A very brief (and incomplete) review of Higgsplosion

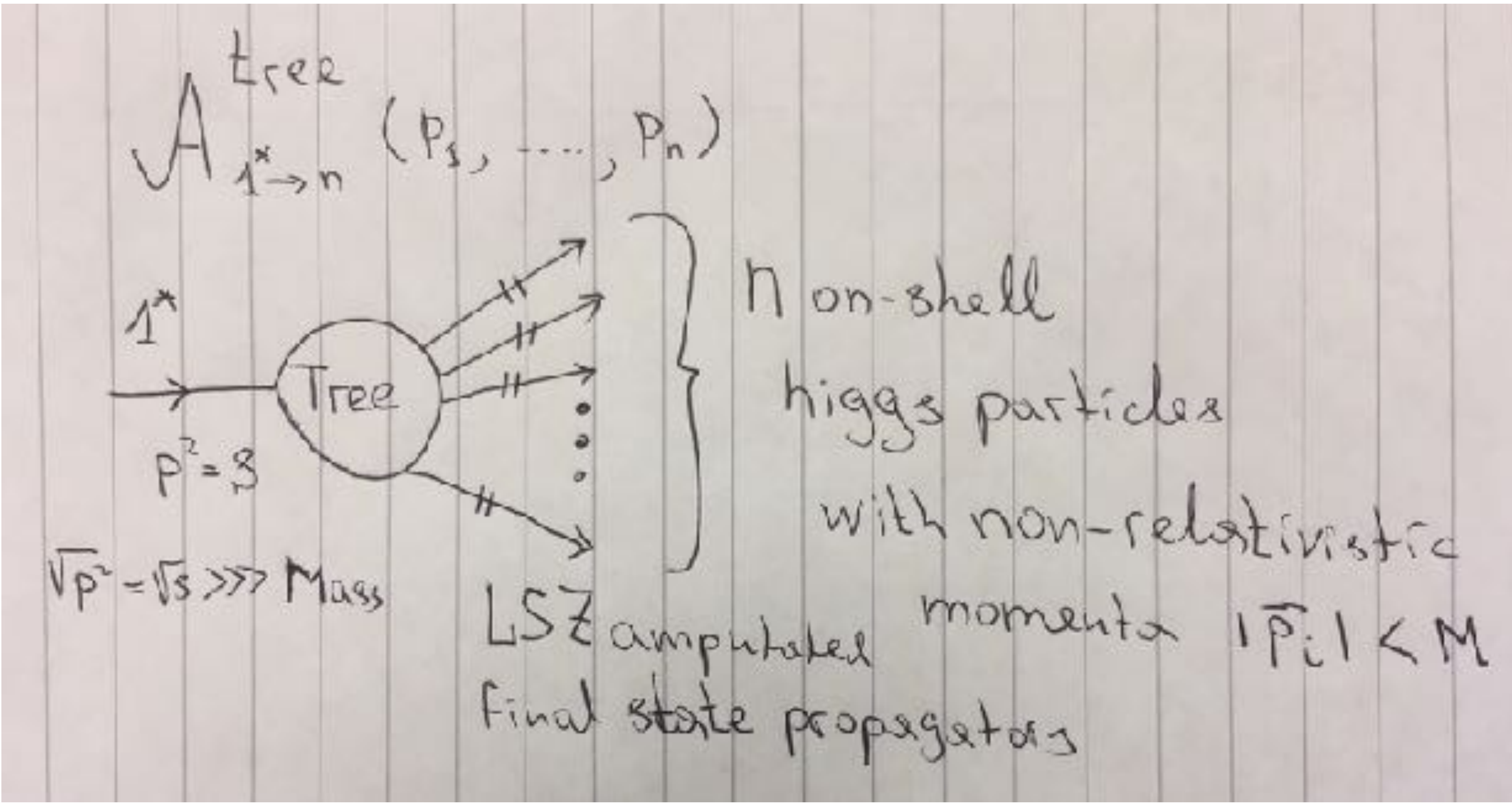
Jakub Scholtz
IPPP, Durham U

MC4BSM, April 21st, Durham

Introduction & Disclaimers

- It appears that there is something odd about spontaneously broken phi^4 theory.
- Namely amplitudes of 1* -> n grow like n! which seems to violate unitarity. Turns out this may be a feature rather than a bug.
- Disclaimer 1: I am confused. So will be you.
- Disclaimer 2: I took a lot of slides from Valya’s talks

Compute 1 -> n amplitudes @LO with non-relativistic final-state momenta:



see classic 1992-1994 papers:
Brown; Voloshin;
Argyres, Kleiss, Papodopoulos
Libanov, Rubakov, Son, Troitski

more recently: Khoze
1411.2925

$$\mathcal{L} = \frac{1}{2}(\partial_\mu h)^2 - \frac{\lambda}{4}(h^2 - v^2)^2$$

prototype of the SM Higgs
in the unitary gauge

Tree-level $1^* \to n$ amplitudes in the limit $\varepsilon \to 0$ for any n are given by

$$\mathcal{A}_n(p_1, \dots p_n) = n! \left(\frac{\lambda}{2M_h^2}\right)^{\frac{n-1}{2}} \left(1 - \frac{7}{6}n\varepsilon - \frac{1}{6}\frac{n}{n-1}\varepsilon + \mathcal{O}(\varepsilon^2)\right)$$

factorial growth

amplitude on the n-particle threshold

$$\varepsilon = \frac{1}{n\,M_h}\,E_n^{\text{kin}} = \frac{1}{n}\frac{1}{2M_h^2}\sum_{i=1}^{n}\vec{p}_i^{\,2}$$

kinetic energy per particle per mass

In the large-n-non-relativistic limit the result is

$$\mathcal{A}_n(p_1, \dots p_n) = n! \left(\frac{\lambda}{2M_h^2}\right)^{\frac{n-1}{2}} \exp\left[-\frac{7}{6}n\varepsilon\right], \quad n \to \infty,\ \varepsilon \to 0,\ n\varepsilon = \text{fixed}$$

