Symmetries of $\mathcal{2HDM}$, $\mathcal{CP}$ violation and potential of Linear Colliders

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Based on papers with
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The simplest extension of the $\mathcal{SM}$ —
a Two Higgs Doublet Model ($2\mathcal{HDM}$):

$$\mathcal{L} = \mathcal{L}_{gf}^{SM} + \mathcal{L}_H + \mathcal{L}_Y\,;$$

$\mathcal{L}_{gf}^{SM}$ – $\mathcal{SM}$ interaction, gauge bosons + fermions

$\mathcal{L}_H \equiv T - V$ – Higgs lagrangian,

T – Higgs kinetic term, V – Higgs potential,

$\mathcal{L}_Y$ – Yukawa interaction of fermions to scalars.

$$T = (D_\mu\phi_1)^\dagger(D^\mu\phi_1) + (D_\mu\phi_2)^\dagger(D^\mu\phi_2)$$
$$+\varkappa(D_\mu\phi_1)^\dagger(D^\mu\phi_2) + \varkappa^*(D_\mu\phi_2)^\dagger(D^\mu\phi_1)\,,$$

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$$V = \frac{\lambda_1}{2}(\phi_1^\dagger\phi_1)^2 + \frac{\lambda_2}{2}(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2)$$
$$+\lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \frac{1}{2}\left[\lambda_5(\phi_1^\dagger\phi_2)^2 + h.c.\right]$$
$$+\left\{\left[\lambda_6(\phi_1^\dagger\phi_1) + \lambda_7(\phi_2^\dagger\phi_2)\right](\phi_1^\dagger\phi_2) + h.c.\right\} + \mathcal{M}(\phi_i)$$
$$\mathcal{M}(\phi_i) = -\frac{1}{2}\left\{m_{11}^2(\phi_1^\dagger\phi_1) + m_{22}^2(\phi_2^\dagger\phi_2)\right.$$
$$\left.+\left[m_{12}^2(\phi_1^\dagger\phi_2) + h.c.\right]\right\}.$$

λ_{5-7}, $\varkappa$, m_{12} — generally complex.

Two fields with identical quantum numbers

$\Downarrow$

Most general $2\mathcal{HDM}$ allow for global transformations which mix ϕ_1, ϕ_2 $\Rightarrow$

Reparameterization invariance

in the space of Lagrangians with coordinates λ_i, m_{ij}^2, $\varkappa$:

The physical reality corresponding to a particular choice of Lagrangian does not change with the change of Lagrangian

under the global transformation $\mathcal{F}$:

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = e^{-i\rho_0} \begin{pmatrix} \cos\theta\, e^{i\rho/2} & \sin\theta\, e^{i\tau} \\ -\sin\theta\, e^{-i\tau} & \cos\theta\, e^{-i\rho/2} \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}$$

accompanied by compensating transformation of λ_i, m_{ij}, $\varkappa$ and renormalization of fields η_i.

It is governed by 3 angles θ, ρ, τ.

Reparameterization transformation and reparameterization representation.

Reparameterization equivalent space

Particular case

at $\theta = 0$ (τ is irrelevant):

Rephasing invariance

under the global rephasing transformation

$$\phi_i \to e^{-i\rho_i}\phi_i, \quad (i=1,2),$$
$$\rho_0 = (\rho_1+\rho_2)/2, \quad \rho = \rho_2 - \rho_1,$$

accompanied by transformation

$$\lambda_{1-4} \to \lambda_{1-4}\,, \quad m_{ii}^2 \to m_{ii}^2\,, \quad m_{12}^2 \to m_{12}^2 e^{i\rho}$$
$$\lambda_5 \to \lambda_5\, e^{2i\rho}\,, \;\; \lambda_{6,7} \to \lambda_{6,7}\, e^{i\rho}\,, \;\; \varkappa \to \varkappa\, e^{i\rho}\,.$$

ρ – rephasing gauge parameter,

ρ_0 – overall phase parameter.

This invariance is extended to the description of a whole system of scalars and fermions by adding of similar transformations for the phases of fermion fields and Yukawa couplings.

Rephasing transformation and rephasing representation.

Rephasing equivalent subspace of the reparameterization equivalent space

(Violated) Z_2 symmetry

The $2\mathcal{HDM}$ with general $\mathcal{L}_Y$ generally give large $\not{C}P$ and $\mathcal{FCNC}$ effects at $\mathcal{EWSB}$.
Experiment: $\not{C}P$ and $\mathcal{FCNC}$ effects are weak.

