The anomalous magnetic moment of the muon and supersymmetry

Dominik Stöckinger,
Durham

- experimental and SM status
- 2-loop corrections in the MSSM and the SM

  [Heinemeyer,Weiglein,DS'03,'04]

Milestones

'49 Schwinger: QED 1L: $\frac{\alpha}{2\pi}$

'57: QED 2L terms: -1.77×10^{-6}

'68 CERN measurement to: $\pm 2.7 \times 10^{-7}$

'72 QED 2L log $\frac{m_\mu}{m_e}$-terms: $+5.9 \times 10^{-6}$

'75 CERN measurement to: $\pm 2.7 \times 10^{-8}$

'76 QED 3L numerically: $+3 \times 10^{-7}$

'76 QED 4L leading: $(4 \pm 2) \times 10^{-9}$

'76 hadronic contributions: $(6.7 \pm .9) \times 10^{-8}$

'78 CERN measurement to: $\pm 1.0 \times 10^{-8}$

Calmet, Narison, Perrottet, and de Rafael:

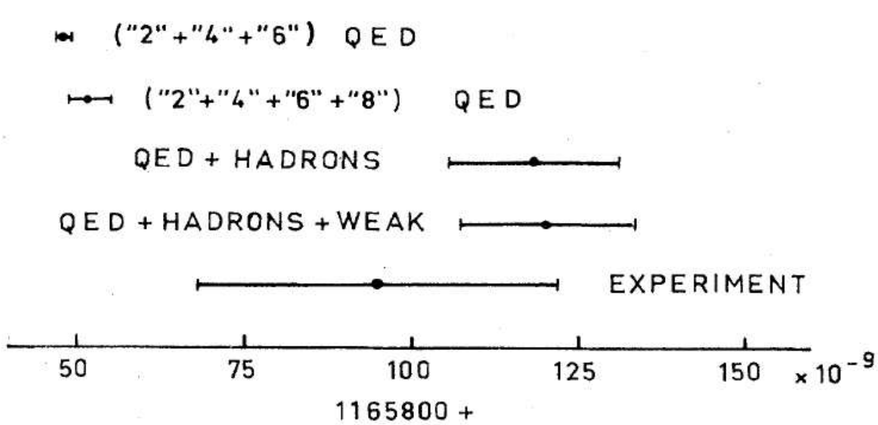


FIG. 20. Final comparison between the theoretical contributions and experiment.

Milestones

QED calculation to $\pm 0.1 \times 10^{-10}$

'75–'96 (Remiddi): 3L analytically
'83–today (Kinoshita): 4L,5L numerically

'96 Electroweak 2L calculation to $\pm 0.5 \times 10^{-10}$

Hadronic light-by-light contributions ($\times 10^{-10}$)

'85 Kinoshita et al	$+4.9 \pm 0.5$
'95 Kinoshita et al	-5.2 ± 1.8
'96 Bijnens et al	-9.2 ± 3.2
'98 Kinoshita et al	-7.9 ± 1.5

'01 BNL experiment: $a_\mu^{\text{exp}} - a_\mu^{\text{th}} = (43 \pm 16)$

sign of new physics?

Milestones

Full light-by-light history

'85 Kinoshita et al	$+5 \pm 1$
'95–'98 three groups	-7 ± 2
'01 BNL experiment	2.6σ discrepancy
'02 Knecht et al	$+8 \pm 4$
'02 other groups	$+8 \pm 3$