Can now integrate over the n-particle phase-space

The cross-section and/or the n-particle partial decay Γ_n

$$\Gamma_n(s) \;=\; \int d\Phi_n \, \frac{1}{n!} \, \left|\mathcal{A}_{h^* \to n \times h}\right|^2$$

The n-particle Lorentz-invariant phase space volume element

$$\int d\Phi_n \;=\; (2\pi)^4 \delta^{(4)}(P_{\rm in} - \sum_{j=1}^{n} p_j) \, \prod_{j=1}^{n} \int \frac{d^3 p_j}{(2\pi)^3 \, 2p_j^0} \,,$$

in the large-n non-relativistic limit with $n\varepsilon_h$ fixed becomes,

$$\Phi_n \;\simeq\; \frac{1}{\sqrt{n}} \left(\frac{M_h^2}{2}\right)^n \, \exp\left[\frac{3n}{2}\left(\log\frac{\varepsilon_h}{3\pi} + 1\right) \,+\, \frac{n\varepsilon_h}{4} \,+\, \mathcal{O}(n\varepsilon_h^2)\right]$$

We find:

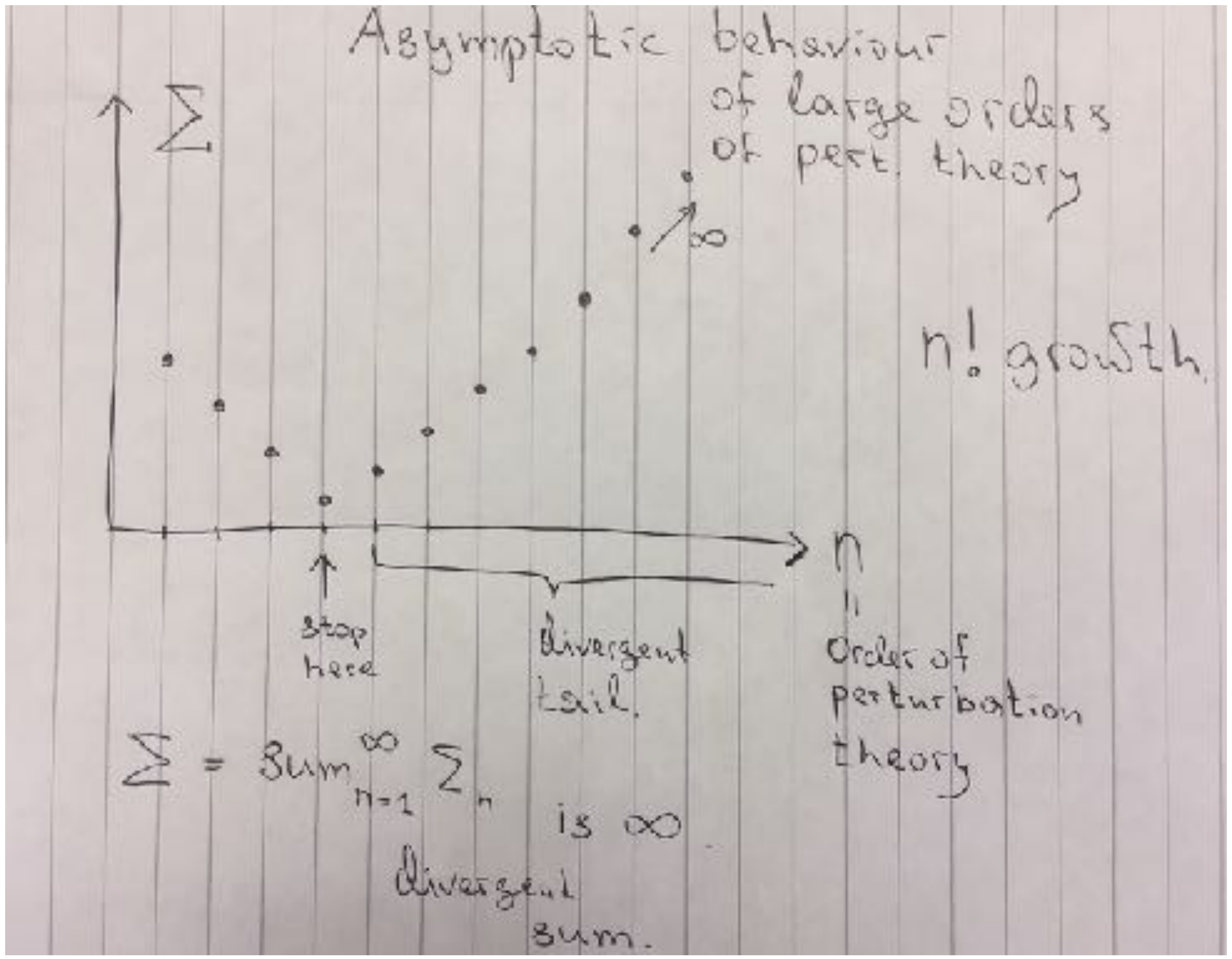
$$\Gamma_n^{\rm tree}(s) \;\sim\; \exp\left[n\left(\log\frac{\lambda n}{4} - 1\right) \,+\, \frac{3n}{2}\left(\log\frac{\varepsilon}{3\pi} + 1\right) \,-\, \frac{25}{12}\, n\varepsilon \,+\, \mathcal{O}(n\varepsilon^2)\right]$$

