The background features a faint, golden-toned illustration of a classical scene. On the left, a woman in a headscarf is seated at a table, looking down. On the right, a man with a laurel wreath stands, facing her. A central column supports a decorative archway with a Greek key pattern. The entire scene is framed by a large, circular Greek key border.

## Higher-order techniques

MC4BSM 2018, IPPP, Durham  
April 20, 2018

Stefan Prestel (Fermilab)

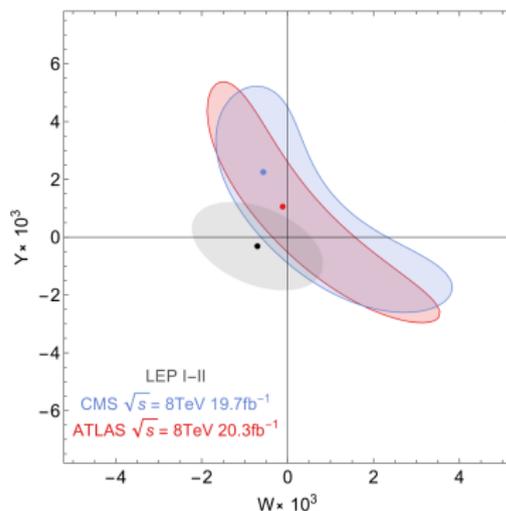
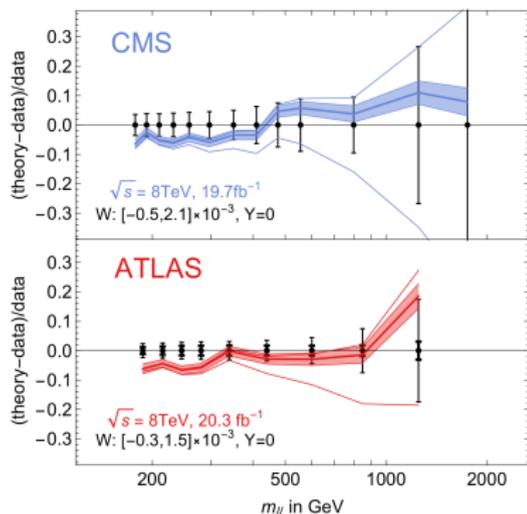
BSM searches at LHC have had one side-effect: Very accurate & precise SM calculations (particularly QCD corrections).

⇒ We can also put strong indirect bounds on new physics by comparing precision calculations with precision measurements.

⇒ Need to rely on reasonable theory error estimates.

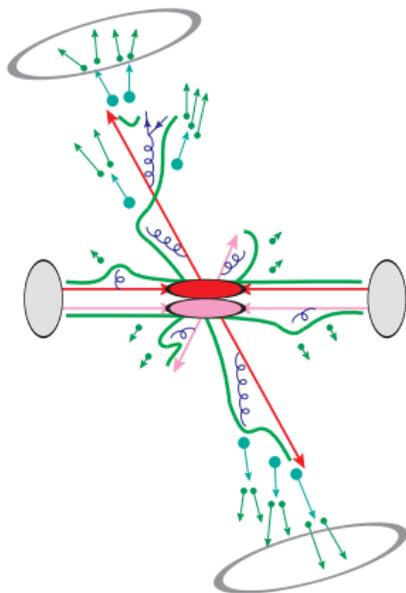
# Indirect bounds from precision measurements

Plots from arXiv:1407.1043, arXiv:1609.08157



Very inclusive measurements (top cross section, Drell-Yan mass spectrum) can already provide useful levers.  
More differential observables require better understanding of theory & theory tools!

# Theory calculations



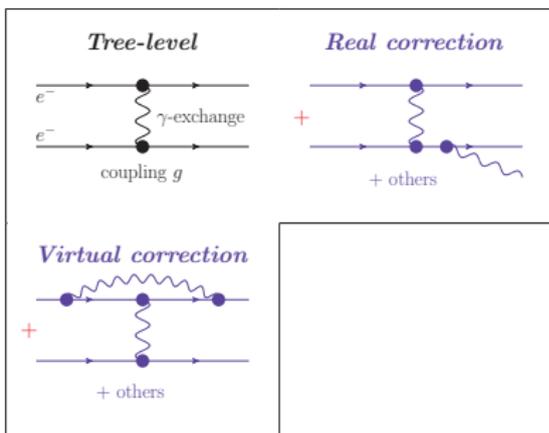
- A. Hard interaction
- B. Radiative cascade
- C. Multiple interactions
- D. Hadron formation & hadron decays