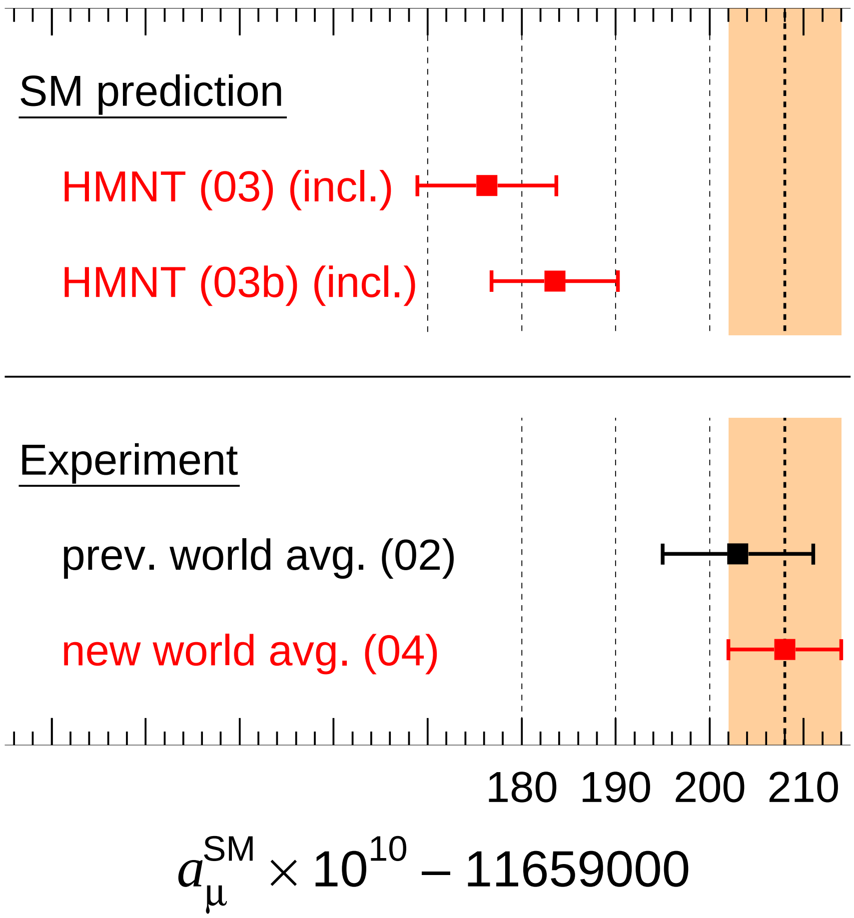
Hadronic vacuum polarization

'03 Davier et al:	discrepancy e^+e^- vs. τ data
'03 Jegerlehner,:	isospin breaking effects
Ghozzi	$\Rightarrow \tau$ data not as reliable

recent developments:

'03 error in QED 4L:	$+1.4$
'03 new LBL-result:	$+5.6$
'04 BNL experiment:	$+5$
	$\Delta a_\mu = (25 \pm 9)$

Measurement of a_μ



Experimental result

$$a_\mu^{\text{exp}} = (11\,659\,208 \pm 6) \times 10^{-10}$$

SM prediction *[Hagiwara, Martin, Nomura, Teubner '03]*
[Melnikov, Vainshtein '03, Kinoshita, Nio '03]

$$a_\mu^{\text{SM}} = (11\,659\,184 \pm 7) \times 10^{-10}$$

Discrepancy

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (24.5 \pm 9.0) \times 10^{-10}$$

Discrepancy

In spite of . . .

many discovered errors

new developments

Discrepancy persists

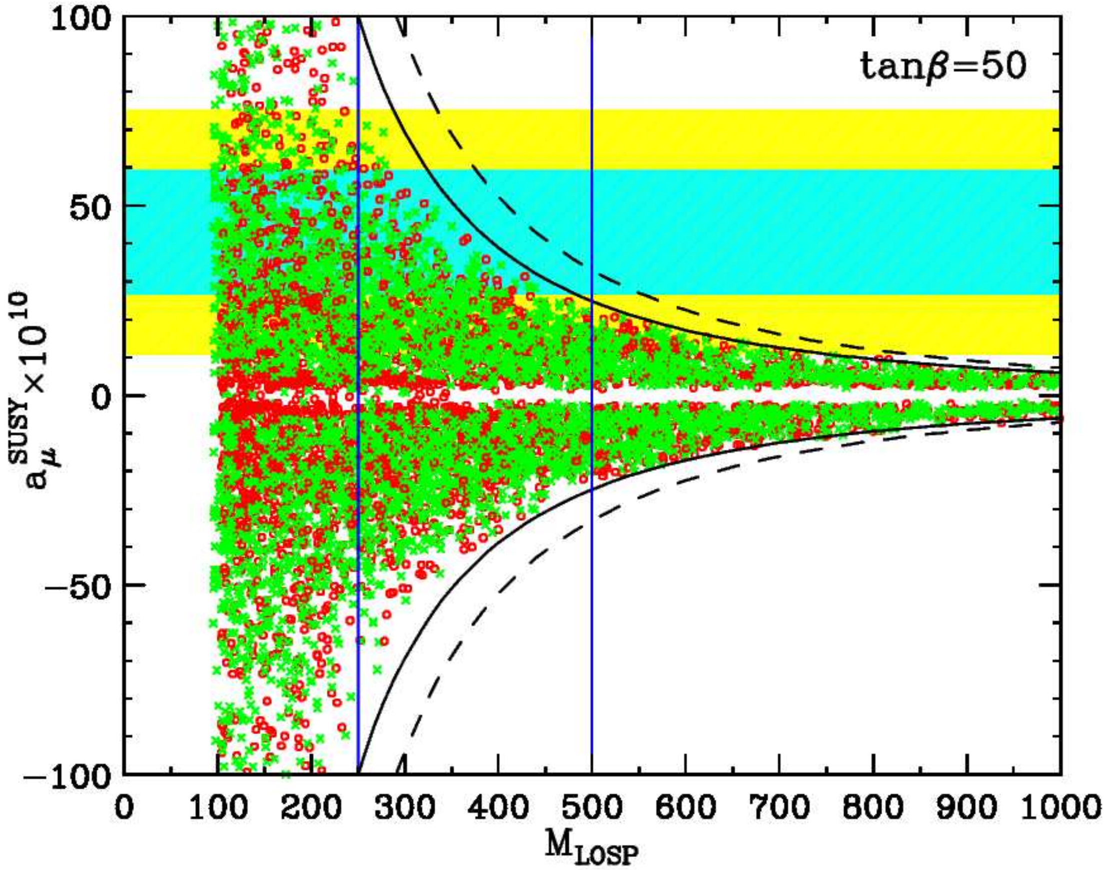
SM prediction too low by $\approx (25 \pm 9)$

