Renormalization scheme of the MSSM

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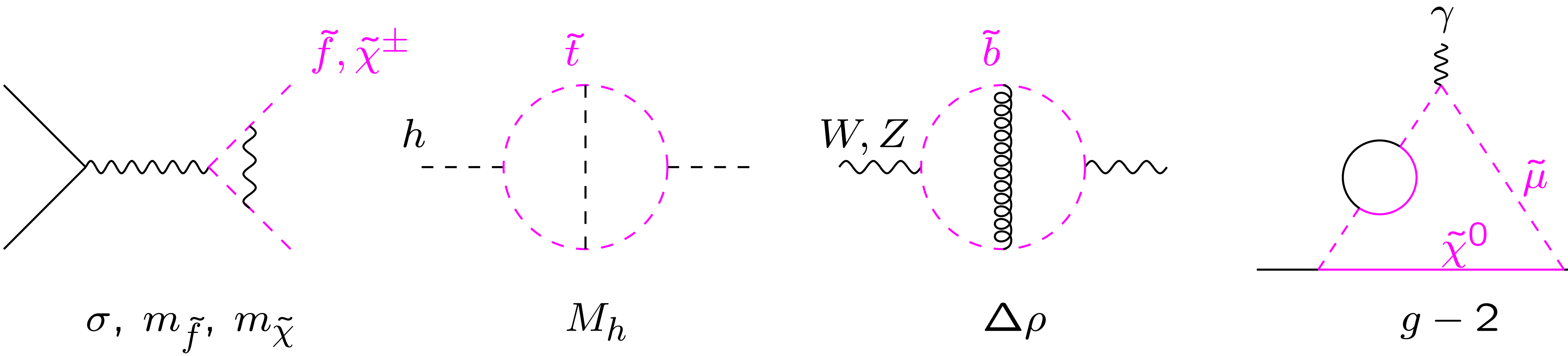
- Motivation — the need for a consistent definition of SUSY parameters
- General problems, our proposal & discussion

Precise measurement of SUSY observables



$\leftrightarrow$ determination of SUSY parameters $\tan\beta,\ \mu,\ \theta_{\tilde{t}},\ A_b, \ldots$

SUSY parameters in $\mathcal{L}$ $\longleftrightarrow$ Observables

$\rightarrow$ SPA project, talk by P. Zerwas

SUSY parameters are
- not directly observable
- not a priori unambiguously defined

Definitions of SUSY parameters
$\Leftrightarrow$ Choice of renormalization scheme

This choice determines:

- relation SUSY parameters in $\mathcal{L}$ $\longleftrightarrow$ Observables
- physical meaning of SUSY parameters
- numerical value of SUSY parameters
- perturbation expansion well- / ill-behaved
- gauge dependence

sample page in particle data book:

$\tan\beta$

$\overline{DR}$ scheme	$\tan\beta = 50$
DCPR scheme	$\tan\beta = 46.4$

[Freitas, DS, m_h^{max} scenario]

A_b

"A_b OS" scheme	$A_b = 2219\text{GeV}$
$\overline{DR}$ scheme	$A_b = 2000\text{GeV}$

[Heinemeyer, Hollik, Rzehak, Weiglein, $\mu = -1\,\text{TeV}$]

→ see talk by H. Rzehak

m_b

$\overline{MS}$ scheme	$m_b = 4.1 \ldots 4.4\text{GeV}$
1S mass	$m_b = 4.6 \ldots 4.9\text{GeV}$

[PDG, today]

(equivalent physics but different values because e.g. $\tan\beta^{\overline{DR}}$ and $\tan\beta^{\text{DCPR}}$ mean different things)

SM: parameters = masses, α, α_s, (CKM matrix)

- parameters closely related to observables
- common definition: on-shell scheme
- but: $\sin^2\theta_W$, m_b, CKM matrix (years of discussions, $\rightarrow$ talk by M. Roth)

MSSM: SUSY parameters: $\tan\beta$, μ, $\theta_{\tilde{t}}$, A_b, ...

- not closely related to any particular observable
- no obvious "best choice" of a def./renorm. scheme
- already now many different definitions exist in the literature

We should converge to one/a few common standards

Situation in Literature:

many different definitions for different applications and different kinds of corrections — particularly elaborate:

Higgs (2-loop): *[Heinemeyer,Hollik,Weiglein; Brignole,Degrassi,Slavich,Zwirner]*
$\tilde{\chi}$: *[Fritzsche,Hollik; Eberl,Kincel,Majerotto,Yamada]*
$\tilde{f}$: *[Hollik,Rzehak; see Higgs-sector]*

- formulations quite different although mainly on-shell
- schemes successfully applied at one-loop/two-loop

Questions:

- Gauge independent for EW corrections?
- Uniform formulation for all sectors?