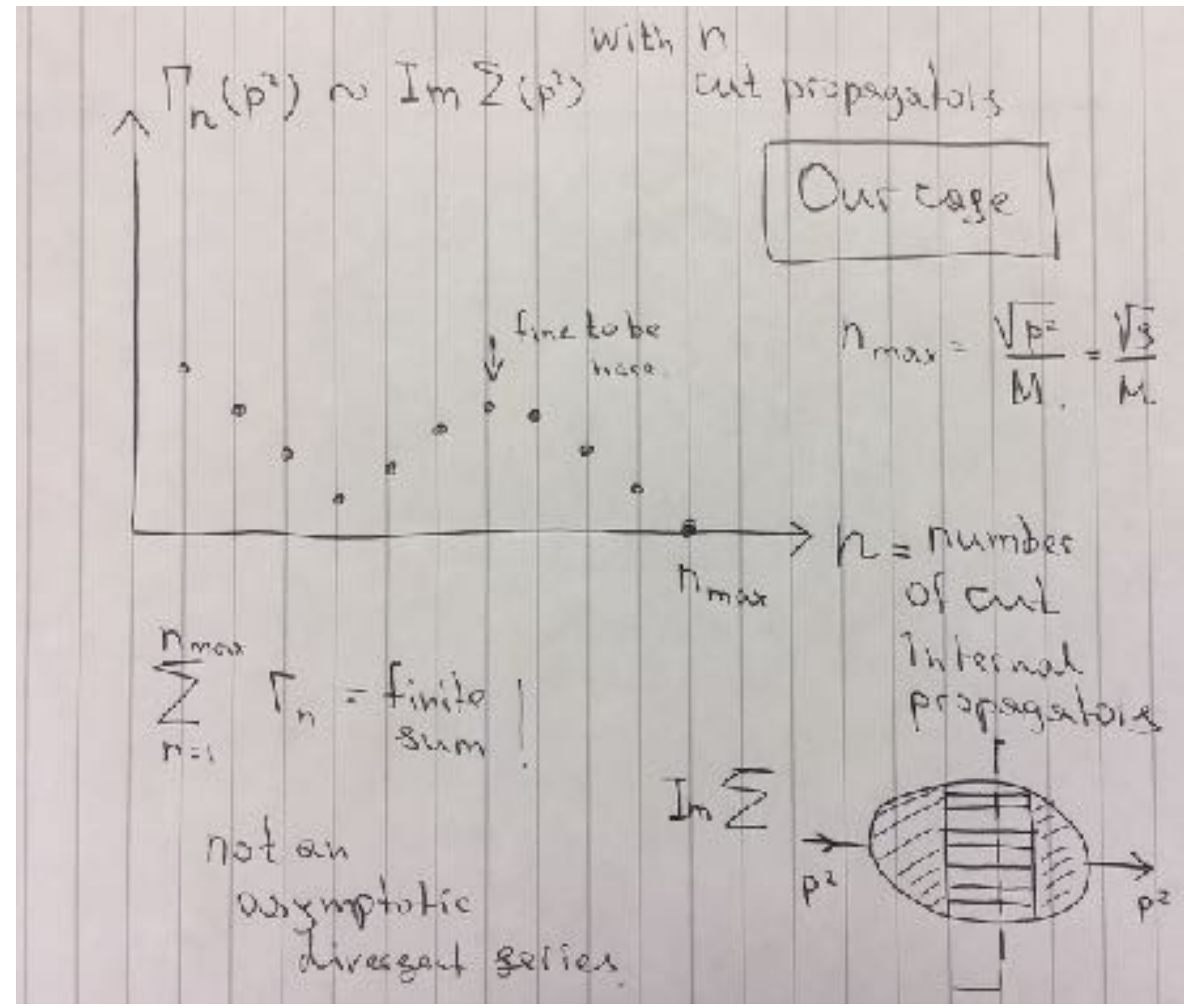
Son 1994;
Libanov, Rubakov, Troitskii 1997; more recently: Khoze 1411.2925

Problems?

- This looks like a perturbation theory breakdown
- Growing Amplitude seems to violate unitarity

Contrast asymptotic growth of higher-order corrections in perturbation theory with the ~n! contributions to Gamma_n(s)





Not the same types of beasts

It is the decay width Gamma_n(s) which is the central object of interest and the driving force of Higgsplosion.

Semi-classical approach for computing the rate R(1->n,E)

- DT Son1995

Multi-particle decay rates Γ_n can also be computed using an alternative semi-classical method. This is an intrinsically non-perturbative approach, with no reference in its outset made to perturbation theory.

The path integral is computed in the steepest descent method, controlled by two large parameters, $1/\lambda \to \infty$ and $n \to \infty$.

$$\lambda \to 0\,, \quad n \to \infty\,, \quad \text{with} \quad \lambda n = \text{fixed}\,, \quad \varepsilon = \text{fixed}\,.$$

The semi-classical computation in the regime where,

$$\lambda n = \text{fixed} \ll 1\,, \quad \varepsilon = \text{fixed} \ll 1\,,$$

reproduces the tree-level perturbative results for non-relativistic final states.

Remarkably, this semi-classical calculation also reproduces the leading-order quantum corrections arising from resumming one-loop effects.