Could SUSY be the explanation?

a_μ in the MSSM

Possible SUSY contributions:

[Matchev,Feng'01]



Scan over μ, M_2, $m_{\tilde{\mu}_L}$, $m_{\tilde{\mu}_R}$.

$\tilde{\mu}$ or $\tilde{\chi}$ =lightest observable sparticle

Dashed: $|A_\mu|$ varied up to 100TeV

$\Rightarrow$ SUSY could easily explain the discrepancy

a_μ in the MSSM

Summary SUSY 1-Loop contributions:

- $a_\mu \propto \tan\beta \, \mathrm{sgn}(\mu)$, large contributions

$$
\begin{aligned}
a_\mu^{1L} &= (-100\ldots+100) \times 10^{-10} \\
a_\mu^{\text{exp}} - a_\mu^{\text{SM}} &= (25 \pm 9) \times 10^{-10}
\end{aligned}
$$

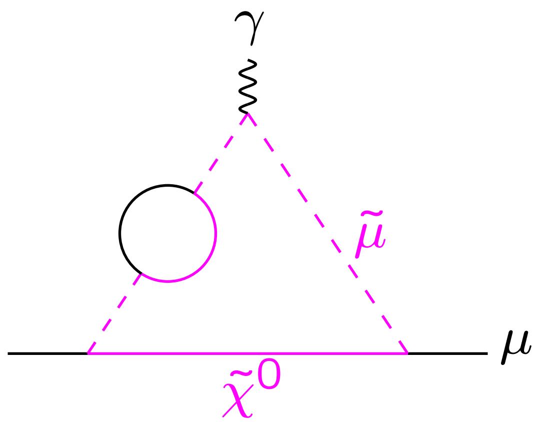
- strong restrictions on SUSY parameter space

Motivation for 2-Loop calculation

- precise knowledge of a_μ to $\approx 1 \times 10^{-10}$ necessary to derive precise bounds on SUSY parameters
- $a_\mu^{\text{2L,SM,EW}} = -23\%$ of $a_\mu^{\text{1L,SM,EW}}$
- $a_\mu^{\text{2L,SUSY}}$ could be large, even if 1-Loop is small

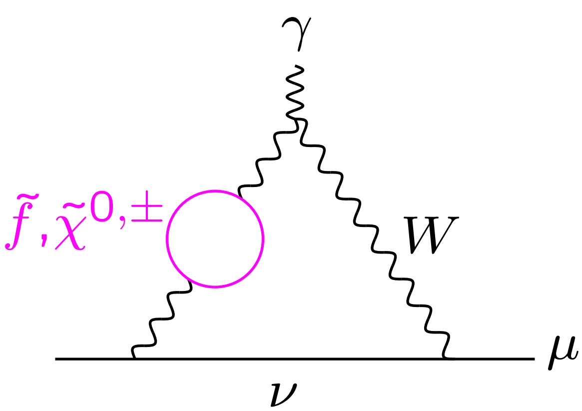
2-Loop contributions

diagrams involving a 1-Loop SUSY diagram



- QED-logs: $\frac{4\alpha}{\pi}\log\frac{m_\mu}{M_{\text{SUSY}}} \times a_\mu^{\text{1L,SUSY}} \approx -9\%$

