Anderson localisation and the clockwork

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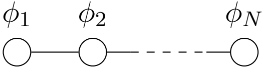
The original clockwork mechanism

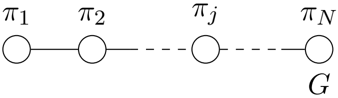
Choi *et al.* 1404.6209; Choi & Im 1511.00132; Kaplan & Rattazzi 1511.01827

Is a quiver theory wherein $U(1)^N \to U(1)$:



$$\mathcal{L} = \sum_{i=1}^{N} |\partial \phi_i|^2 + \sum_{i=1}^{N-1} (\epsilon \phi_i^\dagger \phi_{i+1}^q + \text{h.c.}) \tag{1}$$

Linearise $\phi_i = f e^{i \frac{\pi_i}{\sqrt{2} f}}$:



$$\mathcal{L} = \frac{1}{2} \sum_{i=1}^{N} (\partial \pi_i)^2 + \frac{1}{2} \epsilon f^{q-1} \sum_{i=1}^{N-1} (\pi_i - q \pi_{i+1})^2 + \frac{1}{4} G^2 + \frac{1}{f} \pi_N G \tilde{G} \tag{2}$$

Find the massless mode

$$\mathcal{L} = \frac{1}{2}\sum_{i=1}^{N}(\partial\pi_i)^2 + \frac{1}{2}\epsilon f^{q-1}\sum_{i=1}^{N-1}(\pi_i - q\pi_{i+1})^2 + \frac{1}{4}G^2 + \frac{1}{f}\pi_N G\tilde{G} \quad (3)$$

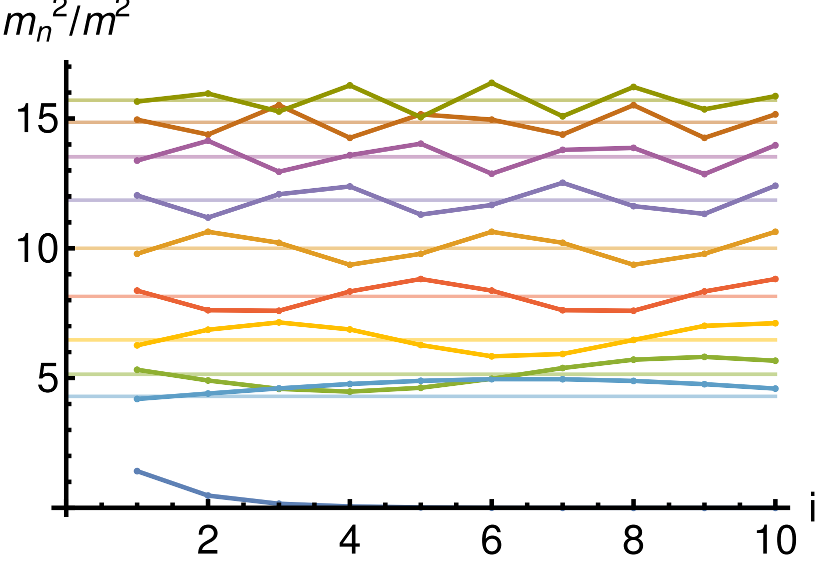
Sub. in $\pi_i = q^{N-i}\pi_{\text{zero}}$

$$\mathcal{L} = \frac{1}{2}\left(\sum_{i=1}^{N} q^{2N-2i}\right)(\partial\pi_{\text{zero}})^2 + \frac{1}{4}G^2 + \frac{1}{f}\pi_{\text{zero}} G\tilde{G} \qquad (4)$$

The coupling to gluons at the end of the lattice is **exponentially suppressed**, *i.e.* the effective decay constant is exponentially enhanced, $F = \sqrt{\left(\sum_{i=1}^{N} q^{2N-2i}\right)} f$.

One localised massless state, a band of extended heavy states

$N = 10,\ q = 3$



The construction works for $U(1)$ gauge theories

Saraswat 1608.06951; Giudice & McCullough 1610.07962

Consider N photons (Lorentz indices suppressed):

$$\mathcal{L} = \sum_{i=1}^{N} \frac{1}{4g^2} F_i^2 + \frac{1}{2} v^2 \sum_{i=1}^{N-1} (A_i - qA_{i+1})^2 + |(\partial + QA_N)\phi|^2 \quad (5)$$

Sub. in $A_i = q^{N-i} A_{\text{zero}}$

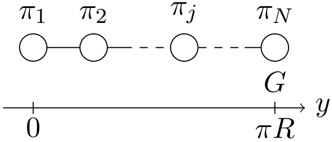
$$\mathcal{L} = \left(\sum_{i=1}^{N} \frac{q^{2N-2i}}{4g^2} \right) F_{\text{zero}}^2 + |(\partial + QA_{\text{zero}})\phi|^2 \quad (6)$$

The coupling to charged matter at the end of the lattice is **exponentially suppressed**. (Due to the suppression in the gauge coupling.)

