Complete fermionic two-loop results to the effective weak mixing angle

Georg Weiglein

IPPP Durham

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Based on collaboration with M. Awramik, M. Czakon and A. Freitas

1. Introduction
2. Evaluation of complete fermionic two-loop corrections to $\sin^2\theta_{\rm eff}$
3. Results
4. Conclusions

1. Introduction

Electroweak precision measurements:

$$
\begin{array}{lcll}
M_{\rm W}[\,\mathrm{GeV}] & = & 80.425 \pm 0.034 & 0.04\% \\
\sin^2\theta_{\rm eff}^{\rm lept} & = & 0.23147 \pm 0.00017 & 0.07\% \\
\Gamma_{\rm Z}[\,\mathrm{GeV}] & = & 2.4952 \pm 0.0023 & 0.09\% \\
 & & & \\
M_{\rm Z}[\,\mathrm{GeV}] & = & 91.1875 \pm 0.0021 & 0.002\% \\
G_\mu[\,\mathrm{GeV}^{-2}] & = & 1.16637(1)\,10^{-5} & 0.0009\% \\
m_{\rm t}[\,\mathrm{GeV}] & = & 178.0 \pm 4.3 & 2.4\% \\
\ldots & & &
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Loop corr. to "pseudo-observables" M_W, $\sin^2\theta_{\mathrm{eff}}$, ...: $\sim \mathcal{O}(1\%)$

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Quantum effects of the theory:

Loop corr. to “pseudo-observables” $M_{\rm W}$, $\sin^2\theta_{\rm eff}$, . . . : $\sim \mathcal{O}(1\%)$

$\Rightarrow$ Constraints on $M_{\rm H}$, . . . — effects of “new physics”?

Theoretical predictions for M_W***,*** $\sin^2\theta_\mathrm{eff}$ ***in the SM:***

Comparison of SM prediction for muon decay with experiment

(Fermi constant G_μ)

$$\Rightarrow \quad M_\mathrm{W}^2\left(1-\frac{M_\mathrm{W}^2}{M_\mathrm{Z}^2}\right)=\frac{\pi\alpha}{\sqrt{2}G_\mu}\left(1+\Delta r\right),$$

$$\Updownarrow$$

loop corrections

$\Rightarrow$ Theo. prediction for M_W in terms of M_Z, α, G_μ, $\Delta r(m_\mathrm{t}, M_\mathrm{H}, \ldots)$

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$\Rightarrow$ Theo. prediction for M_{W} in terms of M_{Z}, α, G_μ, $\Delta r(m_{\mathrm{t}}, M_{\mathrm{H}}, \ldots)$

Effective couplings at the Z resonance:

$$\Rightarrow \quad \sin^2\theta_{\mathrm{eff}}=\frac{1}{4}\left(1-\mathrm{Re}\,\frac{g_V}{g_A}\right)=\left(1-\frac{M_{\mathrm{W}}^2}{M_{\mathrm{Z}}^2}\right)\mathrm{Re}\,\kappa_l(s=M_{\mathrm{Z}}^2)$$

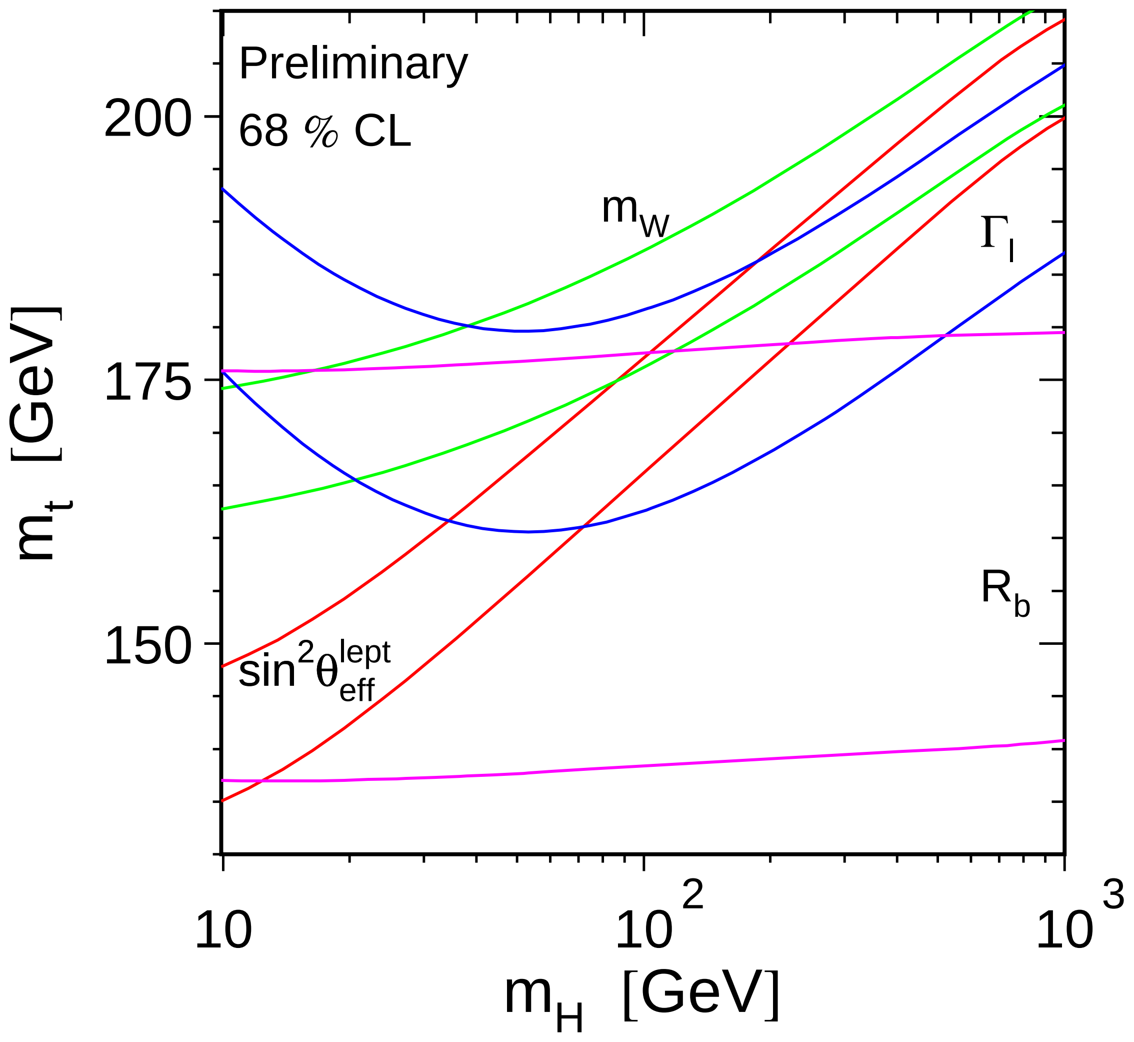
Theoretical uncertainties

- Unknown higher-order corrections $\Rightarrow$ "blue band"

- experimental error of input parameters: m_{t}, $\Delta\alpha_{\mathrm{had}}$, $\ldots$

