High-Precision Predictions for the MSSM Higgs Sector at $\mathcal{O}(\alpha_b \alpha_s)$

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Outline

1. **Higgs-bosons in the MSSM**
2. **the lightest Higgs-boson mass M_{h^0}**
3. **two-loop bottom-sbottom-corrections to M_{h^0}**
4. **conclusions**

Higgs-bosons in the MSSM

physical mass eigenstates: H^0, h^0, A^0, $H^\pm$

- Higgs-boson **masses** are not independent:

  common: A^0-boson mass M_A as free parameter

Existence of one light Higgs boson h^0 in the MSSM:

- **Upper** theoretical **Born** mass limit: $M_{h^0} \leq M_Z$
- quantum corrections of **higher orders** $\Rightarrow M_{h^0} \lesssim 140$ GeV

  depend on MSSM-parameters

Discovery of the Higgs-boson:

- accurate measurement & precise prediction of the mass:

  $\Longrightarrow$ strong bounds on the MSSM-parameters

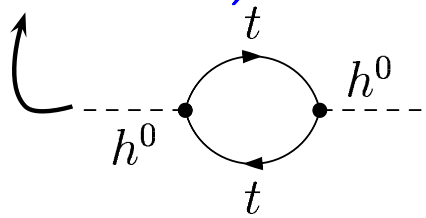
  at the LC: $\Delta M_{h^0}^{\text{exp}} = 0.05$ GeV $\Rightarrow$ small theoretical uncertainty needed

Before the discovery: Constraints on the parameter space

Determination of the Higgs mass

Two-point vertex function:

$$\Gamma = \begin{pmatrix} k^2 - M^2_{H^0_{\text{tree}}} + \hat{\Sigma}_{HH}(k^2) & \hat{\Sigma}_{Hh}(k^2) \\ \hat{\Sigma}_{hH}(k^2) & k^2 - M^2_{h^0_{\text{tree}}} + \hat{\Sigma}_{hh}(k^2) \end{pmatrix}$$



determining the zero of $\det(\Gamma) \Rightarrow M_{h^0}$, M_{H^0}, α_{eff}

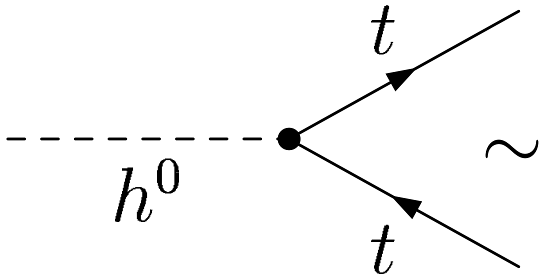
$\uparrow$

eff. mixing angle of H^0, h^0

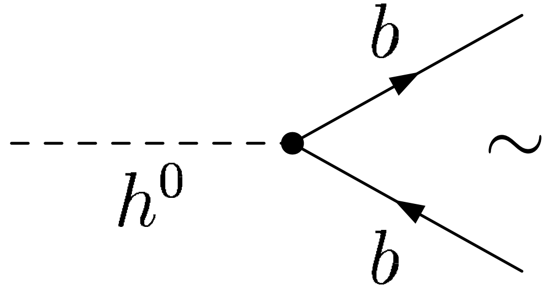
Approximation of the self energy Σ:

- 2-loop-level:
  - vanishing external momentum $\hat{\Sigma}^{(2)}(0)$
  - we consider only terms of order $\mathcal{O}(\alpha_s \alpha_t)$ and $\mathcal{O}(\alpha_s \alpha_b)$

Why calculate these 2-loop corrections?



$\sim \quad \lambda_t \sim \frac{m_t}{\sin\beta}$



$\sim \quad \lambda_b \sim \frac{m_b}{\cos\beta}$

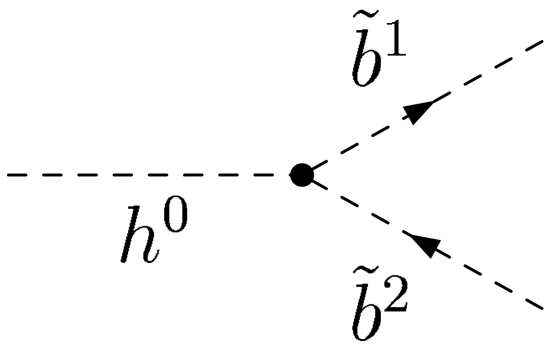
with: $$\frac{\lambda_b}{\lambda_t} = \frac{m_b}{m_t}\tan\beta$$

(λ: Yukawa coupling, $\alpha_t \sim \lambda_t^2$, $\alpha_b \sim \lambda_b^2$)

($\tan\beta = \frac{v_2}{v_1}$; v_1, v_2: Higgs vac. exp. values)

Large contribution: – from the top sector

– from the bottom sector for large $\tan\beta$



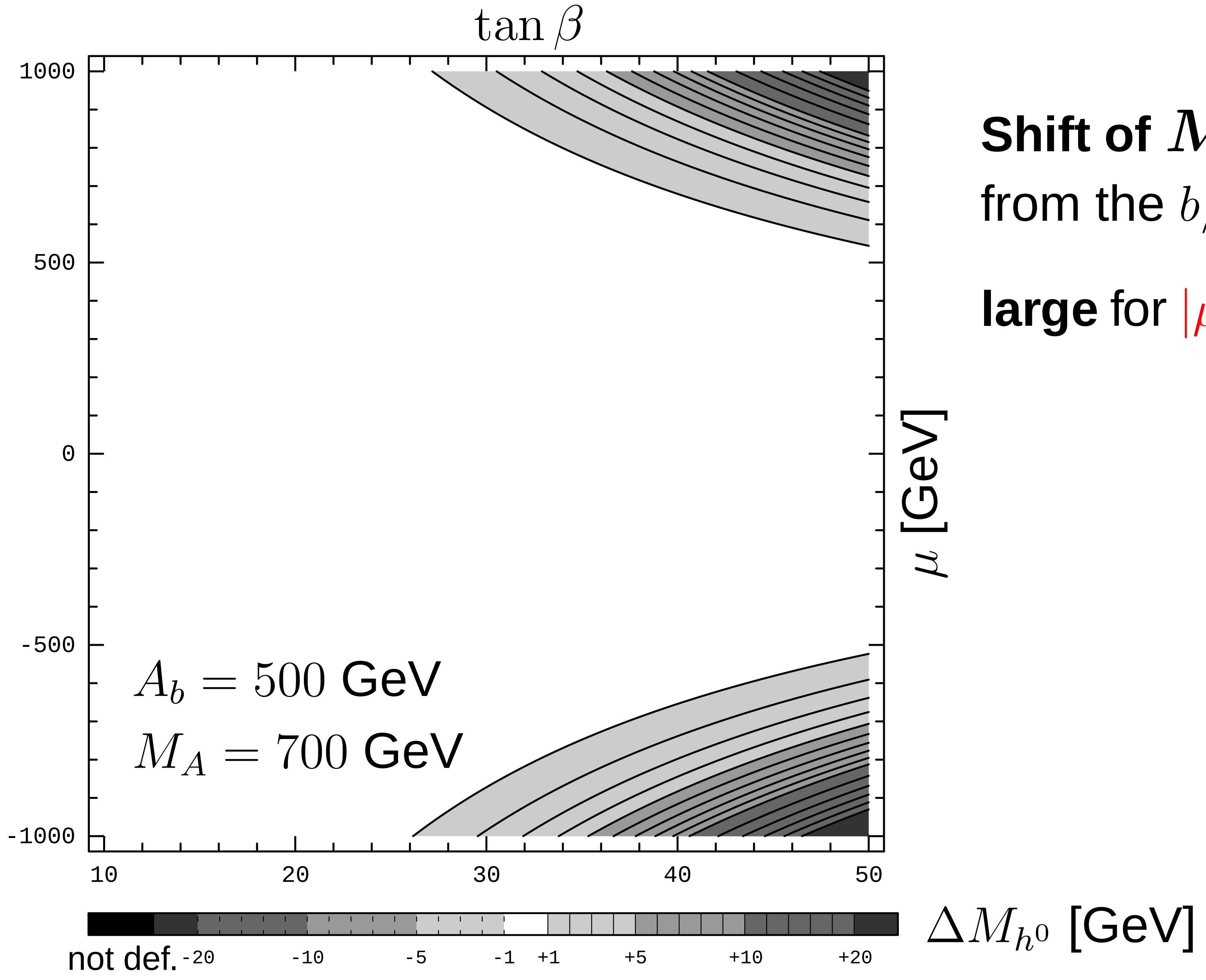
- Yukawa part:

α: mixing angle of h^0, H^0
A_b: SUSY-breaking parameter
μ: Higgsino mass term

$$\sim \lambda_b (A_b^* \sin\alpha + \mu\cos\alpha)$$

$\Rightarrow$ bottom-contribution **large** for μ and $\tan\beta$ large

One-loop corrections in the $b/\tilde{b}$ sector



Shift of M_{h^0} arising from the $b/\tilde{b}$ sector:

large for $|\mu|$ and $\tan\beta$ large

Scheme dependence

Within the 2-loop calculation:

- **parameters of the top/bottom sector** are defined at **one-loop**:

  different choices of schemes are possible

- investigation of **scheme dependence**

  $\Rightarrow$ **information** about **size** of **missing higher order** contributions

  $\Rightarrow$ theoretical error estimate

Here: top sector: only one scheme (masses/mixing angle on-shell)

bottom sector: 4 different schemes

Different schemes

Bottom sector: in all schemes: $\tilde{b}$-masses: $m_{\tilde{b}_1}$ dep., $m_{\tilde{b}_2}$ on-shell

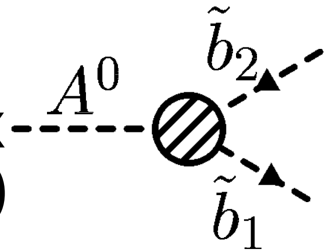
SU(2)-invariance

scheme	b-mass m_b	A_b	mixing angle $\theta_{\tilde{b}}$
m_b $\overline{\text{DR}}$	running ($\overline{\text{DR}}$)	running ($\overline{\text{DR}}$)	dep.
A_b, $\theta_{\tilde{b}}$ OS	dep.	on-shell	on-shell
A_b, $\theta_{\tilde{b}}$ $\overline{\text{DR}}$	dep.	running ($\overline{\text{DR}}$)	running ($\overline{\text{DR}}$)
m_b OS	on-shell	dep.	on-shell

analog. top sector

$$\delta\theta_{\tilde{b}} = \frac{\text{Re}\Sigma_{\tilde{b}_{12}}(m^2_{\tilde{b}_1}) + \text{Re}\Sigma_{\tilde{b}_{12}}(m^2_{\tilde{b}_2})}{2(m^2_{\tilde{b}_1} + m^2_{\tilde{b}_2})}$$

determined via the vertex
(similar in Brignole et. al.)



Bottom quark mass m_b

As $\alpha_b \sim \lambda_b^2 \sim m_b^2$:

a **precise** value of m_b is **necessary** for a **good** prediction of M_{h^0}

value for $\overline{\text{DR}}$-bottom quark mass as **input**:

$$m_b^{\overline{\text{DR}}} = \frac{m_b^{\text{pole}} + \Sigma_b^{t_\beta\text{non.-enh.}}\big|_{\text{fin.}}}{1 + \Delta m_b}$$