SM/2HDM 1-Loop diagram + SUSY insertion

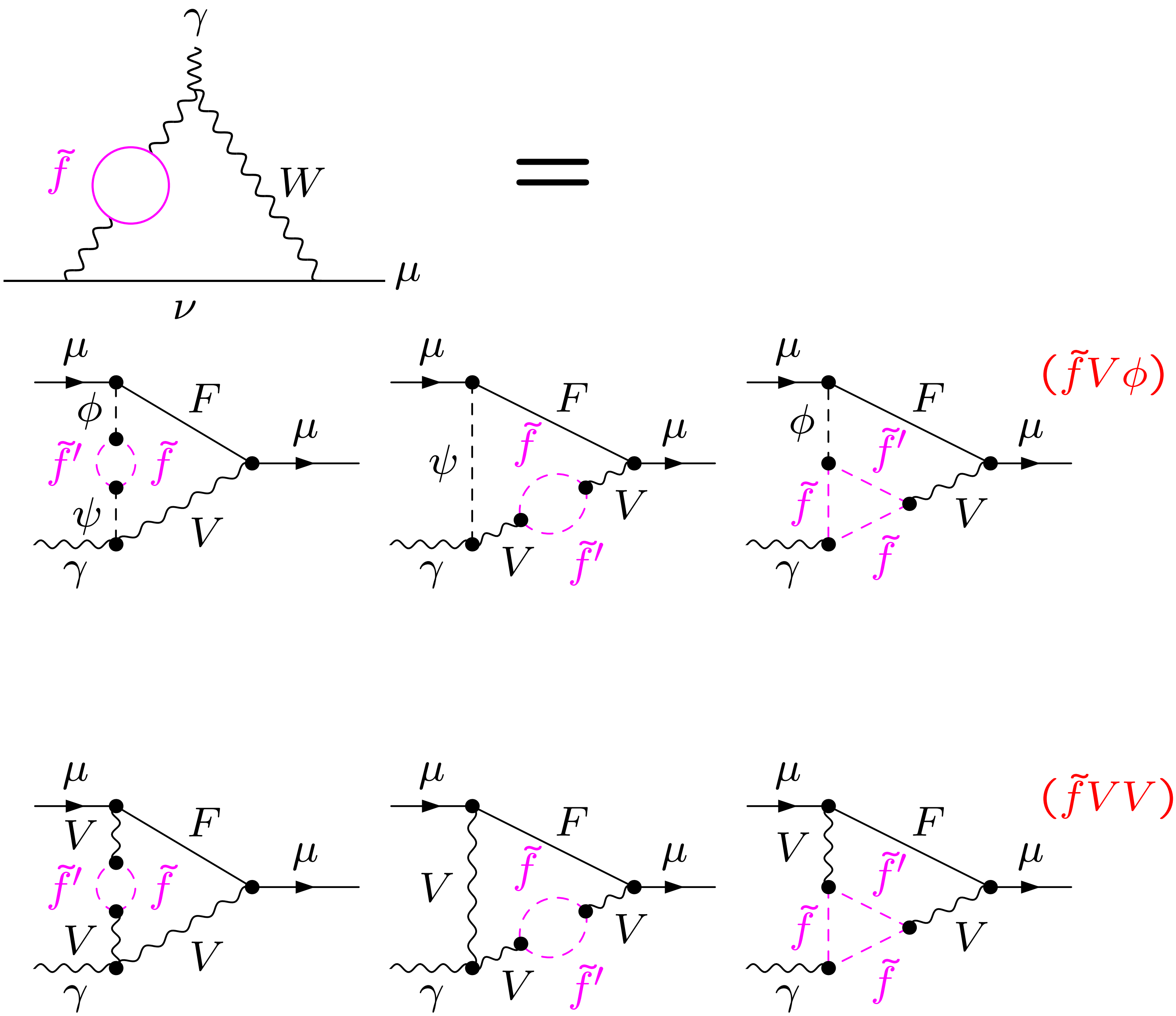


- could be large even if 1-Loop contribution is small (e.g. heavy smuons, charginos but light stops)

$\longrightarrow$ our calculation: $\tilde{f}$ and $\tilde{\chi}^{0,\pm}$ insertions and 2HDM diagrams: f and boson insertions

