SPS Benchmarks and 3-loop RGEs

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Outline

1. N=1 supersymmetry
2. Soft supersymmetry breaking
3. Results for β-functions
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N = 1 Supersymmetry

A general $N = 1$ theory is described by the superpotential:

$$W(\Phi) = \frac{1}{6} Y^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2} \mu^{ij} \Phi_i \Phi_j$$

The corresponding Lagrangian is:

$$L_{\mathrm{SUSY}} = L_G + L_M$$

where

$$L_G = -\frac{1}{4} G^{\mu\nu} G_{\mu\nu} + i \lambda^\alpha \sigma^\mu_{\alpha\dot{\alpha}} D_\mu \overline{\lambda}^{\dot{\alpha}} + \frac{1}{2} D^2$$

and

$$\begin{aligned} L_M &= i\psi\sigma.D\overline{\psi} + \left|D_\mu\phi\right|^2 + F^i F_i + F^i W_i + F_i W^i \\ &- \frac{1}{2} W_{ij} \psi^i \psi^j - \frac{1}{2} W^{ij} \psi_i \psi_j \\ &+ g \left(R^a\right)^i{}_j \left[D^a \phi_i \phi^j - \sqrt{2} \phi_i \lambda^a \psi^j - \sqrt{2} \phi^j \overline{\psi}_i \overline{\lambda}^a \right] \end{aligned}$$

where $W^i \equiv \frac{\partial W}{\partial \phi_i}$ etc.

Soft Supersymmetry Breaking

The following standard terms are usually added to break supersymmetry:

$$
\begin{aligned}
L^{(1)}_{\rm SOFT} &= (m^2)^j{}_i \phi^i \phi_j \\
&+ \left(\tfrac{1}{6} h^{ijk} \phi_i \phi_j \phi_k + \tfrac{1}{2} b^{ij} \phi_i \phi_j + \tfrac{1}{2} M \lambda\lambda + \mathrm{h.c.}\right).
\end{aligned}
$$

but there is no reason not to have the following *non-standard* terms as well (unless there are gauge singlet fields):

$$
L^{(2)}_{\rm SOFT} = \tfrac{1}{2} r_i^{jk} \phi^i \phi_j \phi_k + \tfrac{1}{2} m_F{}^{ij} \psi_i \psi_j + m_A{}^{ia} \psi_i \lambda_a + \mathrm{h.c.}
$$

None of these terms introduce quadratic divergences so we say they preserve naturalness. But it's usual to ignore $L^{(2)}$.

The Minimal Supersymmetric Standard Model

The superpotential is :

$$W = H_2 Q Y_t t^c + H_1 Q Y_b b^c + H_1 L Y_\tau \tau^c + \mu H_1 H_2$$

The soft breaking terms are:

$$\begin{aligned}
L^{(1)}_{\mathrm{SOFT}} &= \sum_\phi m_\phi^2 \phi^* \phi + \left[m_3^2 H_1 H_2 + \sum_{i=1}^{3} \tfrac{1}{2} M_i \lambda_i \lambda_i + \mathrm{h.c.} \right] \\
&+ \left[H_2 Q h_t t^c + H_1 Q h_b b^c + H_1 L h_\tau \tau^c + \mathrm{h.c.} \right] \\
L^{(2)}_{\mathrm{SOFT}} &= m_\psi \psi_{H_1} \psi_{H_2} + H_1^* Q \hat{h}_t t^c \\
&+ H_2^* Q \hat{h}_b b^c + H_2^* L \hat{h}_\tau \tau^c + \mathrm{h.c.}
\end{aligned}$$

NB the susy limit when $m_4 = \mu$, $\hat{h}_t = \mu Y_t$, $\hat{h}_b = -\mu Y_b$, $\hat{h}_\tau = -\mu Y_\tau$, $m^2_{H_1} = m^2_{H_2} = \mu^2$.

In most supersymmetry-breaking models the $L^{(2)}_{\mathrm{SOFT}}$ terms have coefficients of $O(m^2_{\mathrm{SUSY}}/M)$.

