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Effective-Theory Approach to Unstable Particle Production

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Overview

- problem with unstable particles

  $\longrightarrow$ $t\bar{t}$ threshold scan
- systematic approach using effective theory techniques

  $\longrightarrow$ toy model
- outlook

Work done in collaboration with:

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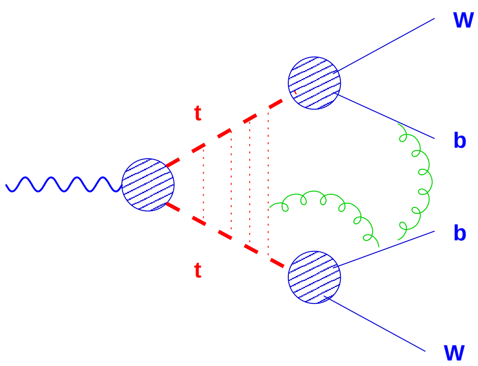
Problem

- Ordinary perturbation theory breaks down near resonance
$$\frac{g^2}{p^2 - M^2} \sim \frac{1}{M^2} \quad \text{if} \quad p^2 - M^2 \sim M\Gamma \sim g^2 M^2$$
- avoid pole: $\frac{1}{p^2 - M^2} \to \frac{1}{p^2 - M^2 + i\, M\Gamma}$. This includes part of higher order corrections (self-energy insertions)
- previous approaches include: pole scheme [Stuart; Aepply et.al.], pinch technique [Papavassiliou et.al.], fermion loop scheme [Argyres et.al.], complex mass scheme [Denner et.al.] ...
- how to systematicially compute order by order in perturbation theory in g^2 and Γ/M (i.e. go beyond pole approximation)
- note: gauge invariance will be automatic

Motivation

- Precision physics involves often unstable particles:
  $Z, W^{\pm}, t, H(?), \mathrm{Susy}(??)$
- At Linear Collider need to go beyond the current status e.g:
  $\Delta m_t \simeq 50(?) \to 25(??)$ MeV
  $\Delta M_W \simeq 6(??)$ MeV
- An interesting problem in QFT **!**



Goal: A systematic approach (order by order) in perturbation theory for processes involving resonant unstable particles.

$t\bar{t}$ Threshold Scan

- exploit $\alpha_s \ll 1$ and $v \ll 1$ $\rightarrow$ double expansion
- effective theory: identify modes (method of regions) hard, soft, potential, usoft [Beneke et.al.]
- QCD (h,s,p,u) $\longrightarrow$ NRQCD (s,p,u)$\longrightarrow$ PNRQCD (p$|_q$,u)
- done to NNLO, can resum $\log(v)$ using RG techniques, done to NLL and most of NNLL [Beneke et.al; Hoang et.al; Melnikov et.al; Yakovlev; Pineda; Manohar et.al; . . .]
- instability of top quarks taken into account only via $E \rightarrow E + i\Gamma$. Non-factorizable corrections are not included [Fadin, Khoze; Melnikov et.al; Beenakker et.al.]
- long-term plan: repeat the construction of the effective theory, taking into account the instability effects

Toy Model

- Lagrangian:

$$\mathcal{L} = (D_\mu \phi)^\dagger D^\mu \phi - \hat{M}^2 \phi^\dagger \phi + \overline{\psi} i \not{D} \psi + \overline{\chi} i \not{\partial} \chi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\xi} (\partial_\mu A^\mu)^2 + y \phi \overline{\psi} \chi + y^* \phi^\dagger \overline{\chi} \psi - \frac{\lambda}{4} \left(\phi^\dagger \phi\right)^2 - \mathcal{L}_{\rm ct}$$

- Process:

