Progress towards
$e^+e^- \to$ 3 jets at NNLO

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ECFA Linear Collider Workshop

Durham September 2004

- Introduction
- Virtual Corrections
- Real Corrections
- Infrared Cancellation
- First Results

Jets

Jet Observables

- major testing ground of QCD in e^+e^- annihilation

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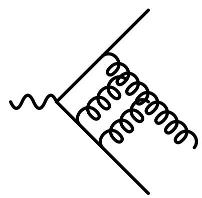
- similar in $ep \to (2+1)j$ and $pp \to 2j$:

$$\begin{aligned} \alpha_s^{\text{ZEUS}}(M_Z) &= 0.1190 \pm 0.0017(\text{stat})^{+0.0049}_{-0.0023}(\text{sys}) \pm 0.0026(\text{th}) \\ \alpha_s^{\text{H1}}(M_Z) &= 0.1186 \pm 0.0030(\text{exp})^{+0.0039}_{-0.0045}(\text{scale}) \pm 0.0023(\text{pdf}) \\ \alpha_s^{\text{CDF}}(M_Z) &= 0.1178 \pm 0.0001(\text{stat})^{+0.0081}_{-0.0095}(\text{sys}) \\ &\quad {}^{+0.0071}_{-0.0047}(\text{scale}) \pm 0.0059(\text{pdf}) \end{aligned}$$

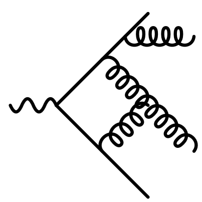
Jet physics at NNLO

Ingredients to NNLO n-jet:

- Two-loop matrix elements

  $|\mathcal{M}|^2_{\text{2-loop},n\text{ partons}}$ 

- One-loop matrix elements

  $|\mathcal{M}|^2_{\text{1-loop},n+1\text{ partons}}$ 

- One-loop one-particle subtraction terms

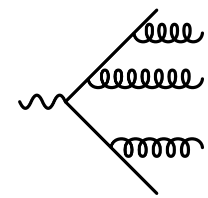
  $\int |\mathcal{M}^{R,1}|^2_{\text{1-loop},n+1\text{ partons}}\, d\Phi_1$

  D. Kosower, P. Uwer

  Z. Bern et al.

  S. Weinzierl; D. Kosower

- Tree level matrix elements

  $|\mathcal{M}|^2_{\text{tree},n+2\text{ partons}}$ 

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  $\int |\mathcal{M}^{R,1}|^2_{\text{tree},n+2\text{ partons}}\,\mathrm{d}\Phi_1$

  W. Giele, N. Glover

  S. Catani, M. Seymour

- Tree-level two-particle subtraction terms

  $\int |\mathcal{M}^{R,2}|^2_{\text{tree},n+2\text{ partons}}\,\mathrm{d}\Phi_2$

  D. Kosower; S. Weinzierl

  remain to be integrated

Virtual Corrections at NNLO

Generic structure of two-loop integrals

$$I_{t,r,s}(p_1,\ldots,p_n) \quad = \quad \int \frac{\mathrm{d}^d k}{(2\pi)^d}\frac{\mathrm{d}^d l}{(2\pi)^d}\frac{1}{D_1^{m_1}\ldots D_t^{m_t}} S_1^{n_1}\ldots S_q^{n_q}$$

D_i	:	massless scalar propagators
S_i	:	scalar products involving loop momenta
t	:	number of different propagators
$r=\sum_i m_i$	:	dimension of denominator
$s=\sum_i n_i$	:	dimension of numerator

Topology of Feynman graph defined by specifying the set of different propagators

$$\{D_1,\ldots,D_t\}$$

Virtual Corrections at NNLO

Identities relating loop integrals

- Integration-by-parts (IBP)
  K. Chetyrkin, F. Tkachov

$$\int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{\mathrm{d}^d l}{(2\pi)^d} \frac{\partial}{\partial a^\mu} \left[b^\mu f(k, l, p_i) \right] = 0$$

with: $a^\mu = k^\mu, l^\mu$ and $b^\mu = k^\mu, l^\mu, p_i^\mu$

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- Lorentz Invariance (LI)
  E. Remiddi, TG

$$\int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{\mathrm{d}^d l}{(2\pi)^d} \delta\varepsilon_\nu^\mu \left(\sum_i p_i^\nu \frac{\partial}{\partial p_i^\mu} \right) f(k,l,p_i) = 0$$

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For each two-loop four-point integral, one has 10 IBP and 3 LI identities.

