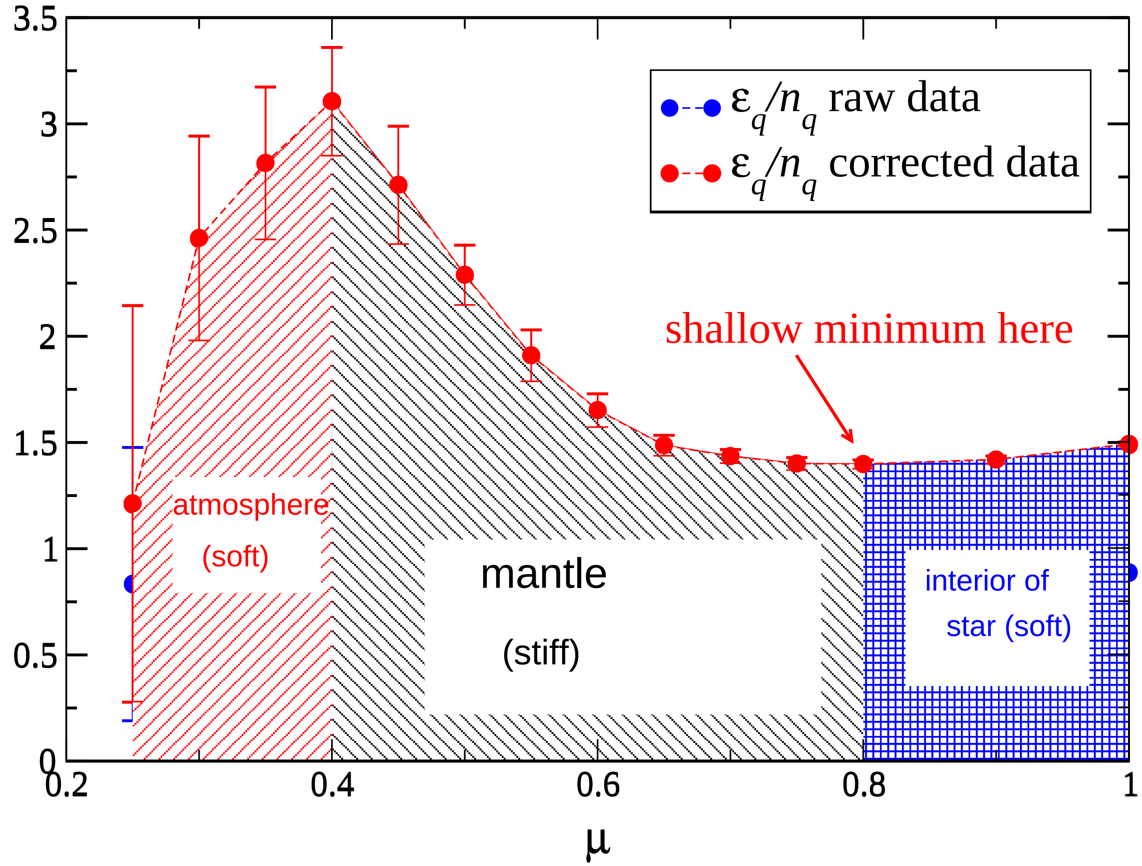
This is significant because in perturbation theory magnetic gluons are not screened in the static limit.

A Star is Born?



Remarkably, ε_q/n_q exhibits a robust minimum for $\mu \gtrsim \mu_d$, implying that macroscopic objects such as Two Color Stars are largely made of quark matter…

A Star is Born?



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Summary & Outlook

- Thermodynamic results support a BEC at intermediate μ, and deconfined BCS superfluid at large μ
- A non-vanishing gluon energy density may be a more reliable indicator of deconfinement than the Polyakov line as $T \to 0$
- In-medium decrease of ρ mass, meson/baryon degeneracy, and kaonic-nuclear bound states
- Non-perturbative screening of gluon propagator in magnetic sector
- Bulk quark matter may be more stable energetically than predicted by χPT
- Future analysis to include: nature of deconfinement, topological excitations. . .