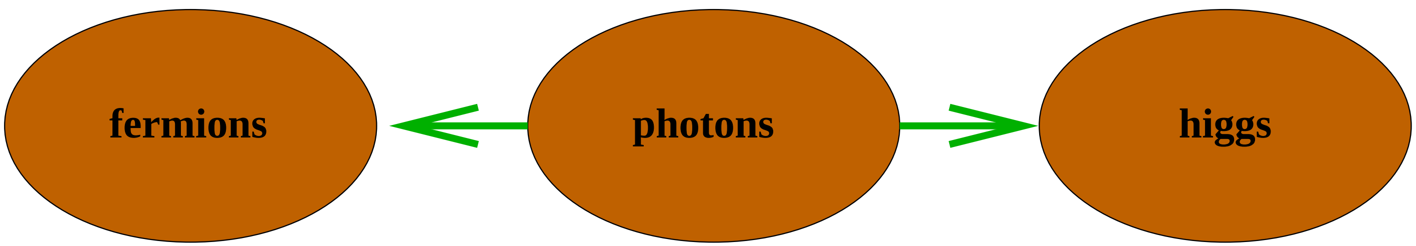
$$\bar{\nu}(p_1) e^-(p_2) \to \phi \to X$$

with $s - \hat{M}^2 \sim M\Gamma$. Use optical theorem and compute $\mathrm{Im}\,\mathcal{T}$

- scales: decay time $1/M$, lifetime $1/\Gamma \gg 1/M$
- expand in α and $\delta \equiv (s - \hat{M}^2)/\hat{M}^2 \sim \Gamma/M \sim \alpha$
- fermions: SCET; scalar (higgs): H"Q"ET

Modes

Soft-Collinear Effective Theory	+	Heavy “Quark” Effective Theory
fermions		higgs
$p^\mu = (n_+p)\frac{n_-}{2} + (n_-p)\frac{n_+}{2} + p_\perp$		$q^\mu = Mv^\mu + k^\mu; \quad q_\top = q^\mu - v^\mu(qv)$
$n_\pm^2 = 0, \quad n_+n_- = 2$		$v^\mu \equiv (q_1^\mu + q_2^\mu)/\sqrt{s}, \quad v^2 = 1$
hard: $p \sim M$		hard: $k^\mu \sim M$
(u)soft: $p \sim M\delta$		soft: $k^\mu \sim \delta$
collinear: $p_\perp \sim M\delta^{1/2}; \; n_+p \sim M; \; n_-p \sim M\delta$		



Effective Theory

underlying theory

$\mathcal{L}(\phi_h, \phi_c, \phi_s)$

dynamical modes:

hard, collinear, soft

integrate out hard modes

factorizable corrections

non-factorizable corrections

effective theory

$\mathcal{L} = \sum_n c_n(\mathrm{h})\, O_n(\phi_c, \phi_s)$

dynamical modes:

collinear, soft

The Effective Lagrangian

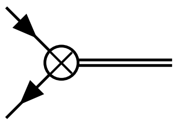
The effective Lagrangian for the NLO line shape:

$$\begin{aligned}
\mathcal{L}_{\rm eff} &= -\frac{1}{4} F_{\rm s}^{\mu\nu} F_{{\rm s}\mu\nu} + 2\hat{M}\,\phi_v^\dagger \left(i(vD_{\rm s}) - \frac{\Delta}{2}\right)\phi_v + 2\hat{M}\,\phi_v^\dagger \left(\frac{iD_{{\rm s}\top}^2}{2\hat{M}} + \frac{\Delta^2}{8\hat{M}}\right)\phi_v \\
&+ \bar{\psi}_{\rm s} i \not{D}_{\rm s} \psi_{\rm s} + \bar{\chi}_{\rm s} i \not{\partial} \chi_{\rm s} + \bar{\psi}_{n-} \left(i n_- D + \not{D}_{{\rm c}\top} \frac{i}{n_+ D_{\rm c}} \not{D}_{{\rm c}\top} \right) \psi_{n-} \\
&+ C \left(y\,\phi_v \bar{\psi}_{n-} \chi_{n+} + y^*\,\phi_v^\dagger \bar{\chi}_{n+} \psi_{n-} \right) + \frac{y y^* B}{4\hat{M}^2} \left(\bar{\psi}_{n-}\chi_{n+}\right)\left(\bar{\chi}_{n+}\psi_{n-}\right) + \ldots
\end{aligned}$$