Virtual Corrections at NNLO

Reduction of two-loop integrals

Process	Two-loop amplitudes
$e^+e^- \to e^+e^-$ $pp \to 2$ Jets	4-point, all legs on-shell
$e^+e^- \to 3$ Jets $ep \to (2+1)$ Jets $pp \to V + 1$ Jet	4-point, one leg off-shell

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Using IBP and LI identities, any two-loop four-point integral can be expressed as linear combination of a small number of two-loop four-point master integrals: S. Laporta

- on-shell: 6 master integrals
- off-shell: 14 master integrals

plus simpler two- and three-point master integrals.

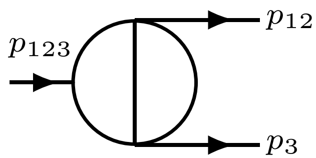
Virtual Corrections at NNLO

Calculation of master integrals

E. Remiddi, TG

Multi-scale master integrals fulfil inhomogeneous differential equations in their external invariants.

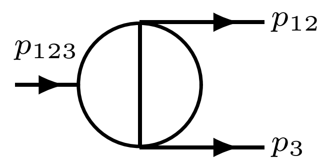
$$s_{123}\frac{\partial}{\partial s_{123}}$$



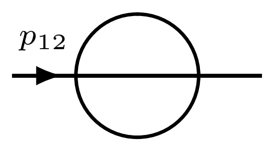
$$=$$

For example:

$$+\frac{d-4}{2}\,\frac{2s_{123}-s_{12}}{s_{123}-s_{12}}$$



$$-\frac{3d-8}{2}\,\frac{1}{s_{123}-s_{12}}$$



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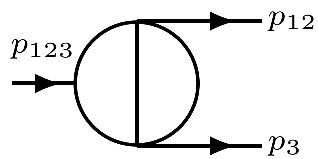

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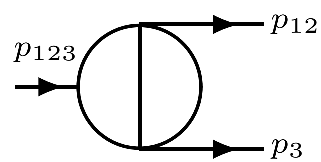

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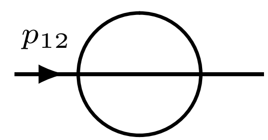

Master integrals are computed by

- integration of differential equations
- subsequent matching of boundary conditions

Virtual Corrections at NNLO

Virtual two-loop matrix elements have recently been computed for:

- Bhabha-Scattering: $e^+e^- \to e^+e^-$
  Z. Bern, L. Dixon, A. Ghinculov
- Hadron-Hadron 2-Jet production: $qq' \to qq'$, $q\bar{q} \to q\bar{q}$, $q\bar{q} \to gg$, $gg \to gg$
  C. Anastasiou, N. Glover, C. Oleari, M. Yeomans-Tejeda
  Z. Bern, A. De Freitas, L. Dixon [SUSY-YM]
- Photon pair production at LHC: $gg \to \gamma\gamma$, $q\bar{q} \to \gamma\gamma$
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- Three-jet production: $e^+e^- \to \gamma^* \to q\bar{q}g$
  L. Garland, N. Glover, A.Koukoutsakis, E. Remiddi, TG
  S. Moch, P. Uwer, S. Weinzierl
- DIS (2+1) jet production: $\gamma^* g \to q\bar{q}$, Hadronic (V+1) jet production: $qg \to Vq$
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- DIS (2+1) jet production: $\gamma^* g \to q\bar{q}$, Hadronic (V+1) jet production: $qg \to Vq$
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- Matrix elements with internal masses: $\gamma^* \to Q\bar{Q}$, $Q\bar{Q} \to Q\bar{Q}$
  W.Bernreuther, R.Bonciani, R.Heinesch, T.Leineweber, P.Mastrolia, E.Remiddi, TG
  R. Bonciani, A. Ferroglia, P. Mastrolia, E. Remiddi, J.J. van der Bij

Real corrections at NNLO

Double real radiation

$$d\sigma^{(n+2)} = |\mathcal{M}_{n+2}|^2 d\Phi_{n+2} \mathcal{F}_n^{(n+2)}(p_1, \ldots, p_{n+2}) \sim \frac{1}{\epsilon^4}$$

with $\mathcal{F}_n^{(n+2)}$ jet definition for combining n+2 partons into n jets

Two approaches:

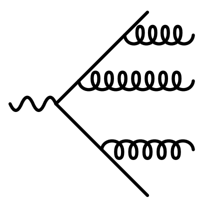
- Direct evaluation
  C. Anastasiou, K. Melnikov, F. Petriello
  - expand $|\mathcal{M}_{n+2}|^2 d\Phi_{n+2}$ in distributions
  - decompose $d\Phi_{n+2}$ into sectors corresponding to different singular configurations (Iterated sector decomposition)
    T. Binoth, G. Heinrich
  - compute sector integrals numerically
- Evaluation with subtraction term

both approaches tested on $e^+e^- \to 2j$

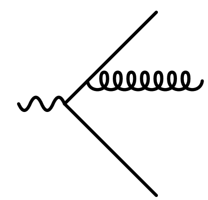
Real corrections at NNLO

Infrared subtraction terms

$n+2$ parton final state forming n jets:



$$\mathrm{d}\Phi_2 \longrightarrow C$$



- Singular configurations:
  J. Campbell, N. Glover; S. Catani, M. Grazzini; A. Frizzo, F. Maltoni, V. Del Duca
  - triple collinear
  - double single collinear
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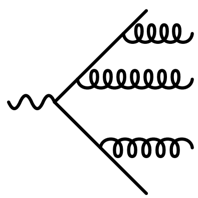
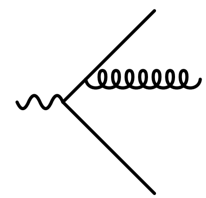
 $d\Phi_2 \longrightarrow C$ 

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  - triple collinear
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- Issue: find subtraction functions which
  - approximate full $n+2$ matrix element in all singular limits
  - are sufficiently simple to be integrated analytically

NLO subtraction

Structure of NLO m-jet cross section

$$\mathrm{d}\sigma_{NLO} = \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\sigma^{R}_{NLO} - \mathrm{d}\sigma^{S}_{NLO}\right) + \left[\int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^{S}_{NLO} + \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\sigma^{V}_{NLO}\right].$$

Dipole subtraction S. Catani, M. Seymour

$$\mathrm{d}\sigma^{R}_{NLO} - \mathrm{d}\sigma^{S}_{NLO} =$$

$$N_{in} \sum_{m+1} \mathrm{d}\Phi_{m+1}(p_1, ..., p_{m+1}, Q) \frac{1}{S_{m+1}} \Bigg[\left|\mathcal{M}_{m+1}(p_1, ...p_{m+1})\right|^2 \mathcal{F}_J^{(m+1)}(p_1, ..., p_{m+1})$$

$$- \sum_{\text{pairs } i,j} \sum_{k \neq i,j} \mathcal{D}_{ijk} \left|\mathcal{M}_m(p_1, ..\tilde{p}_{ij}, \tilde{p}_k, ..., p_{m+1})\right|^2 \mathcal{F}_J^{(m)}(p_1, ..\tilde{p}_{ij}, \tilde{p}_k, ..., p_{m+1}) \Bigg]$$

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$$N_{in} \sum_{m+1} \mathrm{d}\Phi_{m+1}(p_1, \ldots, p_{m+1}, Q) \frac{1}{S_{m+1}} \left[|\mathcal{M}_{m+1}(p_1, \ldots p_{m+1})|^2 \, \mathcal{F}_J^{(m+1)}(p_1, \ldots, p_{m+1}) \right.$$

$$\left. - \sum_{\mathrm{pairs}\ i,j} \sum_{k \neq i,j} \mathcal{D}_{ijk} \, |\mathcal{M}_m(p_1, ..\tilde{p}_{ij}, \tilde{p}_k, \ldots, p_{m+1})|^2 \, \mathcal{F}_J^{(m)}(p_1, ..\tilde{p}_{ij}, \tilde{p}_k, \ldots, p_{m+1}) \right]$$

For special case of two jets

$$\begin{aligned} |\mathcal{M}_2|^2 &= 1 \\ \sum_{\mathrm{pairs}\ i,j} \sum_{k \neq i,j} \mathcal{D}_{ijk} &\sim |\mathcal{M}_3|^2 \end{aligned}$$

NLO subtraction

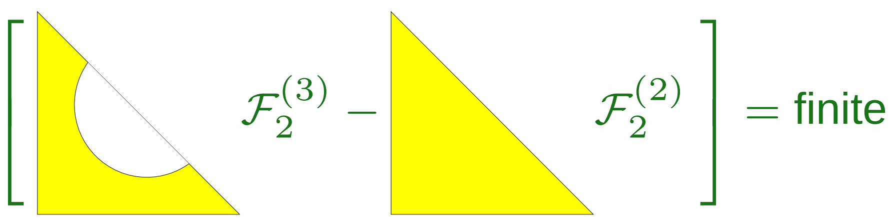
Two-jet cross section

$$\mathrm{d}\sigma^{2j}_{\mathrm{NLO}} = \mathrm{d}\Phi_3 \, |\mathcal{M}_3(p_1,...p_3)|^2 \left[(\mathcal{F}_2^{(3)} - \mathcal{F}_2^{(2)}) \right]$$

$$+\mathrm{d}\Phi_2 \, |\mathcal{M}_2|^2 \, \mathcal{F}_2^{(2)} \left(\int_{\mathrm{d}\Phi_D} |M_3|^2 + |M_2^{V,1}|^2 \right)$$