2-Loop contributions

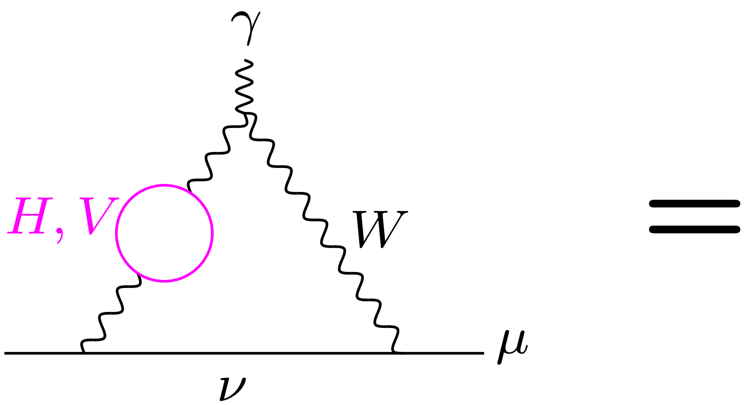
First three classes similar:

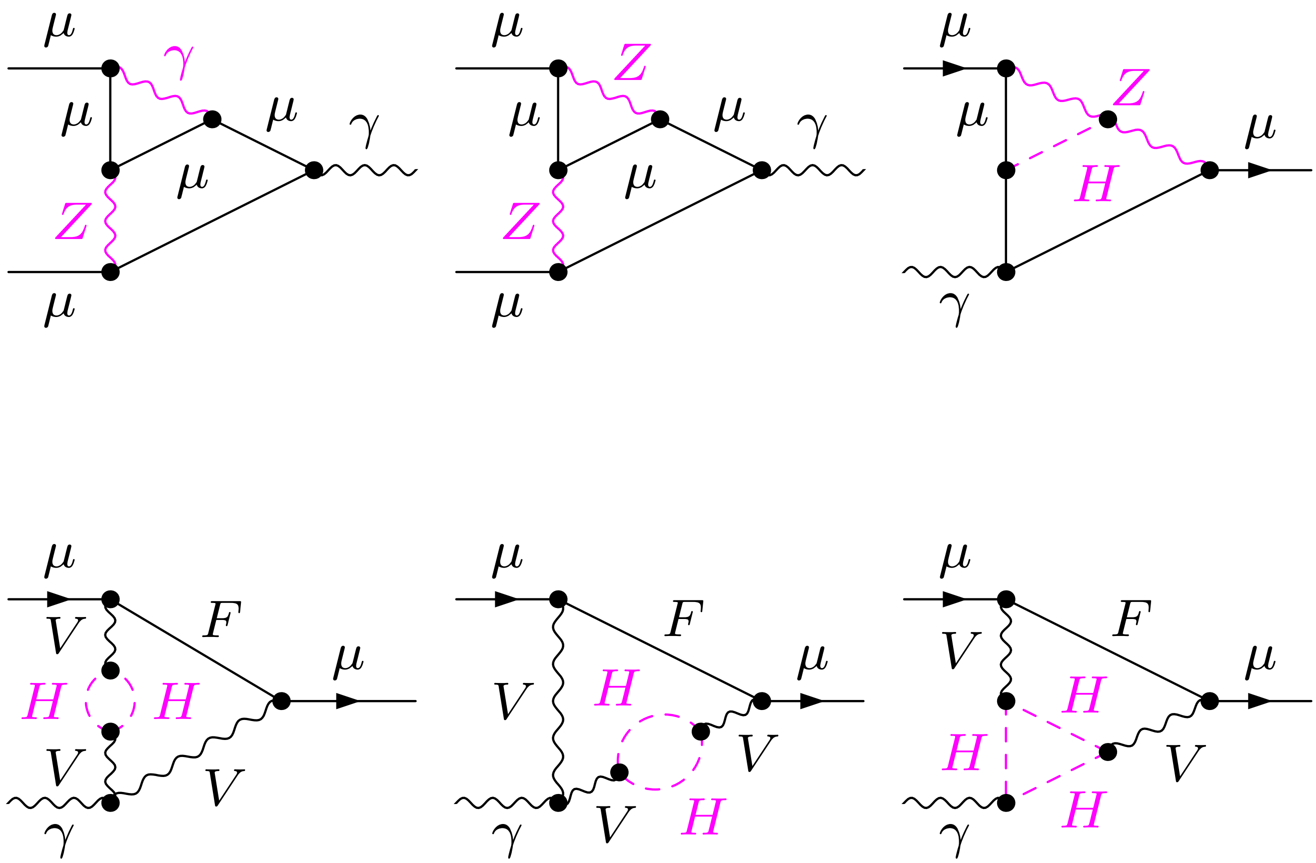


Similar for $\tilde{f}$, $\tilde{\chi}$, f

2-Loop contributions

Fourth class different
(bosonic 2HDM diagrams):