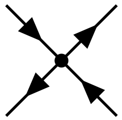
Matching coefficients (contain hard effects)

- $\Delta \equiv (\bar{s} - \hat{M}^2)/\hat{M} = \alpha\,\Delta^{(1)} + \alpha^2 \Delta^{(2)} + \ldots$
  In the pole scheme: $\Delta = -i\Gamma$

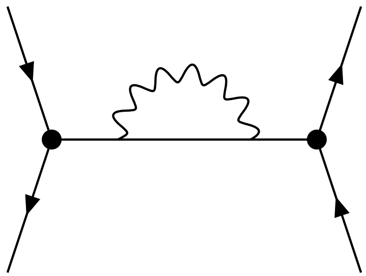
- $C = 1 + \alpha\, C^{(1)} + \ldots$



- $B = 1 + \alpha B^{(1)} + \ldots$



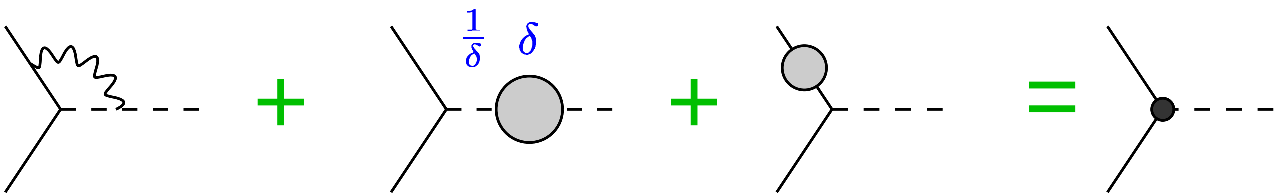
Matching

Consider self-energy diagrams  + higher orders

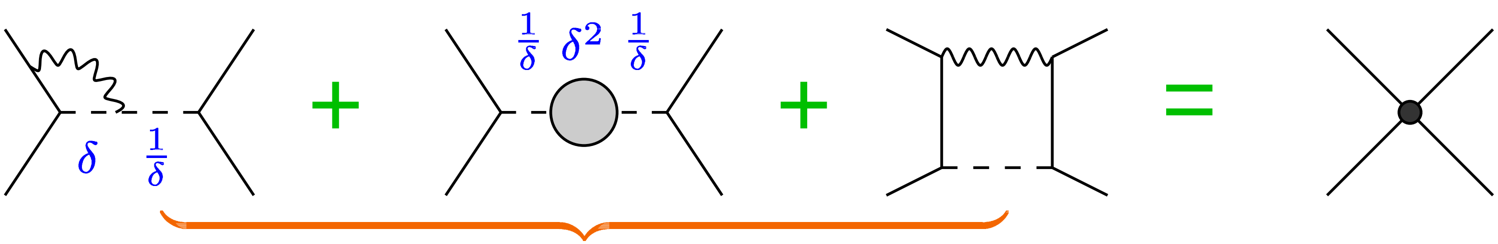
split self-energy into hard and soft part $\Pi(s) = \Pi_h(s) + \Pi_s(s)$ and expand the hard part of the self energy $\Pi_h(s) = \hat{M}^2 \sum \alpha^k \delta^l \Pi^{(k,l)}$

- $\Pi^{(1,0)}$ (gauge independent) $\rightarrow$ $\Delta^{(1)}$ (LO, Propagator)
- $\Pi^{(1,1)}$ (gauge dependent) $\rightarrow$ $C^{(1)}$ (NLO)
- $\Pi^{(1,2)}$ (gauge dependent) $\rightarrow$ $B^{(1)}$ (NNLO)
- $\Pi^{(2,0)}$ and $\Pi^{(1,0)}\Pi^{(1,1)}$ (separately gauge dependent) $\rightarrow$ $\Delta^{(2)}$ (NLO, gauge independent)
- Π_s (gauge dependent) $\rightarrow$ diagram in effective theory (NLO)