Semi-classical approach for computing the rate R(1->n,E)

$$\Gamma_n(s) \propto \mathcal{R}(\lambda; n, \varepsilon)$$

The semiclassical approach is equally applicable and more relevant to the realisation of the non-perturbative Higgsplosion case where,

$$\lambda n = \text{fixed} \gg 1\,, \quad \varepsilon = \text{fixed} \ll 1\,.$$

This calculation was carried out for the spontaneously broken theory with the result given by,

- Khoze 1705.04365

$$\mathcal{R}_n(\lambda; n, \varepsilon) = \exp\left[\frac{\lambda n}{\lambda}\left(\log\frac{\lambda n}{4} + 0.85\sqrt{\lambda n} + \frac{1}{2} + \frac{3}{2}\log\frac{\varepsilon}{3\pi} - \frac{25}{12}\varepsilon\right)\right],$$

Higher order corrections are suppressed by $\mathcal{O}(1/\sqrt{\lambda n})$ and powers of ε.

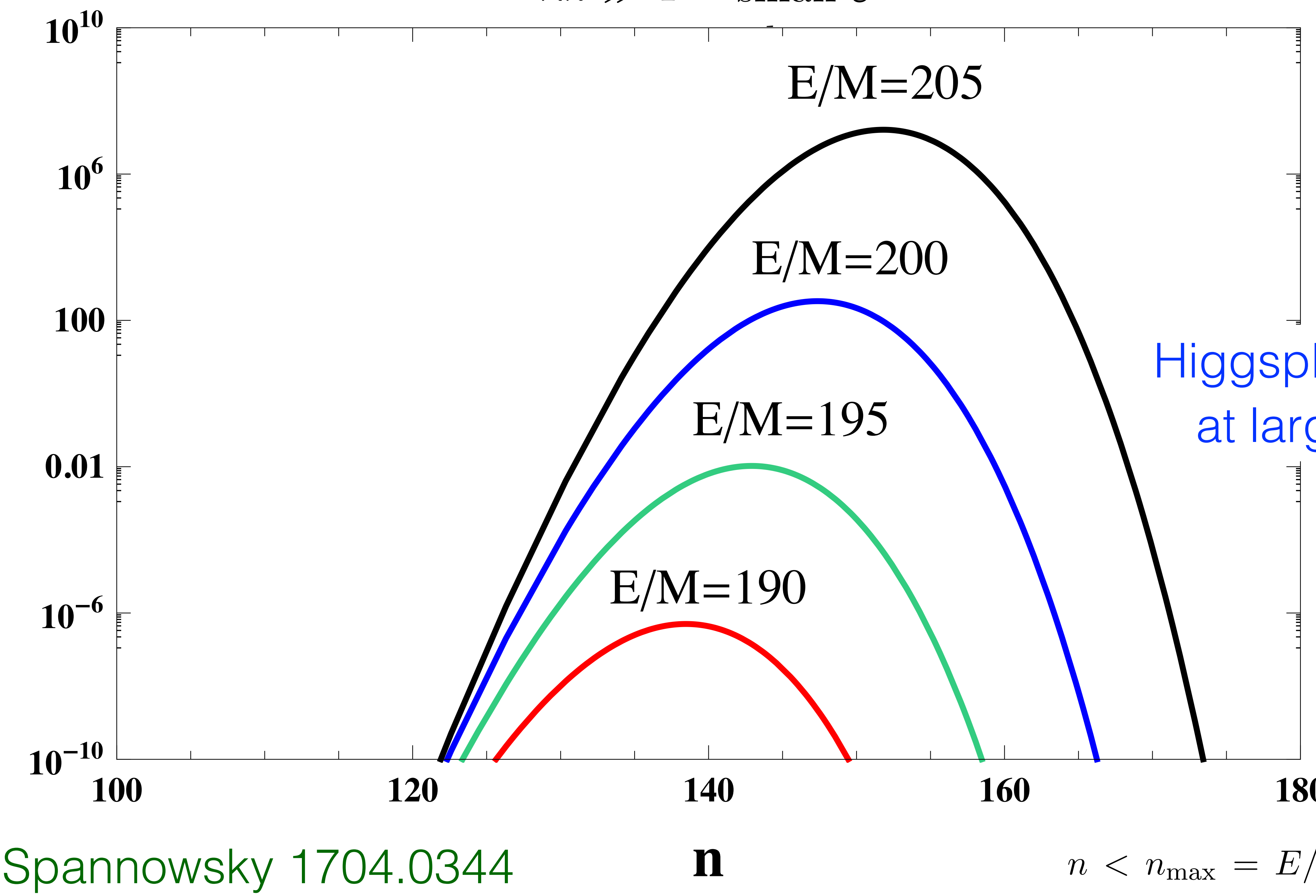
Thus we have computed the rate R in the large lambda n limit:

using the semi-classical approach and the thin-wall approximation

Khoze 1705.04365

$$\mathcal{R} = \exp\left[\frac{\lambda n}{\lambda}\left(\log\frac{\lambda n}{4} + 3.02\sqrt{\frac{\lambda n}{4\pi}} - 1 + \frac{3}{2}\left(\log\frac{\varepsilon}{3\pi}+1\right) - \frac{25}{12}\varepsilon\right)\right]$$