$\Rightarrow$ The natural analysis of $2\mathcal{HDM}$ should start with the lagrangian having additional symmetry which forbids a $\not{C}P$ and $\mathcal{FCNC}$ effects. $\Rightarrow$

That is Z_2 symmetry under independent transformations for both fields

$$\phi_1 \to -\phi_1, \; \phi_2 \to \phi_2,$$
$$\phi_1 \to \phi_1, \; \phi_2 \to -\phi_2, \qquad ,$$

which forbids (ϕ_1, ϕ_2) mixing.
This symmetry can be weakly broken to open door for weak $\not{C}P$ and $\mathcal{FCNC}$ effects.

Z_2 conserving case: $m_{12} = \lambda_6 = \lambda_7 = \varkappa = 0$ – 8 parameters of $\mathcal{L}$.
Soft violation of Z_2: dim. 2 operator with m_{12} (retained unmixed ϕ_i fields at small distances) – 10 parameters of $\mathcal{L}$.
Hard violation of Z_2: + dim. 4 operators with $\lambda_6, \lambda_7, \varkappa$. (ϕ_1, ϕ_2) are mixed at small distances – 14/16 parameters of $\mathcal{L}$.

(Hidden) softly violated Z_2 symmetry

It looks natural to assume: $2\mathcal{HDM}$ appears as low energy limit of some underlying theory where fields ϕ_i have different quantum numbers, without mixing $\Rightarrow$ the basic lagrangian has no mixed kinetic term, this term cannot be generated in the perturbation theory – softly violated Z_2 symmetry, $\lambda_6 = \lambda_7 = \varkappa = 0$. The reparameterization makes $\lambda_6, \lambda_7 \neq 0 \Rightarrow$ the mixed kinetic term appears in the perturbation but it can be eliminated with backward reparameterization (hidden softly violated Z_2 symmetry) – 12 parameters of $\mathcal{L}$: 10 $+\theta$, τ (ρ was among 10).

Transformation to the observable Higgs fields h_i, etc. gives terms like $\lambda_{6,7}$ in the obtained potential. The correlations between quartic couplings in the case of soft Z_2 symmetry (or in its hidden form) prevent running mixing between fields ϕ_i at small distances.

Hardly violated Z_2 symmetry

The case of hidden softly violated Z_2 symmetry mimic hard violation of Z_2 symmetry BUT case with hard Z_2 symmetry contains 2 additional parameters in potential

(+ 2 extra parameters, $Re\varkappa$, $Im\varkappa$).

One can eliminate mixed kinetic terms by the nonunitary transformation, like

$$(\phi_1', \phi_2') \to \left(\frac{\sqrt{\varkappa^*}\phi_1 + \sqrt{\varkappa}\phi_2}{2\sqrt{|\varkappa|(1+|\varkappa|)}} \pm \frac{\sqrt{\varkappa^*}\phi_1 - \sqrt{\varkappa}\phi_2}{2\sqrt{|\varkappa|(1-|\varkappa|)}}\right). \quad \blacklozenge$$

Starting from the case $\varkappa = 0$, $\lambda_{6,7} \neq 0$, the renormalization of quadratically divergent, non-diagonal two-point functions leads to $\varkappa \neq 0$

$\Rightarrow$ λ_6, λ_7, $\varkappa$ are running

$\Rightarrow$ all of these terms should be included in Lagrangian on the same footing

$\Rightarrow$ the treatment of the hard violation of Z_2 symmetry without $\varkappa$ terms (as in most of papers considering this "most general $2\mathcal{HDM}$ potential") is inconsistent.

The minimum of the potential

defines the v.e.v.'s $\langle \phi_i \rangle$ via

$$\frac{\partial V}{\partial \phi_i}(\phi_1 = \langle \phi_1 \rangle,\ \ \phi_2 = \langle \phi_2 \rangle) = 0$$

with $\quad \langle \phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 e^{i\xi} \end{pmatrix};$