The β-functions

The renormalisation of a supersymmetric theory is governed by the gauge β-function(s) $\beta_g(g, Y, Y^*)$ and the matter multiplet anomalous dimension $\gamma^i{}_j(g, Y, Y^*)$; the latter governs both mass and Yukawa β-functions.

$$
\begin{aligned}
\beta_Y^{ijk} &= Y^{p(ij}\gamma^{k)}{}_p = Y^{ijp}\gamma^k{}_p + (k \leftrightarrow i) + (k \leftrightarrow j) \\
\beta_\mu^{ij} &= \mu^{p(i}\gamma^{j)}{}_p
\end{aligned}
$$

In DRED (Dimensional Reduction) β_g has been calculated through four loops and γ^i_j through three loops in general and through four loops in the ungauged case. (In QCD four loops marks the first appearance in β_g of higher order group invariants; these cancel in the supersymmetric case).

Retaining the top Yukawa only:

$$
\begin{aligned}
\beta_{y_t} &= 6y_t^3 - 22y_t^5 + [102 + 36\zeta(3)]y_t^7 \\
&- [678 + 696\zeta(3) - 216\zeta(4) + 1440\zeta(5)]\, y_t^9.
\end{aligned}
$$

Note the increasing coefficients, and the sign alternation.

The gauge β-function

$$\beta_g^{\mathrm{NSVZ}} = \frac{g^3}{16\pi^2}\left[\frac{Q - 2r^{-1}\mathrm{tr}[\gamma C(R)]}{1 - 2g^2 C(G)(16\pi^2)^{-1}}\right],$$

where $Q = N_f - 3N_c$ for SQCD. For the case $\gamma = 0$:

$$\beta_g^{\mathrm{NSVZ}} = -3\frac{g^3 C(G)}{16\pi^2} - 6\frac{g^5 C(G)^2}{(16\pi^2)^2} - 12\frac{g^7 C(G)^3}{(16\pi^2)^3} + \cdots$$

while

$$\beta_g^{\mathrm{DRED}} = -3\frac{g^3 C(G)}{16\pi^2} - 6\frac{g^5 C(G)^2}{(16\pi^2)^2} - 21\frac{g^7 C(G)^3}{(16\pi^2)^3} + \cdots$$

Evidently β_g^{NSVZ} has a finite radius of convergence when $\gamma = 0$. We can reconcile these results with a redefinition of g:

$$\delta g = \tfrac{3}{2} g^5 C(G)^2 (16\pi^2)^{-2}.$$

$$\delta\beta_g = \beta_g^{(1)} . \frac{\partial}{\partial g}\delta g - \delta g . \frac{\partial}{\partial g}\beta_g^{(1)} = -9 g^7 C(G)^3 (16\pi^2)^{-3}.$$

β_g at Three Loops

$$\begin{aligned}\beta_g^{(3)\mathrm{DRED}} &= \frac{g}{r}\{3X_1 + 6X_3 + X_4 - 6g^6 Q\mathrm{tr}[C(R)^2] \\ &- 4g^4 C(G)\mathrm{tr}[PC(R)]\} \\ &+ g^7 QC(G)[4C(G) - Q]\end{aligned}$$

$$\begin{aligned}\beta_g^{(3)\mathrm{NSVZ}} &= \frac{g}{r}\{2X_1 + 4X_3 - 4g^6 Q\mathrm{tr}[C(R)^2] \\ &- 4g^4 C(G)\mathrm{tr}[PC(R)]\} + 4g^7 C(G)^2 Q\end{aligned}$$

where $\beta_g^{(1)} \sim Q$, $\gamma^{(1)} \sim P$ and
$X_1 = g^2 Y^{klm} P^n{}_l C(R)^p{}_m Y_{knp}$, etc. The redefinition

$$\delta g = \tfrac{1}{2} g^3 \left[r^{-1} \mathrm{tr}\left[PC(R)\right] - g^2 QC(G) \right]$$

is essentially *unique* apart from the overall constant, and leads to

$$\delta\beta_g = \frac{g}{r}\left\{X_1 + 2X_3 + X_4 - 2g^6 Q\mathrm{tr}[C(R)^2]\right\} - g^7 Q^2 C(G).$$

The soft β-functions

The non-standard soft β-functions have been calculated thru 2 loops, and the β-function of the Fayet-Iliopoulos term thru 3 loops.