⇒ Stable hadrons, photons etc. as measured in detector

Usually not part of MCEG: Beam spectrum, nuclear & detector effects

	Impact	Uncertainties	Talks to...	Higher orders?
A	Normalization, correlations	Scales, PDF	B, C	SM@NLO ✓
B	Jet evolution	Scales, PDF, cut-off	B, C, D	Tough but possible
C	Overall activity	PDF, tuning, model	B, D	Maybe long-term?
D	Observable spectrum	Tuning, model, data	B	Yedi level

## Short-distance cross section: NLO calculations



Problem: Regularizing IR divergences in 4D. Solve by:

**“Slicing”**

$$\sigma = [c + \ln(\text{cut})] f(0) + \int_{\text{cut}} dz \frac{f(z)}{z}$$

**“Subtraction”**

$$\sigma = [c + ct] f(0) + \int dz \frac{f(z) - f(0)}{z}$$

Subtraction methods dominate @ NLO.  
4D regularization allows NLO “plots”.

Note: Apart from the most complicated cases (NLO for loop-induced processes), all one-loop integrals for QCD are known.

## Short-distance cross section: NLO matching

Frixione:2002ik, Nason:2004rx, Frixione:2007vw, Frixione:2010ra, Torrielli:2010aw, Alioli:2010xd  
Hoeche:2010pf, Hoeche:2011fd, Platzer:2011bc, Alwall:2014hca, Jadach:2015mza, Czakon:2015cla

NLO calculation after IR regularization:

$$\langle \mathcal{O} \rangle^{\text{NLO}} = \int [B_n + V_n + \int d\Phi_{\text{rad}} D_{n+1}] \mathcal{O}(\Phi_n) d\Phi_n + \int [B_{n+1} \mathcal{O}(\Phi_{n+1}) - D_{n+1} \mathcal{O}(\Phi'_n)] d\Phi_{n+1}$$

New challenges: No NLO “events”. Real & virtual corrections overlap with subsequent shower. **Can be solved simultaneously by adding zeros!**

$$\begin{aligned} \langle \mathcal{O} \rangle^{\text{NLO}} &= \int \left[ B_n + V_n + I_n + \int d\Phi_{\text{rad}} (B'_{n+1} - D_{n+1}) \right] \mathcal{O}(\Phi_n) d\Phi_n \\ &+ \int (B_{n+1} - B'_{n+1}) \mathcal{O}(\Phi_{n+1}) \\ &+ \int (B'_{n+1} \mathcal{O}(\Phi_{n+1}) - B'_{n+1} \mathcal{O}(\Phi_n)) \leftarrow \text{That's the } \mathcal{O}(\alpha_s) \text{ of PS acting on } B_n \end{aligned}$$

$\Rightarrow$  Red term can be generated by PS in 4D. Remaining terms can be grouped into events. Need first order expansion of PS.

## Parton showers as all-order subtractions

Parton showers distribute fixed-order cross sections  $B$  over higher-multiplicity phase space, according to Sudakov factors  $\Pi$ .

This allows to resum collinear leading logarithms to all orders using MC hit&miss techniques.

Probability conservation means that PS is an “all-order” finite subtraction:

$$\begin{aligned} \mathbf{PS} [B_0] &= \underbrace{B_0 \Pi_0 \mathcal{O}_0}_{\text{no emission}} + \int_1 B_0 P \Pi_0 \mathcal{O}_1 + \dots \\ &\quad \underbrace{\hspace{10em}}_{\text{at least 1 emission}} \\ &\equiv B_0 \mathcal{O}_0 - \int_1 B_0 P \Pi_0 \mathcal{O}_0 + \int_1 B_0 P \Pi_0 \mathcal{O}_1 + \dots \end{aligned}$$

Subtraction removes any overlap between different multiplicities.

⇒ Can be used to “stack” fixed-order calculations.

## Short-distance cross section: Higher multiplicities

Catani:2001cc, Mangano:2001xp, Mrenna:2003if, Alwall:2007fs, Hamilton:2009ne, Hamilton:2010wh, Hoche:2010kg, Lavesson:2008ah, Lonnblad:2012ng, Lonnblad:2001iq, Lavesson:2005xu, Lonnblad:2011xx, Platzer:2012bs, Gehrmann:2012yg, Hoeche:2012yf, Lonnblad:2012ix, Frederix:2012ps, Alioli:2012fc, Bellm:2017ktr

Fixed-order calculations for different multiplicities overlap.

⇒ Reweight (inclusive) fixed-order as if it had been generated by parton shower, thus enforcing an “all-order subtraction” of overlaps.