Interpretation: subtraction by subtracting and adding three parton inclusive contribution

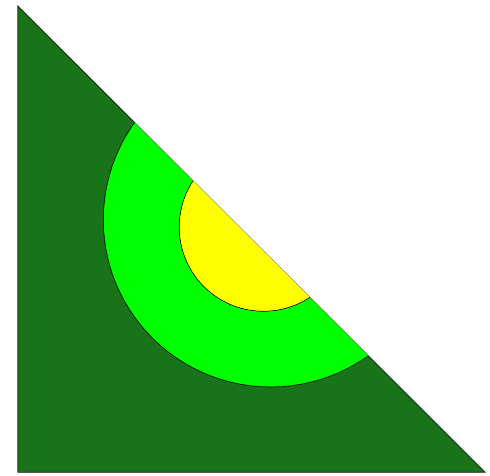
$\gamma^* \to q\bar{q}g$



NNLO subtraction

NNLO two-jet cross section

A. Gehrmann-De Ridder, N. Glover, TG



$$
\begin{aligned}
\mathrm{d}\sigma_{NNLO} &= \left[\mathrm{d}\sigma^{R}_{NNLO} - \mathrm{d}\sigma^{S,0}_{NNLO} + \mathrm{d}\sigma^{S,1}_{NNLO}\right] \\
&+ \left[\mathrm{d}\sigma^{V,1}_{NNLO} - \mathrm{d}\sigma^{VS,1}_{NNLO} - \mathrm{d}\sigma^{S,1}_{NNLO}\right] \\
&+ \left[\mathrm{d}\sigma^{V,2}_{NNLO} + \mathrm{d}\sigma^{S,0}_{NNLO} + \mathrm{d}\sigma^{VS,1}_{NNLO}\right] \\
&= \mathrm{d}\Phi_4 \left[|\mathcal{M}_4|^2 \left(\mathcal{F}_2^{(4)} - \mathcal{F}_2^{(2)}\right) + \sum_{ijk} |\mathcal{M}_3|^2 D_{ijk} \mathcal{F}_3^{(3)}\right] \\
&+ \mathrm{d}\Phi_3 \left[|\mathcal{M}_3^{V,1}|^2 \left(\mathcal{F}_2^{(3)} - \mathcal{F}_2^{(2)}\right) - \sum_{ijk} |\mathcal{M}_3|^2 \left(\int_{\mathrm{d}\Phi_D} D_{ijk}\right) \mathcal{F}_3^{(3)}\right] \\
&+ \mathrm{d}\Phi_2\, |\mathcal{M}_2|^2 \left[|M_2^{V,2}|^2 + \int_{\mathrm{d}\Phi_T} |M_4|^2 + \int_{\mathrm{d}\Phi_D} |M_3^{V,1}|^2\right] \mathcal{F}_2^{(2)} \,.
\end{aligned}
$$

where: $\mathrm{d}\Phi_2 \mathrm{d}\Phi_D = \mathrm{d}\Phi_3$, $\mathrm{d}\Phi_2 \mathrm{d}\Phi_T = \mathrm{d}\Phi_4$, $|\mathcal{M}_2|^2 |M_i|^2 = |\mathcal{M}_i|^2$

Phase space at NNLO

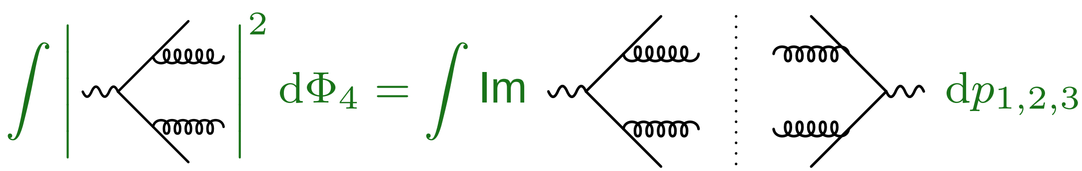
Four-particle phase space integrals $\int \mathrm{d}\Phi_T |\mathcal{M}|_4^2$