Similar coupling suppression can be achieved in a lattice of fermions, with a Dirac mass matrix M chosen such that $M^\dagger M$ equals the photon mass matrix above.

Continuum clockwork[1]

Giudice & McCullough 1610.07962



'Linear dilaton background metric':
$\mathrm{d}s^2 = e^{-\frac{4}{3}k(|y|-\pi R)}(\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}y^2)$

$$S = \int \mathrm{d}^4x\mathrm{d}y\sqrt{|g|}\left((\partial_M\pi)(\partial^M\pi) + \left[G_{MN}G^{MN} + \frac{\pi}{f^{\frac{3}{2}}}G_{MN}\tilde{G}^{MN}\right]\frac{\delta(y-\pi R)}{\sqrt{g_{55}}}\right) \tag{7}$$

$$S = \int \mathrm{d}^4x\mathrm{d}y\left(e^{-\frac{6}{3}k(|y|-\pi R)}(\partial\pi)^2 + e^{-\frac{6}{3}k(|y|-\pi R)}(\partial_y\pi)^2 + \left[G^2 + \frac{\pi}{f^{\frac{3}{2}}}G\tilde{G}\right]\delta(y-\pi R)\right) \tag{8}$$

Sub in $\pi(x,y) = \pi_{\text{zero}}(x)$

$$S = \int \mathrm{d}^4x\left(\left(\int \mathrm{d}y e^{-\frac{6}{3}k(|y|-\pi R)}\right)(\partial\pi_{\text{zero}})^2 + \left[G^2 + \frac{\pi_{\text{zero}}}{f^{\frac{3}{2}}}G\tilde{G}\right]\right) \tag{9}$$

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[1] Disclaimer: certain properties of the discrete clockwork model and possible continuous analogues are different, see Choi *et al.* 1711.06228. The 5D model here is called clockwork in the literature, and its phenomenological interest is unaffected by these subtleties.

A 'new' class of warped extra dimensional models to explore

Antoniadis *et al.* 1102.4043; Baryakhtar 1202.6674; Cox & Gherghetta 1203.5870

The clockwork models have reawakened interest in linear dilaton theories.

$$\text{LD: } \mathrm{d}s^2 = e^{\frac{4}{3}k|y|}(\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + \mathrm{d}y^2)$$

$$\text{RS: } \mathrm{d}s^2 = e^{\frac{4}{3}k|y|}(\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu) + \mathrm{d}y^2$$

Just like RS, the warping of the extra dimension induces a large hierarchy between the 4D and 5D Planck masses.[2]

Just like RS, you can put all kinds of fields in the bulk of/on branes in the extra dimension (but with very different phenomenological consequences).

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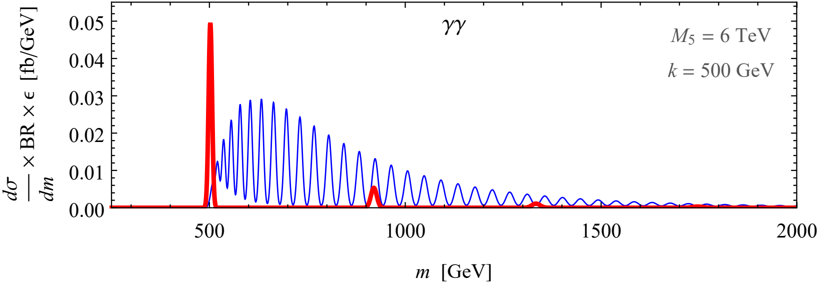
[2] Thereby solving the hierarchy problem, because 5D Planck scale $\sim \mathrm{O}(\text{TeV})$

There are novel collider signatures

Choi *et al.* 1711.06228; (fig. from) Giudice *et al.* 1711.08437



A compressed band of many weakly coupled states can lead to:

- periodicity/continuity in $\sqrt{\hat{s}}$;
- high multiplicity final states;
- long lived particles;
- and much, much more.