Aim and outline of the following:

Three general problems:

1.) QCD vs EW / Gauge dependence
2.) On-shell scheme
3.) Parameters $\tan\beta$, mixing angles $\theta_{\tilde{t}}$, ...

Propose convenient renormalization scheme for the MSSM

- uniform
- gauge independent
- easy to use
- numerically well-behaved

Examples and discussion

1.) Charginos 2.) Higgs sector: $\tan\beta$ 3.) Masses

Problem 1: QCD vs EW corrections

QCD: large corrections

- scheme should not give rise to artificially large corrections

EW: spontaneous symmetry breaking

- many schemes are gauge dependent

$\rightarrow$ Conflict

Criterion for gauge independence

Theory with spontaneous symmetry breaking:

symmetric parameters: $g, g', y_t, \mu, M_{\text{soft}}, \ldots$

VEVs: v_1, v_2

param. of broken theory: $M_W^2 = \frac{g^2(v_1^2+v_2^2)}{2}$, $m_t = y_t v_2$, $\tan\beta = \frac{v_2}{v_1}, \ldots$

Criterion:

relation		c.t.s for symm. param.:
parameters in $\mathcal{L}$ $\longleftrightarrow$ Observables	$\Leftrightarrow$	$\delta y_t, \delta\mu, \delta M_{\text{soft}}, \ldots$
gauge independent		gauge independent

e.g. $\overline{DR}$ for y_t, μ, M_{soft} is gauge independent,

while $\overline{DR}$ for M_W, m_t is not

Problem 2: On-shell scheme for masses

$$m^2 = \text{pole of propagator}, \qquad \delta m^2 = \Sigma(p^2 = m^2)$$

$\rightarrow$ Gauge independent

SM: all masses independent $\Rightarrow$ on-shell scheme possible

MSSM: Mass relations , $m_{\tilde{b}_1} = f(m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{b}_2})$, $M_h, m_{\chi^0_{2,3,4}} = f(\ldots)$

$\Rightarrow$ on-shell scheme only for 3 out of 4 masses possible

$\Rightarrow$ on-shell scheme not possible for all masses

$\Rightarrow$ unsymmetric selection necessary

but remain good alternatives since gauge independent and physically motivated

Problem 3: Definition of "difficult parameters"

$\tan\beta$, A_b, mixing angles $\theta_{\tilde{t}}, \theta_{\tilde{b}}, \ldots$

Many different definitions in literature, e.g.

$$\delta\theta_{\tilde{t}} = \frac{\Sigma_{12}(m_1) + \Sigma_{12}(m_2)}{2(m_1^2 - m_2^2)}, \qquad \overline{DR}: \ \delta\theta_{\tilde{t}}^{\text{fin}} = 0, \qquad \delta\tan\beta \propto \frac{\Sigma_{A^0 Z}(M_A^2)}{\cos^2\beta}$$

$\Rightarrow$ no direct relation to physical quantity

These and many other schemes: gauge dependent for EW corrections

Possibility:

$$\underbrace{\begin{pmatrix}\text{all masses and}\\ \text{mixing angles}\end{pmatrix}}_{\text{N parameters}} = \text{function}(\underbrace{\mu,\ \text{soft breaking parameters}}_{\text{M parameters}})$$

M renormalization conditions at our disposal

Impose $\overline{DR}$ scheme directly on $(\mu,$ soft breaking parameters $)$

This implies a certain renormalization for all masses and mixing angles

$\Rightarrow$ uniform treatment of all masses and mixing angles,
gauge independent

such a scheme even works if generation-mixing is taken into account

(6×6 mixing matrix for $\tilde{t}_{1,2}$, $\tilde{c}_{1,2}$, $\tilde{u}_{1,2}$)

Renormalization scheme: Proposal

SM parameters	e, $M_{W,Z}$, m_f, α_s:	as in the SM
Higgs parameters	$\tan\beta$:	$\overline{DR}$
	M_A:	on-shell
	Tadpoles:	$T + \delta t = 0$
Sfermion/Cha/Neu	μ, soft parameters :	$\overline{DR}$ scheme
	($M_{1,2,3}$, $M_{Q,U,D,L,E}$, A_f)	

Renormalization scheme: Properties

SM parameters	e, $M_{W,Z}$, m_f, α_s:	as in the SM
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Sfermion/Cha/Neu	μ, soft parameters :	$\overline{DR}$ scheme
	$(M_{1,2,3},\ M_{Q,U,D,L,E},\ A_f)$	

- uniform, implicit definition of susy masses / mixing angles
- gauge independent even for EW corrections (except $\tan\beta$)
- easy to use, numerically o.k.