A. Gehrmann-De Ridder, G. Heinrich, TG

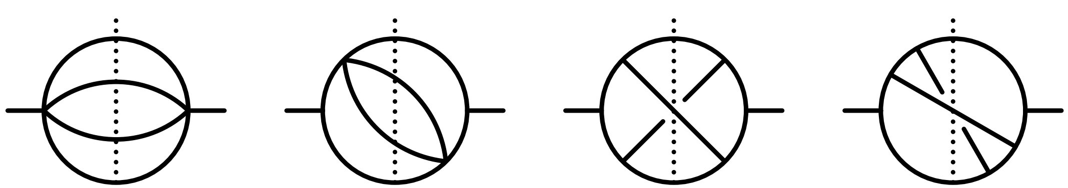
- use I-duality (C. Anastasiou, K. Melnikov)

$$\frac{\mathrm{d}^{d-1}p}{2E} = d^d p \delta_+(p^2) = \frac{1}{2\pi i}\mathrm{d}^d p \left(\frac{1}{p^2 + i\epsilon} - \frac{1}{p^2 - i\epsilon} \right)$$

to convert to cuts of three-loop propagator integrals



- use IBP to reduce to master integrals



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compute master integrals

- by direct integration

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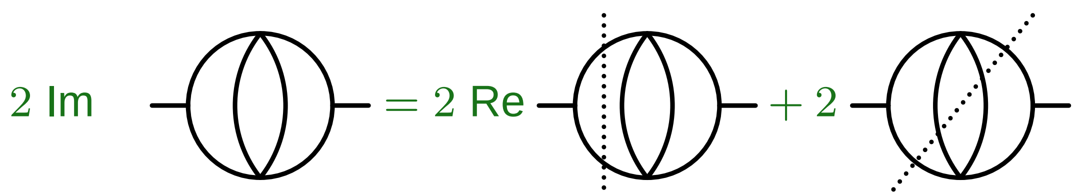
- from unitarity



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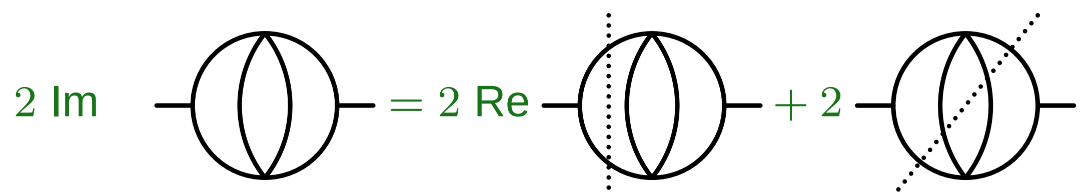

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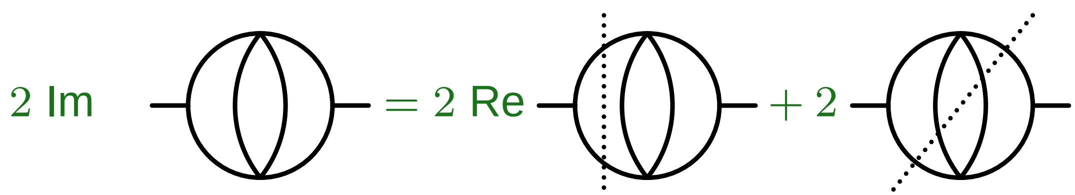

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same approach yields $\int \mathrm{d}\Phi_D |\mathcal{M}^{V,1}|_3^2$

Infrared structure at NNLO

Contributions to $\gamma^* \to 2j$ at NNLO

Two parton contributions

$$\mathrm{d}\Phi_2 \, |\mathcal{M}_2|^2 \left[|M_2^{V,2}|^2 + \int_{\mathrm{d}\Phi_T} |M_4|^2 + \int_{\mathrm{d}\Phi_D} |M_3^{V,1}|^2 \right] \mathcal{F}_2^{(2)}$$

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Infrared singularity operator

S. Catani

$$\boldsymbol{I}^{(1)}(\epsilon) = -\frac{e^{\epsilon\gamma}}{2\Gamma(1-\epsilon)} \left[\frac{N^2-1}{2N} \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + c_1 \right) \left(-\frac{\mu^2}{q^2} \right)^{\epsilon} \right]$$

Infrared structure at NNLO

Infrared poles of virtual two-loop corrections

S. Catani

$$\mathcal{P}oles_{q\bar{q}}^{(2\times 0)} = 2\Re\Bigg[-\frac{1}{2}\langle\mathcal{M}^{(0)}|\boldsymbol{I}^{(1)}(\epsilon)\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(0)}\rangle - \frac{\beta_0}{\epsilon}\langle\mathcal{M}^{(0)}|\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(0)}\rangle$$

$$+\langle\mathcal{M}^{(0)}|\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(1)}\rangle$$

$$+e^{-\epsilon\gamma}\frac{\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)}\left(\frac{\beta_0}{\epsilon}+K\right)\langle\mathcal{M}^{(0)}|\boldsymbol{I}^{(1)}(2\epsilon)|\mathcal{M}^{(0)}\rangle$$

