# Next-to-leading Order Calculations with Parton Showers

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(University of Edinburgh)

work done in collaboration with Dave Soper and Steve Mrenna

see M. Krämer and D.E. Soper, Phys. Rev. D66:054017 (2002) and Phys. Rev. D69:054019 (2004); D.E. Soper, Phys. Rev. D69:054020 (2004); M. Krämer, S. Mrenna and D.E. Soper, in preparation

# Motivation

- N$^k$LO calculations
  - + allow precision test of QFTs
  - — break down for certain kinematic configurations
  - — do not provide realistic final states
  - — are limited to IR safe observables

- Parton shower Monte Carlo programs
  - + include certain summations (soft/collinear emissions)
  - + provide realistic final states
  - — do not include hard emissions
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- Aim: Perform NLO calculations with (LL) summation of soft/collinear logarithms and realistic hadronic final states

  $\Rightarrow$ match NLO calculations with parton shower Monte Carlo programs

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- Consider event shape variables in $e^+e^- \to 3$ jets

Examine $df_3/dM$ where $f_3$ is the fraction of events that have three jets and $M$ is the mass of a jet (Durham algorithm, $y_{cut} = 0.05$)

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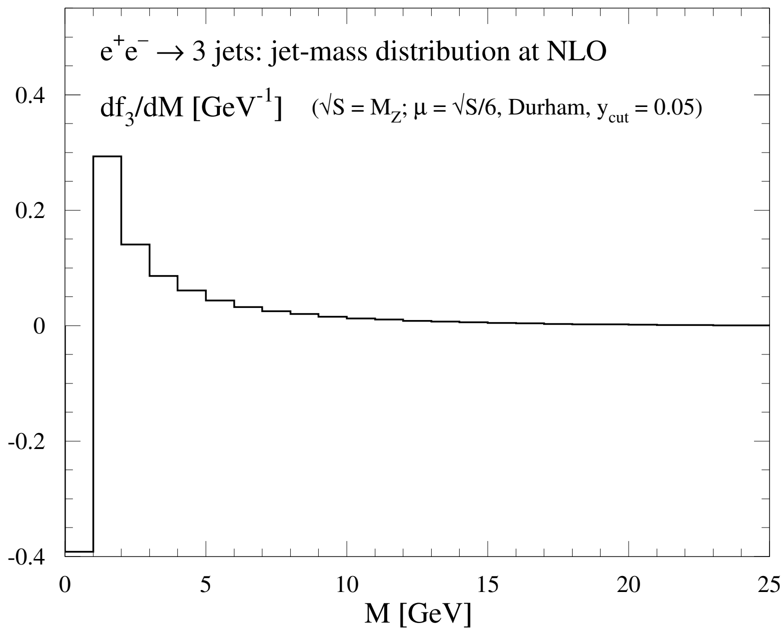
- NLO calculation

