Precision of the top mass and vacuum stability

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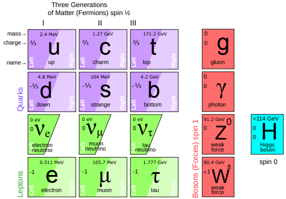
Lesson from LHC so far – Standard Model is good



- SM works in all laboratory/collider experiments (electroweak, strong)
- LHC 2012 – final piece of the model discovered – Higgs boson
  - Mass measured $\sim 125\,\mathrm{GeV}$ – weak coupling! Perturbative and predictive for high energies[1]

[1] An exciting twist here – Saturday talk by V.Khoze

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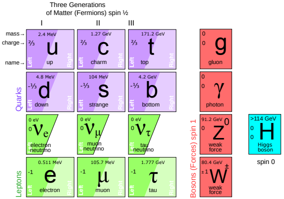

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Einstein gravity

- SM works in all laboratory/collider experiments (electroweak, strong)
- LHC 2012 – final piece of the model discovered – Higgs boson
  - Mass measured $\sim 125\,\text{GeV}$ – weak coupling! Perturbative and predictive for high energies[1]
- Add gravity
  - get cosmology
  - get Planck scale $M_P \sim 1.22 \times 10^{19}\,\text{GeV}$ as the highest energy to worry about

[1] An exciting twist here – Saturday talk by V.Khoze

Many things in cosmology are not explained by SM

Experimental observations

- Dark Matter
- Baryon asymmetry of the Universe
- Inflation (nearly scale invariant spectrum of initial density perturbations)

Laboratory also asks for SM extensions

- Neutrino oscillations

Nothing really points to a definite scale above EW

- Neutrino masses and oscillations (absent in SM)
  - Right handed neutrino between 1 eV and 10^{15} GeV
- Dark Matter (absent in SM)
  - Models exist from 10^{-5} eV (axions) up to 10^{20} GeV (Wimpzillas, Q-balls)
- Baryogenesys (absent in SM)
  - Leptogenesys scenarios exist from $M \sim 10$ MeV up to 10^{15} GeV

Important disclaimer

This can be easily changed by experiment, if we are lucky

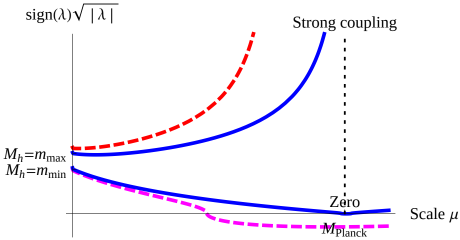
What happens at the scales between Electroweak 200 GeV and Planck 10^{19} GeV?

- Is SM consistent everywhere there?
- Does any problems appear?
- If yes, does it point to any scale?

Standard Model self-consistency and Radiative Corrections

- Higgs self coupling constant λ changes with energy due to radiative corrections.

$$(4\pi)^2\beta_\lambda = 24\lambda^2 - 6y_t^4 + \frac{3}{8}(2g_2^4 + (g_2^2 + g_1^2)^2) + (-9g_2^2 - 3g_1^2 + 12y_t^2)\lambda$$

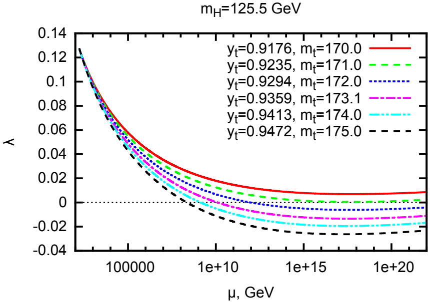


- Behaviour is determined by the masses of the Higgs boson $m_H = \sqrt{2\lambda}v$ and other heavy particles (top quark $m_t = y_t v/\sqrt{2}$)
- If Higgs is heavy $M_H > 170$ GeV – the model enters *strong coupling* at some low energy scale – new physics required.