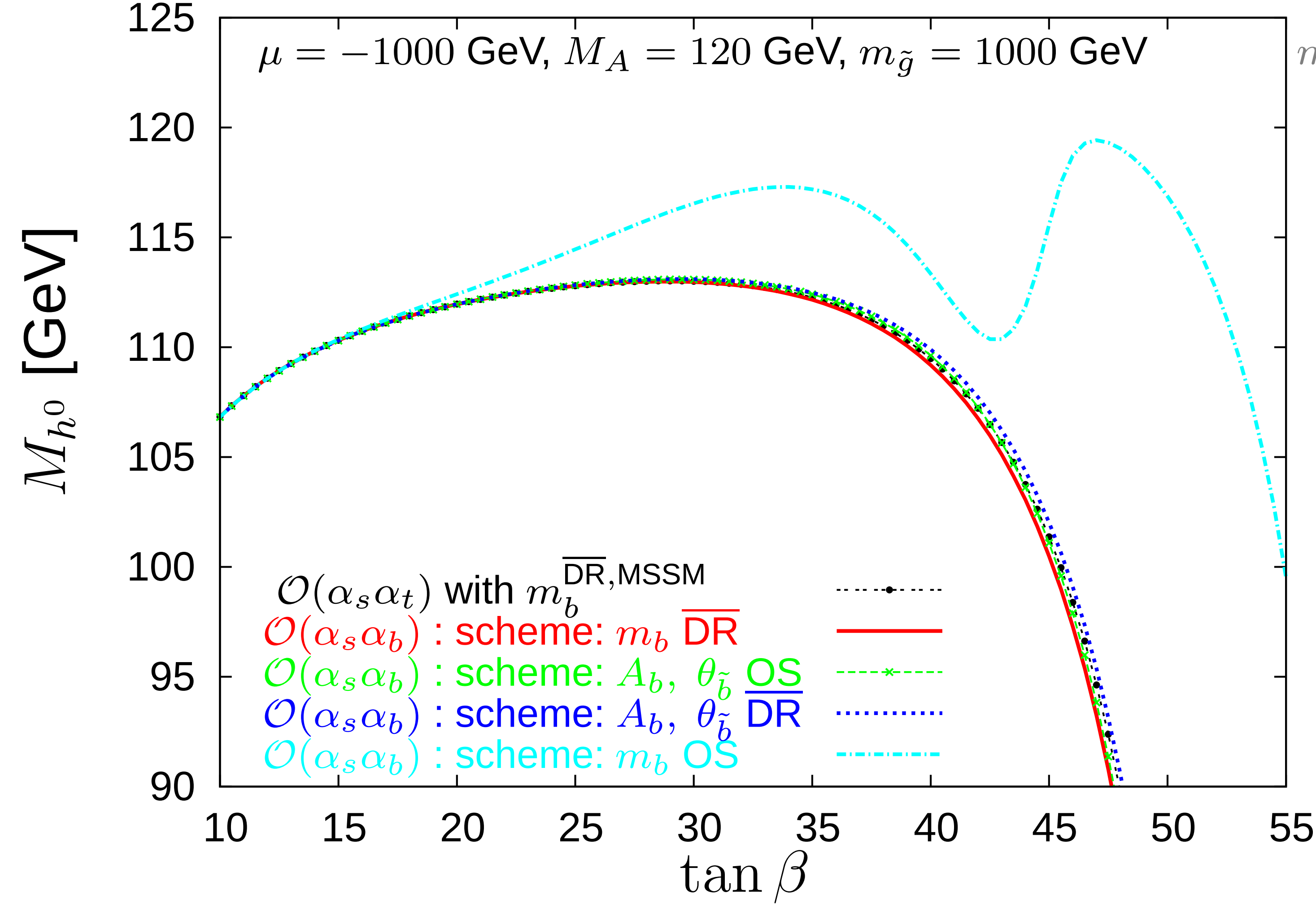
with

$$\Delta m_b \sim \alpha_s \mu \, m_{\tilde{g}} \tan\beta$$

$\Sigma_b^{t_\beta\text{non.-enh.}}\big|_{\text{fin.}}$: $\tan\beta$-non-enhanced terms

$\Rightarrow$ **large higher order** corrections are **included** via one-loop ($\tan\beta$-enhanced)

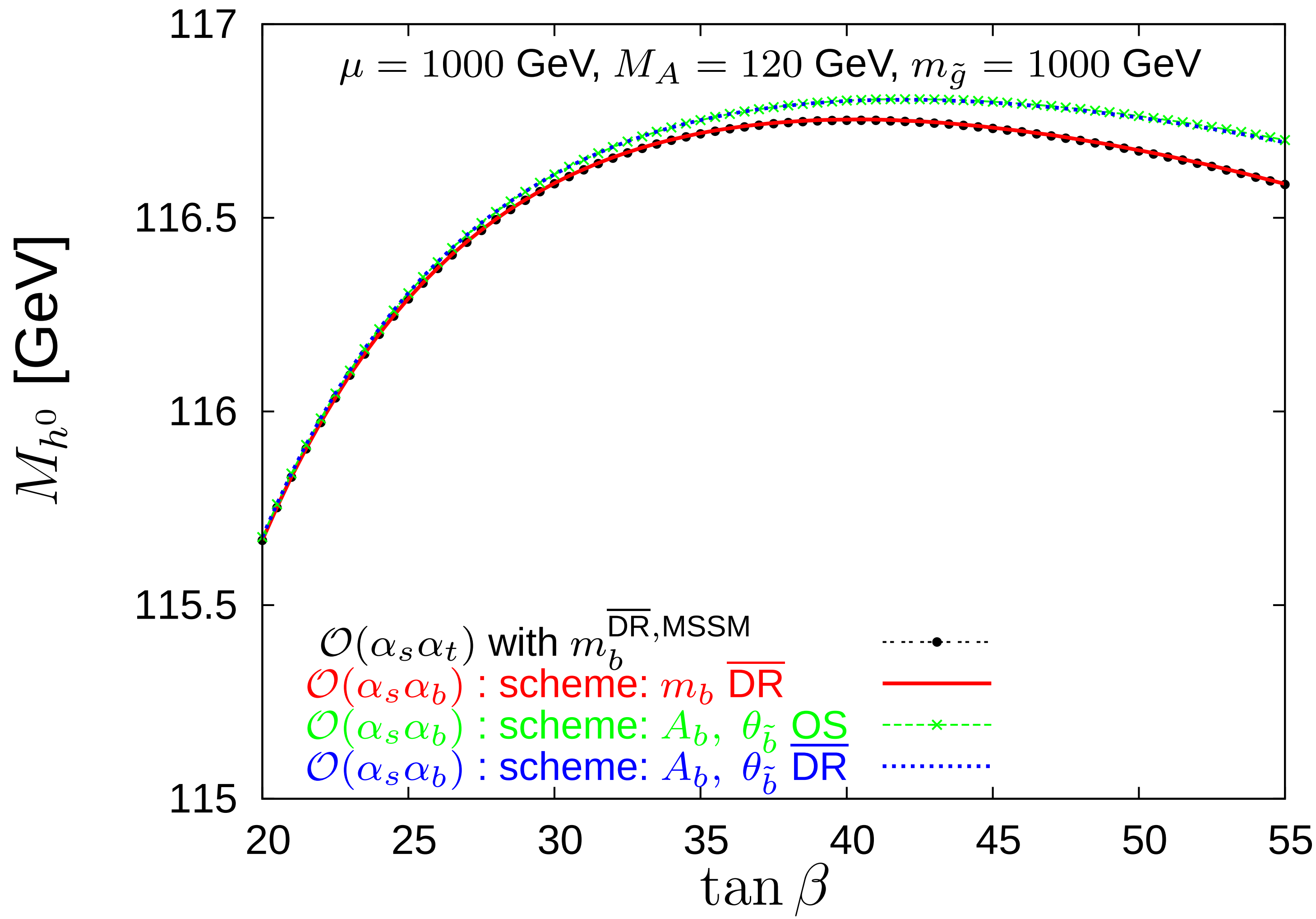
Results: $\tan\beta$-dependence (μ negative)