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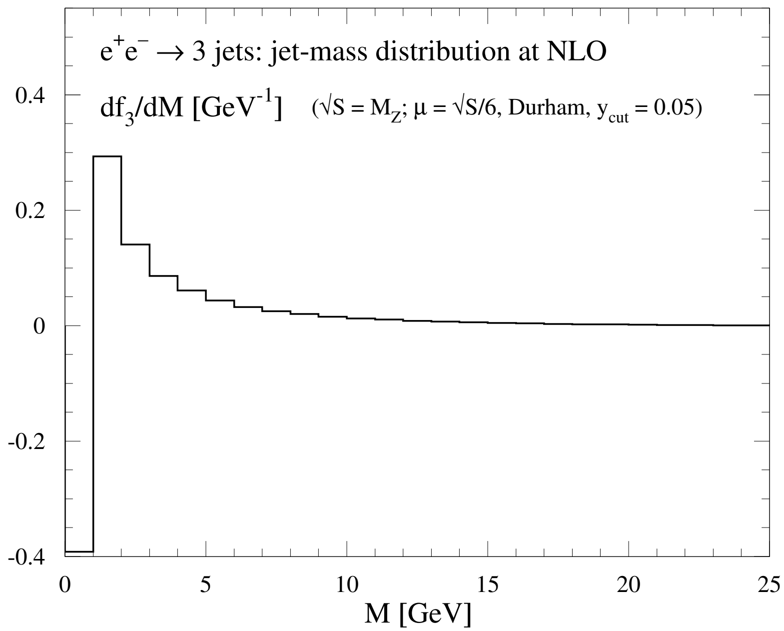
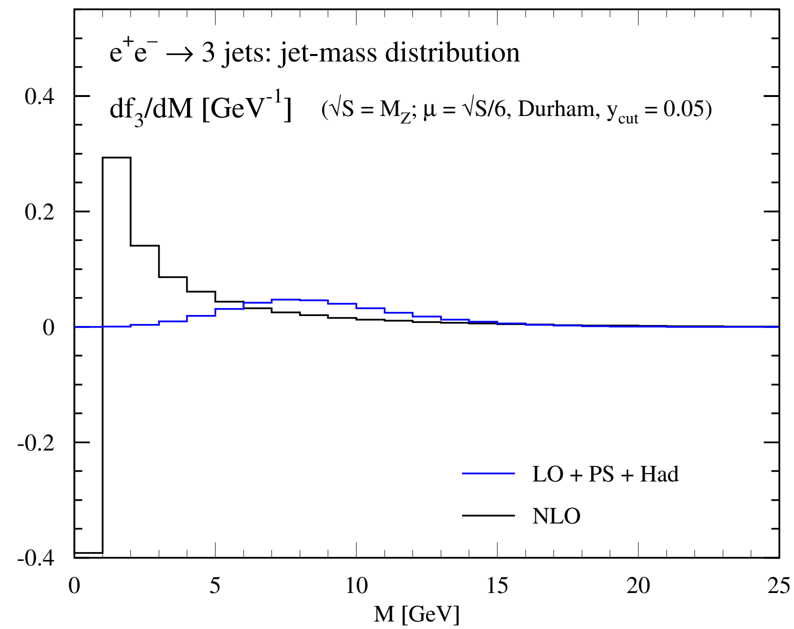



$\Rightarrow$ wrong jet structure for $M \to 0$

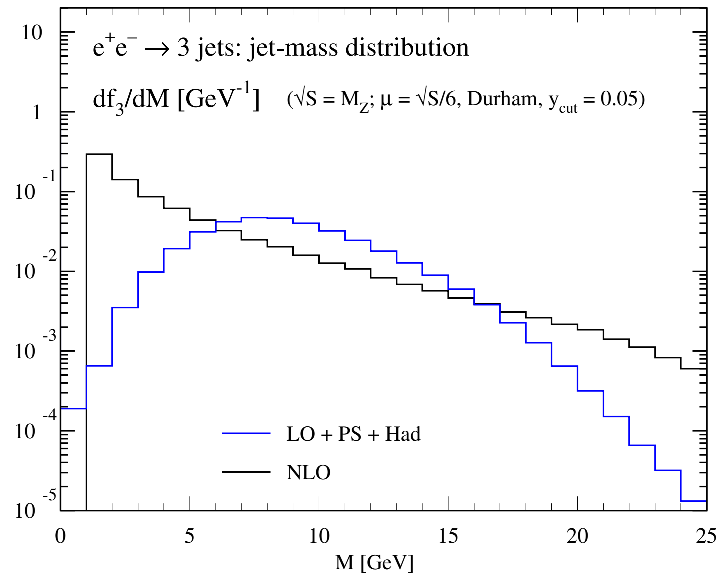
- LO parton shower calculation




$\Rightarrow$ realistic jet structure for $M \to 0$

# Motivation

- Comparison between NLO and LO parton shower calculations




## NLO calculation

+ NLO estimate of $\int dM \, d\sigma/dM$
+ correct hard scattering kinematics
  $\longrightarrow$ realistic jet structure at large $M$
– large collinear logs
  $\longrightarrow$ wrong jet structure at small $M$

