QCD corrections to top quark decay at NNLO

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in collaboration with

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Outline

1. Introduction & motivation
2. Methods: optical theorem, asymptotic expansions, recurrence relations
3. Results
4. Summary & outlook

Motivation for studying top decay

- Properties of the top:
  - Very short lifetime:

$$\frac{1}{\tau_t} = 175\ \mathrm{MeV} \left(\frac{m_t}{m_W}\right)^3 \simeq 1.5\ \mathrm{GeV} \gg \Lambda_{QCD}$$

    much shorter than typical confinement scale:

    top behaves almost like a free quark !

  - t — bW dominant decay channel: $|V_{tb}| \simeq 1$

Can shed light on new physics but high accuracy SM predictions needed !

- What is known so far from theory side
  - Tree level: $\Gamma_0 \simeq 1.5\ \mathrm{GeV}$
  - NLO QCD corrections: $\simeq -8.4\,\%\ \Gamma_0$

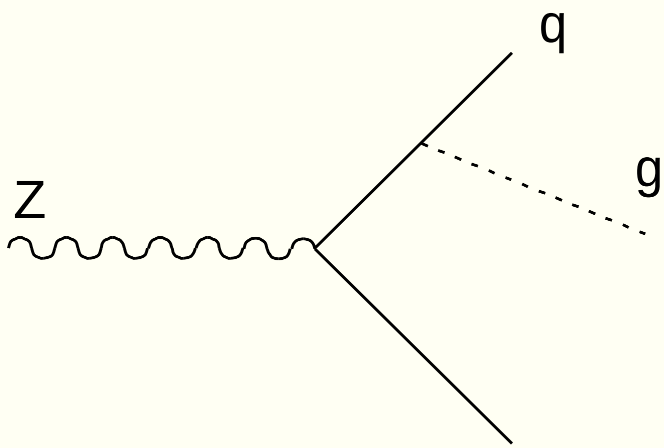
    (Jezabek, Kuhn 1989)
  - NLO electroweak: $< +2\,\%\ \Gamma_0$

    (Denner, Sack; Eilam *et al.* 1991)

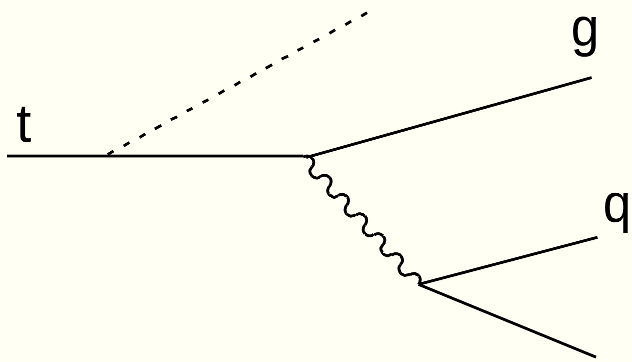
Up to now theoretical uncertainty mainly due to NNLO QCD contributions.

Numerically $\simeq -2\,\%\ \Gamma_0$ (Czarnecki, Melnikov 1999; Chetyrkin *et al.* 1999)