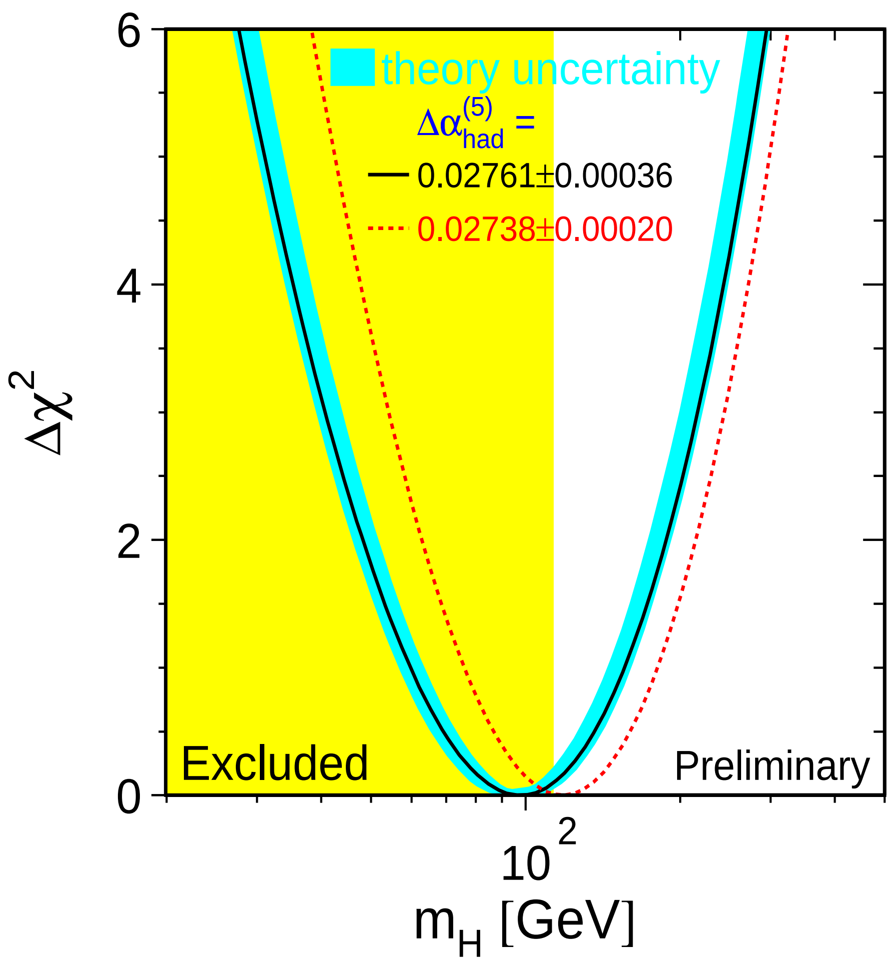
Sensitivity of pseudo-observables to $M_{\rm H}$

[*LEPEWWG '04*]



$\Rightarrow$ highest sensitivity from $\sin^2\theta_{\rm eff}$ and $M_{\rm W}$

Global fit to all data in the SM: Winter ’01



[*LEPEWWG ’01*]

$\Rightarrow M_{\mathrm{H}} = 98^{+58}_{-38}$ GeV

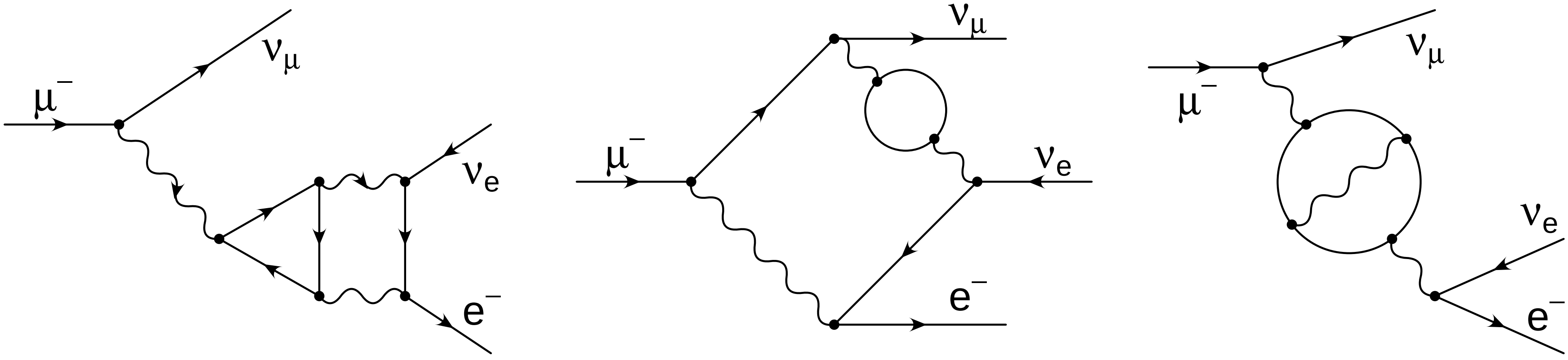
$M_{\mathrm{H}} < 212$ GeV, 95% C.L.

Main changes in global fit: Winter '01 $\longrightarrow$ Winter '04

- Complete fermionic two-loop contributions to M_{W}:

  [*A. Freitas, W. Hollik, W. Walter, G.W. '00, '02*]

  [*M. Awramik, M. Czakon '03*]



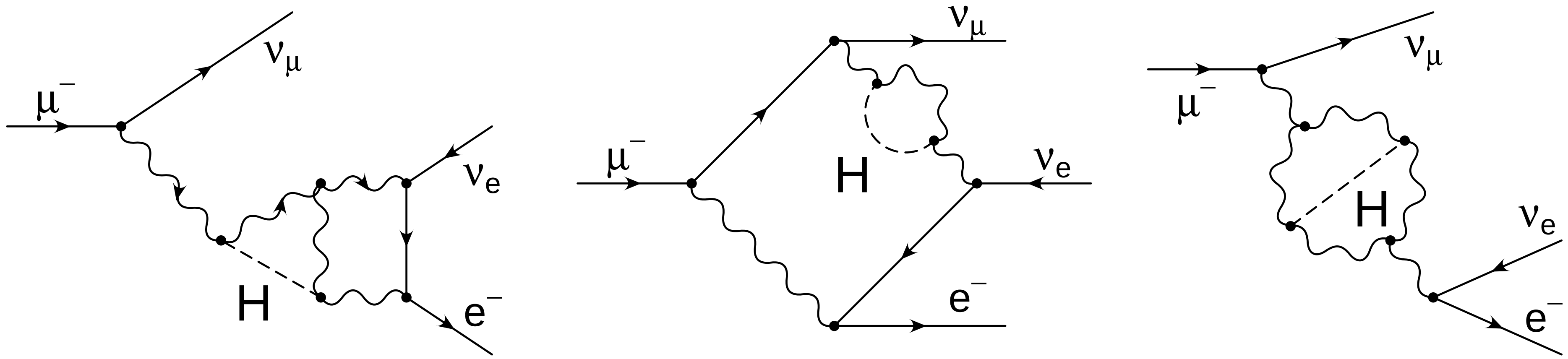
$\Rightarrow$ full dependence on m_{t}, complete light fermion contributions

$\Rightarrow$ improved error estimate of $\sin^2\theta_{\mathrm{eff}}$

Main changes in global fit: Winter '01 → Winter '04

- Purely bosonic two-loop contributions to M_W:

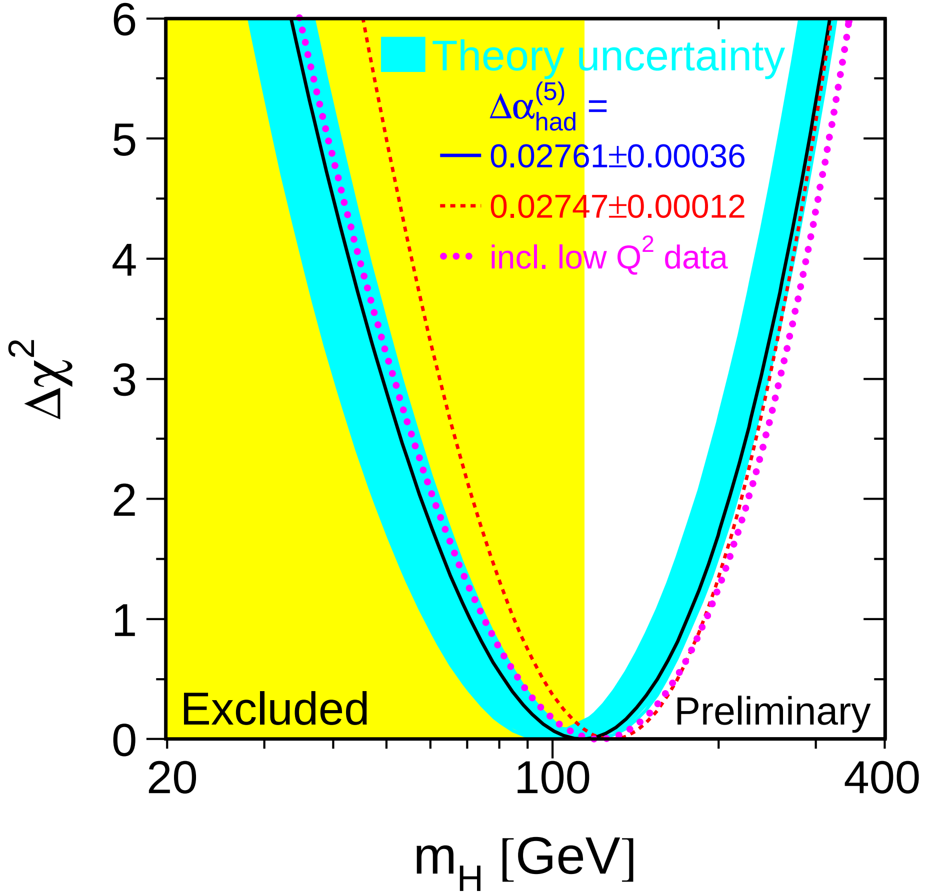
  [*M. Awramik, M. Czakon '02*] [*A. Onishchenko, O. Veretin '02*]



  $\Rightarrow \Delta M_\mathrm{W} \lesssim \pm 1$ MeV

- New m_t value: $m_\mathrm{t} = 178.0 \pm 4.3$ GeV [*Tevatron EWWG '04*]

Global fit to all data in the SM: Winter '04



[*LEPEWWG '04*]

$\Rightarrow M_{\rm H} = 117^{+67}_{-45}$ GeV

$M_{\rm H} < 251$ GeV, 95% C.L.

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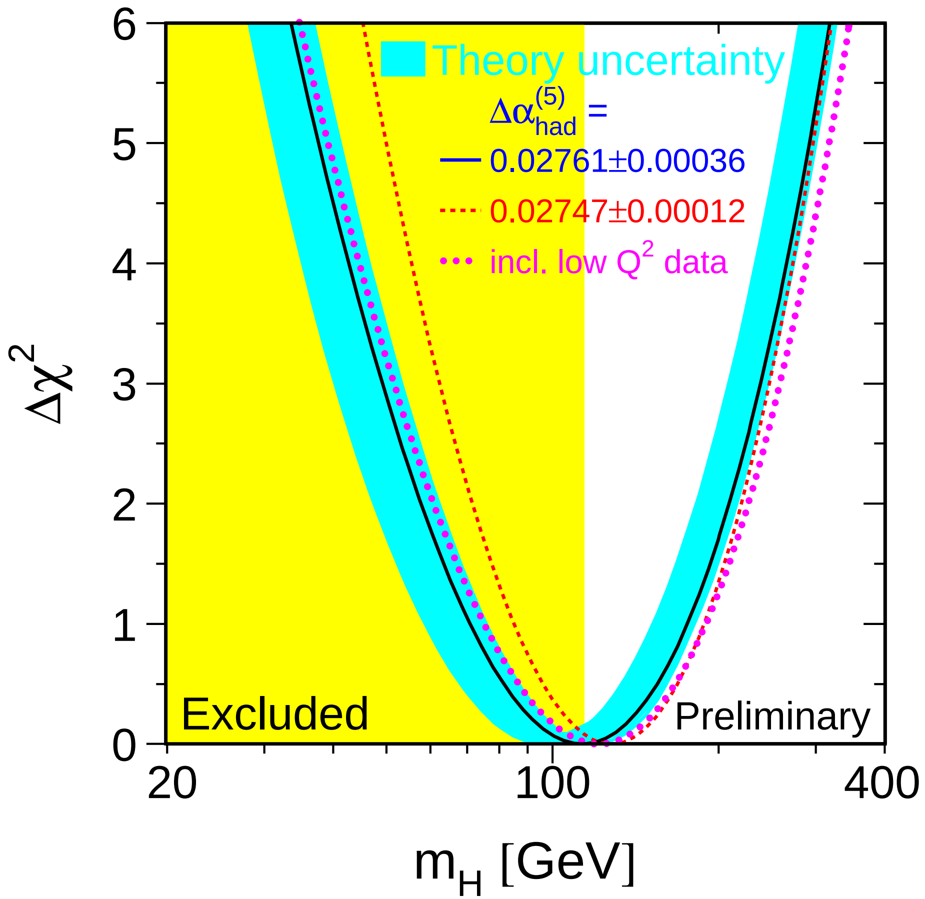

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⇒ "Blue band" has widened due to improved estimate of theoretical uncertainties; main effect from uncertainty of $\sin^2\theta_{\rm eff}$

[*M. Awramik, M. Czakon, A. Freitas, G.W. '03*]

Status of higher-order corrections to M_{W} and $\sin^2\theta_{\mathrm{eff}}$

- Electroweak two-loop corrections:

  M_{W}: complete electroweak two-loop result
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  $\sin^2\theta_{\mathrm{eff}}$: next-to-leading order terms in expansion in m_{t}, $\mathcal{O}(G_\mu^2 m_{\mathrm{t}}^2 M_{\mathrm{Z}}^2)$
  [*G. Degrassi, P. Gambino, A. Sirlin '97*]

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  $\sin^2\theta_{\rm eff}$: next-to-leading order terms in expansion in $m_{\rm t}$,
  $\mathcal{O}(G_\mu^2 m_{\rm t}^2 M_{\rm Z}^2)$
  [*G. Degrassi, P. Gambino, A. Sirlin '97*]