## LO parton shower calculation

– LO estimate of $\int dM \, d\sigma/dM$
– no hard gluon emission
  $\longrightarrow$ wrong jet structure at large $M$
+ summation of collinear logs
  $\longrightarrow$ realistic jet structure at small $M$

# Motivation

- Add NLO calculations and parton showers such that
  - NLO results are recovered upon expansion in $\alpha_s$
  - hard emissions are treated as in NLO calculations
  - soft/collinear emissions are treated as in parton shower MC calculations
  - an exclusive set of events is generated
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See also:

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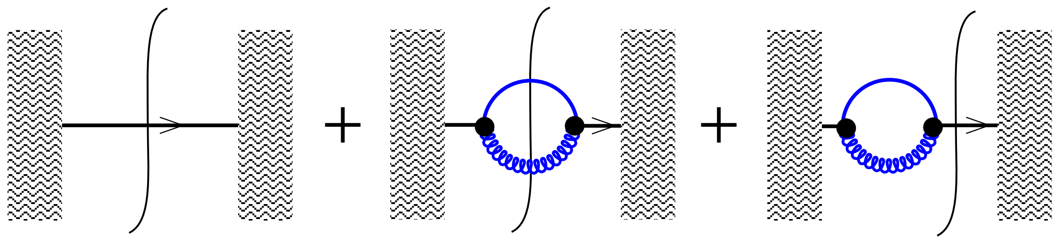
Note: related program of matching multi-leg real emission graphs and parton shower calculations does not include virtual corrections and therefore does not reproduce NLO accuracy

# Adding parton showers to a NLO calculation

- The NLO calculation: collinear singularities (Coulomb gauge)



$$\mathcal{I}[\mathrm{Born}] + \mathcal{I}[\mathrm{real}] + \mathcal{I}[\mathrm{virtual}] \quad =$$

$$\int \frac{d\vec{q}}{2|\vec{q}\,|}\, \mathrm{Tr}\Big\{ \not{q}\, R_0 + \int_0^\infty \frac{d\vec{q}^{\,2}}{\vec{q}^{\,2}} \int_0^1 dx \int_{-\pi}^{\pi} \frac{d\phi}{2\pi}$$

$$\times \left[ \frac{\alpha_s}{2\pi}\, \mathcal{M}_{g/q}(\vec{q}^{\,2}, x, \phi) R(\vec{q}^{\,2}, x, \phi) - \frac{\alpha_s}{2\pi}\, \mathcal{P}_{g/q}(\vec{q}^{\,2}, x) \not{q}\, R_0 \right] \Big\}$$

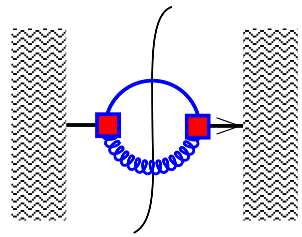
– $R(\vec{q}^{\,2}, x, \phi)$ and $R_0$ represent the rest of the graph. $R(\vec{q}^{\,2}, x, \phi) \to R_0$ for $\vec{q}^{\,2} \to 0$

– For $\vec{q}^{\,2} \to 0$: $\mathcal{M}_{g/q}(\vec{q}^{\,2}, x, \phi) \to \not{q}\tilde{P}_{g/q}(x)$ and $\mathcal{P}_{g/q}(\vec{q}^{\,2}, x) \to \tilde{P}_{g/q}(x)$

$\to \vec{q}^{\,2} \to 0$ singularities cancel

# Adding parton showers to a NLO calculation

- The parton shower way



$$\mathcal{I}[\text{shower}] \;=\; \int \frac{d\vec{q}}{2|\vec{q}|}\,\text{Tr}\Big\{\int_0^\infty \frac{d\bar{q}^2}{\bar{q}^2}\int_0^1 dx\int_{-\pi}^{\pi}\frac{d\phi}{2\pi}\,\mathcal{M}_{g/q}(\bar{q}^2,x,\phi)R(\bar{q}^2,x,\phi)$$
$$\times\exp\left(-\int_{\bar{q}^2}^\infty \frac{d\bar{l}^2}{\bar{l}^2}\int_0^1 dz\,\frac{\alpha_s}{2\pi}\,\mathcal{P}_{g/q}(\bar{l}^2,z)\right)\Big\}$$