1.) Example: Charginos

Renormalization transformation:

$$
\begin{aligned}
m_i &\to m_i + \delta m_i \qquad i = 1, 2 \\
U &\to (1 + \delta u) U \\
V &\to (1 + \delta v) V
\end{aligned}
$$

The proposed scheme implies:

$$
X = \begin{pmatrix} M_2 & \sqrt{2} M_W s_\beta \\ \sqrt{2} M_W c_\beta & \mu \end{pmatrix}, \qquad \delta X^{\text{fin}} = \sqrt{2} \delta M_W^{\text{fin}} \begin{pmatrix} 0 & s_\beta \\ c_\beta & 0 \end{pmatrix}
$$

Evaluation of renormalization constants

$$
\begin{aligned}
\delta m_i &= \left(\text{Re} U^* \delta X V^\dagger \right)_{ii}, \\
(\delta u)^*_{ij} &= \frac{1}{m_i^2 - m_j^2} \left(U^* \delta (X X^\dagger) U^T \right)_{ij}, \\
(\delta v)_{ij} &= \frac{1}{m_i^2 - m_j^2} \left(V \delta (X^\dagger X) V^\dagger \right)_{ij}
\end{aligned}
$$

2.) Higgs sector: tan β

[A. Freitas, DS '02]

Many gauge independent schemes: numerically ill-behaved

Proposed scheme: $\overline{DR}$

+ numerically stable, easy to use, partially gauge independent

− generally gauge dependent

Worse scheme: tan $\beta \leftrightarrow \Sigma_{A^0 Z}(M_A^2) = 0$

[Dabelstein '95; Chankowski et al '94]

− gauge dependent

tan $\beta(\overline{DR})$, M_A(on-shell), $T + \delta t = 0$:
good compromise between
numerical stability, gauge independence, convenience

3.) $\overline{DR}$ scheme for soft parameters vs $\overline{DR}$ for masses

E.g. $\tilde{t}_L$: $M^2_{\tilde{t}_L}$ (physical mass) $\leftrightarrow$ $m^2_{\tilde{t}_L} = M^2_Q + m^2_t + D_L$

In $\overline{DR}$ scheme for $m^2_{\tilde{t}_L}$, we have $(\delta m^2_{\tilde{t}_L})^{\text{fin}} = 0$, and the physical $\tilde{t}$ mass is

$$M^2_{\tilde{t}_L} = m^2_{\tilde{t}_L} - \Sigma^{\text{fin}}_{\tilde{t}_L}(m_{\tilde{t}_L}) = \text{gauge dependent}$$

Proposed scheme:

In the $\overline{DR}$ scheme for M^2_Q, we have $(\delta m^2_{\tilde{t}_L})^{\text{fin}} = (\delta m^2_t + \delta D_L)^{\text{fin}}$ and

$$M^2_{\tilde{t}_L} = m^2_{\tilde{t}_L} + [\delta m^2_t + \delta D_L - \Sigma_{\tilde{t}_L}(m_{\tilde{t}_L})]^{\text{fin}} = \text{gauge independent}$$

In agreement with general statement on gauge (in)dependence

Conclusions

Consistent definition ($\Leftrightarrow$ renormalization scheme)
for all SUSY parameters necessary

Problems:

- not all masses independent, parameters like mixing angles, $\tan\beta$: no obvious "best choice"

- Gauge dependence of many schemes ($\leftrightarrow$ QCD vs EW corrections)

Proposal: SM, M_A on-shell; $\tan\beta$, μ, soft parameters: $\overline{DR}$

- uniform, gauge independent, easy to use, numerically well-behaved

- examples: charginos, $\tan\beta$, masses

Outlook: $\rightarrow$ set of 2–3 complementary standard schemes (e.g. this scheme + on-shell schemes)