$$+\langle\mathcal{M}^{(0)}|\boldsymbol{H}^{(2)}(\epsilon)|\mathcal{M}^{(0)}\rangle\Bigg]$$

$$\mathcal{P}oles_{q\bar{q}}^{(1\times 1)} = \Re\left[2\langle\mathcal{M}^{(1)}|\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(0)}\rangle - \langle\mathcal{M}^{(0)}|\boldsymbol{I}^{(1)\dagger}(\epsilon)\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(0)}\rangle\right]$$

with $\boldsymbol{H}^{(2)}(\epsilon) \sim 1/\epsilon$

Infrared structure at NNLO

Infrared poles of one-loop subtraction term

$$
\begin{aligned}
\mathcal{P}oles^{(1\times 0)}_{q\bar{q}g} \quad = \quad & 2\mathcal{R}\Bigg[-\langle \mathcal{M}^{(0)} | \boldsymbol{I}^{(1)}(\epsilon) | \mathcal{M}^{(1)} \rangle + \frac{\beta_0}{\epsilon} \langle \mathcal{M}^{(0)} | \boldsymbol{I}^{(1)}(\epsilon) | \mathcal{M}^{(0)} \rangle \\
& -\langle \mathcal{M}^{(1)} | \boldsymbol{I}^{(1)}(\epsilon) | \mathcal{M}^{(0)} \rangle - \langle \mathcal{M}^{(0)} | \boldsymbol{H}^{(2)}_V(\epsilon) | \mathcal{M}^{(0)} \rangle + \frac{1}{2} \langle \mathcal{M}^{(0)} | \boldsymbol{S}^{(2)}_V(\epsilon) | \mathcal{M}^{(0)} \rangle \Bigg]
\end{aligned}
$$

Infrared structure at NNLO

Infrared poles of one-loop subtraction term

$$\mathcal{P}oles^{(1\times 0)}_{q\bar{q}g} = 2\mathcal{R}\Bigg[-\langle \mathcal{M}^{(0)}|\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(1)}\rangle + \frac{\beta_0}{\epsilon}\langle \mathcal{M}^{(0)}|\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(0)}\rangle$$

$$-\langle \mathcal{M}^{(1)}|\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(0)}\rangle - \langle \mathcal{M}^{(0)}|\boldsymbol{H}^{(2)}_V(\epsilon)|\mathcal{M}^{(0)}\rangle + \frac{1}{2}\langle \mathcal{M}^{(0)}|\boldsymbol{S}^{(2)}_V(\epsilon)|\mathcal{M}^{(0)}\rangle \Bigg]$$

- must fix finite constant $c_1 = 43/4 - \pi^2/3$ in $\boldsymbol{I}^{(1)}(\epsilon)$

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$$
\begin{aligned}
\mathcal{P}oles^{(1\times 0)}_{q\bar{q}g} \quad = \quad & 2\mathcal{R}\Bigg[-\langle \mathcal{M}^{(0)} | \boldsymbol{I}^{(1)}(\epsilon) | \mathcal{M}^{(1)} \rangle + \frac{\beta_0}{\epsilon} \langle \mathcal{M}^{(0)} | \boldsymbol{I}^{(1)}(\epsilon) | \mathcal{M}^{(0)} \rangle \\
& -\langle \mathcal{M}^{(1)} | \boldsymbol{I}^{(1)}(\epsilon) | \mathcal{M}^{(0)} \rangle - \langle \mathcal{M}^{(0)} | \boldsymbol{H}^{(2)}_V(\epsilon) | \mathcal{M}^{(0)} \rangle + \frac{1}{2} \langle \mathcal{M}^{(0)} | \boldsymbol{S}^{(2)}_V(\epsilon) | \mathcal{M}^{(0)} \rangle \Bigg]
\end{aligned}
$$

- must fix finite constant $c_1 = 43/4 - \pi^2/3$ in $\boldsymbol{I}^{(1)}(\epsilon)$
- contribution from one-loop correction to soft gluon current
  S. Catani, M. Grazzini

$$
\langle \mathcal{M}^{(0)} | \boldsymbol{S}^{(2)}_V(\epsilon) | \mathcal{M}^{(0)} \rangle =
$$

$$
-\langle \mathcal{M}^{(0)} | \mathcal{M}^{(0)} \rangle \frac{e^{2\epsilon\gamma}}{1+\epsilon} \left[(N^2-1) \frac{1}{\epsilon^2} \frac{\Gamma^4(1-\epsilon)\Gamma^3(1+\epsilon)}{\Gamma^2(1-2\epsilon)\Gamma(1+2\epsilon)} \right] \int \mathrm{d}\Phi_D \left(\frac{q^2}{s_{13}s_{23}} \right)^{1+\epsilon}
$$