– The collinear singularity at $\bar{q}^2 \to 0$ is damped by a Sudakov exponential with behaviour $\exp\left(-\alpha_s c \log^2(\bar{q}^2)\right)$ for $\bar{q}^2 \to 0$

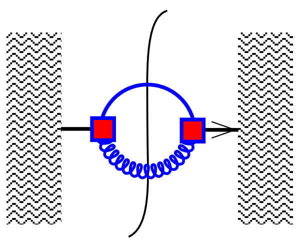
– This is similar to what parton shower MC programs use

# Adding parton showers to a NLO calculation

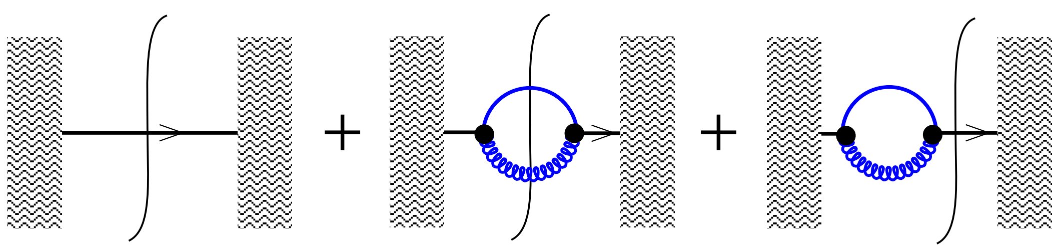
- The connection

$$\mathcal{I}[\text{shower}] = (\mathcal{I}[\text{Born}] + \mathcal{I}[\text{real}] + \mathcal{I}[\text{virtual}]) \times \left(1 + \mathcal{O}(\alpha_s^2)\right)$$