(Semi-)random, tridiagonal mass matrices have localised eigenvectors too

$$\mathcal{L}_\pi = \frac{1}{2}\sum_{i=1}^{N}(\partial\pi_i)^2 - \frac{1}{2}\sum_{i=1}^{N}\epsilon_i\pi_i^2 - \frac{1}{2}\sum_{i=1}^{N-1}t(\pi_i - \pi_{i+1})^2 \qquad (10)$$

Sample ϵ_i uniformly from $[0, W]$. If $\epsilon_i > 0, \forall i$, all mass squareds are positive.

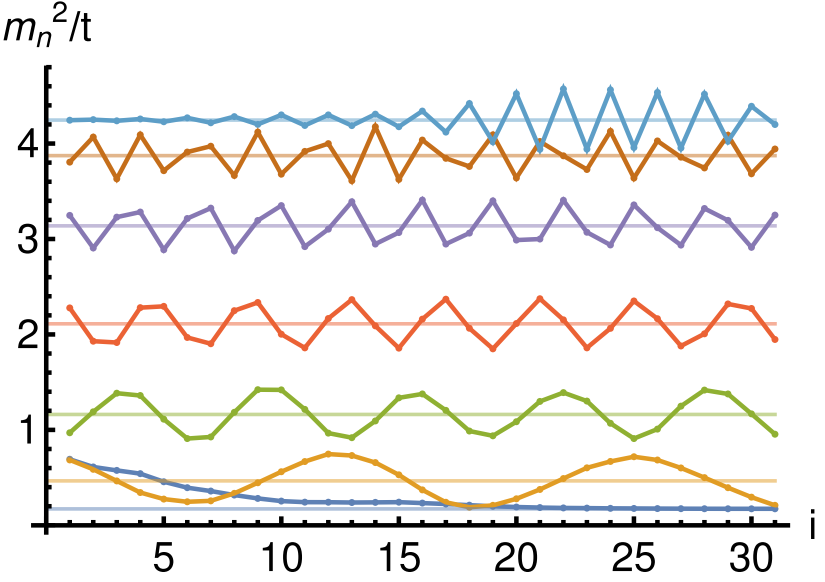
Save for edge effects, the mass matrix is a constant shift away from the Anderson tight binding model Hamiltonian.

$$\begin{pmatrix} t+\epsilon_1 & -t & 0 & \cdots \\ -t & 2t+\epsilon_2 & -t & \cdots \\ 0 & -t & 2t+\epsilon_3 & \ddots \\ \vdots & \vdots & \ddots & \ddots \end{pmatrix} \qquad (11)$$

(Coset rep lagrangian $\Phi_i = f\exp(\frac{i\pi_i}{f\sqrt{2}})$:

$$\mathcal{L} = \sum \partial\Phi_i^\dagger\partial\Phi_i - (\sum \frac{1}{4}\epsilon_i\Phi_i\Phi_i + \sum t\Phi_i^\dagger\Phi_{i+1} + \text{h.c.}) - \sum V(|\Phi_i|)\ .) \qquad (12)$$

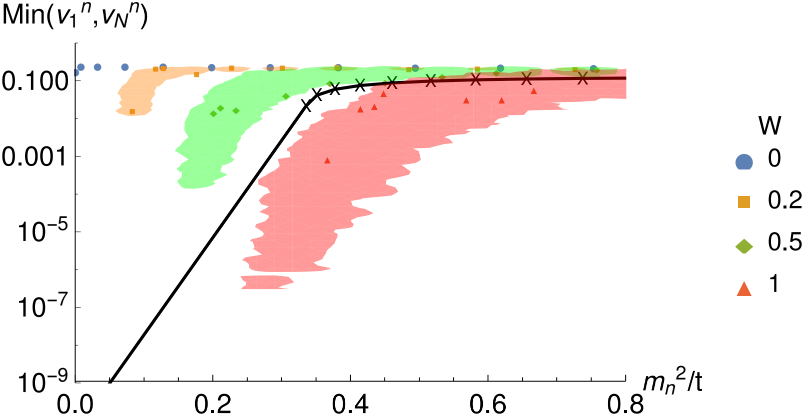
$N = 31$, $W = 0.4$, $t = 1$



(Every 5th eigenvalue/vector shown.)

Similar (but distinct) effects to clockwork

Craig & DS, 1710.01354



$(N = 35,\ m^2 = \frac{t}{3},\ t = 1,\ q = 2)$

In summary

Clockwork uses a lattice of $\mathrm{O}(N)$ fields to generate $\mathrm{O}(e^N)$ hierarchies in couplings.

The compressed band of states in these models — or more often their extra dimensional analogues — leads to novel collider signatures, such as periodicity in s-channel spectra or high-multiplicity cascade decays.

Similar effects can be realised in a lattice with random parameters, in a manner akin to Anderson localisation along a disordered 1D wire.