$m_{\tilde{g}}$: gluino mass

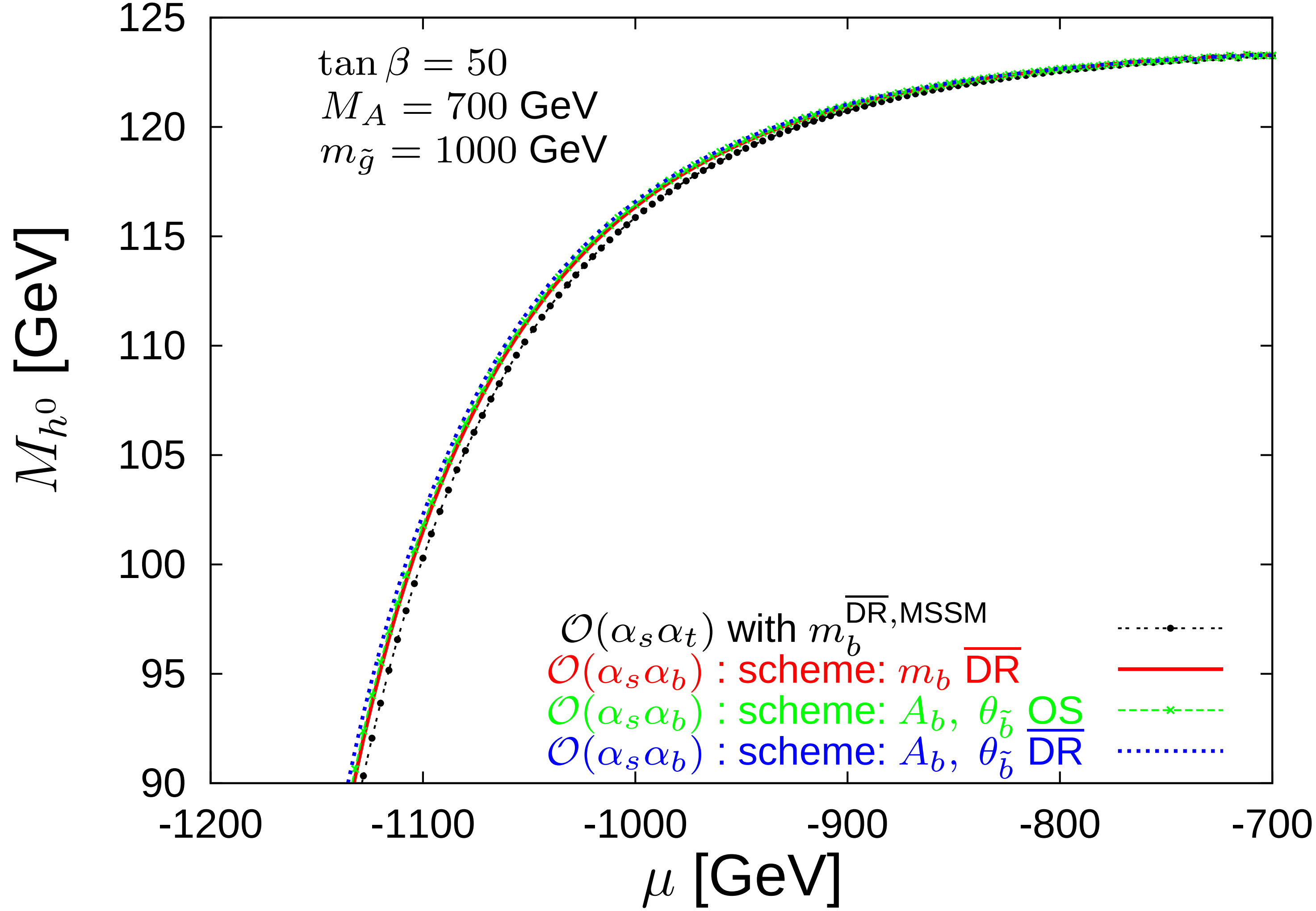
- scheme m_b OS: very large corrections, unpractical scheme
- other schemes: sizeable differences, up to $\mathcal{O}(1$ GeV$)$, for large $\tan\beta$

Results: $\tan\beta$-dependence (μ positive)



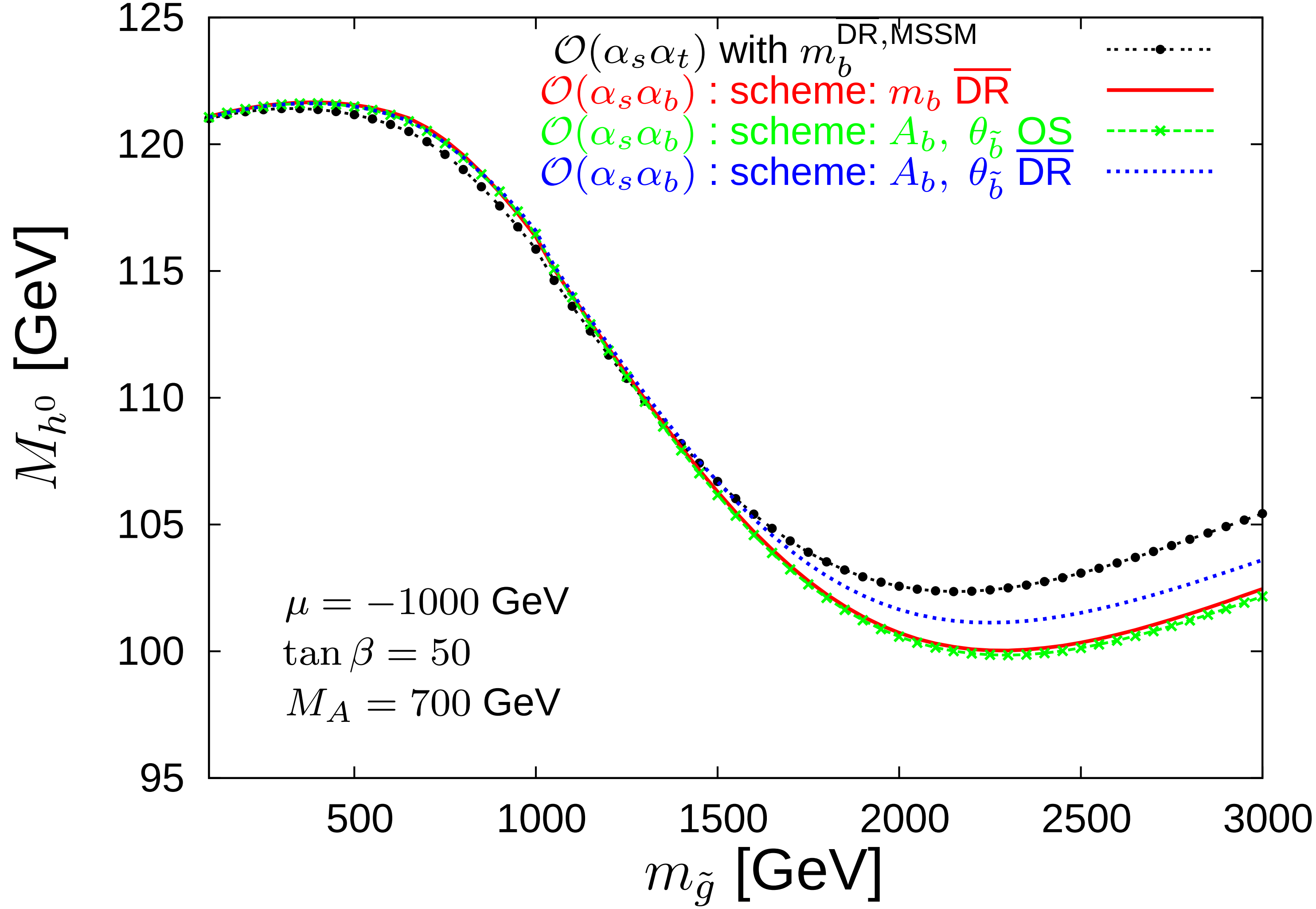
- tiny differences between schemes, max. $\mathcal{O}(0.1$ GeV$)$