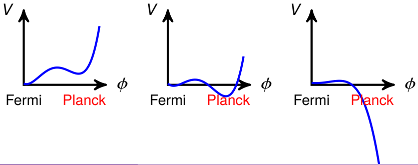
Lower Higgs masses: RG corrections push Higgs coupling to negative values

- For Higgs masses $M_H < M_{\text{critical}}$ coupling constant is negative above some scale μ_0.
- The Higgs potential may become negative!
  - Our world is not in the lowest energy state!
  - Problems at some scale $\mu_0 > 10^8$ GeV?

Coupling λ evolution:

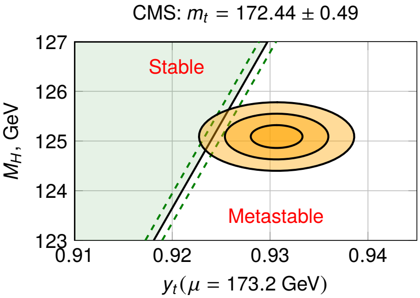


Higgs potential $V(\phi) \simeq \lambda(\phi)\frac{\phi^4}{4}$

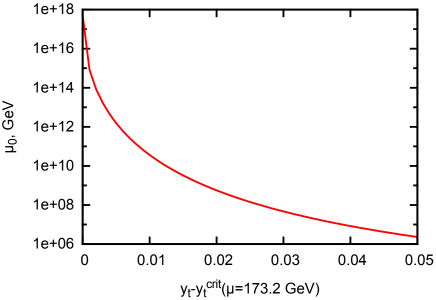


Experiment: SM probably has metastable SM vacuum

Experimental values for y_t



Scale μ_0 for $\lambda(\mu_0) = 0$



We live close to the metastability boundary!

Precision goal for y_t – better than 0.5%

FB, Kalmykov, Kniehl, Shaposhnikov’12; Buttazo et.al.’13, FB, Shaposhnikov’14, Bednyakov et.al.’15

top quark mass

PDG'17 average

$$m_t = 173.1 \pm 0.6 \text{ GeV}$$

ATLAS

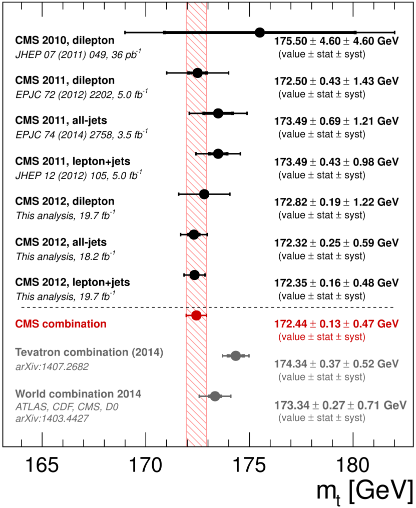
172.84 ± 0.34 ± 0.61 GeV

CMS

172.44 ± 0.13 ± 0.47 GeV

Tevatron

174.30 ± 0.35 ± 0.54 GeV



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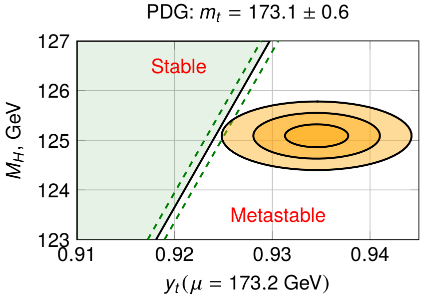

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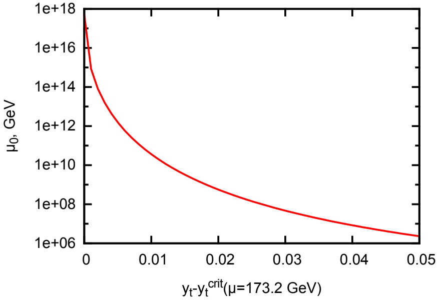

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Note about precisions

- Experiment – m_t^{MC}
  $\Downarrow$ Error $\delta m/m \sim 0.5\%$
- Pole mass m_t
  $\Downarrow$ $O(\alpha^2), O(\alpha\alpha_S), O(\alpha_S^4)$ Error $\delta m/m \sim 0.2\%$
- $\overline{MS}$ Yukawa coupling $y_t(\mu = M_t)$
  $\Downarrow$ 3-loop RG z Error $\delta y/y \sim 0.02\%$
- stability constraint