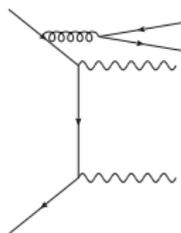
⇒ Replace the shower approximation  $B_0 \cdot P(z_1) \cdot P(z_2) \dots$  with complete results  $B_n$ :

$$\begin{aligned} \mathbf{ME+PS} [B_0] &= B_0 \mathcal{O}_0 - \int_1 B_1 \Pi_0 \mathcal{O}_0 + \int_1 B_1 \Pi_0 \mathcal{O}_1 \\ &\quad - \int_2 B_2 \Pi_0 \Pi_1 \mathcal{O}_1 + \int_2 B_2 \Pi_0 \Pi_1 \mathcal{O}_2 + \dots \end{aligned}$$

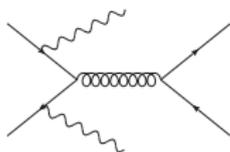
Extension to NLO (and some simple NNLO cases) possible by expanding the subtractions to remove overlap with virtual/real corrections.

# Resonances and all that

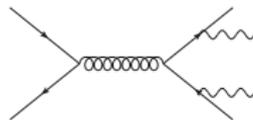
Jezo:2015aia, Kallweit:2015dum, Jezo:2016ujg, Frederix:2016rdc, Nejad:2016bci, Kallweit:2017khh



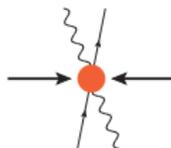
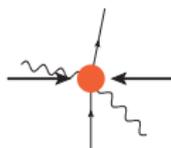
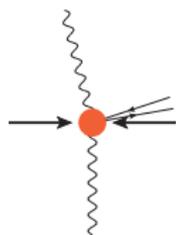
$W^+$  and  $W^- \approx$  back-to-back  
Potentially large QCD Sudakov logs



$W^+$  and  $b$  in different hemispheres  
Potentially large EW Sudakov logs



$W^+$  and  $b$  in same hemisphere



Subtraction/matching/merging require physical intermediate states.

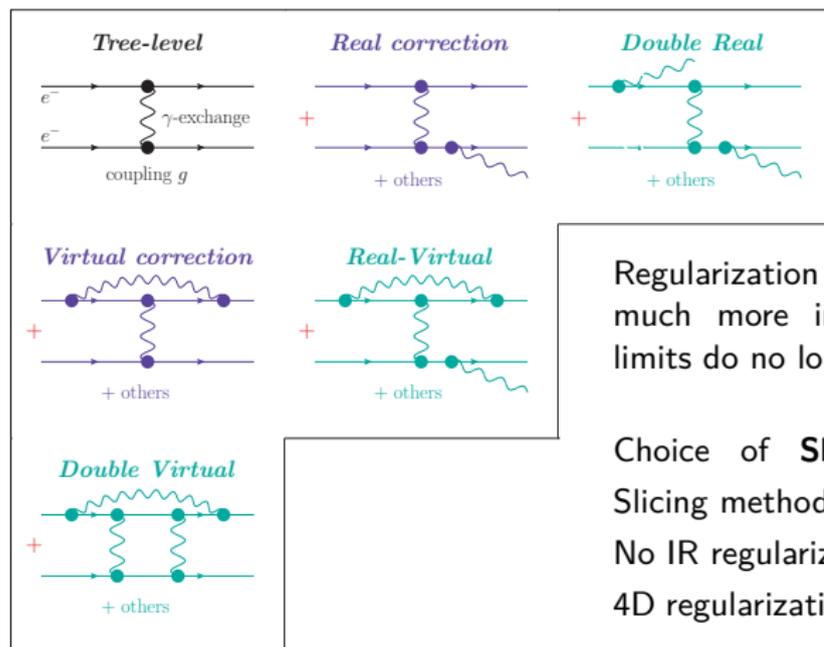
Resonances in SM (e.g. top) complicate power counting.

$\Rightarrow$  Resonance-aware subtraction + PS starting conditions

BSM: Same problems, but much more severe. “Inclusive” approaches impractical, i.e. need diagram removal/subtraction. Not automated.

# Short-distance cross section: NNLO calculations

GehrmannDeRidder:2005cm, Somogyi:2006da, Czakon:2010td, GehrmannDeRidder:2012ja, Caola:2017dug  
Catani:2007vq, Boughezal:2015dva, Gaunt:2015pea



Regularization of IR divergences in 4D  
much more involved: Soft/collinear  
limits do no longer commute.