- Three-loop QCD corrections to $M_{\rm W}$, $\sin^2\theta_{\rm eff}$
  [*L. Avdeev, J. Fleischer, S.M. Mikhailov, O. Tarasov '94*]
  [*K. Chetyrkin, J. Kühn, M. Steinhauser '95*]

Status of higher-order corrections to $M_{\rm W}$ *and* $\sin^2\theta_{\rm eff}$

- Pure fermion-loop contributions up to 4-loop order
  [*A. Stremplat '98*], [*G.W. '98*]

- $\mathcal{O}(G_\mu^3 m_{\rm t}^6)$, $\mathcal{O}(G_\mu^2 \alpha_{\rm s} m_{\rm t}^4)$ terms
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⇒ Theoretical uncertainty of M_W from unknown higher-order corrections: [*M. Awramik, M. Czakon, A. Freitas, G.W. '03*]

$$\Delta M_\mathrm{W} \approx \pm 4 \text{ MeV}$$

2. Evaluation of complete fermionic two-loop corrections to $\sin^2\theta_{\rm eff}$

$\sin^2\theta_{\rm eff}$, $M_{\rm W}$, ...: pseudo-observables

$\Rightarrow$ need deconvolution procedure (unfolding) in order to determine $\sin^2\theta_{\rm eff}$, $M_{\rm W}$, etc. from measured cross sections

Expansion of amplitude around complex pole:

$$\mathcal{A}(e^+e^- \to f\bar{f}) = \frac{R}{s - \mathcal{M}_{\rm Z}^2} + S + (s - \mathcal{M}_{\rm Z}^2)\,S' + \cdots$$

$$\mathcal{M}_{\rm Z} = M_{\rm Z}^2 - iM_{\rm Z}\Gamma_{\rm Z}$$

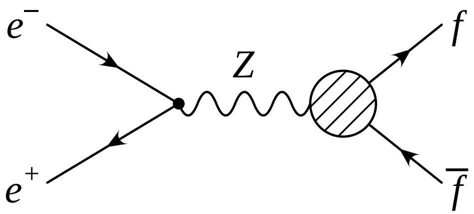
Expanding up to $\mathcal{O}(\alpha^2)$ using $\mathcal{O}(\Gamma_{\rm Z}/M_{\rm Z}) = \mathcal{O}(\alpha)$

$\Rightarrow$ electroweak form factors

Electroweak form factors

$$\kappa_f = \frac{1 - v_f/a_f}{1 - v_f^{(0)}/a_f^{(0)}};\qquad (0) = \text{tree-level}$$

where v_f, a_f are the vector, axial-vector $Z f \bar{f}$ couplings

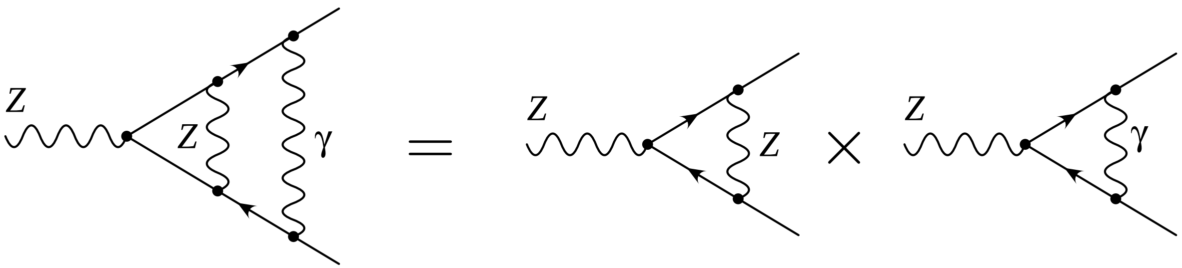


⇒ Two-loop contribution:

$$\kappa_l^{(2)} = \left. \frac{a_l^{(2)} v_l^{(0)} a_l^{(0)} - v_l^{(2)} \left(a_l^{(0)}\right)^2 - \left(a_l^{(1)}\right)^2 v_l^{(0)} + a_l^{(1)} v_l^{(1)} a_l^{(0)}}{\left(a_l^{(0)}\right)^2 \left(a_l^{(0)} - v_l^{(0)}\right)} \right|_{s=M_Z^2}$$

Two-loop contributions to κ_l:

- Interplay between 2-loop terms and products of 1-loop terms to cancel IR-divergencies



+ finite

$\Rightarrow \sin^2\theta_{\text{eff}}$ is IR-safe

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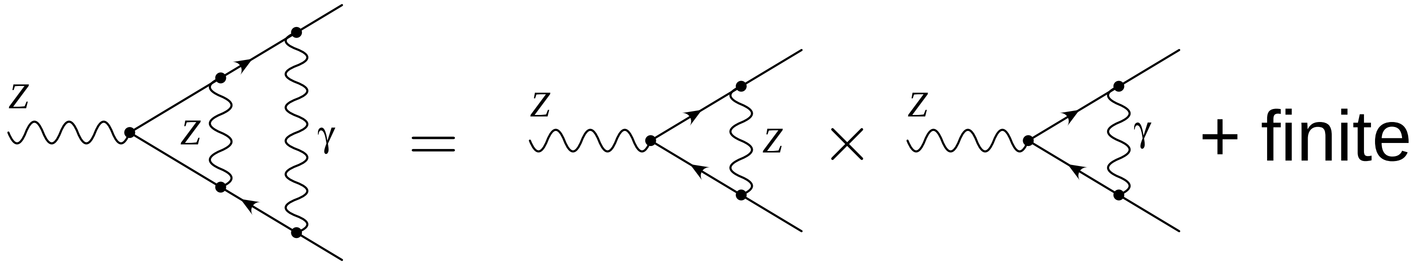

$\Rightarrow \sin^2\theta_{\mathrm{eff}}$ is IR-safe

- Genuine 2-loop contributions contain products of imaginary parts of 1-loop terms

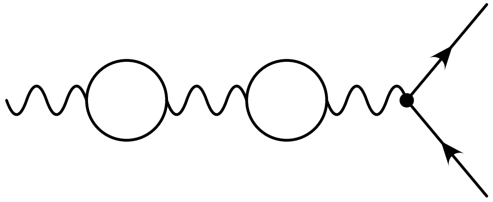

Diagram classes:

- Renormalisation requires on-shell two-loop propagators,

  e.g. weak-mixing angle counterterm

$$\delta s_{\mathrm{W}(2)} = \frac{M_{\mathrm{W}}^2}{2 s_{\mathrm{W}} M_{\mathrm{Z}}^2} \left[\frac{\Sigma^{\mathrm{Z}}_{\mathrm{T}(2)}(M_{\mathrm{Z}}^2)}{M_{\mathrm{Z}}^2} - \frac{\Sigma^{\mathrm{W}}_{\mathrm{T}(2)}(M_{\mathrm{W}}^2)}{M_{\mathrm{W}}^2} \right] + \text{(1-loop terms)}$$