Careful study of different parton shower generators, and further study of NLO generators is required

Or/and

Build a e^+e^- collider at 350 GeV

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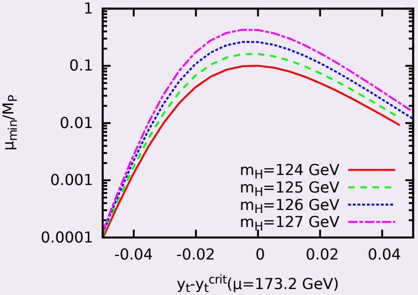
eg. Frixione, Mitov'14, Corcella'17

One more coincidence

Without reason and explanation

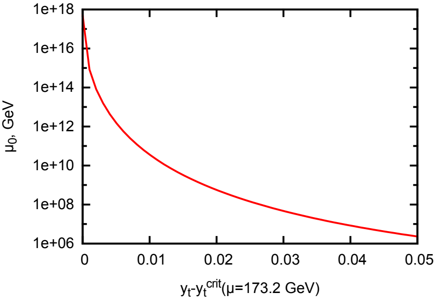
μ scale of *minimum* λ is close to Planck



Are there problems with metastable SM vacuum?

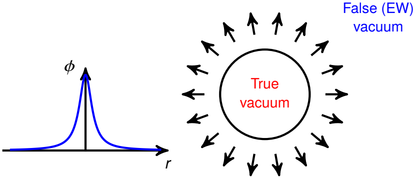
- Stable Electroweak vacuum – quite safe
  - Some low scale/hidden sector BSM physics is enough (e.g. νMSM)
- Metastable –
  - Problems at μ_0? (somewhere above 10^8 GeV)

Scale μ_0 for $\lambda(\mu_0) = 0$



What to do if we are metastable?

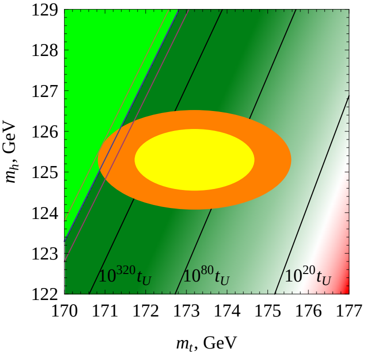
Vacuum decays by creating bubbles of true vacuum, which then expand very fast ($v \to c$)



Tunneling suppression:

$$p_{\text{decay}} \propto e^{-S_{\text{bounce}}} \sim e^{-\frac{8\pi^8}{3|\lambda(h)|}}$$

Lifetime $\gg$ age of the Universe!



Note on Planck corrections

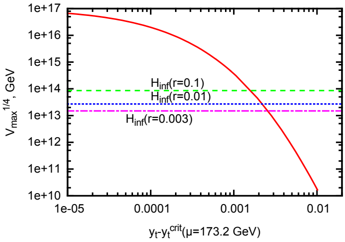
- Critical bubble size $\sim$ Planck scale
- Potential corrections $V_{\text{Planck}} = \pm \frac{\phi^n}{M_P^{n-4}}$ change lifetime!
  - Only + sign is allowed for Planck scale corrections!

As far as we are "safe" now (i.e. at low energies), what about Early Universe? What happens with the Higgs boson at inflation?

- if Higgs boson is completely separate from inflation
- if Higgs boson interacts with inflaton/gravitation background
- if Higgs boson drives inflation

Metastable vacuum during inflation *is* dangerous

- Let us suppose Higgs is not at all connected to inflationary physics (e.g. R^2 inflation)
- All fileds have vacuum fluctuation
- Typical momentum $k \sim H_{\rm inf}$ is of the order of Hubble scale



- If typical momentum is greater than the potential barrier – SM vacuum would decay if

$$H_{\rm inf} > V_{\rm max}^{1/4}$$

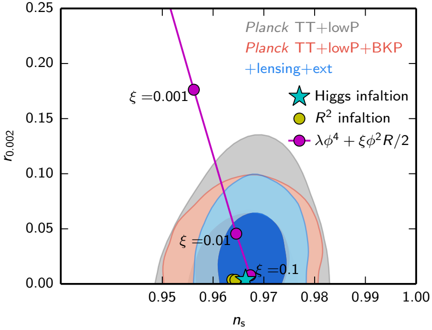
Most probably, fluctuations at inflation lead to SM vacuum decay. . .