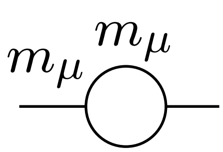


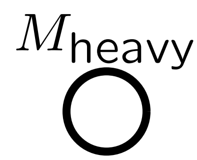
no closed loop necessary

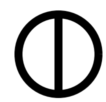
2-Loop contributions

Large mass expansion with closed heavy loop

$$\Gamma \to \sum_{\gamma \supset \text{heavy subgraph}} \Gamma/\gamma \quad \star \quad \mathcal{T}(\gamma)$$

either 

$\star$ 

or  $\star$ 

Reduction of numerators/powers of denominators

- Numerators: Passarino/Veltman
- Denominators: Integration by parts

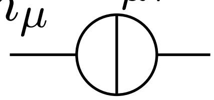
Result:

- A_0, B_0, 2-loop vacuum master integral
- Coefficients: rational functions in masses, couplings

2-Loop contributions

Large mass expansion without closed heavy loop

$$\Gamma \to \sum_{\gamma \supset \text{heavy subgraph}} \Gamma/\gamma \quad \star \quad \mathcal{T}(\gamma)$$

can be m_μ m_μ,0  $\star$ M_{heavy} $\bullet$

In addition: two-loop two-point integral with one mass scale

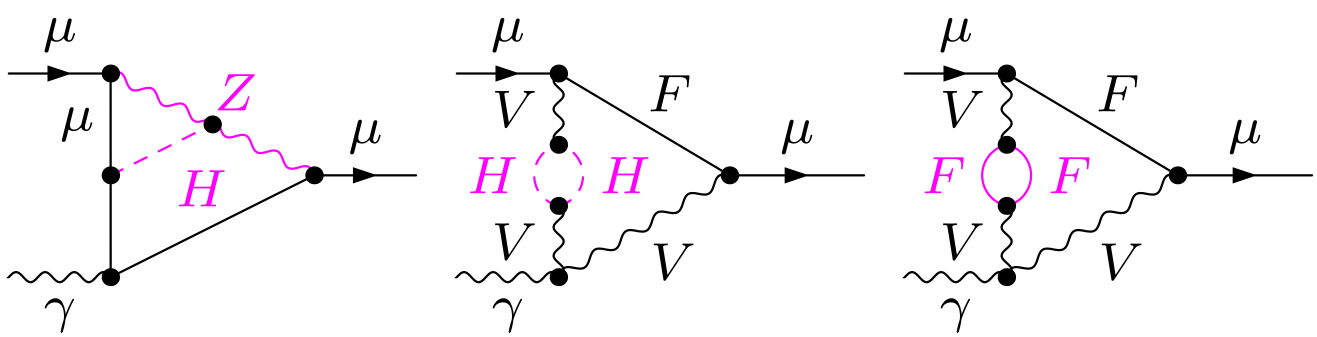
- Reduction to master integrals more involved
- Can involve spurious $1/(D-4)^3$ divergences

$$Y^1_{2345} = -\frac{1}{2(D-4)}\Big\{4A_0(m_\mu)\ B_0(m_\mu^2, 0, m_\mu) + (D-2)\Big[\frac{A_0(m_\mu)^2}{m_\mu^2(D-3)} - 2T_{234}\Big]\Big\}.$$

Pure 2HDM diagrams

First question

- cross check with known SM results
- calculate 2HDM contributions



SM/2HDM diagrams with bosonic/fermionic loops

Pure 2HDM diagrams

Pure 2HDM diagrams

- no SUSY particles
- no fermion loops
- "bosonic contributions"

Difference MSSM vs SM: more Higgs bosons

SM: recalculation

- agreement with *[Czarnecki, Krause, Marciano '96]*

$$a_\mu^{\text{bos,2L}} = \frac{5}{3}\frac{G_\mu m_\mu^2}{8\pi^2\sqrt{2}}\frac{\alpha}{\pi}\left(c_L^{\text{bos,2L}} \log\frac{m_\mu^2}{M_W^2} + c_0^{\text{bos,2L}}\right)$$

- log-piece:

$$c_L^{\text{bos,2L}} = \frac{1}{30}\left[107 + 23(1-4s_W^2)^2\right] \approx 3.6$$