$\Rightarrow$ Well known and tested for two-loop corrections to M_{W}

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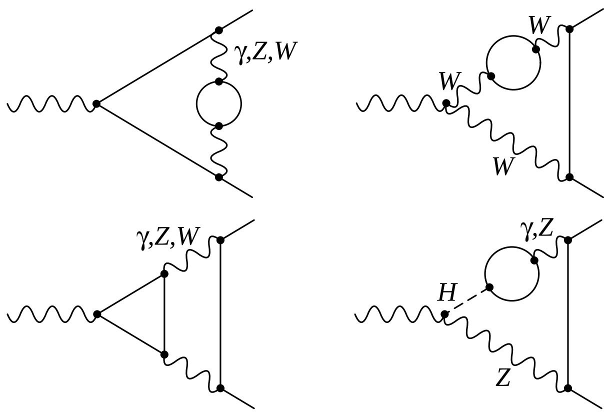
- New complication:
  two-loop vertex diagrams

  two classes:
  top, light fermions



Different methods for evaluation

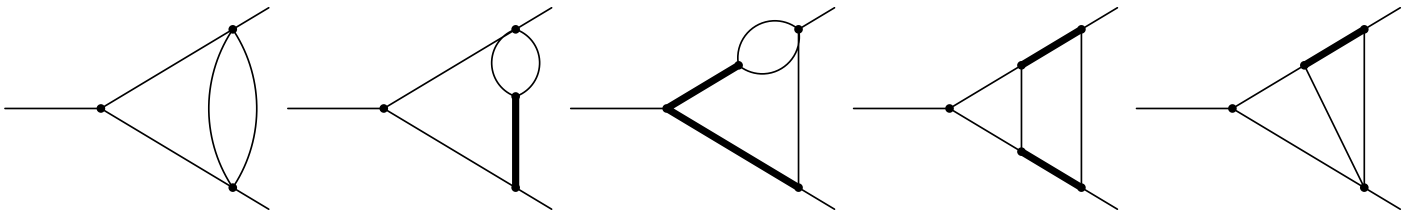
- Top-quark contributions: expansion in M_Z^2/m_t^2

  $\Rightarrow$ expansion up to $(M_Z^2/m_t^2)^5$ yields intrinsic precision of 10^{-7}

  Light fermion contributions:

  depend on only one variable $\Rightarrow$ reduction to master integrals using integration by parts and Lorentz invariance identities

  [*Chetyrkin, Tkachov '81*] [*Gehrmann, Remiddi '00*] [*Laporta '00*]



Analytical results for master integrals via differential equations

Different methods for evaluation

- Numerical integrations of master integrals for top-quark and light fermion contributions

  Self-energy subloop: dispersion representation

$$B_0(p^2, m_1^2, m_2^2) = -\int_{(m_1+m_2)^2}^{\infty} \mathrm{d}s \, \frac{\Delta B_0(s, m_1^2, m_2^2)}{s - p^2}$$

  $\Rightarrow$ insertion in 2-loop integral yields N-point one-loop function that can be integrated

  $\Rightarrow$ one-dimensional integral representation [*S. Bauberger '95*]

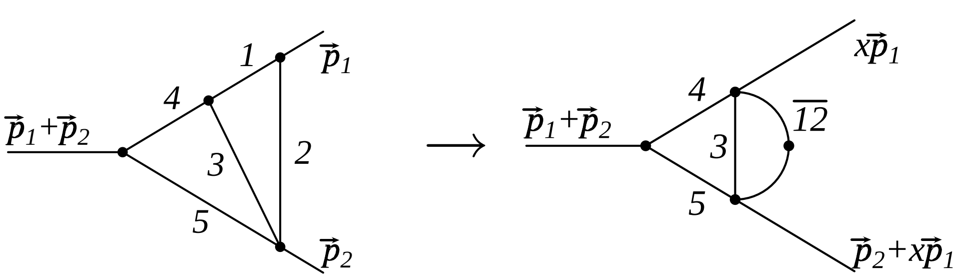
Different methods for evaluation

Triangle subloop: Feynman parameters [*J. v.d.Bij, A. Ghinculov '94*]

$$\frac{1}{(q+p_1)^2 - m_1^2} \, \frac{1}{(q+p_2)^2 - m_2^2} = \int_0^1 \mathrm{d}x \, \frac{1}{[(q+\bar{p})^2 - \overline{m}^2]^2}$$

$$\bar{p} = x\,p_1 + (1-x)p_2, \qquad \overline{m} = x\,m_1 + (1-x)m_2 - x(1-x)(p_1-p_2)^2$$

⇒ triangle reduced to self-energy subloop



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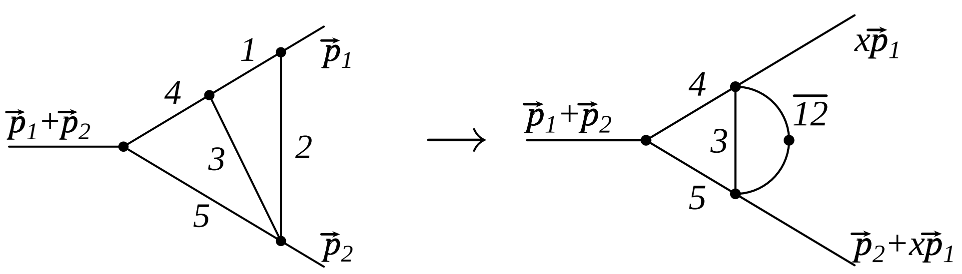

⇒ At most three-dimensional numerical integration

⇒ Independent calculations with different methods

3. Results

Simple parametrisation of full result for $\sin^2\theta_{\rm eff}$ (contains all known corrections): [*M. Awramik, M. Czakon, A. Freitas, G.W. '04*]

$$\sin^2\theta_{\rm eff} = \sin^2\theta^0_{\rm eff} + c_1\,{\rm dH} + c_2\,{\rm dH}^2 + c_3\,{\rm dH}^4 + c_4\,({\rm dh}^2 - 1) + c_5\,{\rm d}\alpha + c_6\,{\rm dt} + c_7\,{\rm dt}^2 + c_8\,({\rm dh} - 1)\,{\rm dt} + c_9\,{\rm d}\alpha_{\rm s} + c_{10}\,{\rm dz}$$

where

$${\rm dH} = \ln\left(\frac{M_{\rm H}}{100\ {\rm GeV}}\right), \quad {\rm dh} = \left(\frac{M_{\rm H}}{100\ {\rm GeV}}\right), \quad {\rm dt} = \left(\frac{m_{\rm t}}{178.0\ {\rm GeV}}\right)^2 - 1$$

$${\rm d}\alpha = \frac{\Delta\alpha}{0.05907} - 1, \quad {\rm d}\alpha_{\rm s} = \frac{\alpha_{\rm s}(M_{\rm Z})}{0.117} - 1, \quad {\rm dz} = \frac{M_{\rm Z}}{91.1875\ {\rm GeV}} - 1$$

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$\Rightarrow$ approximates full result within 4.5×10^{-6} for $10\ {\rm GeV} \leq M_{\rm H} \leq 1\ {\rm TeV}$, 2σ variations

implemented in ZFITTER 6.40 [*D. Bardin et al. '04*]

Estimate of remaining theoretical uncertainty

Unknown higher-order corrections:

- $\mathcal{O}(\alpha^2\alpha_s)$ beyond leading m_t^4 term
- $\mathcal{O}(\alpha^3)$ beyond leading m_t^6 term and pure fermion-loop contributions
- $\mathcal{O}(\alpha\alpha_s^3)$
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- $\mathcal{O}(\alpha\alpha_{\rm s}^3)$
- $\mathcal{O}(\alpha^2\alpha_{\rm s}^2)$
- $\mathcal{O}(\alpha^2)$ purely bosonic corrections