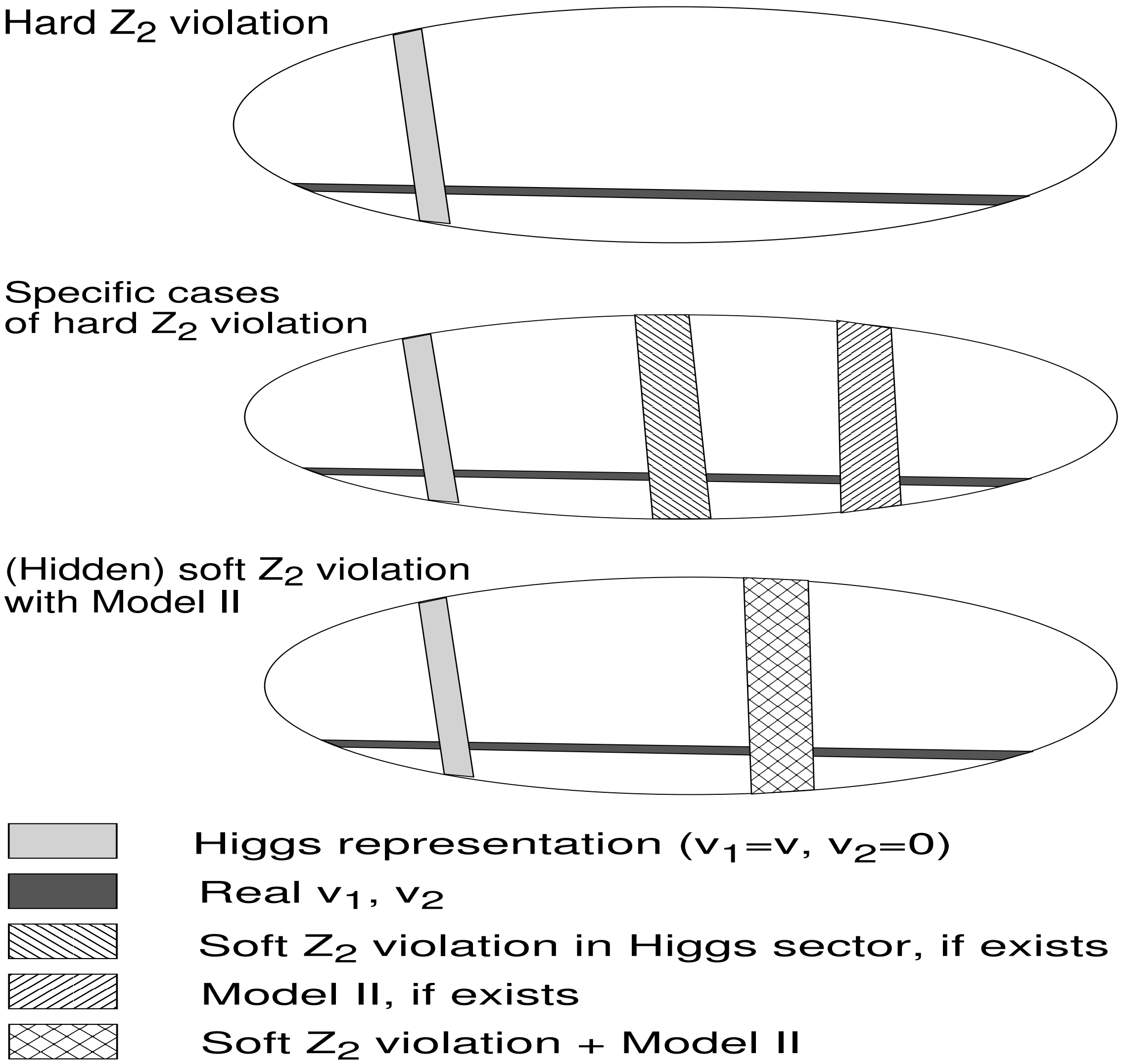
$$v_1 = v \cos\beta\,, \ \ v_2 = v \sin\beta\,, \ \ \beta \in (0, \pi/2).$$

The $\mathcal{SM}$ constraint $v = \left(G_F \sqrt{2}\right)^{-1/2} = 246$ GeV.

At the rephasing transformation $\boxed{\xi \to \xi - \rho}$

$\Rightarrow$ Phase shift ξ can be eliminated by suitable rephasing transformation.

In different problems different reparameterization representations are more useful.



The reparemetrization equivalent space
FOR THE SAME PHYSICAL REALITY
in different cases
and specific series of representations,
"vertical" domains present some rephasing
equivalent subspace

λ_i, ξ, $\tan\beta$ are reparameterization non invariant.