- remainder: exact expression in M_H, but

$$|a_\mu(c_0^{\text{bos,2L}})| < 0.2 \times 10^{-10}$$

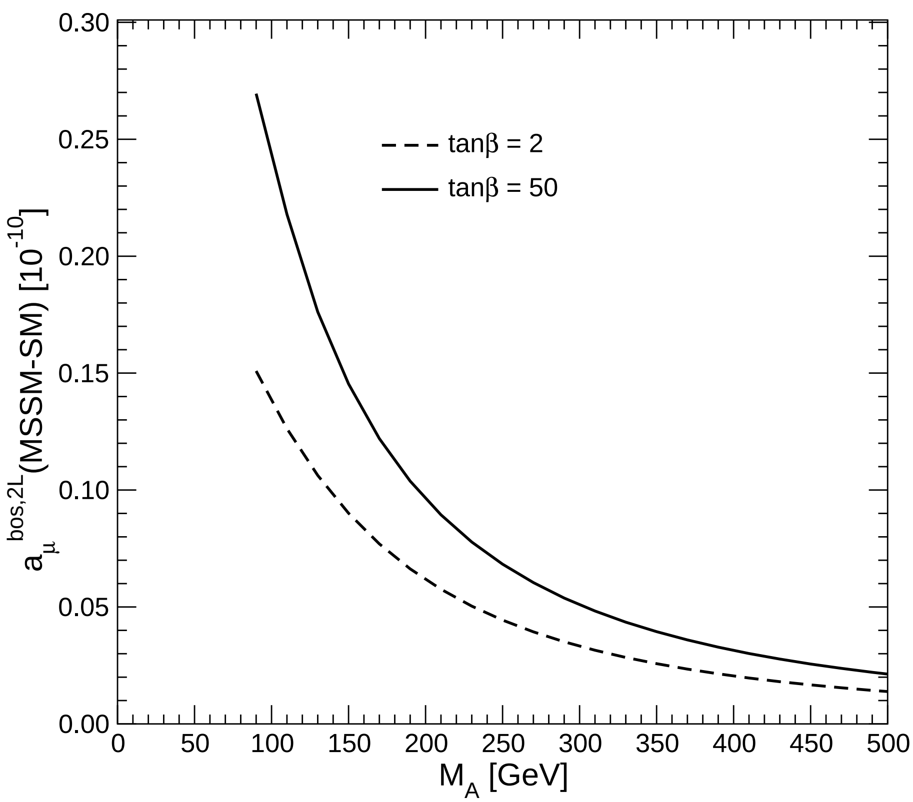
Pure 2HDM diagrams

MSSM: new calculation of “bosonic contributions”

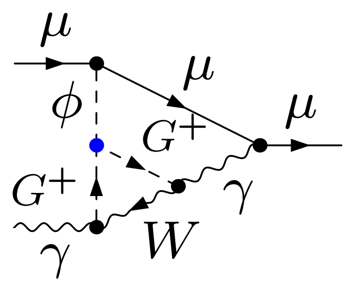
⇔ 2HDM diagrams

Result:

- log-piece identical to SM
- remainder: depends on $\tan\beta$, M_A



Pure 2HDM diagrams



Logarithms: M_H dependence

SM: $\bullet \propto \frac{M_H^2}{M_W} \quad \Rightarrow \frac{M_H^2}{M_H^2}$

MSSM: $\bullet \propto M_W \quad \Rightarrow \frac{c_{2\beta}M_Z^2}{c_\beta}\left[\frac{c_\alpha c_{\alpha+\beta}}{M_H^2} + \frac{s_\alpha s_{\alpha+\beta}}{M_h^2}\right]$

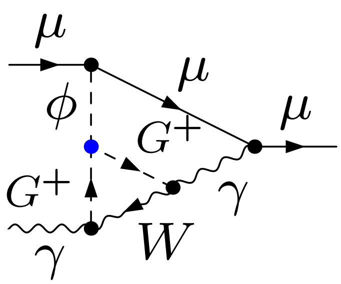
RG argument *[Degrassi,Giudice]*: $\log\frac{m_\mu}{M_W}$ terms from bosonic diagrams should be identical in the SM and MSSM

Background field gauge, unitary gauge, . . . : above diagrams=0

at tree level: $\frac{c_{2\beta}M_Z^2}{c_\beta}\left[\frac{c_\alpha c_{\alpha+\beta}}{M_H^2} + \frac{s_\alpha s_{\alpha+\beta}}{M_h^2}\right] = 1$

tree-level Higgs masses should be used

Pure 2HDM diagrams



Using loop-corrected Higgs masses...