All the standard soft β-functions can be expressed exactly in terms of the $\beta_g(g, Y, Y^*)$ and the $\gamma(g, Y, Y^*)$ of the unbroken theory:

$$\begin{aligned}
\beta_h^{ijk} &= \gamma_l^{(i} h^{jk)l} - 2\Gamma_l^{(i} Y^{jk)l} \\
\beta_b^{ij} &= \gamma_l^{(i} b^{j)l} - 2\Gamma_l^{(i} \mu^{j)l} \\
\beta_M &= 2\mathcal{O}\left(\frac{\beta_g}{g}\right)
\end{aligned}$$

where

$$\begin{aligned}
\mathcal{O} &= \left(Mg^2\frac{\partial}{\partial g^2} - h\frac{\partial}{\partial Y}\right), \\
(\Gamma)^i{}_j &= \mathcal{O}\gamma^i{}_j.
\end{aligned}$$

The soft scalar mass β-function

The β-function for the scalar m^2 is:

$$\beta_{m^2} = \left[2\mathcal{O}\mathcal{O}^* + 2|M|^2 g^2 \frac{\partial}{\partial g^2} + \left(\tilde{Y} \frac{\partial}{\partial Y} + cc \right) + X \frac{\partial}{\partial g} \right] \gamma$$

where $Y_{lmn} = (Y^{lmn})^*$, and $\tilde{Y}^{ijk} = Y^{l(jk}(m^2)^{i)}{}_l$

Here

$$X_{\text{NSVZ}} = -2 \frac{g^3}{16\pi^2} \frac{r^{-1}\text{tr}[m^2 C(R)] - MM^* C(G)}{1 - 2C(G) g^2 (16\pi^2)^{-1}}.$$

$X_{\text{DRED}'}$ is known through three loops.

This means that we can write down the soft β-functions through three loops. For the MSSM the three loop βs have now been calculated.

Three Loop Results

Here's the three loop result for $\beta_{m^2_{Q_t}}$ in the approximation that we retain only y_t and α_3, in the MSSM with 3 generations and n_5, n_{10} additional $5, 10$ multiplets:

$$\beta^{(1)}_{m^2_{Q_t}} = 2y_t(\Sigma_t + A_t^2) - 32\alpha_3 M_3^2/3$$

$$\begin{aligned}
\beta^{(2)}_{m^2_{Q_t}} &= 16\alpha_3^2 M_3^2(n_5 + 3n_{10} - \tfrac{8}{3}) - 20y_t^2(\Sigma_t + 2A_t^2) \\
&+ \tfrac{16}{3}\alpha_3^2[2m^2_{Q_t} + m^2_{t^c} + m^2_{b^c} \\
&+ (n_{10}+2)(m^2_{u^c} + 2m^2_{Q_u}) + (n_5+2)m^2_{d^c}]
\end{aligned}$$

where $\Sigma_t = m^2_{Q_t} + m^2_2 + m^2_{t^c}$,

$$
\begin{aligned}
\beta^{(3)}_{m^2_{Q_t}} &= [(1280k + \tfrac{20512}{9} + 16n_5^2 + 96n_{10}n_5 + 144n_{10}^2 \\
&+ (\tfrac{6224}{9} + \tfrac{320}{3}k)(n_5 + 3n_{10}))M_3^2 \\
&+ (\tfrac{320}{9} - \tfrac{16}{3}(n_5 + 3n_{10}))(m^2_{t^c} + m^2_{b^c} + 2m^2_{Q_t}) \\
&+ (2m^2_{Q_u} + m^2_{u^c})(\tfrac{640}{9} - \tfrac{32}{3}n_5 + \tfrac{32}{9}n_{10} \\
&- \tfrac{16}{3}n_5 n_{10} - 16n_{10}^2) \\
&+ m^2_{d^c}(\tfrac{640}{9} + \tfrac{224}{9}n_5 - 32n_{10} - 16n_5n_{10} - \tfrac{16}{3}n_5^2)]\alpha_3^3 \\
&- [(288 + \tfrac{544}{3}k + 48(n_5 + 3n_{10}))M_3^2 \\
&- (192 + \tfrac{1088}{9}k + 32(n_5 + 3n_{10}))A_t M_3 \\
&+ (\tfrac{272}{9}k + \tfrac{176}{3} + 8(n_5 + 3n_{10}))(\Sigma_t + A_t^2)]y_t\alpha_3^2 \\
&+ (\tfrac{160}{3} + 32k)\left[M_3^2 - 2A_tM_3 + \Sigma_t + 2A_t^2)\right] y_t^2\alpha_3 \\
&+ (6k + 90)(\Sigma_t + 3A_t^2)y_t^3
\end{aligned}
$$