• We use the zero rephasing representation – point in the "horizontal zone" in figure, all λ's and m_{ij} in this representation are overlined, and

$$\overline{\lambda}_{345} = \lambda_3 + \lambda_4 + Re(\overline{\lambda}_5), \;\; \overline{\lambda}_{67} = \frac{v_1}{v_2}\overline{\lambda}_6 + \frac{v_2}{v_1}\overline{\lambda}_7,$$

$$\tilde{\lambda}_{67} = \frac{1}{2}\left(\frac{v_1}{v_2}\overline{\lambda}_6 - \frac{v_2}{v_1}\overline{\lambda}_7\right),$$

Than we express mass term of potential via v.e.v.'s v_i plus these $\overline{\lambda}$'s:

$$\overline{m}_{11}^2 = \underbrace{\overline{\lambda}_1 v_1^2 + v_2^2\Big[\overline{\lambda}_{345}}_{Z_2\ sym} \;\underbrace{-2\nu}_{soft} + \underbrace{Re\left(\overline{\lambda}_{67} + \tilde{\lambda}_{67}\right)\Big]}_{hard},$$

$$\overline{m}_{22}^2 = \underbrace{\overline{\lambda}_2 v_2^2 + v_1^2\big[\overline{\lambda}_{345}}_{Z_2\ sym} \;\underbrace{-2\nu}_{soft} + \underbrace{Re\left(\overline{\lambda}_{67} - \tilde{\lambda}_{67}\right)\big]}_{hard},$$

$$\boxed{\overline{m}_{12}^2 = 2v_1v_2(\nu + i\delta)\,,}$$

$$\boxed{\delta = Im\big(\underbrace{\overline{\lambda}_5/2}_{soft} + \underbrace{\overline{\lambda}_{67}/2}_{hard}\big).}$$

There are no limitation for quantity ν,
while δ is expressed via $Im(\overline{\lambda}_{5-7})$.
(The simple form of this subdivision is property of zero rephasing representation.)

The standard decomposition of the fields ϕ_i in terms of reparemerization noninvariant "physical" fields (but in zero rephasing representation):

$$\phi_i = \begin{pmatrix} \varphi_i^+ \\ \frac{1}{\sqrt{2}}(v_i + \eta_i + i\chi_i) \end{pmatrix} \quad (i = 1, 2).$$

Reparameterization invariant fields:

Goldstone boson fields

$$G^0 = c_\beta \, \chi_1 + s_\beta \, \chi_2,$$
$$G^\pm = c_\beta \, \varphi_1^\pm + s_\beta \, \varphi_2^\pm.$$

$$c_\beta = \cos\beta, \qquad s_\beta = \sin\beta.$$

Charged Higgs boson fields

$H^\pm = s_\beta \, \varphi_1^\pm + c_\beta \, \varphi_2^\pm$ with

$$M_{H^\pm}^2 = v^2 \left[\nu - \frac{1}{2} Re(\lambda_4 + \overline{\lambda}_5 + \overline{\lambda}_{67}) \right].$$

Neutral Higgs sector is diagonalized as

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \\ c_\beta \chi_2 - s_\beta \chi_1 \equiv A \end{pmatrix}$$

in two steps. First step is identical to that in CP conserving case. The CP violating mixing is described by two terms in neutral mass matrix,

$$M_{13} = -\left(\delta + Im\tilde{\lambda}_{67}\right) s_\beta v^2,$$
$$M_{23} = -\left(\delta - Im\tilde{\lambda}_{67}\right) c_\beta v^2.$$

Relative couplings of Higgs boson h_i:

$$\chi_a^i \stackrel{def}{=} g_a^i / g_a^{SM}, \qquad a = q,\, \ell,\, V(= Z,\, W)$$

Yukawa interaction

To have only soft violation of Z_2 symmetry (to keep separate fields ϕ_i at small distances), each right-handed fermion should couple to only one field, either ϕ_1 or ϕ_2.

Otherwise, e.g. in Model III, hard violation of Z_2 symmetry appears via one–loop corrections.