Infrared structure at NNLO

Infrared poles of one-loop subtraction term

$$\mathcal{P}oles^{(1\times 0)}_{q\bar{q}g} = 2\mathcal{R}\Bigg[-\langle \mathcal{M}^{(0)}|\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(1)}\rangle + \frac{\beta_0}{\epsilon}\langle \mathcal{M}^{(0)}|\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(0)}\rangle$$

$$-\langle \mathcal{M}^{(1)}|\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(0)}\rangle - \langle \mathcal{M}^{(0)}|\boldsymbol{H}_V^{(2)}(\epsilon)|\mathcal{M}^{(0)}\rangle + \frac{1}{2}\langle \mathcal{M}^{(0)}|\boldsymbol{S}_V^{(2)}(\epsilon)|\mathcal{M}^{(0)}\rangle \Bigg]$$

- must fix finite constant $c_1 = 43/4 - \pi^2/3$ in $\boldsymbol{I}^{(1)}(\epsilon)$
- contribution from one-loop correction to soft gluon current
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  $\langle \mathcal{M}^{(0)}|\boldsymbol{S}_V^{(2)}(\epsilon)|\mathcal{M}^{(0)}\rangle =$

  $$-\langle \mathcal{M}^{(0)}|\mathcal{M}^{(0)}\rangle \frac{e^{2\epsilon\gamma}}{1+\epsilon}\left[(N^2-1)\frac{1}{\epsilon^2}\frac{\Gamma^4(1-\epsilon)\Gamma^3(1+\epsilon)}{\Gamma^2(1-2\epsilon)\Gamma(1+2\epsilon)}\right]\int \mathrm{d}\Phi_D \left(\frac{q^2}{s_{13}s_{23}}\right)^{1+\epsilon}$$
- partial contribution $\boldsymbol{H}_V^{(2)}(\epsilon) \sim 1/\epsilon$ to $\boldsymbol{H}^{(2)}(\epsilon)$

Infrared structure at NNLO

Infrared poles of two-particle subtraction term

$$
\begin{aligned}
\mathcal{P}oles^{(0\times 0)}_{q\bar{q}(ij)} \quad = \quad \Re\Bigg[& \langle \mathcal{M}^{(0)} | \boldsymbol{I}^{(1)}(\epsilon) \boldsymbol{I}^{(1)}(\epsilon) | \mathcal{M}^{(0)} \rangle + \langle \mathcal{M}^{(0)} | \boldsymbol{I}^{(1)\dagger}(\epsilon) \boldsymbol{I}^{(1)}(\epsilon) | \mathcal{M}^{(0)} \rangle \\
& -2 e^{-\epsilon\gamma} \frac{\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)} \left(\frac{\beta_0}{\epsilon} + K \right) \langle \mathcal{M}^{(0)} | \boldsymbol{I}^{(1)}(2\epsilon) | \mathcal{M}^{(0)} \rangle - 2 \langle \mathcal{M}^{(0)} | \boldsymbol{H}^{(2)}_{R}(\epsilon) | \mathcal{M}^{(0)} \rangle \\
& - \langle \mathcal{M}^{(0)} | \boldsymbol{S}^{(2)}_{V}(\epsilon) | \mathcal{M}^{(0)} \rangle \Bigg]
\end{aligned}
$$

Infrared structure at NNLO

Infrared poles of two-particle subtraction term

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\mathcal{P}oles^{(0\times 0)}_{q\bar{q}(ij)} \quad = \quad & \Re\Bigg[\langle \mathcal{M}^{(0)} | \boldsymbol{I}^{(1)}(\epsilon) \boldsymbol{I}^{(1)}(\epsilon) | \mathcal{M}^{(0)} \rangle + \langle \mathcal{M}^{(0)} | \boldsymbol{I}^{(1)\dagger}(\epsilon) \boldsymbol{I}^{(1)}(\epsilon) | \mathcal{M}^{(0)} \rangle \\
& -2e^{-\epsilon\gamma} \frac{\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)} \left(\frac{\beta_0}{\epsilon} + K \right) \langle \mathcal{M}^{(0)} | \boldsymbol{I}^{(1)}(2\epsilon) | \mathcal{M}^{(0)} \rangle - 2 \langle \mathcal{M}^{(0)} | \boldsymbol{H}^{(2)}_{R}(\epsilon) | \mathcal{M}^{(0)} \rangle \\
& - \langle \mathcal{M}^{(0)} | \boldsymbol{S}^{(2)}_{V}(\epsilon) | \mathcal{M}^{(0)} \rangle \Bigg]
\end{aligned}
$$