Matching of $C = 1 + \frac{\alpha_y}{4\pi}\left[\ln\frac{M^2}{\mu^2} - \frac{1}{4} - \frac{i\pi}{2}\right] +$
$\frac{\alpha_g}{4\pi}\left[-\frac{1}{\bar{\epsilon}^2} + \frac{1}{\bar{\epsilon}}\left(\ln\frac{M^2}{\mu^2} - \frac{5}{2}\right) - \frac{1}{2}\ln^2\frac{M^2}{\mu^2} + \frac{7}{4}\ln\frac{M^2}{\mu^2} - \frac{15}{4} - \frac{\pi^2}{12}\right]$ (in $\overline{\mathrm{MS}}$ scheme)



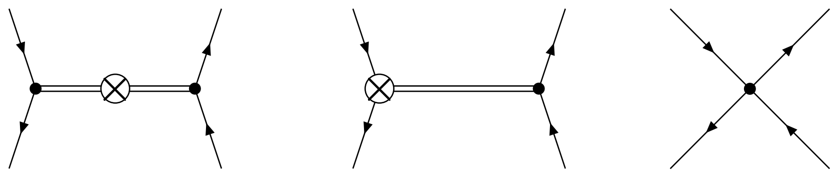
Matching of B at order α (contributes at NNLO)



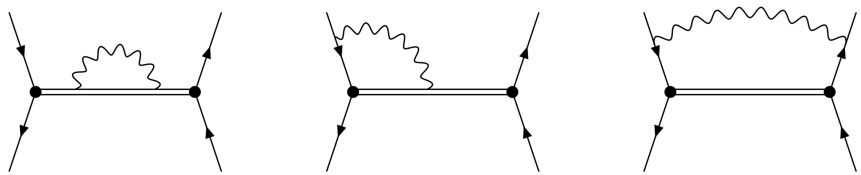
gauge dependence cancels

Results

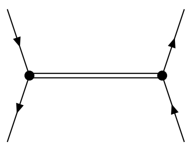
Forward scattering amplitude at NLO:



$$i\,\mathcal{T}_h^{(1)} = i\,\mathcal{T}^{(0)} \times \left(2\,C^{(1)} - \frac{[\Delta^{(1)}]^2}{8\mathcal{D}\hat{M}} + \frac{\Delta^{(2)}}{2\mathcal{D}} - \frac{\mathcal{D}}{2\hat{M}}\right)$$



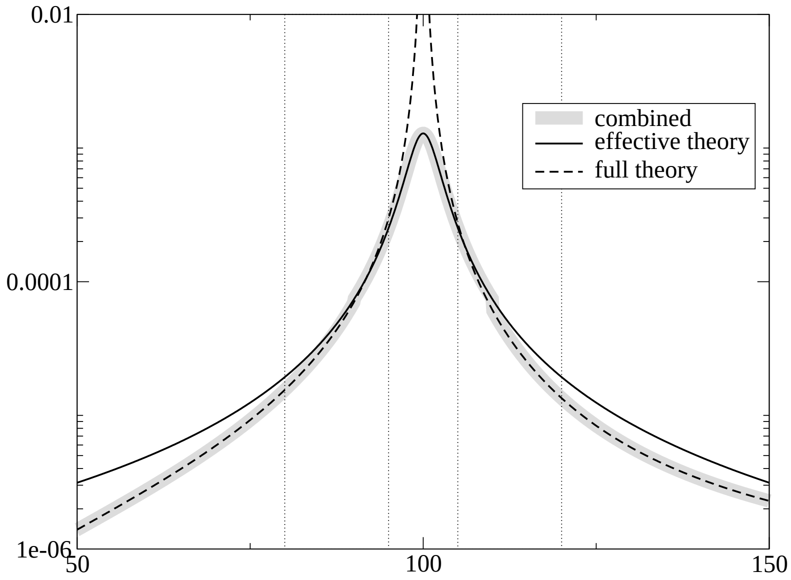
$$i\,\mathcal{T}_s^{(1)} = i\,\mathcal{T}^{(0)}\,\frac{\alpha_g}{4\pi}\left(\frac{-2\mathcal{D}}{\mu}\right)^{-2\epsilon} \times \left(\frac{2}{\epsilon^2} + \frac{2}{\epsilon} + 4 + \frac{5\pi^2}{6}\right)$$