Model II

$$-\mathcal{L}_Y^{II} = \sum_{k=1,2,3} g_{dk} \bar{Q}_{Lk} \phi_1 d_{Rk} + \sum_{k=1,2,3} g_{uk} \bar{Q}_{Lk} \tilde{\phi}_2 u_{Rk} + \sum_{k=1,2,3} g_{\ell k} \bar{\ell}_{Lk} \phi_1 \ell_{Rk} + \text{h.c.}$$

For the physical Higgs fields it result in (for two-component spinors)

$$\chi_u^{(i)} = \frac{1}{\sin\beta}[R_{i2} - i\cos\beta\, R_{i3}],$$
$$\chi_d^{(i)} = \frac{1}{\cos\beta}[R_{i1} - i\sin\beta\, R_{i3}].$$

The unitarity of the mixing matrix R result in

pattern relation, sum rules, etc.

- Pattern relation among the basic relative couplings of each neutral Higgs particle h_i (GKO):

$$(\chi_u^{(i)} + \chi_d^{(i)})\chi_V^{(i)} = 1 + \chi_u^{(i)}\chi_d^{(i)}, \qquad (pr)$$

- Horizontal sum rule for each neutral Higgs boson h_i (Gunion et al)

$$|\chi_u^{(i)}|^2 \sin^2\beta + |\chi_d^{(i)}|^2 \cos^2\beta = 1. \qquad (hsr)$$

- Vertical sum rule for each basic relative coupling χ_j to all three neutral Higgs bosons h_i (Gunion et al):

$$\sum_{i=1}^{3} (\chi_j^{(i)})^2 = 1 \qquad (j = V, d, u). \qquad (vsr)$$

- Linear relation (lr) (GK):

$$\begin{aligned} \chi_V^{(i)} &= Re\left(\cos^2\beta\chi_d^{(i)} + \sin^2\beta\chi_u^{(i)}\right), \\ Im\left(\cos^2\beta\chi_d^{(i)} - \sin^2\beta\chi_u^{(i)}\right) &= 0\,. \end{aligned} \qquad (lr)$$

- New reparameterization independent relation (GK):

$$(1-|\chi_d^{(i)}|^2)\, Im\chi_u^{(i)} + (1-|\chi_u^{(i)}|^2)\, Im\chi_d^{(i)} = 0\,.(nlr)$$

• $\tan\beta$ – a basic parameter of the $2\mathcal{HDM}$, defined in the Model II reparametrization representations can be determined via measurable basic couplings to one of neutral Higgs bosons:

$$\tan^2\beta = \frac{(\chi_V^{(i)} - \chi_d^{(i)})^*}{\chi_u^{(i)} - \chi_V^{(i)}} = \frac{Im\chi_d^{(i)}}{Im\chi_u^{(i)}} = \frac{1 - |\chi_d^{(i)}|^2}{|\chi_u^{(i)}|^2 - 1}.$$

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In the $\mathcal{CP}$ conserving case for $\phi = h$ or H

$$\chi_{H^\pm}^{(\phi)} \equiv -\frac{v g_{hH^+H^-}}{2M_{H^\pm}^2}$$

$$= \left(1 - \frac{M_\phi^2}{2M_{H^\pm}^2}\right)\chi_V^{(\phi)} + \frac{M_\phi^2 - \nu v^2}{2M_{H^\pm}^2}(\chi_u^{(\phi)} + \chi_d^{(\phi)}).$$

List of problems, solvable with above results

Many analyses of $2\mathcal{HDM}$ assume that the lightest Higgs boson h_1 is similar to the Higgs boson of the $\mathcal{SM}$, all other Higgs bosons are very heavy (with mass $\sim M$).
Usual additional hidden requirement (?!?):
The theory must have explicit decoupling property: the mention features remain valid at $M \to \infty$ (decoupling property).
In fact, the mentioned physical picture can be realized in the $2\mathcal{HDM}$ both with and without decoupling property.

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- We analysed in detail decoupling limit corresponding $\nu \gg |\lambda_i|$ with possible strong $\mathcal{CP}$ violation for heaviest Higgs bosons h_2, h_3.
- We consider in detail many other realizations of a $\mathcal{SM}$ –like scenario, which can take place for a natural set of parameters of $2\mathcal{HDM}$.

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- For a natural set of parameters of $2\mathcal{HDM}$ we show that the measurements of Higgs boson production at Photon Collider (in $\gamma\gamma$ and $e\gamma$ collisions) can distinguish reliably the $\mathcal{SM}$ from $2\mathcal{HDM}$ (see Maria's talk).

■ We plan to study whether the well known form of MSSM with loop corrections describe hard violation of Z_2 symmetry or it can be considered as hidden soft violation of Z_2 symmetry.