Results: μ-dependence



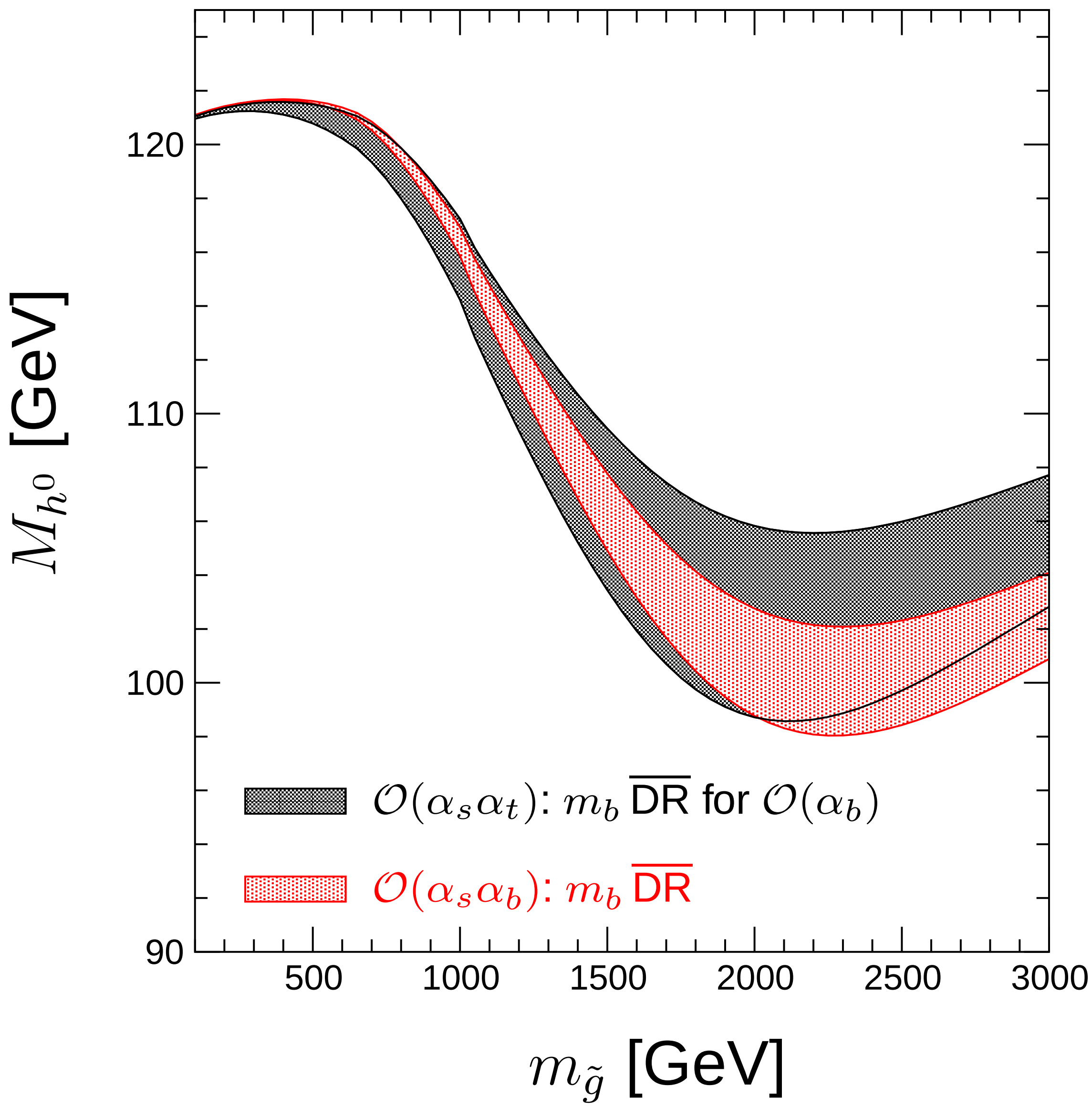
- subleading corrections of $\mathcal{O}(1\text{ GeV})$ for $|\mu| > 1000$ GeV
- differences between schemes of same order, though a bit smaller

Results: $m_{\tilde{g}}$-dependence



- subleading corrections up to $\mathcal{O}(3$ GeV$)$
- scheme differences of the order of $\mathcal{O}(2$ GeV$)$