Different methods:

geometric progression from lower orders, multiplication of possible enhancement factors by coefficients of $\mathcal{O}(1)$, ...

$$\Rightarrow \quad \Delta \sin^2\theta_{\rm eff} \approx \pm 5 \times 10^{-5}$$

Comparison with previous result

Previous version of ZFITTER was based on $\mathcal{O}(G_\mu^2 m_t^2 M_Z^2)$ result

[*G. Degrassi, P. Gambino, A. Sirlin '97*]

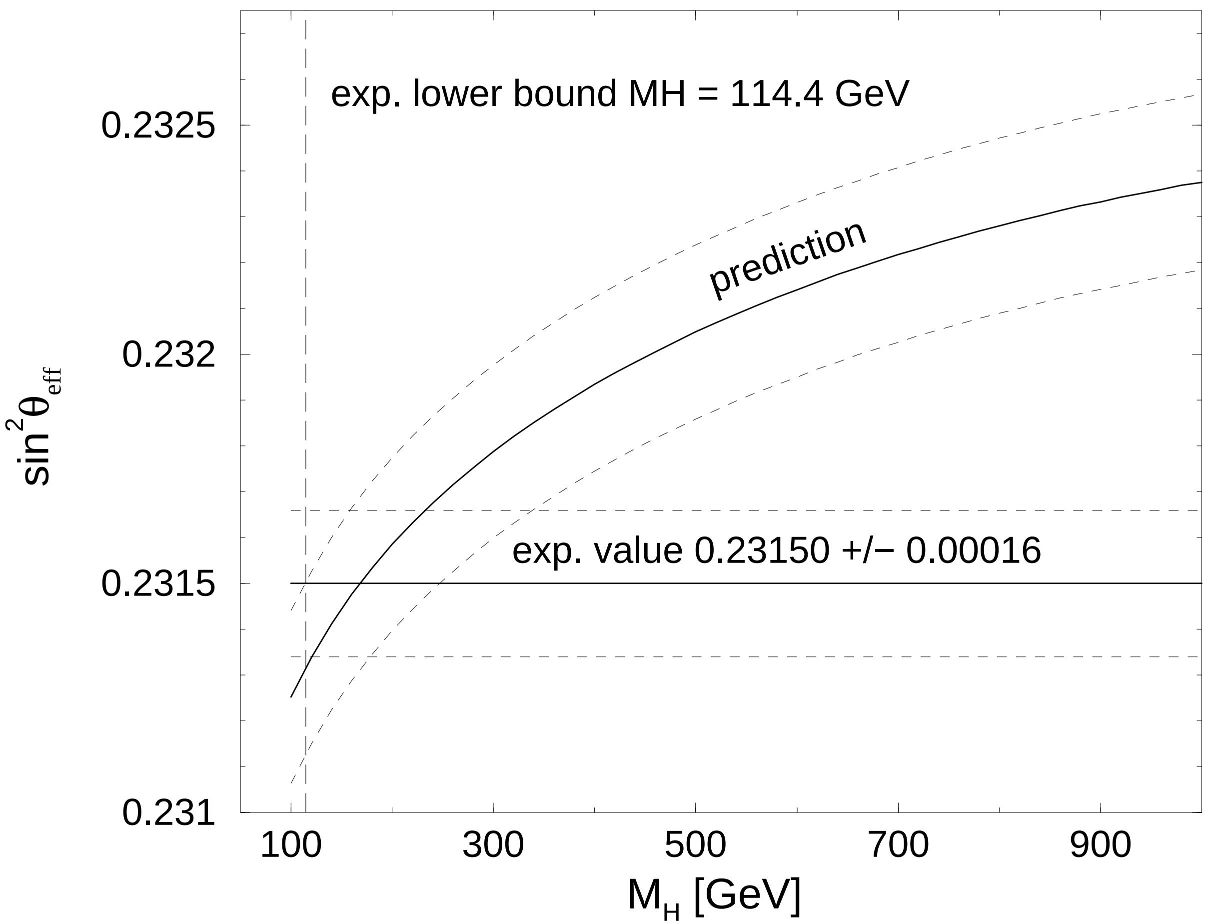
Comparison of new result for $\sin^2\theta_{\rm eff}$ with previous version of ZFITTER:

$M_H = 100$ GeV $\Rightarrow$ downward shift by 4.5×10^{-5}

$M_H = 300$ GeV $\Rightarrow$ downward shift by 8.5×10^{-5}

$M_H = 600$ GeV $\Rightarrow$ downward shift by 11.7×10^{-5}

SM prediction for $\sin^2\theta_{\rm eff}$ *vs. experimental result*



[*M. Awramik, M. Czakon, A. Freitas, G.W. '04*]

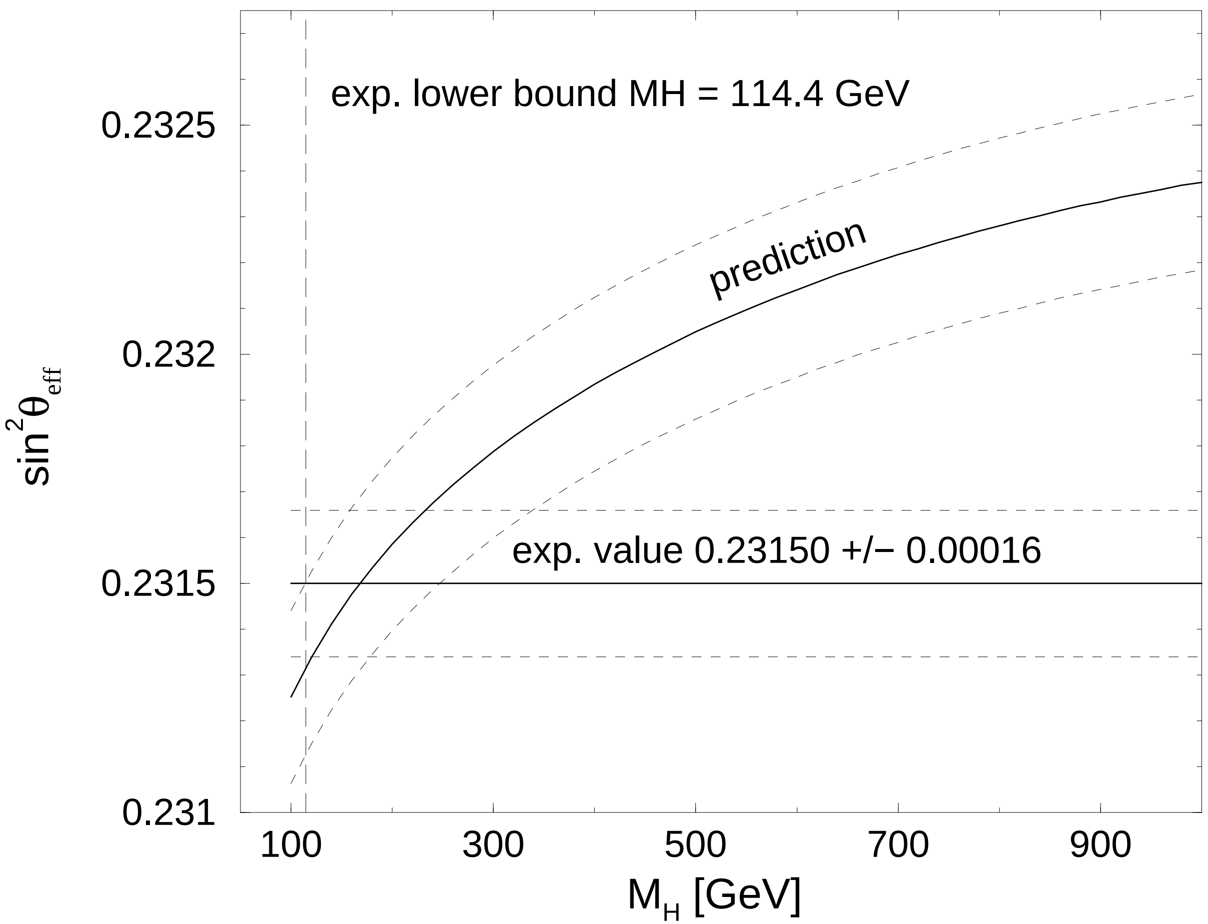
Main source of theoretical uncertainty:

$\delta m_{\rm t} = 4.3$ GeV

$\Rightarrow \Delta \sin^2\theta_{\rm eff}^{\rm para} \approx 14\times 10^{-5}$

$\Rightarrow$ Preference for light Higgs

SM prediction for $\sin^2\theta_{\rm eff}$ ***vs. experimental result***



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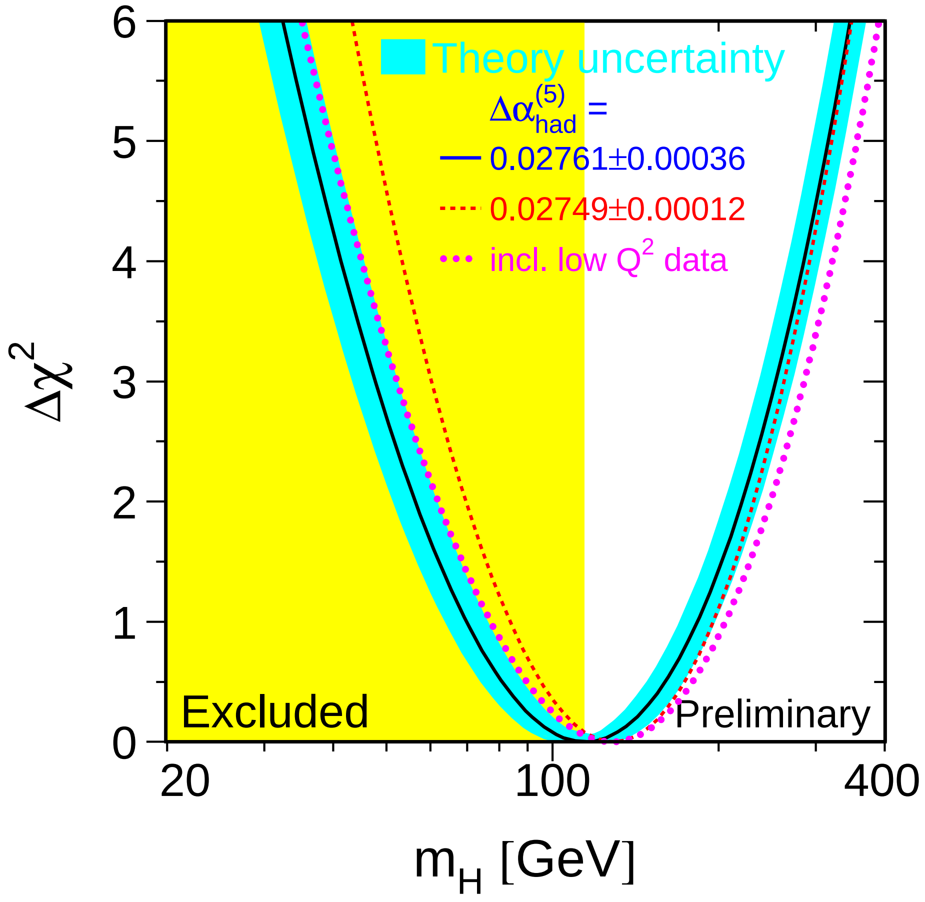
Downward shift of theory prediction for $\sin^2\theta_{\rm eff}$

$\Rightarrow$ larger $M_{\rm H}$ values allowed

Global fit to all data in the SM: Summer '04

Global fi t with new result for $\sin^2\theta_{\text{eff}}$:



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$M_{\text{H}} < 260$ GeV, 95% C.L.

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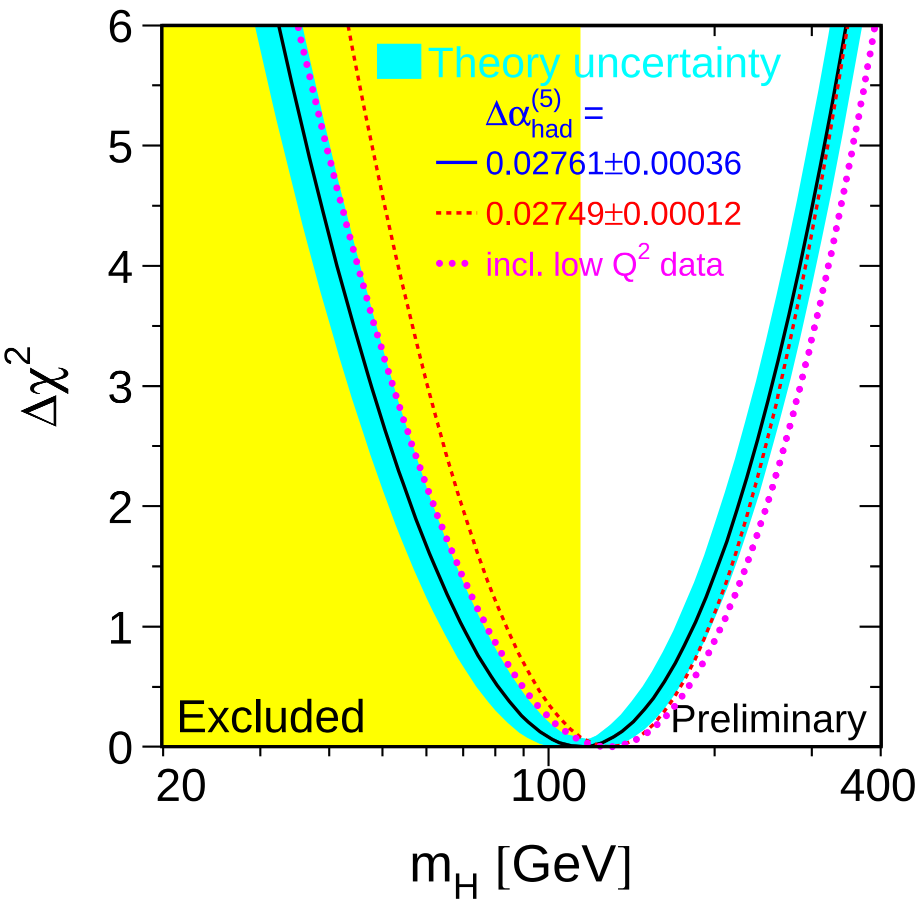

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$M_{\text{H}} < 260$ GeV, 95% C.L.