- Neutral vs. charged particle decays



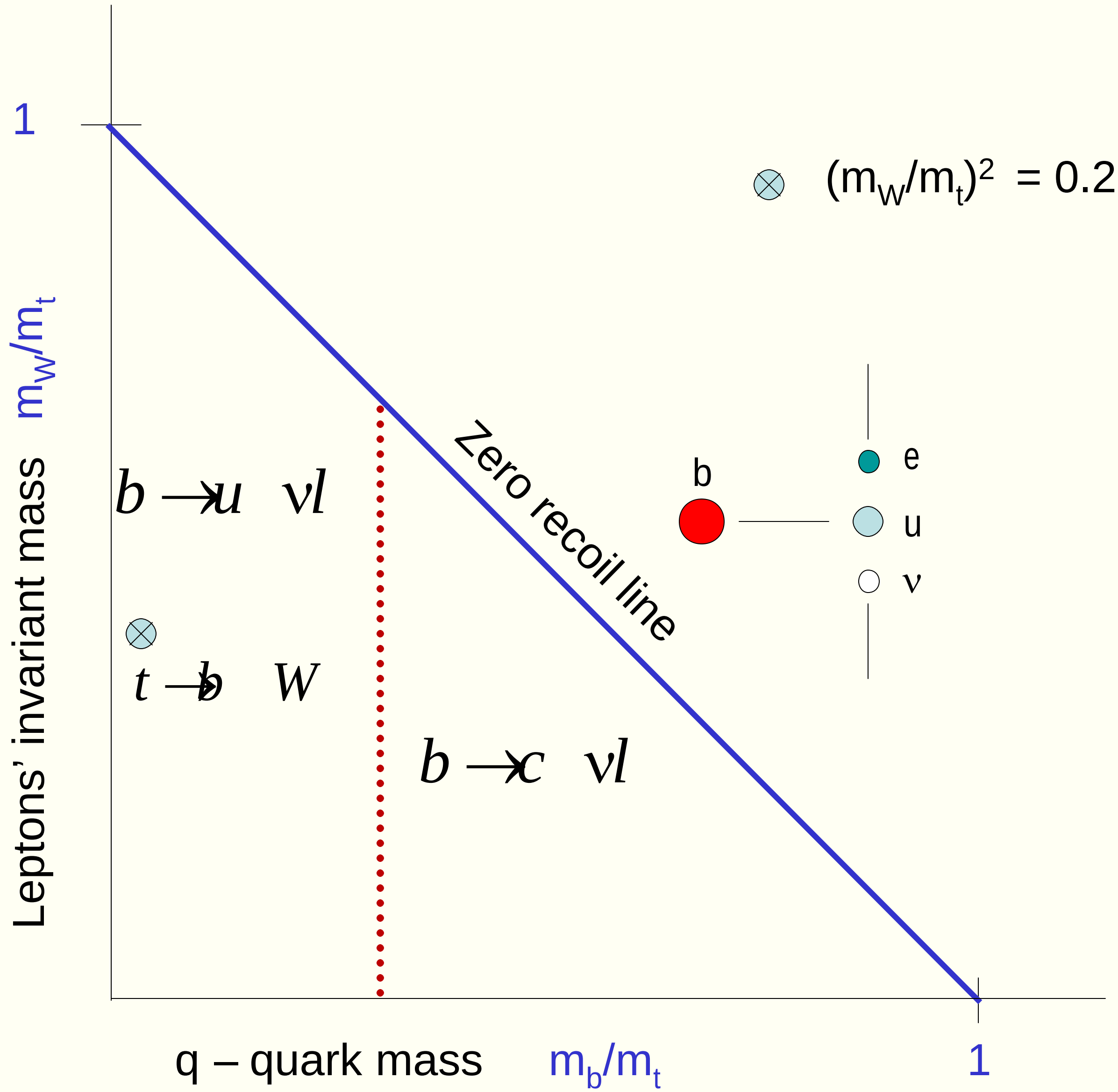
Neutral particle decay: radiators can be treated as massless