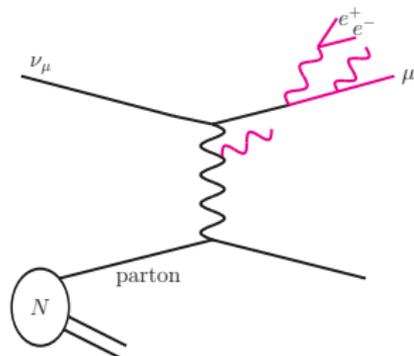
Choice of **Slicing** or **Subtraction**.  
Slicing methods dominate @ NNLO.  
No IR regularization automated.  
4D regularization allows NNLO “plots”.

Note: Not all two-loop integrals for QCD known analytically.  
Note: Cannot interface parton shower to plots - need events :(

Observable exhibits regularization dependence?

→ Transition between Born and real phase space.

→ Fixed-order unsatisfactory.



- ▶ Jet structure @ LHC?  
→ QCD shower
- ▶ Lepton energy @  $\nu$ ?  
→ QED shower
- ▶ Energy loss to dark sector?  
→ Dark shower

Parton showers mandatory to describe the details of the final state.

PS generates renormalization group running of structure functions

$$\frac{d f_a(x, t)}{d \ln t} = \sum_{b=q,g} \int_0^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} [P_{ab}(z)]_+ f_b\left(\frac{x}{z}, t\right)$$

“+” turns into a Sudakov factor, the rest into emission spectrum.

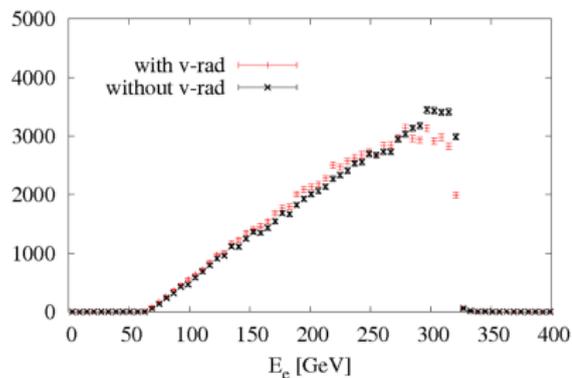
PS attempts to resum double- and single QCD logarithms of its evolution variable  $t$ .

⇒ Reasonable control over wider class of multi-scale observables,  
*e.g. jet rates & separation, exclusive states...*

# Parton showers: Higher orders for light BSM

Carloni:2010tw, Carloni:2011kk

If BSM couples to QCD, it couples to QCD shower.



But PS techniques have wider applicability: Theory with light fermions/bosons will have QCD-like leading logarithms.

PS allows to resum light BSM collinear enhancements (e.g in lepton energy) in DGLAP style.

Warning: No soft-correct coherent shower for light BSM exists.  
Why? Concerns about subleading pieces!

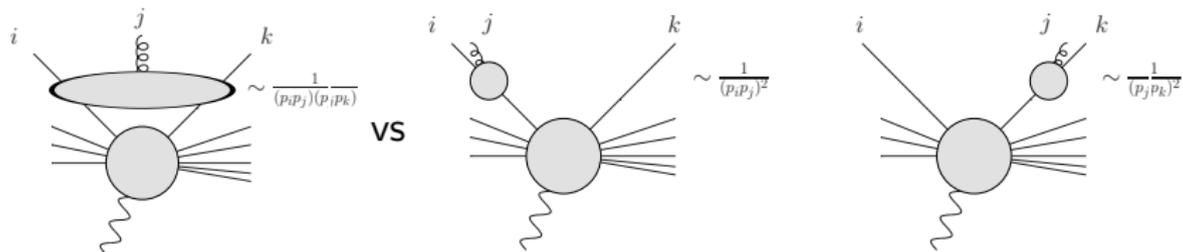
# Parton showers: Subleading corrections

Soft/collinear cross-talk

Running coupling

NLO PDFs

Momentum conservation



How to isolate (hard) collinear from soft physics?

Coupling running different per process? How is it distributed over phase space?

How are NLO PDFs allowed if the shower is only LO?

Interplay of recoil and soft-gluon summation?

⇒ Need to think about PS beyond leading order.