- Observation of any tensor-to-scalar ratio r by CMB polarization missions would mean great danger for metastable SM vacuum!

Measurement of primordial tensor modes determines scale of inflation

$$H_{\text{inf}} = \sqrt{\frac{V_{\text{infl}}}{3M_P^2}} \sim 8.6 \times 10^{13}\,\text{GeV} \left(\frac{r}{0.1}\right)^{1/2}$$

Task for cosmology observations!



Does inflation contradict metastable EW vacuum?

- Higgs interacting with inflation can cure the problem. Examples
  - Higgs (ϕ)–inflaton (χ) interaction may stabilize the Higgs
    $$L_{\text{int}} = -\alpha\phi^2\chi^2$$
  - Higgs-gravity *negative* non-minimal coupling stabilizes Higgs in de-Sitter (inflating) space
    $$L_{\text{nm}} = \xi\phi^2 R$$
    (However, destabilises EW vacuum after inflation)
- New physics *below* μ_0 may remove Planck scale vacuum and make EW vacuum stable – many examples

New physics *above* μ_0 may solve the problem

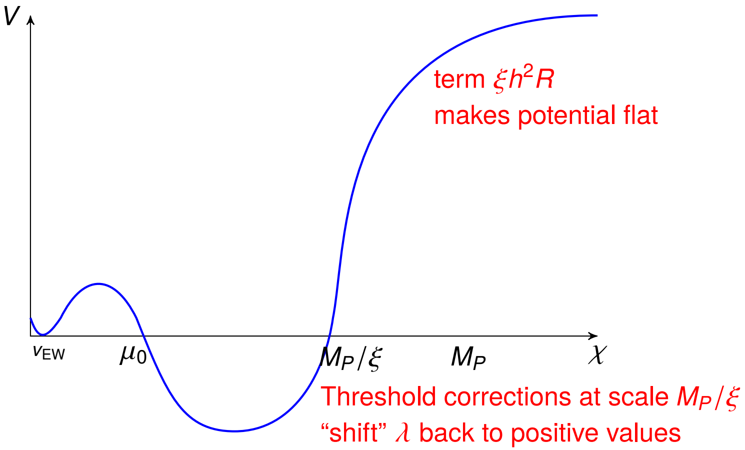
Requirements

- Minimum at Planck scale should be removed (but can remain near $\mu_0 \sim 10^{10}$ GeV)
- Reheating after inflation should be fast.

No need for new physics at "low" ($< \mu_0$) scales!
Example: Higgs inflation with threshold corrections at M_p/ξ

Higgs inflation and radiative corrections

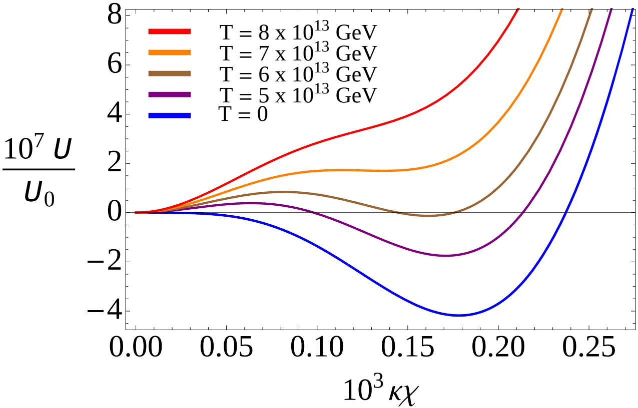
$$S_J = \int d^4x \sqrt{-g} \left\{ -\frac{M_P^2}{2} R - \xi \frac{h^2}{2} R + g_{\mu\nu} \frac{\partial^\mu h \partial^\nu h}{2} - \frac{\lambda}{4}(h^2 - v^2)^2 \right\}$$