Results: scale dependence



$M_A = 700$ GeV

$\mu = -1000$ GeV

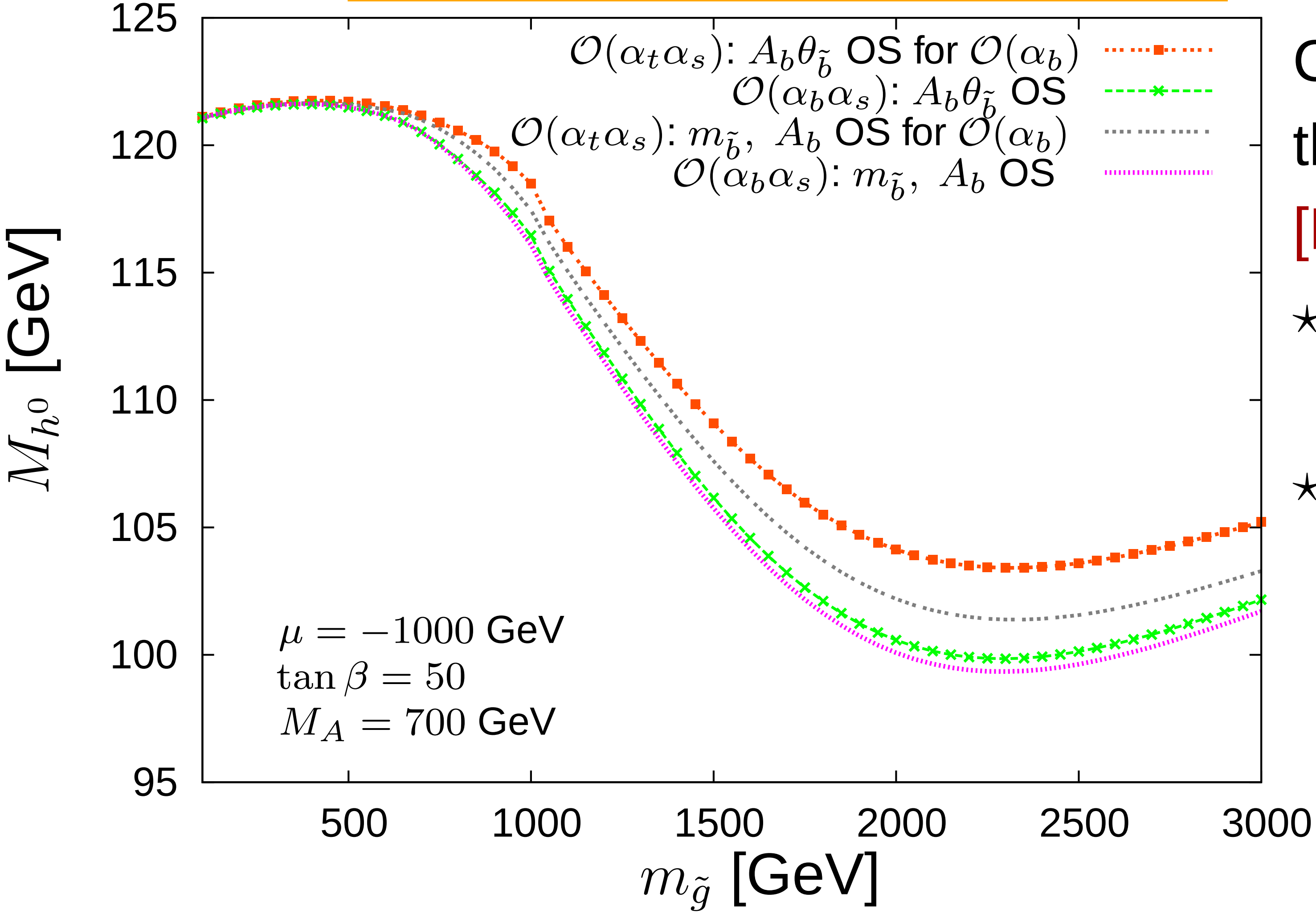
$\tan\beta = 50$ GeV

$m_t/2 \leq \mu^{\overline{\text{DR}}} \leq 2m_t$

$\mu^{\overline{\text{DR}}}$: renormalization scale

- scale dependence up to $\mathcal{O}(\pm 2$ GeV$)$ for large $m_{\tilde{g}}$

Comparison with existing calculations



Comparison with the results of [Brignole et. al.]:

⋆ different treatment of $m_{\tilde{b}_1}$ (on-shell)

⋆ different treatment of $\tan\beta$ ($\tan\beta \to \infty$)

- one-loop results including resummation differ by up to 2 GeV
- two-loop results including subleading $\mathcal{O}(\alpha_s\alpha_b)$ corrections differ only by 0.5 GeV for large $m_{\tilde{g}}$

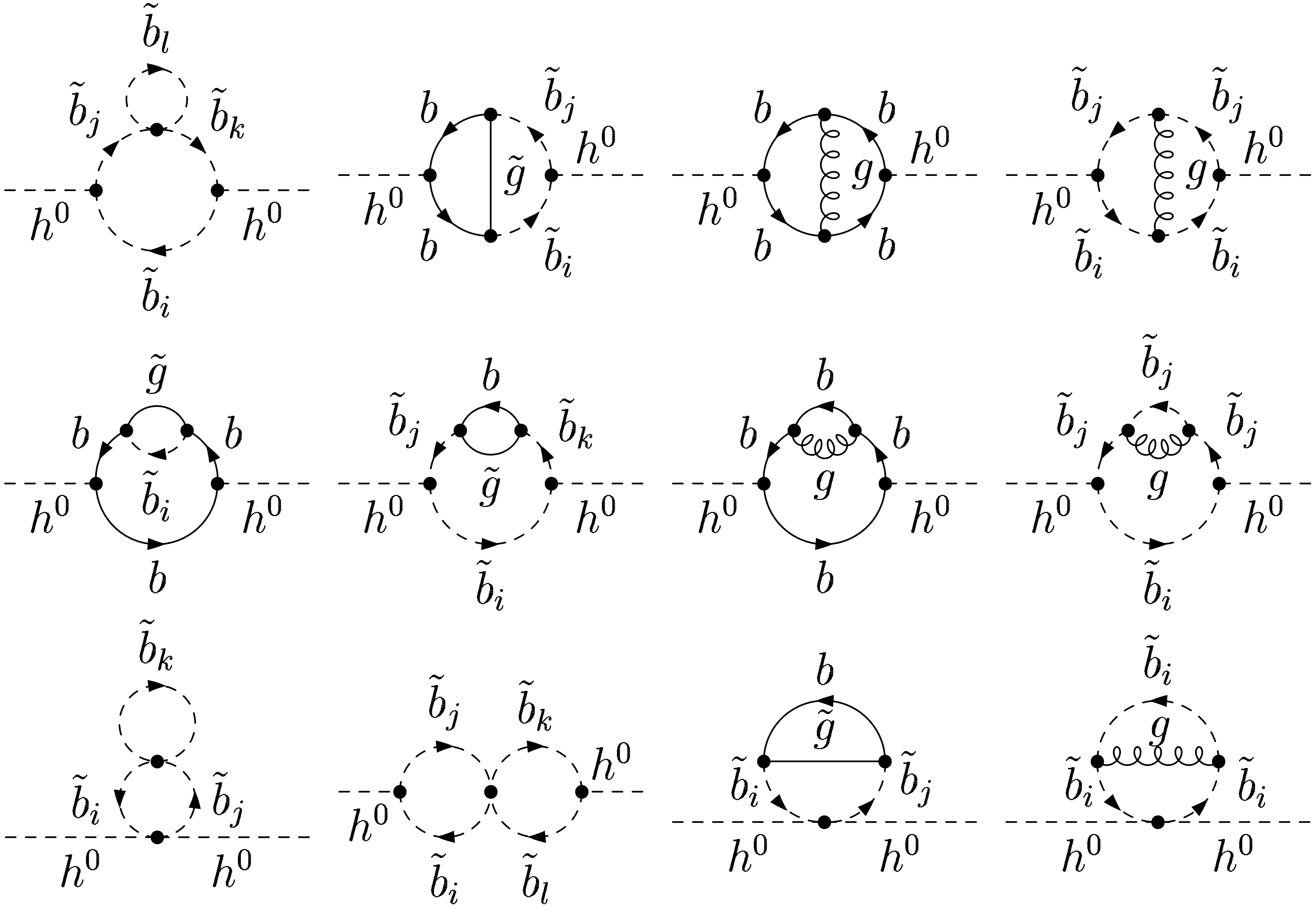
Summary

- The **knowledge** of quantum corrections is **necessary** for **precise** theoretical predictions of M_{h^0}.