$\lambda n \gg 1$ small ε



Higgsplosion realised at large lambda n

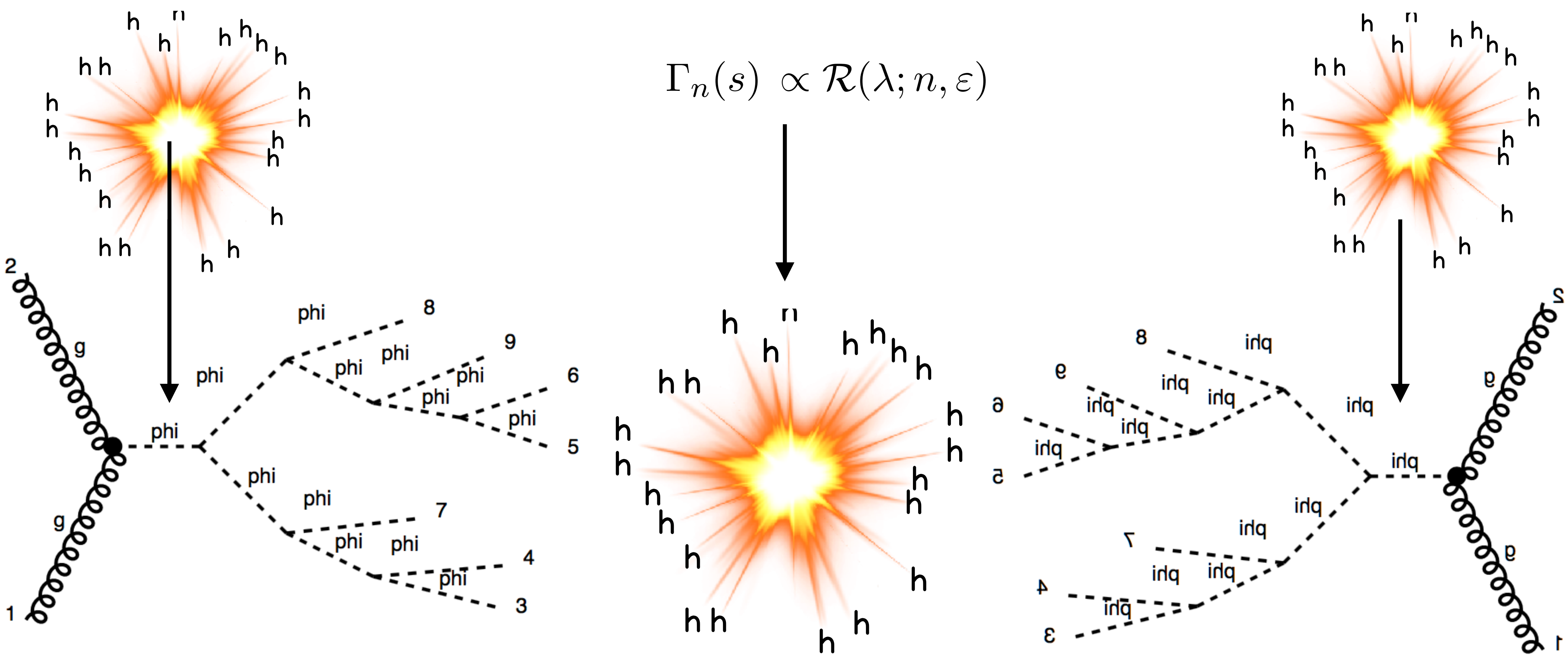
Khoze & Spannowsky 1704.0344

$n < n_{\max} = E/M_h$

HIGGSPLOSION and HIGGSPERSION

$$\mathcal{M}_{gg\to h^*} \times \frac{i}{p^2 - m_h^2 - Re\tilde{\Sigma}(p^2) + i m_h \Gamma(p^2)} \times \mathcal{M}_{h^* \to n\times h}$$

Include self-energy



$$\sigma^{\Delta}_{gg\to n\times h} \sim y_t^2 \frac{m_t^2}{m_h} \log^4\left(\frac{m_t}{\sqrt{s}}\right) \times \frac{1}{(s - Re\Sigma(s))^2 + m_h^2 \Gamma^2(s)} \times \Gamma_n(s)$$

Khoze & Spannowsky 1704.0344

- Unitarity restored!

Higgsplosion

At energy scales above E_* the dynamics of the system is changed:

1. Distance scales below $|x| \lesssim 1/E_*$ cannot be resolved in interactions;
2. UV divergences are regulated;
3. The theory becomes asymptotically safe;
4. And the Hierarchy problem of the Standard Model is therefore absent.

Consider the scaling behaviour of the propagator of a massive scalar particle

$$\Delta(x) \;:=\; \langle 0|T(\phi(x)\,\phi(0))|0\rangle \;\sim\; \begin{cases} m^2\, e^{-m|x|} & : \;\text{for } |x| \gg 1/m \\ 1/|x|^2 & : \;\text{for } 1/E_* \ll |x| \ll 1/m \\ E_*^2 & : \;\text{for } |x| \lesssim 1/E_* \end{cases} ,$$

where for $|x| \lesssim 1/E_*$ one enters the Higgsplosion regime.

This is a non-perturbative criterium. Can in principle be computed on a lattice.