- cancel all remaining terms from two-parton and three-parton final states

$$
\mathcal{P}oles^{(2\times 0)}_{q\bar{q}} + \mathcal{P}oles^{(1\times 1)}_{q\bar{q}} + \mathcal{P}oles^{(1\times 0)}_{q\bar{q}g} + \mathcal{P}oles^{(0\times 0)}_{q\bar{q}(ij)} = 0
$$

Infrared structure at NNLO

Infrared poles of two-particle subtraction term

$$\mathcal{P}oles^{(0\times 0)}_{q\bar{q}(ij)} = \Re\Bigg[\langle\mathcal{M}^{(0)}|\boldsymbol{I}^{(1)}(\epsilon)\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(0)}\rangle + \langle\mathcal{M}^{(0)}|\boldsymbol{I}^{(1)\dagger}(\epsilon)\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(0)}\rangle$$
$$-2e^{-\epsilon\gamma}\frac{\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)}\left(\frac{\beta_0}{\epsilon}+K\right)\langle\mathcal{M}^{(0)}|\boldsymbol{I}^{(1)}(2\epsilon)|\mathcal{M}^{(0)}\rangle - 2\,\langle\mathcal{M}^{(0)}|\boldsymbol{H}^{(2)}_{R}(\epsilon)|\mathcal{M}^{(0)}\rangle$$
$$-\langle\mathcal{M}^{(0)}|\boldsymbol{S}^{(2)}_{V}(\epsilon)|\mathcal{M}^{(0)}\rangle\Bigg]$$

- cancel all remaining terms from two-parton and three-parton final states

$$\mathcal{P}oles^{(2\times 0)}_{q\bar{q}} + \mathcal{P}oles^{(1\times 1)}_{q\bar{q}} + \mathcal{P}oles^{(1\times 0)}_{q\bar{q}g} + \mathcal{P}oles^{(0\times 0)}_{q\bar{q}(ij)} = 0$$

- recover two-loop result for R_{had}

$$\mathcal{F}inite^{(2\times 0)}_{q\bar{q}} + \mathcal{F}inite^{(1\times 1)}_{q\bar{q}} + \mathcal{F}inite^{(1\times 0)}_{q\bar{q}g} + \mathcal{F}inite^{(0\times 0)}_{q\bar{q}(ij)} = R^{NNLO}_{had}$$

3 jet obsevables at NNLO

${\alpha_s}^3 C_F^3$ – contribution to $\langle 1-T \rangle$ at NNLO

A. Gehrmann-De Ridder, E.W.N. Glover, TG

$$\begin{aligned}\langle 1-T \rangle &= \int (1-T) \frac{1}{\sigma_0} \frac{\mathrm{d}\sigma}{\mathrm{d}T} \\ &= C_F \left[\left(\frac{\alpha_s}{2\pi}\right) A + \left(\frac{\alpha_s}{2\pi}\right)^2 B + \left(\frac{\alpha_s}{2\pi}\right)^3 C + \ldots \right]\end{aligned}$$

where $A = 1.57$, $B = 32.3$

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- implemented in EERAD2 ($e^+e^- \to 4j$ at NLO)
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- obtained numerically finite 5 parton and 4 parton contributions

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- constructed subtraction terms
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- obtained numerically finite 5 parton and 4 parton contributions
- observe cancellation of all analytic $1/\epsilon^n$ poles in 4 parton and 3 parton contributions

3 jet obsevables at NNLO

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A. Gehrmann-De Ridder, E.W.N. Glover, TG

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- constructed subtraction terms
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  J. Campbell, M. Cullen, E.W.N. Glover
- obtained numerically finite 5 parton and 4 parton contributions
- observe cancellation of all analytic $1/\epsilon^n$ poles in 4 parton and 3 parton contributions
- First (preliminary) result: $C|_{C_F^2} = -20.4 \pm 4$

Summary

Several technical developments render NNLO calculations of jet observables feasible:

- Two-loop virtual corrections
  - IBP/LI identities
  - Laporta algorithm
  - differential equations
- Real radiation corrections
  - I-duality
  - sector decomposition
  - matrix element subtraction

Summary

Several technical developments render NNLO calculations of jet observables feasible:

- Two-loop virtual corrections
  - IBP/LI identities
  - Laporta algorithm
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- Real radiation corrections
  - I-duality
  - sector decomposition
  - matrix element subtraction

First results:

- NNLO corrections to $e^+e^- \to 2j$
- NNLO corrections to $e^+e^- \to 3j$ (ongoing)