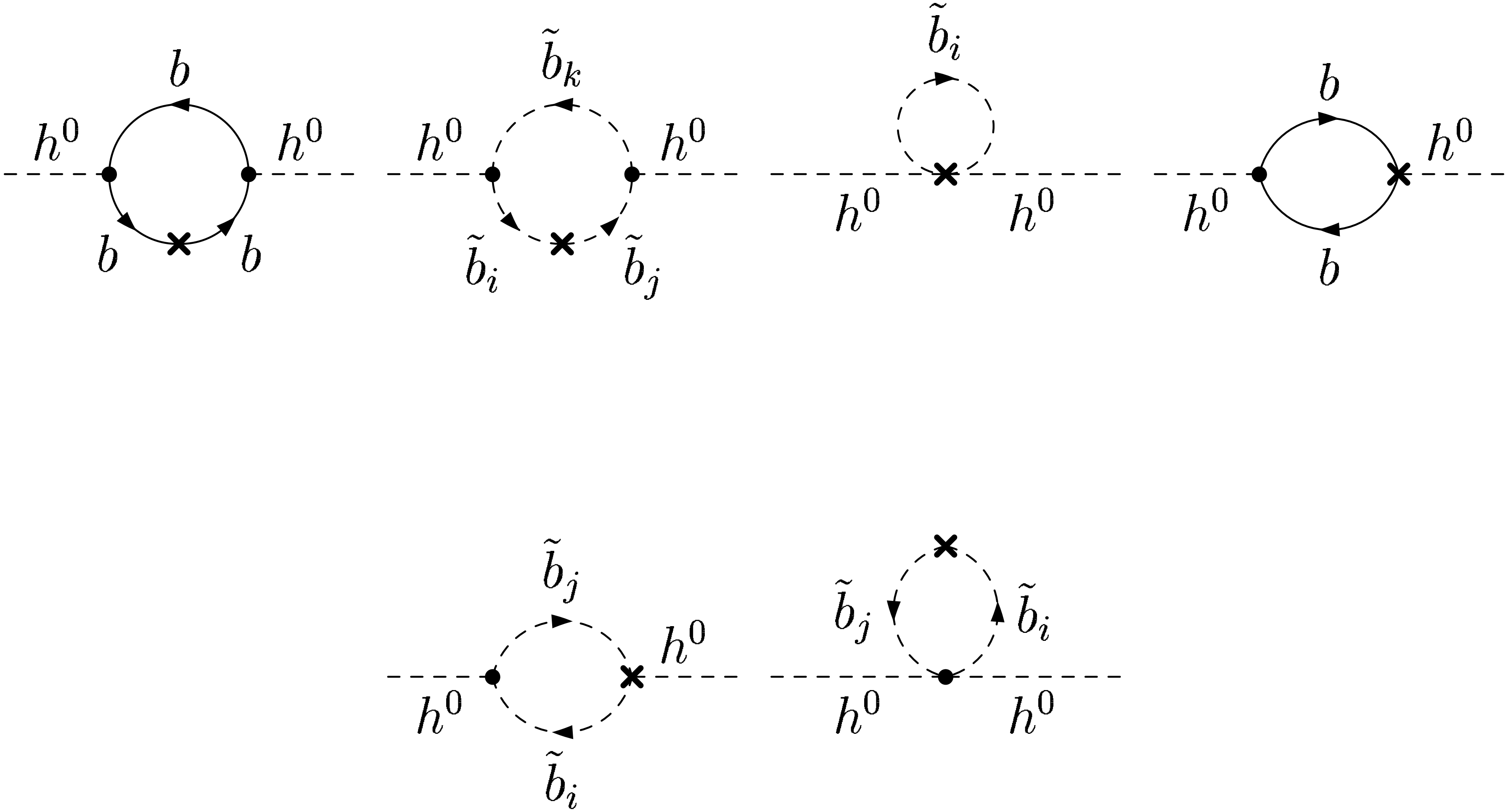
- bottom-quark/squark-corrections:
  - $\star$ relevant for **large** $\boldsymbol{\mu}$ and $\boldsymbol{\tan\beta}$
  - $\star$ subleading two-loop contributions of $\mathcal{O}(\alpha_s\alpha_b)$ can yield shifts up to 3 GeV.

- **first comparison** between different schemes for $\mathcal{O}(\alpha_s\alpha_b)$:
  - $\star$ for positive μ: corrections are under control
  - $\star$ for negative μ: differences between schemes are of the order of $\mathcal{O}(\pm 2$ GeV$)$
  - $\star$ corrections will be included into FeynHiggs (*www.feynhiggs.de* [Heinemeyer et. al.])

Feynman diagrams $\mathcal{O}(\alpha_s \alpha_b)$:



with counterterm insertions $\mathcal{O}(\alpha_s \alpha_b)$:



Default scheme

scheme	b-mass m_b	A_b	mixing angle $\theta_{\tilde{b}}$
m_b $\overline{\text{DR}}$	running ($\overline{\text{DR}}$)	running ($\overline{\text{DR}}$)	dep.