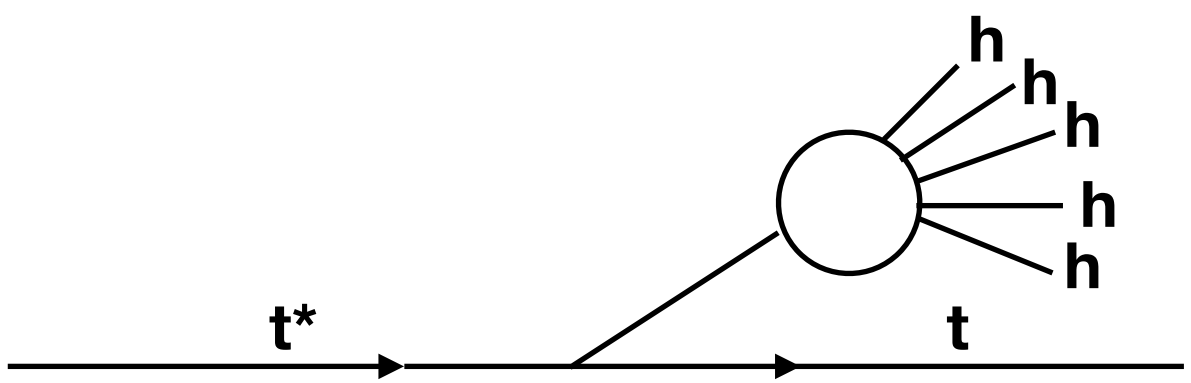
Higgsplosion

Loop integrals are effectively cut off at E_* by the exploding width $\Gamma(p^2)$ of the propagating state into the high-multiplicity final states.

The incoming highly energetic state decays rapidly into the multi-particle state made out of soft quanta with momenta $k_i^2 \sim m^2 \lll E_*^2$.

The width of the propagating degree of freedom becomes much greater than its mass: it is no longer a simple particle state.

In this sense, it has become a composite state made out of the n soft particle quanta of the same field ϕ.

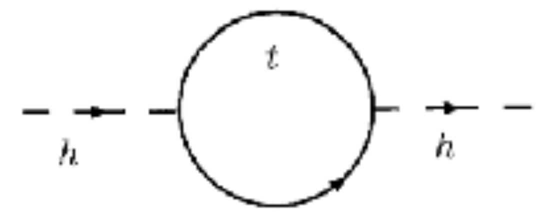


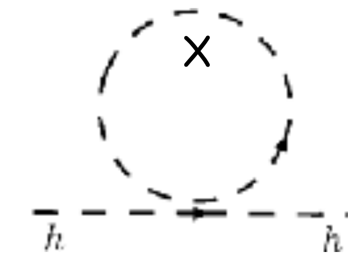
- Khoze & Michael Spannowsky 1704.03447, 1707.01531

Higgsplosion & the Hierarchy problem

X=heavy state

$$\mathcal{L}_X = \frac{1}{2}\partial^\mu X\,\partial_\mu X - \frac{1}{2}M_X^2\,X^2 - \frac{\lambda_P}{4}\,X^2h^2 - \mu X h^2$$





$$\Delta M_h^2 \sim \lambda_P \int \frac{d^4p}{16\pi^4}\,\frac{1}{p^2 + M_X^2 + \Sigma_X(p^2)} \propto \lambda_P\,\frac{E_\star^2}{M_X^2}\,E_\star^2 \quad \ll \lambda_P M_X^2\,.$$

Due to Higgsplosion the multi-particle contribution to the width of X explode at $p^2 = s_\star$ where $\sqrt{s_\star} \simeq \mathcal{O}(25)\mathrm{TeV}$

→ It provides a sharp UV cut-off in the integral, possibly at $s_\star \ll M_X^2$

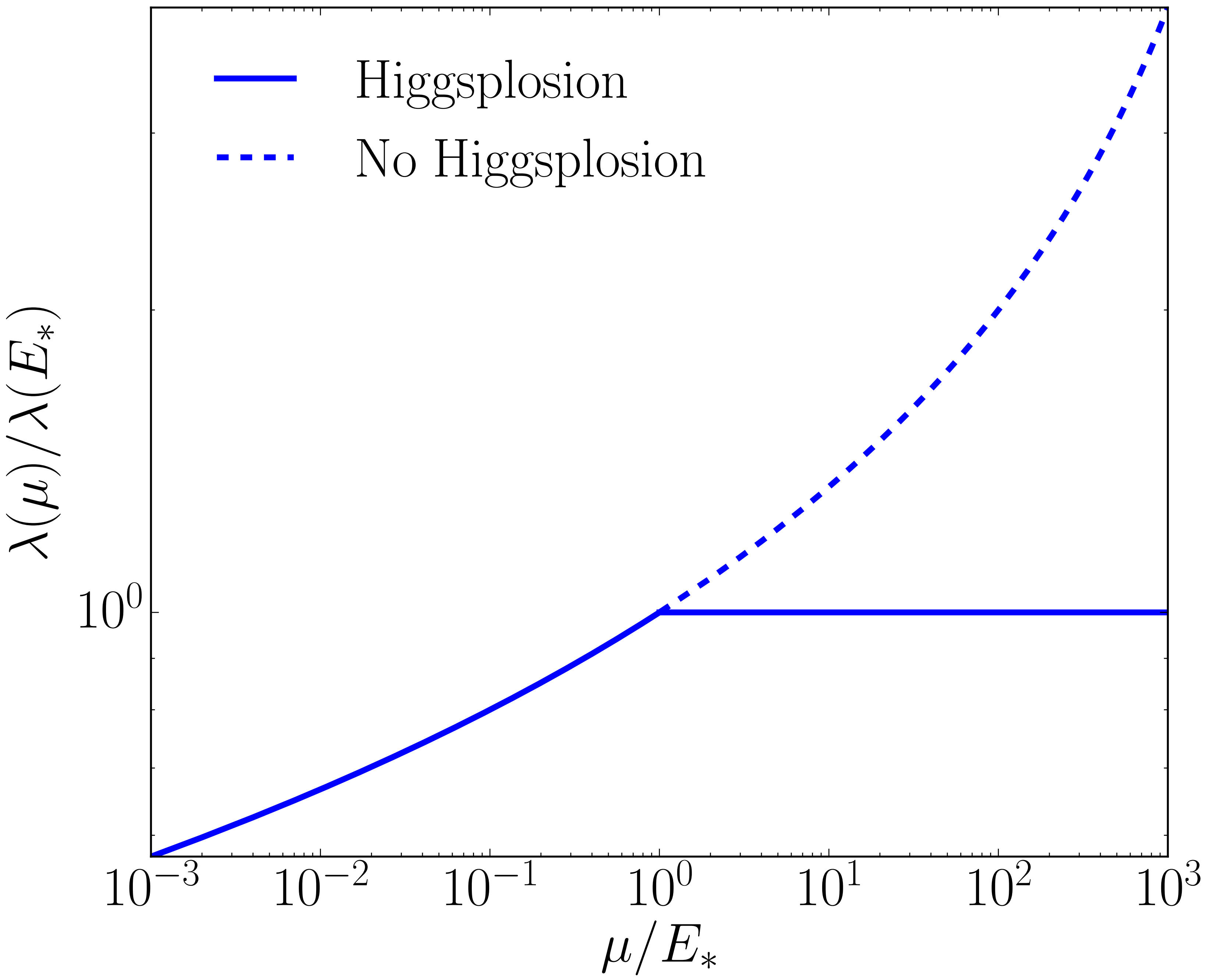
Hence, the contribution to the Higgs mass amounts to