- includes large 3-loop corrections from ϕ-self energy
- does not include large 3-loop corrections from $\phi G^+ G^-$ vertex

...does not consistently take into account large 3-loop effects

...useful in other sectors but not here

Pure 2HDM diagrams

Results for pure 2HDM diagrams

- SM: check of old calculation
- check of our code
- Difference MSSM-SM $< 0.3 \times 10^{-10}$

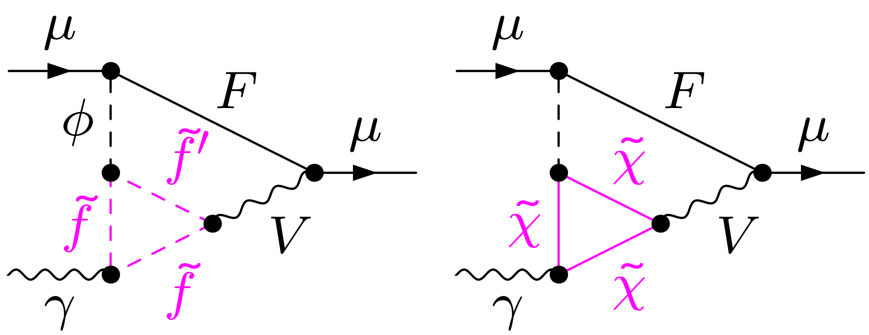
Similarly: results for f loop diagrams

- SM: agreement with known result
- Difference MSSM-SM $< 0.6 \times 10^{-10}$

SUSY 2-Loop contributions

Second question

- results of genuine SUSY contributions



2HDM 1-Loop diagrams with
$\tilde{f}$ or $\chi^{0,\pm}$ loop insertion

2-Loop contributions

Discussion of genuine SUSY results

Sfermion-loop contributions: μ, $\tan\beta$; $m_{\tilde{f}}$, A_f

- complicated dependence
- experimental constraints very important
- e.g. μ=coupling, $\mu \nearrow$: $a_\mu^{\tilde{f}} \nearrow$

Chargino/Neutralino contributions: μ, $\tan\beta$; M_2

- straightforward dependence
- e.g. μ=mass, $\mu \nearrow$: $a_\mu^{\tilde{\chi}} \searrow$

2-Loop contributions

Discussion of genuine SUSY results

Sfermion-loop contributions:

- $a_\mu^{\tilde{f}} =$ up to 20×10^{-10}
  (constraints on susy parameters ignored)
- $a_\mu^{\tilde{f}} < 5 \times 10^{-10}$
  (constraints from $M_h > 114$GeV and from B decays taken into account)

Chargino/Neutralino contributions:

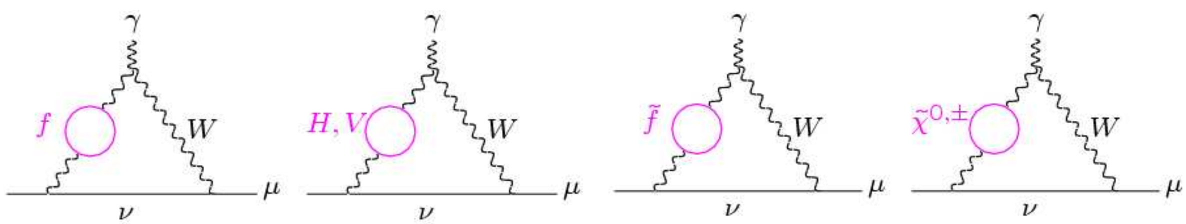
- $a_\mu^{\chi} \approx 13 \times 10^{-10} \left(\frac{\tan\beta}{50}\right) \text{sign}(\mu) \left(\frac{100\text{GeV}}{M_{\text{SUSY}}}\right)^2$
- $a_\mu^{\chi} = 5 \times 10^{-10}$ possible

Conclusions

Final result: $a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (25 \pm 9) \times 10^{-10}$

If SUSY is realized, it has significant impact on a_μ

$$a_\mu^{\text{SUSY,1L}} = (-100 \ldots +100) \times 10^{-10}$$



Confirmation of SM 2-Loop results

MSSM − SM $< [0.6(ferm), 0.3(bos)] \times 10^{-10}$

$a_\mu^{\text{SUSY,2L}} =$ up to 5×10^{-10}