where  $= i\,\mathcal{T}^{(0)} = \frac{-y y^* s}{4\hat{M}\mathcal{D}}$ with $\mathcal{D} \equiv \sqrt{s} - \hat{M} - \frac{\Delta^{(1)}}{2}$

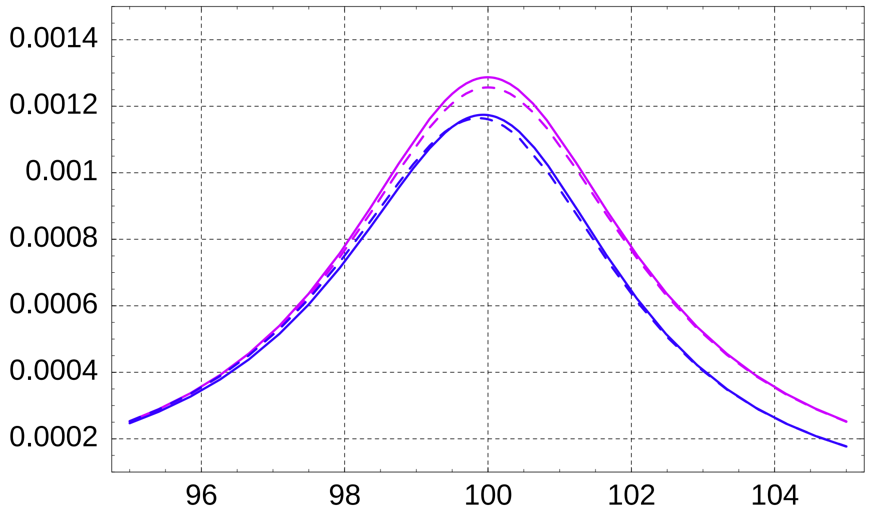
poles $1/\epsilon$ cancel when adding soft and hard contributions (up to initial state collinear singularity)

Results

Partonic cross section for $M = 100$ GeV as a function of $\sqrt{s}$.



full range of $\sqrt{s}$: matching of resonant to off-resonant cross section



resonant region: LO vs. NLO for pole and $\overline{\text{MS}}$ scheme

Outlook

- more realistic processes:
  - higgs $\to$ fermion (t) : H"Q"ET $\to$ HQET
  - higgs $\to$ gauge boson (W, Z) . With $p^\mu = M v^\mu + k^\mu$ we get $p^2 - \xi M^2 = (1-\xi) M^2 + 2M(vk) + k^2$ and the propagator:

$$\frac{i}{p^2 - M^2}\left(-g^{\mu\nu} + (1-\xi)\frac{p^\mu p^\nu}{p^2 - \xi M^2}\right) \to \frac{i}{2M(vk)}\left(-g^{\mu\nu} + v^\mu v^\nu\right)$$

  massive field, 3 polarizations, gauge invariant
- pair production near threshold $t\bar{t};\ W^+W^-$: HQET $\to$ NRQ(C/E)D. Due to potential gluons/photons $(vk) \sim k^2$ since $k^\mu_{\rm pot} \sim (M\nu^2, m\vec{\nu})$
- more exclusive final states: expand also phase-space integrals
- resummation of $\log(\Gamma/M)$ via standard RGE techniques

Summary

- systematic, perturbative, order-by-order approach; power counting in α and δ
- breaks calculations/corrections into well defi ned pieces (factorizable/non-factorizable split is only part of it)
- gauge invariance automatic
- identify and compute minimal amount needed for desired accuracy (kinematic simplifi cations)
- still: diffi cult and tedious calculations are required