(Not really to scale)

FB, Shaposhnikov'14

After inflation symmetry is restored in preheating

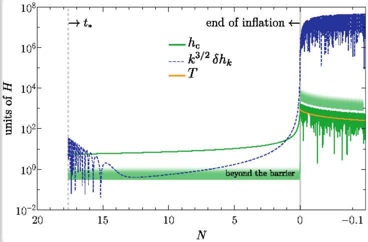


- Thermal potential removes the high scale vacuum
- Universe cools down to EW vacuum

Slightly metastable vacuum can be even useful

SM is metastable, Higgs not coupled to inflaton

- During inflation Higgs fluctuation can go slightly beyond the barrier
- After inflation thermal Higgs potential returns the field to the EW vacuum
- In some regions large fluctuations (instability) creates Primordial Black holes!



Espinosa, Racco, Riotto'17

Conclusions: Higgs potential stability

what is good and what is bad?

Bad

We do not know what is the instability scale (if at all unstable)

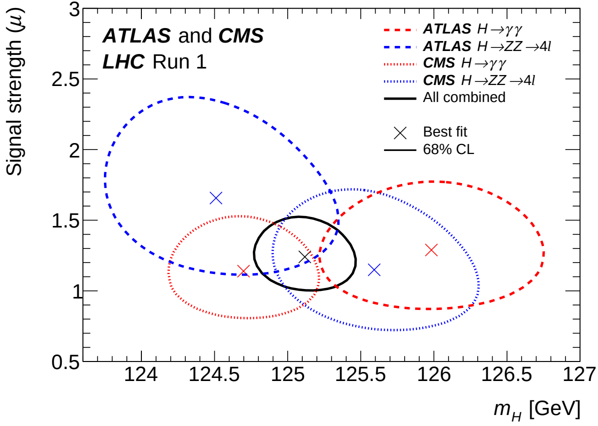
Good

Would be interesting to learn!

Backup

Higgs mass

PDG'17, PRL 114(2015)191803

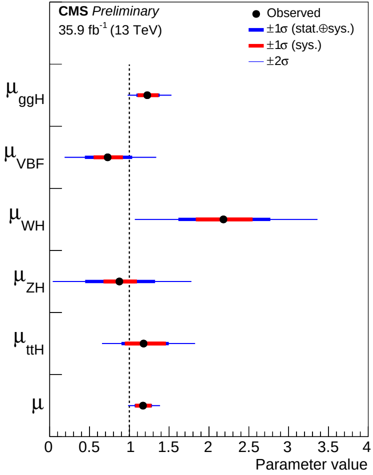


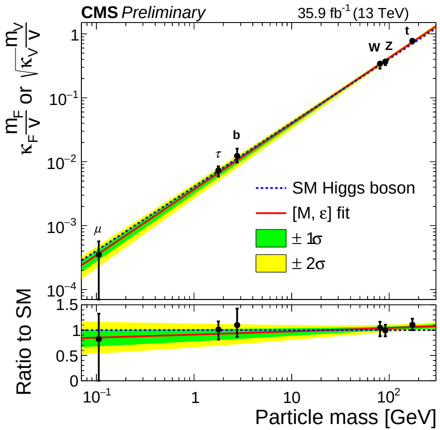
Combined CMS and ATLAS, at 7 and 8 TeV

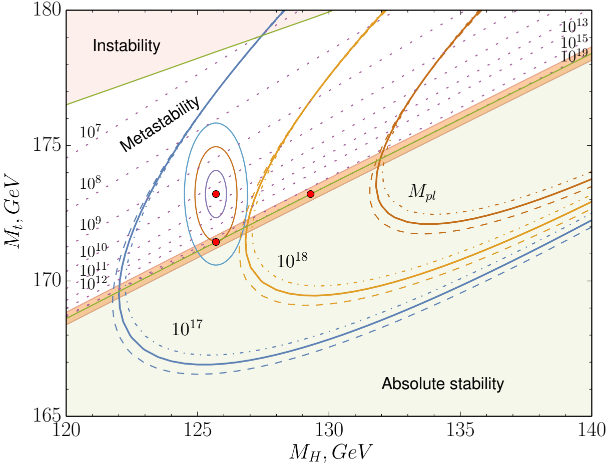
$$m_H = 125.09 \pm 0.21 \pm 0.11 \text{ GeV}$$