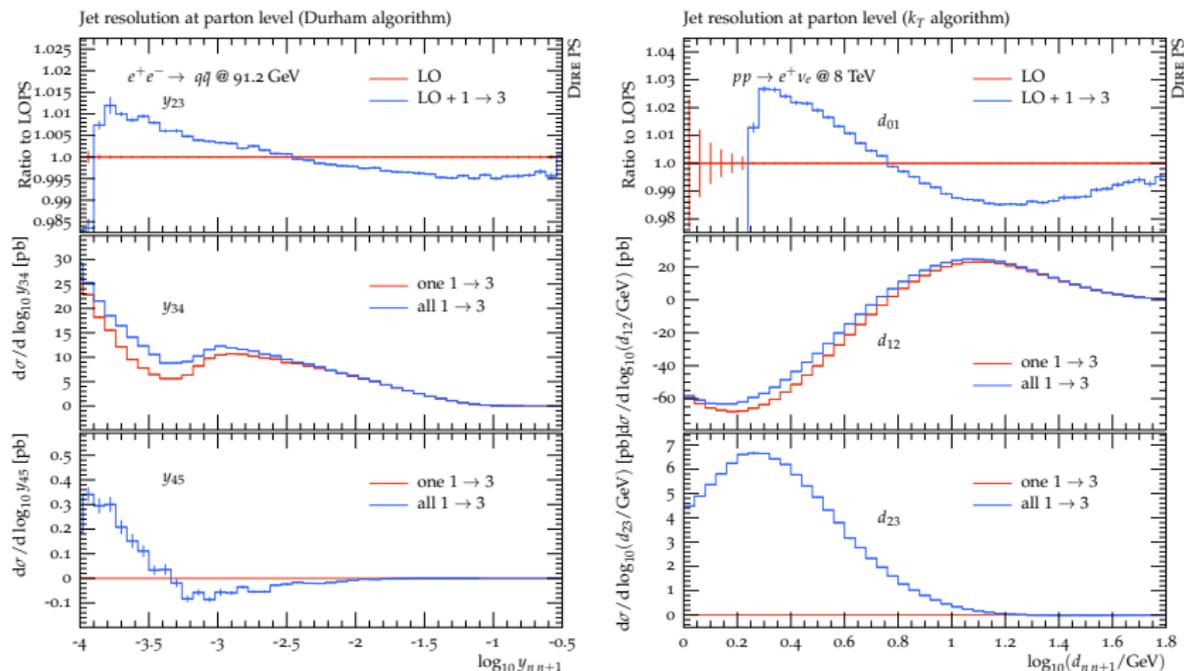
For a LO shower implementation with NLO ambitions, we need

- ... to understand the LO single- and double-emission rate and phase space in  $D$  dimensions.
- ... analytically and numerically manageable calculation.
- ... algorithms that can exponentiate negative (e.g. NLO DGLAP) kernels,
- ... a high pain threshold.

- ↔ LO showers should correspond to local NLO subtractions
- ↔ PS needs to be spin- and color-correlated with hard process

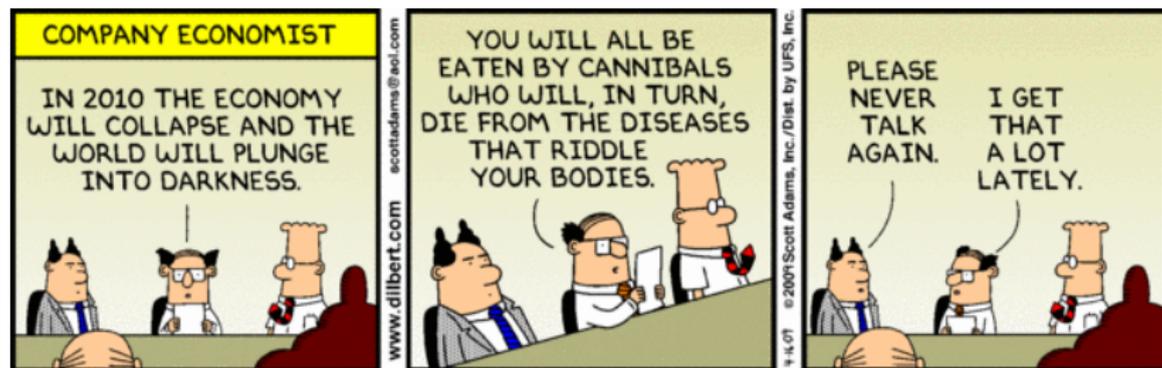


Given a sensible LO shower, what is the impact of “genuine” NLO?



$\Rightarrow$  Irrelevant for simultaneous coherent quark-pair emission.

## Summary & Outlook



- ◇ Higher-order methods are crucial for SM and exotic physics.
- ◇ Higher order calculations calculations rely on IR regularization.
- ◇ Matching of higher-order calculations to PS mandates events.
- ◇ Systematic PS calculations are necessary for any scheme beyond LO.
- ◇ Resonances are tough, both in SM and beyond.
- ◇ Collinear (non-coherent) PS algorithms for light BSM available.