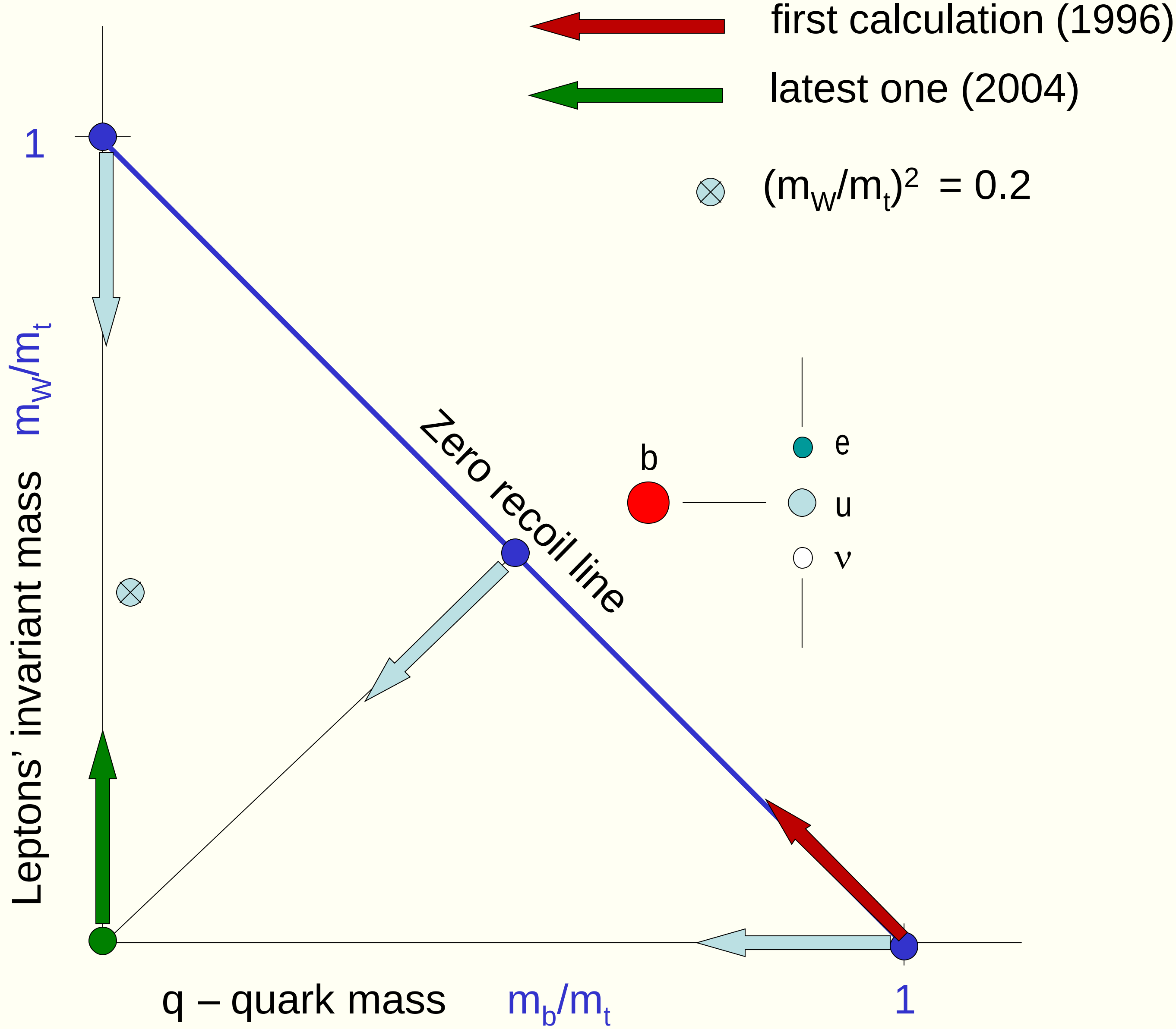
Charged: b mass cannot be neglected. Challenge for loop calculations.

- Many applications where the same technology can be used

Kinematic boundary of semileptonic decays



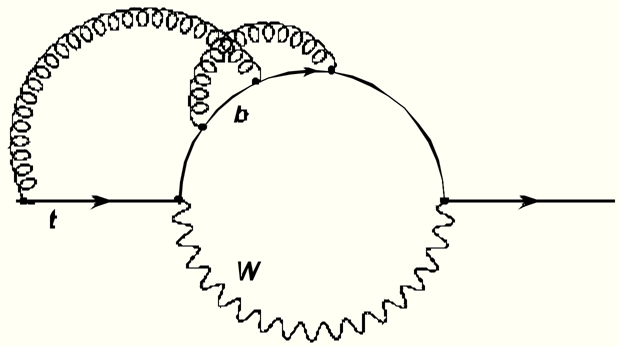
Known two-loop corrections: expansions



Methods

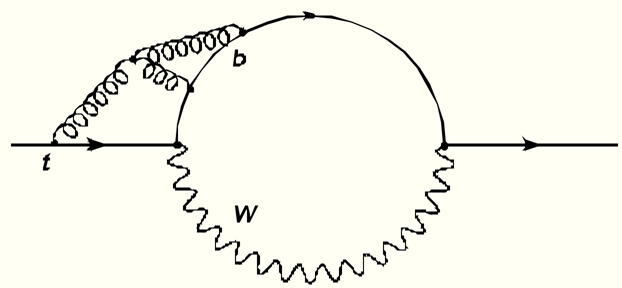
(a) Diagrams to be computed for $O(\alpha_s^2)$