Combined Higgs boson couplings

CMS'18



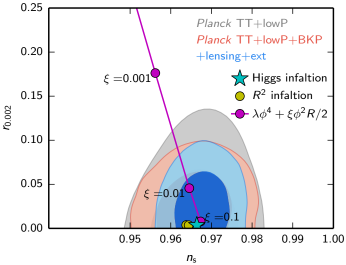




Bednyakov, Kniel, Pikelner, Veretin'15

Stable EW vacuum – mostly anything works

- No problems throughout the whole thermal evolution of the Universe.
- Adding inflation – many examples
  - R^2 inflation
  - Separate scalar inflaton interacting with the Higgs boson
  - non-minimally coupled Higgs inflation



Higgs inflation at tree level

Scalar part of the (Jordan frame) action

$$S_J = \int d^4x\sqrt{-g}\left\{-\frac{M_P^2}{2}R - \xi\frac{h^2}{2}R + g_{\mu\nu}\frac{\partial^\mu h\partial^\nu h}{2} - \frac{\lambda}{4}(h^2 - v^2)^2\right\}$$

- h is the Higgs field; $M_P \equiv \frac{1}{\sqrt{8\pi G_N}} = 2.4\times 10^{18}\mathrm{GeV}$
- SM higgs vev $v \ll M_P/\sqrt{\xi}$ – can be neglected in the early Universe
- At $h \gg M_P/\sqrt{\xi}$ *all* masses are proportional to h – scale invariant spectrum!

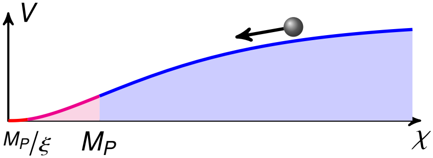
Higgs inflation at tree level

Scalar part of the (Jordan frame) action

$$S_J = \int d^4x\sqrt{-g}\left\{-\frac{M_P^2}{2}R - \xi\frac{h^2}{2}R + g_{\mu\nu}\frac{\partial^\mu h\partial^\nu h}{2} - \frac{\lambda}{4}(h^2-v^2)^2\right\}$$

To get observed $\delta T/T \sim 10^{-5}$

$$\frac{\sqrt{\lambda}}{\xi} = \frac{1}{49000}$$

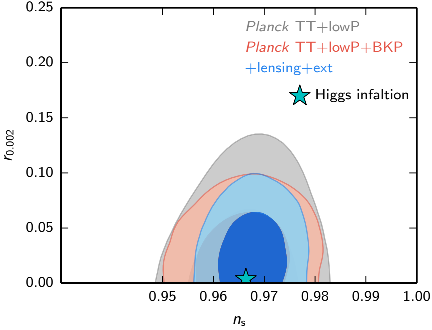


Mathematical trick – conformal transformation

$$g_{\mu\nu} \to \hat{g}_{\mu\nu} = \sqrt{1+\frac{\xi\phi^2}{M_P^2}}\, g_{\mu\nu},$$

leads to flattened potential: $V(\phi) \to \hat{V}(\chi) = \frac{\lambda M_P^4}{4\xi^2}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)^2$

CMB parameters are predicted



For large ξ Higgs inflation

spectral index $\quad n \simeq 1 - \frac{8(4N+9)}{(4N+3)^2} \simeq 0.97$

tensor/scalar ratio $\quad r \simeq \frac{192}{(4N+3)^2} \simeq 0.0033$

$$\delta T/T \sim 10^{-5} \quad \Longrightarrow \quad \frac{\xi}{\sqrt{\lambda}} \simeq 47000$$

Note: for very near critical top quark/Higgs masses results change and allow for larger r