$$\text{For} \quad \Gamma(s_\star) \simeq M_X \quad \text{at} \quad s_\star \ll M_X^2 \quad \Longrightarrow \quad \Delta M_h^2 \propto \lambda_P\,\frac{s_\star}{M_X^2}\,s_\star \ll \lambda_P\,M_X^2$$

and thus mends the Hierarchy problem by $\left(\frac{\sqrt{s_\star}}{M_X}\right)^4 \simeq \left(\frac{25\,\mathrm{TeV}}{M_X}\right)^4$

Asymptotic Safety

For all parameters of the theory (running coupling constants, masses, etc):

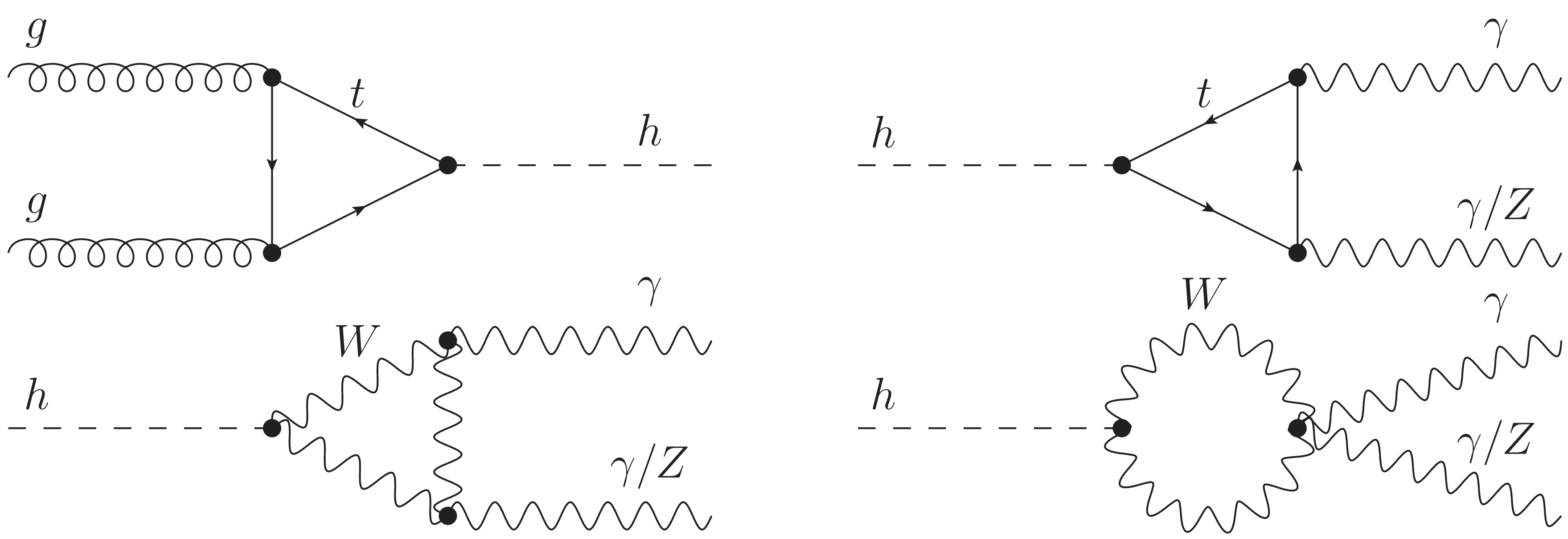


UV fixed point

Effects of Higgsplosion on Precision Observables

- Khoze, J Reiness, M Spannowsky, P Waite 1709.08655

Here focus on a class of observables which have no tree-level contributions



At LHC energies effects of Higgsplosion are small (next slide).

However O(1) effects can be achieved for these loop-induced processes if the interactions are probed close to ~ 2E*.