Abelian



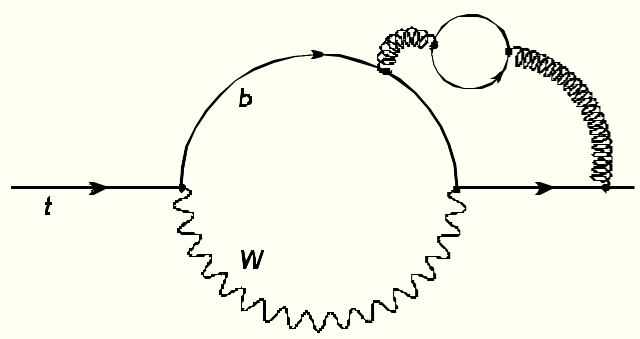
+ 18 terms

Non-abelian



+ 10 terms

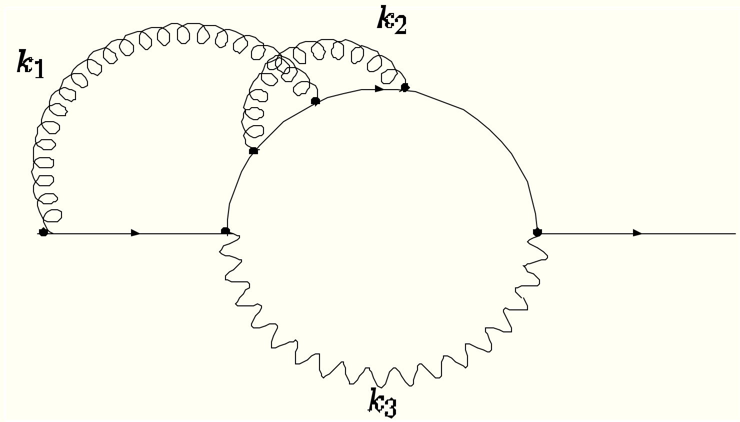
Vacuum polarization



+ 5 terms

(b) Asymptotic expansion

- two scales in the problem: m_t, m_W



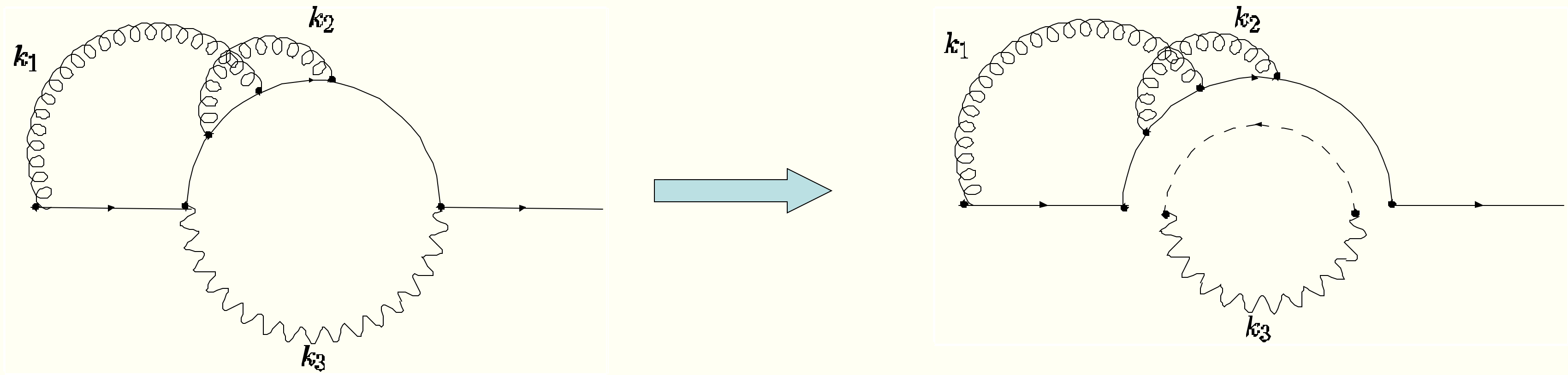
$$k_1, k_2 \sim m_t$$

- Hard region: $k_3 \sim m_t$

W propagator can be expanded as a series in powers of $(m_W/m_t)^2$:

$$\frac{k_\mu k_\nu - m_W^2 g_{\mu\nu}}{k^2 - m_W^2} = \frac{k_\mu k_\nu}{k^2} + \frac{m_W^2}{k^2}\left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\right) + \ldots$$

• Soft region: $k_3 \sim m_W$



• problem factorizes – much simpler than hard part

• does not arise in the leading order $(m_w/m_t)^2$

We end up with single-scale integrals

(c) Lorenz algebra, traces of γ matrices

- Performed automatically, each diagram reduced to a linear combination of scalar integrals
- 9 basic topologies in our problem

(d) Scalar integrals reduced to master integrals (MI) using recurrence relations

$$\int \frac{d^D k}{(2\pi)^D} \frac{\partial}{\partial k_\mu} \left[l_\mu f(p; k_1, \ldots, k_n) \right] = 0$$

How to solve the system of recurrence relations ?

- Traditional method “by inspection” – very time consuming

  - ➢ We programmed reduction procedures for all 9 basic topologies in FORM

- Fully automated and process independent approach – the Laporta algorithm (2001)

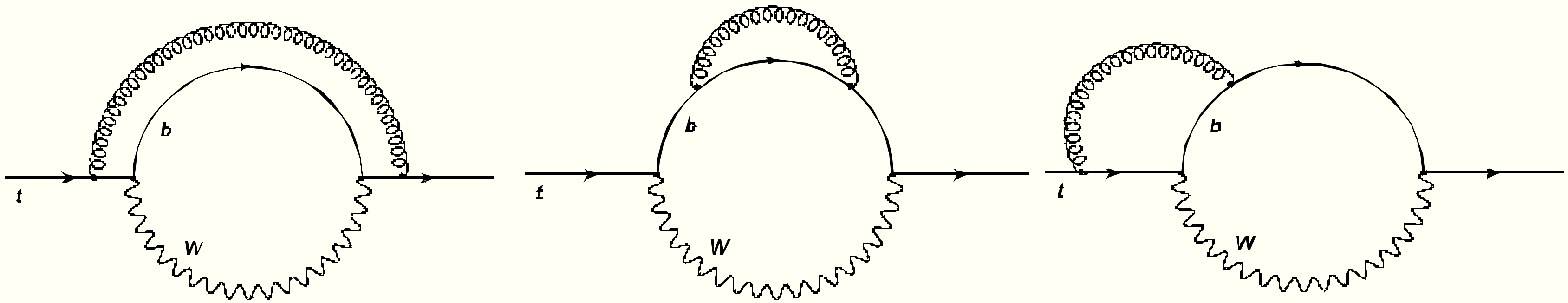
  - ➢ Generate integration-by-parts identities for all possible combinations of propagators

  - ➢ Solve large system of linear equations using Gauss elimination with a given ordering function

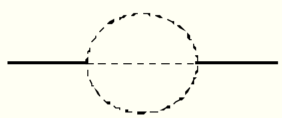
  - ➢ Modified version of the Laporta algorithm in dedicated computer algebra system PolarBear

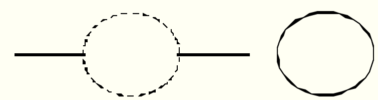
http://www.inr.ac.ru/~ftkachov/projects/bear/

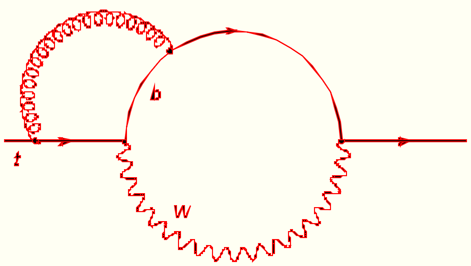
Example: NLO QCD Correction



- Can be reduced to 2 simple master integrals:

M_1 = 

M_2 = 

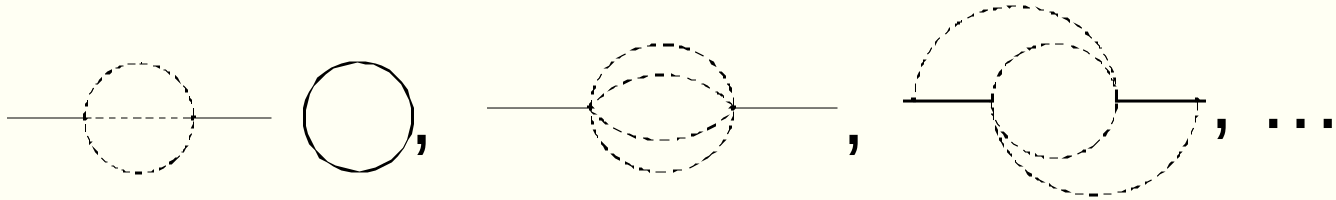


$$= \frac{1}{2\varepsilon^2} M_2 + \frac{1}{4\varepsilon}\left(M_1 - 5M_2\right) + \frac{1}{6} M_2 + \left(\frac{3}{4} M_1 - \frac{5}{12} M_2\right)\varepsilon + O(\varepsilon^2)$$

$$D = 4 - 2\varepsilon$$

Results

- Any integral in the hard region expressed in terms of 24 MI



- t —bW decay rate:

$$\Gamma(t \to bW) = \Gamma_0 \left[X_0 + \frac{\alpha_s}{\pi} X_1 + \left(\frac{\alpha_s}{\pi}\right)^2 X_2 \right]$$

$$\Gamma_0 \equiv \frac{G_F m_t^3 |V_{tb}|^2}{8\sqrt{2}\pi}$$

X_0, X_1 known

- NNLO QCD contribution

$$X_2 = C_F\left(T_R N_L X_L + T_R N_H X_H + C_F X_A + C_A X_{NA}\right)$$

$$T_R = \frac{1}{2} \quad C_F = \frac{4}{3}, \quad C_A = 3, \quad N_L = 5, \quad N_H = 1$$

- Leading coefficients compared with numerical prediction

$X_L = -\frac{4}{9} + \frac{23}{108}\pi^2 + \zeta_3 + ... \quad (\simeq 2.8594...)$ Num. 2.85(7)

$X_H = \frac{12991}{1296} - \frac{53}{54}\pi^2 - \frac{1}{3}\zeta_3 + ... \quad (\simeq -0.06359...)$ Num. -0.06360(1)

$X_F = 5 - \frac{119}{48}\pi^2 - \frac{11}{720}\pi^4 + \frac{19}{4}\pi^2\log 2 - \frac{53}{8}\zeta_3 + ... \quad (\simeq 3.575...)$ Num. 3.5(2)

$X_A = \frac{521}{576} + \frac{505}{864}\pi^2 + \frac{11}{1440}\pi^4 - \frac{19}{8}\pi^2\log 2 + \frac{9}{16}\zeta_3 + ... \quad (\simeq -8.154...)$ Num. -8.15(7)

Expansion parameter: $\omega = (m_W/m_t)^2$

We expanded up to ω^5

- Final result for top decay rate:

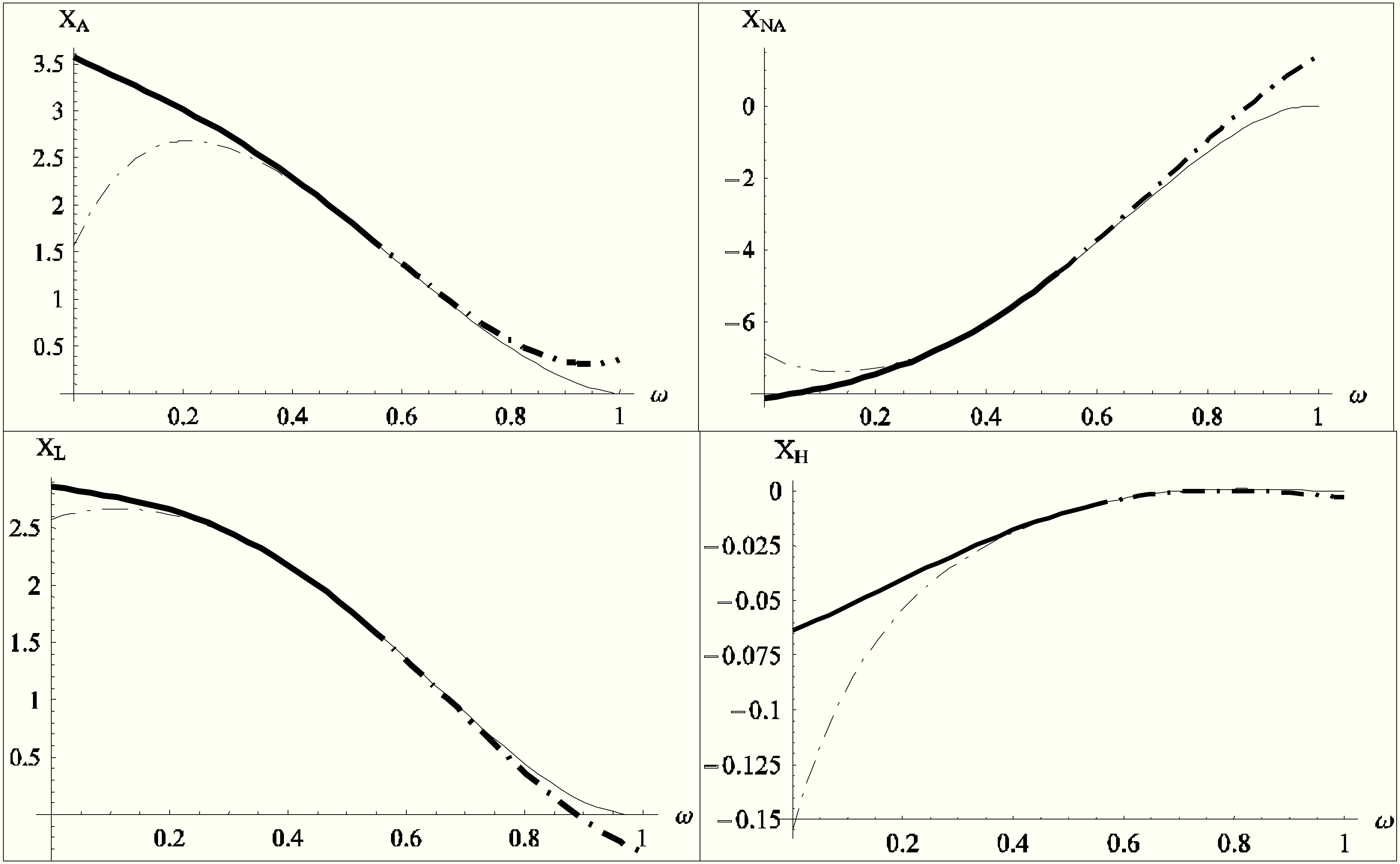
$$\omega \simeq 0.213 \longrightarrow X_2 = -15.5(1)$$

Error almost entirely due to inaccurate determination of m_t

- theoretical uncertainty 20 times smaller

- NNLO QCD correction to t decay $\simeq -2.15\% \;\; \Gamma_0 X_0$

- Matching with expansion around $\omega = 1$



$\omega = 1$ - thin line

$\omega = 0$ - thick line

Summary and outlook

- Two-loop accuracy has been obtained for top decay
- Distributions possible, if required
- New symbolic manipulation software developed
- Many applications of the new technology:

    - Semileptonic heavy-to-light decays
    - Matrix elements for $b \to s\gamma$
    - B mixings
    - Problem of b leptonic branching ratio: NNLO mandatory