Adding required counterterms to the action

- In principle – HI is not renormalizable, all counterterms appear at some loop order
- Let us try to add only the *required* counterterms at each order in loop expansion

$$\mathcal{L} = \frac{(\partial\chi)^2}{2} - \frac{\lambda}{4}F^4(\chi) + i\bar{\psi}_t\partial\!\!\!/\psi_t + \frac{y_t}{\sqrt{2}}F(\chi)\bar{\psi}_t\psi_t$$

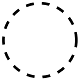
$$F(\chi) \equiv \frac{h(\chi)}{\Omega(\chi)} \approx \left\{ \begin{array}{ll} \chi & , \chi < \frac{M_P}{\xi} \\ \frac{M_P}{\sqrt{\xi}}\left(1 - e^{-\sqrt{2/3}\chi/M_P}\right)^{1/2} & , \chi > \frac{M_P}{\xi} \end{array} \right\}$$

Doing quantum calculations we should add

$$\mathcal{L} + \mathcal{L}_{\text{1-loop}} + \delta\mathcal{L}_{\text{1-loop c.t.}} + \cdots$$

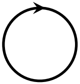
Counterterms: λ modification

Calculating vacuum energy



$$= \frac{1}{2}\,\mathrm{Tr}\ln\left[\Box - \left(\frac{\lambda}{4}(F^4)''\right)^2\right]$$

$$= \frac{9\lambda^2}{64\pi^2}\left(\frac{2}{\bar{\epsilon}} - \ln\frac{\lambda(F^4)''}{4\mu^2} + \frac{3}{2}\right)\left(F'^2 + \frac{1}{3}F''F\right)^2 F^4,$$



$$= -\,\mathrm{Tr}\ln\left[i\partial\!\!\!/ + y_t F\right]$$

$$= -\frac{y_t^4}{64\pi^2}\left(\frac{2}{\bar{\epsilon}} - \ln\frac{y_t^2F^2}{2\mu^2} + \frac{3}{2}\right)F^4$$

Counterterms: λ modification

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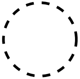
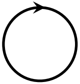


$$= \frac{1}{2}\,\mathrm{Tr}\ln\left[\Box - \left(\frac{\lambda}{4}(F^4)''\right)^2\right]$$

$$\delta\mathcal{L}_{ct} = \frac{9\lambda^2}{64\pi^2}\left(\frac{2}{\bar{\epsilon}} + \delta\lambda_{1a} \qquad\qquad\qquad \right)\left(F'^2 + \frac{1}{3}F''F\right)^2 F^4,$$



$$= -\,\mathrm{Tr}\ln\left[i\partial\!\!\!/ + y_t F\right]$$

$$\delta\mathcal{L}_{ct} = -\frac{y_t^4}{64\pi^2}\left(\frac{2}{\bar{\epsilon}} + \delta\lambda_{1b} \qquad\qquad \right)F^4$$

Small χ : $F'^4F^4 \sim \chi \sim F^4$

Large χ : $F'^4F^4 \sim e^{-4\chi/\sqrt{6}M_P}$, and $F^4 \sim M_P^4/\xi^2$

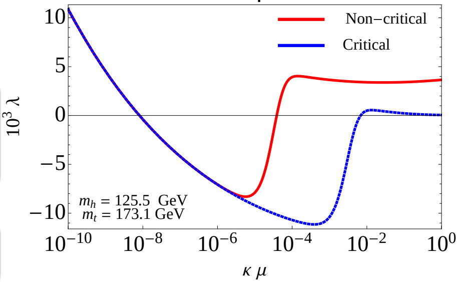
$\delta\lambda_{1b}$ – just λ redefinition, while $\delta\lambda_{1a}$ is not!

Modified λ evolution can make the potential positive again

$$\lambda(\mu) \to \lambda(\mu) + \delta\lambda\left[\left(F'^2 + \frac{1}{3}F''F\right)^2 - 1\right]$$

$$y_t(\mu) \to y_t(\mu) + \delta y_t\left[F'^2 - 1\right]$$

Effect on the potential



(Red curve: $\xi = 1500$, $\delta y_t = 0.025$, $\delta\lambda = -0.015$)