$\Rightarrow$ Smaller blue band; $\sin^2\theta_{\text{eff}}$ still the dominant uncertainty

Upper M_{H} limit increased by ≈ 10 GeV

Electroweak precision measurements at the LC

Precision measurements at the

Z resonance / WW threshold (GigaZ) and the $\mathrm{t}\bar{\mathrm{t}}$ threshold:

$$\Rightarrow \quad \delta \sin^2\theta_{\mathrm{eff}} \approx 1 \times 10^{-5},\ \delta M_{\mathrm{W}} \approx 7\ \mathrm{MeV},\ \delta m_{\mathrm{t}} \approx 0.1\ \mathrm{GeV}$$

$\Rightarrow$ Need further improvement in theoretical uncertainties of $\sin^2\theta_{\mathrm{eff}} = \left(1 - \frac{M_{\mathrm{W}}^2}{M_{\mathrm{Z}}^2}\right) \mathrm{Re}\,\kappa_l$ and M_{W} in order to match the GigaZ experimental accuracy

Comparison of future parametric uncertainties

$$\delta m_{\rm t} = 1.5~\text{GeV} \quad \Rightarrow \quad \Delta M_{\rm W}^{\rm para} \approx 9~\text{MeV}, \quad \Delta \sin^2 \theta_{\rm eff}^{\rm para} \approx 4.5 \times 10^{-5}$$

$$\delta m_{\rm t} = 0.1~\text{GeV} \quad \Rightarrow \quad \Delta M_{\rm W}^{\rm para} \approx 1~\text{MeV}, \quad \Delta \sin^2 \theta_{\rm eff}^{\rm para} \approx 0.3 \times 10^{-5}$$

$$\delta(\Delta\alpha_{\rm had}) = 5 \times 10^{-5} \quad \Rightarrow \quad \Delta M_{\rm W}^{\rm para} \approx 1~\text{MeV}, \quad \Delta \sin^2 \theta_{\rm eff}^{\rm para} \approx 1.8 \times 10^{-5}$$

$$\delta M_{\rm Z} = 2.1~\text{MeV} \quad \Rightarrow \quad \Delta M_{\rm W}^{\rm para} \approx 2.5~\text{MeV}, \quad \Delta \sin^2 \theta_{\rm eff}^{\rm para} \approx 1.4 \times 10$$

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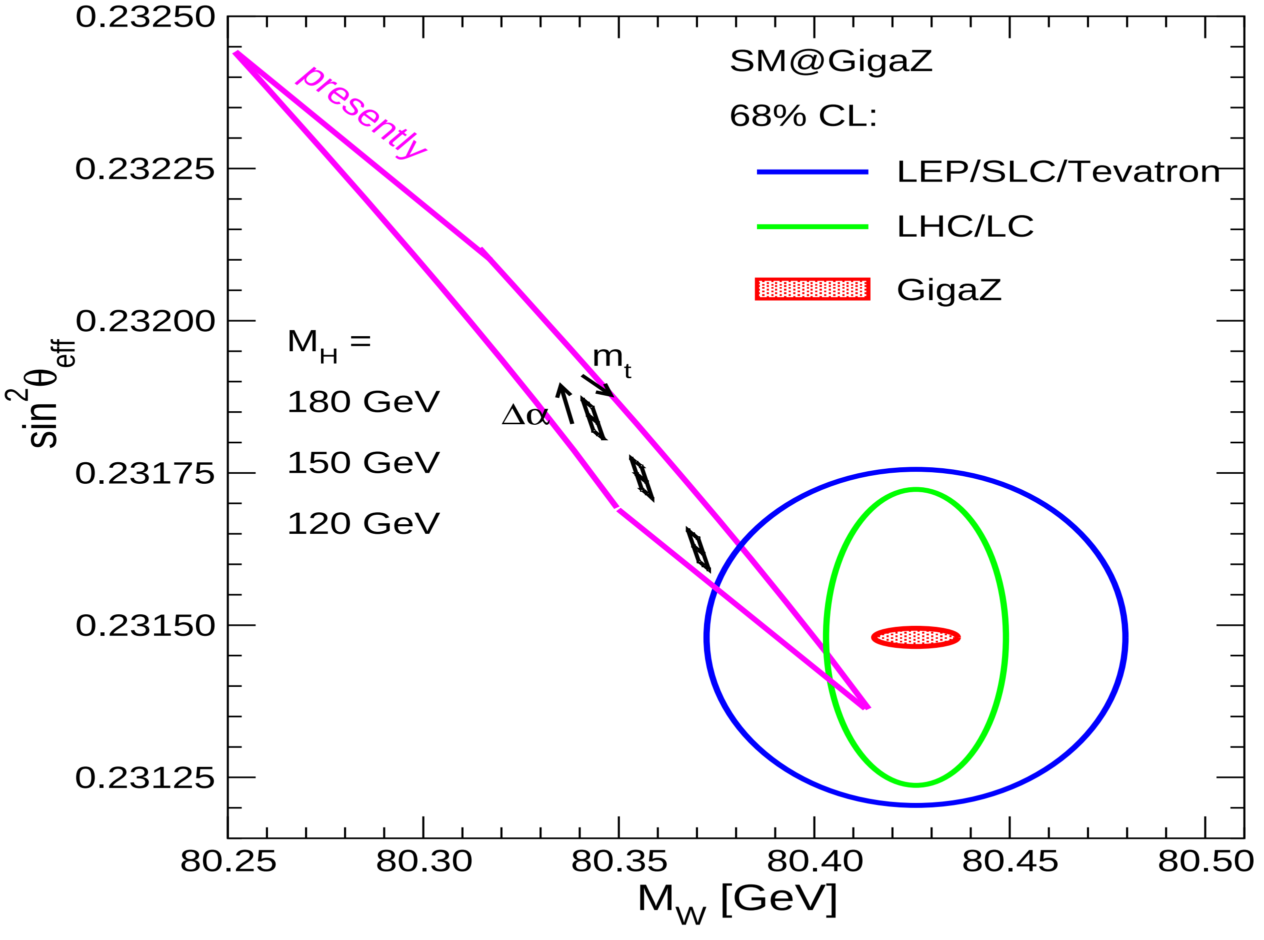
$\Rightarrow$ With LC accuracy on $m_{\rm t}$:

parametric uncertainties can be reduced to the level of GigaZ experimental errors

SM prediction for M_{W} and $\sin^2\theta_{\mathrm{eff}}$ vs. current experimental result (LEP2/Tevatron) and prospective accuracies at the LHC and a LC with low-energy option (GigaZ):

[*J. Erler, S. Heinemeyer, W. Hollik, G.W., P. Zerwas '00*]



$\Rightarrow$ Highly sensitive test of electroweak theory: improved accuracy of observables and input parameters

4. Conclusions

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  Simple parametrisation; implemented in ZFITTER 6.40

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- At LC ($+$ GigaZ): improved accuracy of precision observables M_W, $\sin^2\theta_{\text{eff}}$, m_h, ... and input parameters m_t, $m_{\tilde{t}}$, ...

  $\Rightarrow$ Very sensitive test of electroweak theory