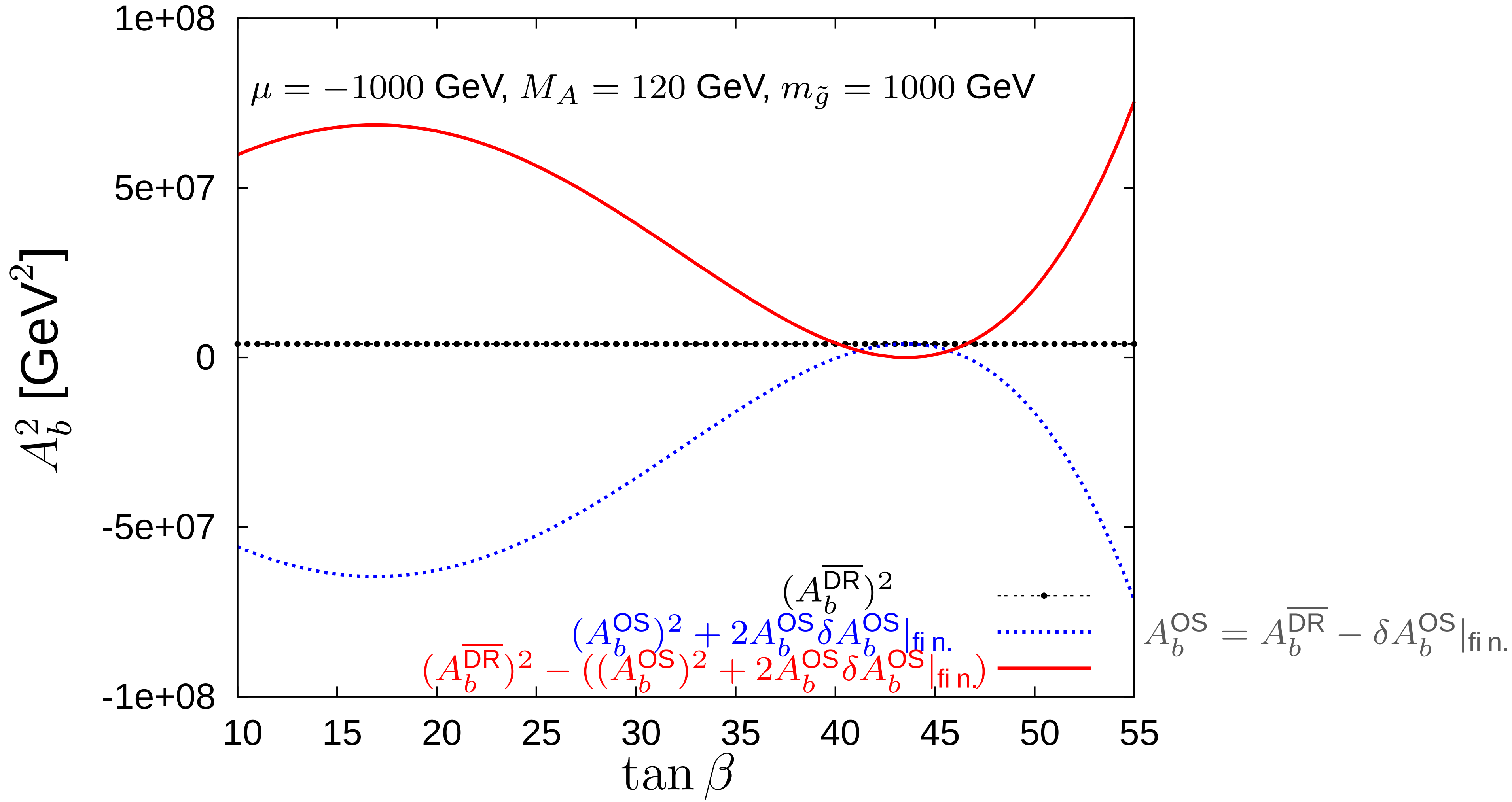
To receive comparable results:

Conversion:

$$m_b^{\text{diff. scheme}} = m_b^{\overline{\text{DR}}} - \delta m_b^{\text{diff. scheme}}\big|_{\text{fin.}}$$

$$A_b^{\text{diff. scheme}} = A_b^{\overline{\text{DR}}} - \delta A_b^{\text{diff. scheme}}\big|_{\text{fin.}}$$

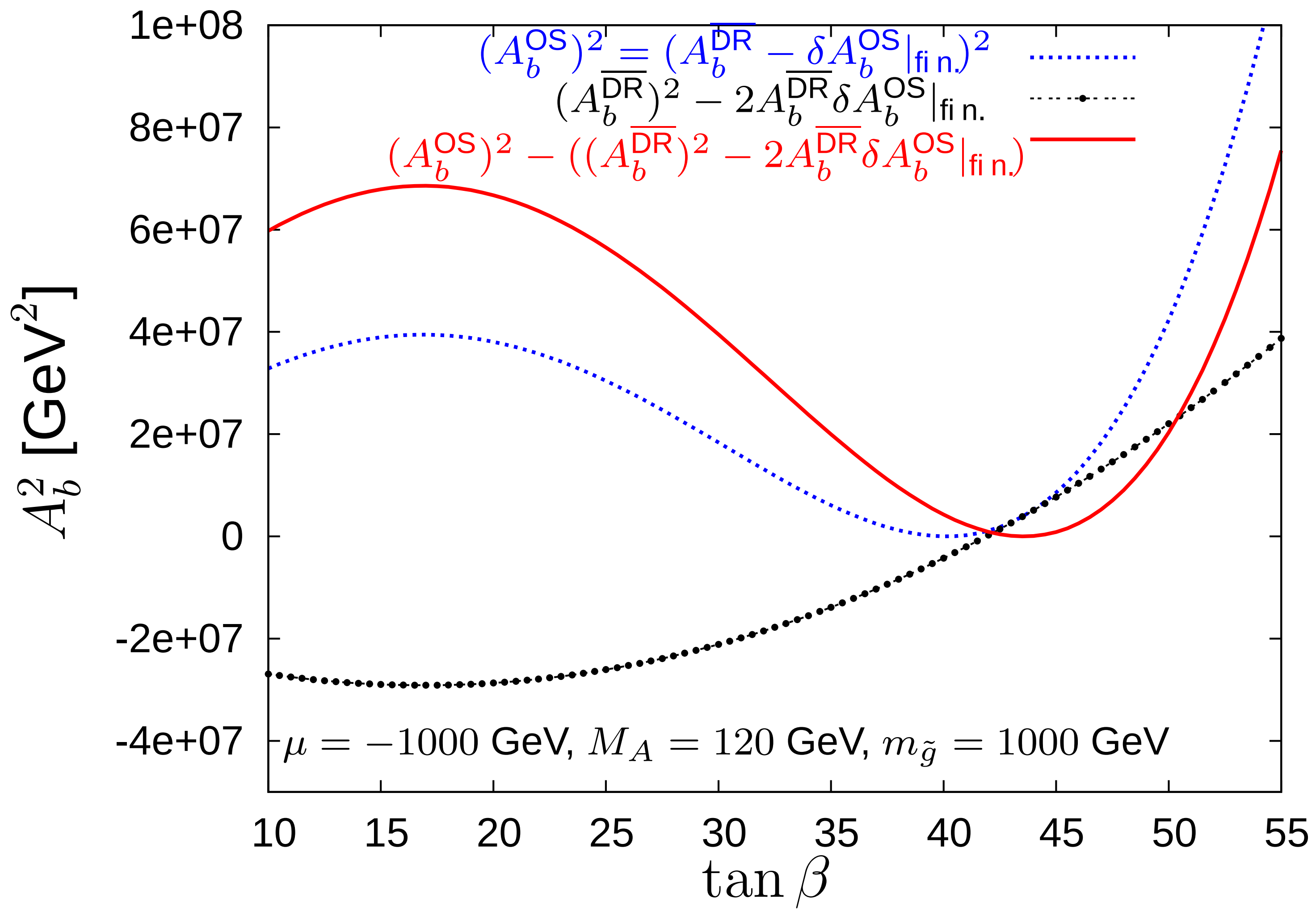
Comparison: A^2 in scheme m_b $\overline{\text{DR}}$ and m_b OS



- large contributions from $(\delta A_b)^2$

  $\Rightarrow$ higher order contributions are **not negligible**

$\left(\text{A-parameter}\right)^2$ in the scheme m_b OS



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  $\Rightarrow$ higher order contributions are **not negligible**