where $k = 6\zeta(3)$. Note the large coefficients, even for $n_{5,10} = 0$.

Precision QFT and the MSSM

Calculations of sparticle spectrum resulting from given assumptions about the underlying theory have become increasingly refined, with several public programs available (ISAJET, SOFTSUSY, SPHENO, SUSPECT...) that incorporate two-loop Renormalisation Group Equations (RGEs) and one-loop radiative corrections (Bagger, Matchev, Pierce, Zhang 1997)

We have repeated this analysis for a selection of MSSM "benchmark" points and extended it to include three-loop β-function corrections. For weakly interacting particles the three loop running corrections are small but for the squark masses they tend to be the same or bigger than the two loop ones.

We have also extended this analysis to incorporate additional matter representations. As the amount of matter is increased the effect of two and three loop corrections becomes more dramatic Kolda, March-Russell (1999).

We have also looked at the impact on the running analysis of R-parity violating soft terms.

Snowmass Benchmark Points

Some CMSSM benchmark points

Point	$\tan\beta$	M	m_0	A	signμ
SPS1a	10	$250\mathrm{GeV}$	$100\mathrm{GeV}$	$-100\mathrm{GeV}$	$+$
SPS1b	30	$400\mathrm{GeV}$	$200\mathrm{GeV}$	0	$+$
SPS3	10	$400\mathrm{GeV}$	$90\mathrm{GeV}$	0	$+$
SPS5	5	$300\mathrm{GeV}$	$150\mathrm{GeV}$	$-1\mathrm{TeV}$	$+$

SPS1a: "Typical" MSUGRA point.

SPS1b: "Typical" MSUGRA point with larger $\tan\beta$.

SPS3: Light stau, almost degenerate with the neutralino LSP.

SPS5: Large A-parameter, leading to a light stop quark.

Examples

SPS5 benchmark point: $(m_t = 174.3\text{GeV})$

Particle	1 loop	2 loops	3 loops	AKP
$\tilde{g}$	743	729	727	718-728
$\tilde{u}_L$	684	677	668	676-684
$\tilde{u}_R$	658	656	646	653-660
$\tilde{t}_2$	243	257	240	232-258
LSP	128	120	120	119-121
h	115	115	115	112-119

SPS5 benchmark point: $(m_t = 178.0\text{GeV})$

Particle	1 loop	2 loops	3 loops	AKP
$\tilde{g}$	743	729	727	719-729
$\tilde{u}_L$	684	677	668	676-685
$\tilde{u}_R$	658	656	646	655-660
$\tilde{t}_2$	265	278	263	258-280
LSP	128	120	120	119-120
h	117	118	118	116-122

The light stop mass is very sensitive to the input top quark mass here.

SPS1a benchmark point:

Particle	1 loop	2 loops	3 loops	AKP
$\tilde{g}$	628	613	611	604-612
$\tilde{u}_L$	573	565	557	565-569
$\tilde{u}_R$	552	548	539	547-549
$\tilde{t}_2$	400	399	391	396-401
LSP	104	97	97	95.6-97.4
h	114	114	114	112-115

Conclusions

The LHC and an e^+e^- linear collider will measure sparticle masses with high accuracy. Very precise theoretical calculations will be required to disentangle the parameters of the underlying theory from the observations, and to distinguish, for example, nonuniversal boundary conditions from extra matter in the Desert or R-parity violation. By LHC-time state-of-the-art calculations will consist of complete two-loop mass-shell/threshold effects (with some three loop effects), plus the three loop running presented here.