Scalar DM: I

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2}\partial_\mu X \partial^\mu X - \frac{1}{2} m_{X,0}^2 X^2 - \frac{\lambda_{\rm X}}{4!} X^4 - \frac{\lambda_{\rm HX}}{2} X^2 \left(H^\dagger H\right)$$

Typically, the mass is uncertain: it's a scalar
Enter Higgsplosion:

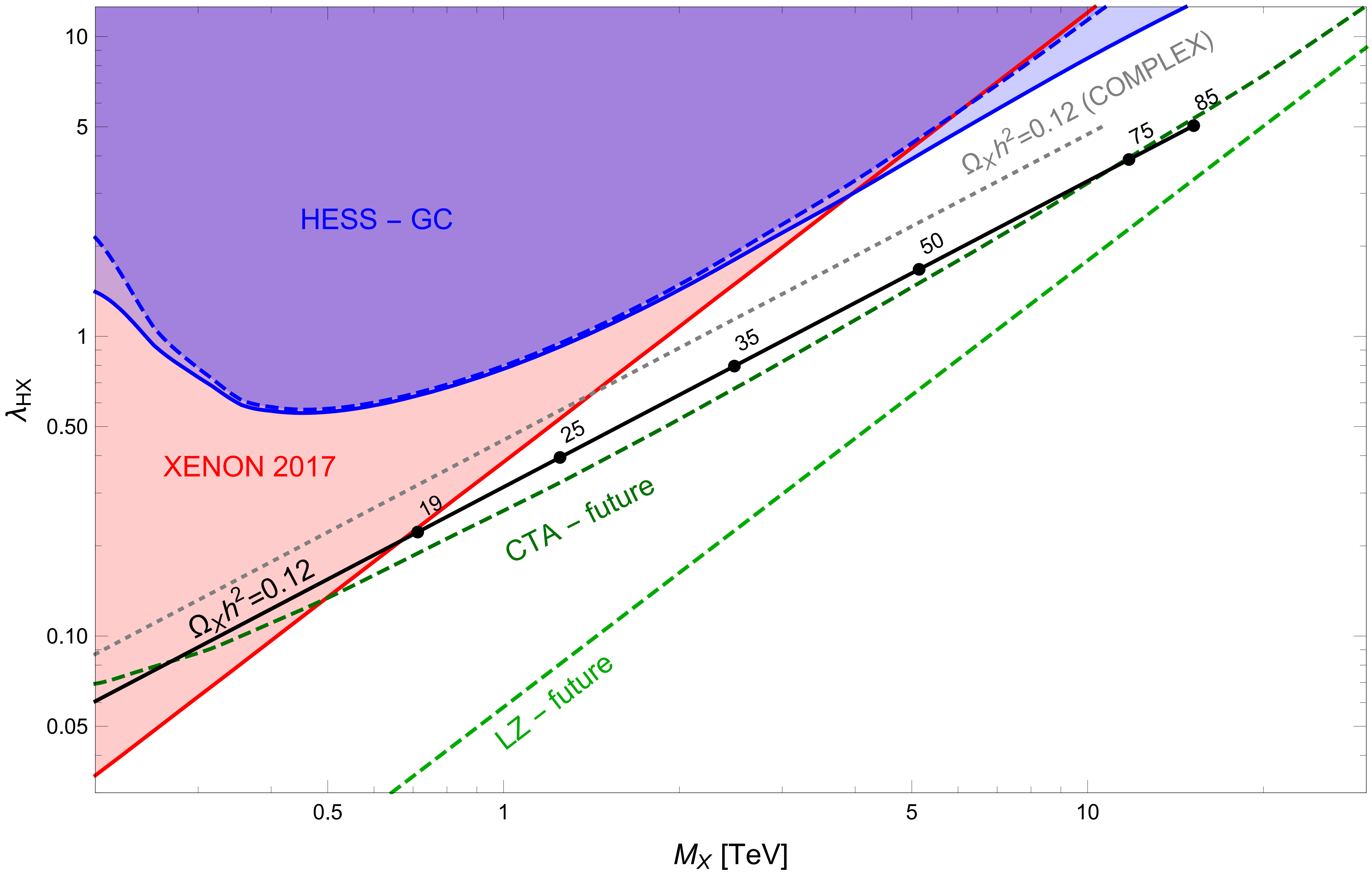
$$m_{\rm X}^2 \approx \lambda_{\rm HX} \frac{E_{\rm H}^2}{16\pi^2}$$

These are also related by the freeze-out condition:

$$\langle \sigma v \rangle \approx \frac{\lambda_{HX}^2}{16\pi m_{\rm X}^2}$$

Khoze, Reiness, Scholtz, Spannowsky: 1803.05441

Scalar DM II


HESS – GC
XENON 2017
$\Omega_X h^2$=0.12 (COMPLEX)
$\Omega_X h^2$=0.12
19
25
35
50
75
85
CTA – future
LZ – future
10
5
1
0.50
0.10
0.05
λ_{HX}
0.5
1
5
10
M_X [TeV]

Scalar DM: summary

- Higgsplosion provides a scale for DM mass.
- This scale is related to its coupling to the SM: once Higgsplosion scale is chosen, everything is fixed.
- In our case the currently favored lower value of E^* is pointing in a reasonable part of the parameter space.

Higgsplosion Future

- Collider production: can we see this at 100TeV?
- Further exploration of the semiclassical method
- What can lattice say?
- Can we learn anything from Hamiltonian truncation methods?
- How about astro signals? There are sources with $E > 25$ TeV.

Higgsplosion Summary

- This n! amplitude growth does not seem to be a perturbation theory breakdown.
- Unitarity is not violated. (Higgspersion)
- Loops of all particles that couple to the Higgs are suppressed after E*. This offers a solution to the large hierarchy problem.
- A new dynamically generate scale can be useful for other physics.