That is



is equivalent to

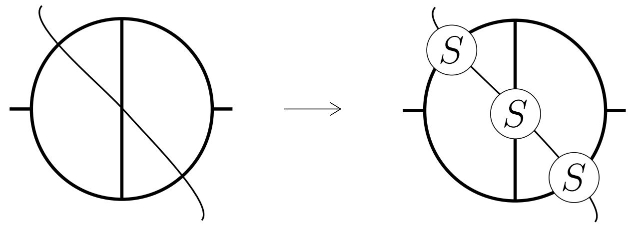


# Adding parton showers to a NLO calculation

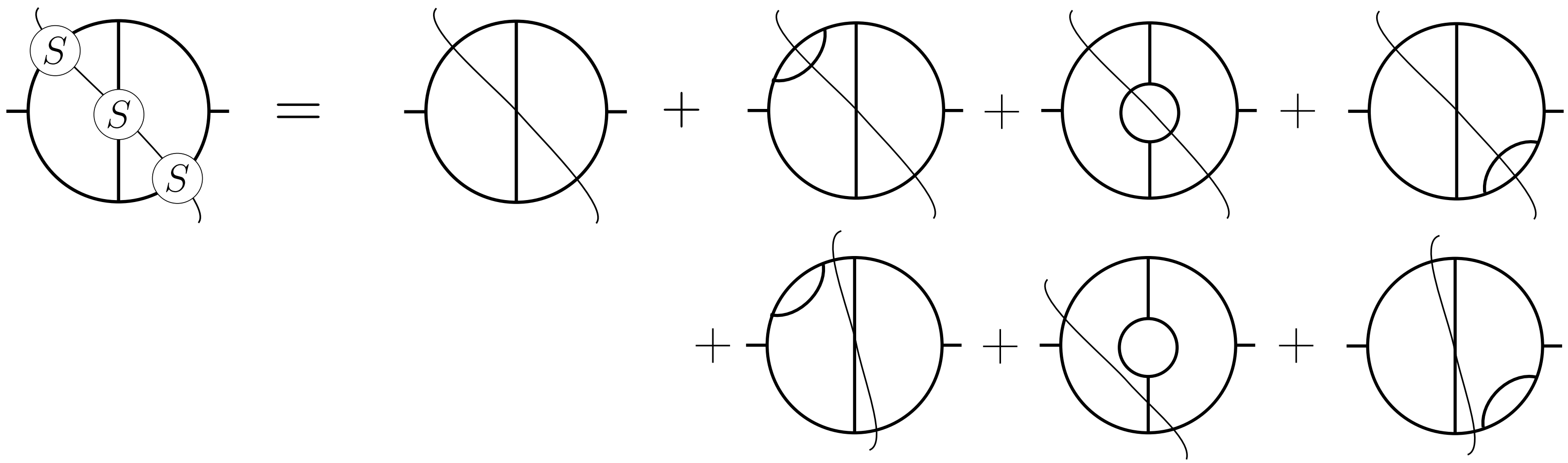
- The basic idea

Start with a $\mathcal{O}(\alpha_s^1)$ cut graph

and replace $\mathcal{I}[\mathrm{Born}]$ by $\mathcal{I}[\mathrm{Shower}]$




expanding gives plus




$\Rightarrow$ delete these $\mathcal{O}(\alpha_s^2)$ graphs from the NLO calculation to avoid double counting

Shower from NLO graphs? $\longrightarrow$ need to treat soft divergences (Soper, Phys.Rev.D69:054020,2004)

# Adding parton showers to a NLO calculation

- A test

Calculate $d\sigma/dt$ at thrust $t = 0.71$ ($\sqrt{s} = M_Z, \mu = \sqrt{s}/6$)

Consider

$$R = \frac{\text{NLO} \oplus \text{Shower} - \text{NLO}}{\text{NLO}}$$

If the calculation is correct,

$$R = \frac{(C_0\alpha_s^B + C_1\alpha_s^{B+1} + C_2\alpha_s^{B+2} + \cdots) - (C_0\alpha_s^B + C_1\alpha_s^{B+1})}{C_0\alpha_s^B + C_1\alpha_s^{B+1}}$$

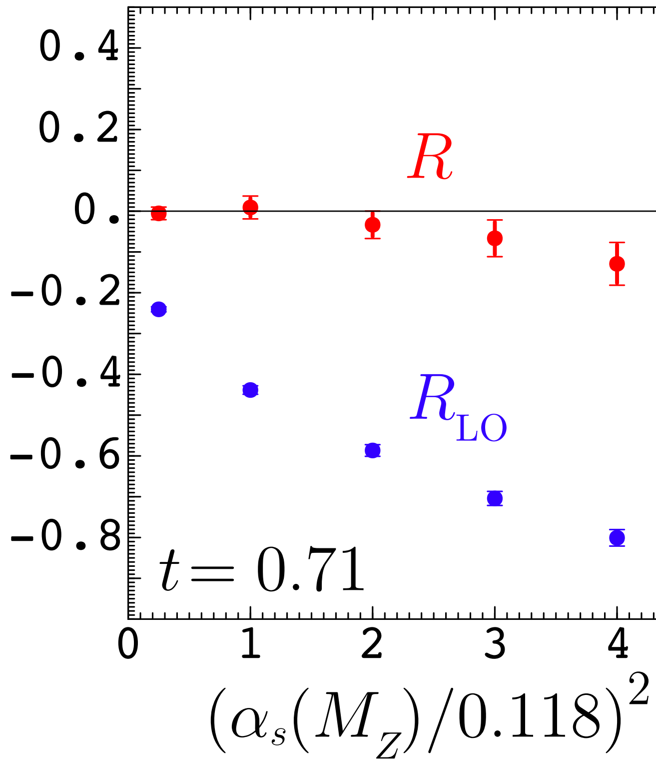
so that

$$R = \frac{C_2}{C_0}\alpha_s^2 + \cdots$$

Note: if the matching is not consistent, then the expansion of $R$ will start at $\mathcal{O}(\alpha_s)$

# Adding parton showers to a NLO calculation

Plot $R$ versus $\alpha_s^2(M_Z)$:



# Adding parton showers to a NLO calculation

- A NLO event generator

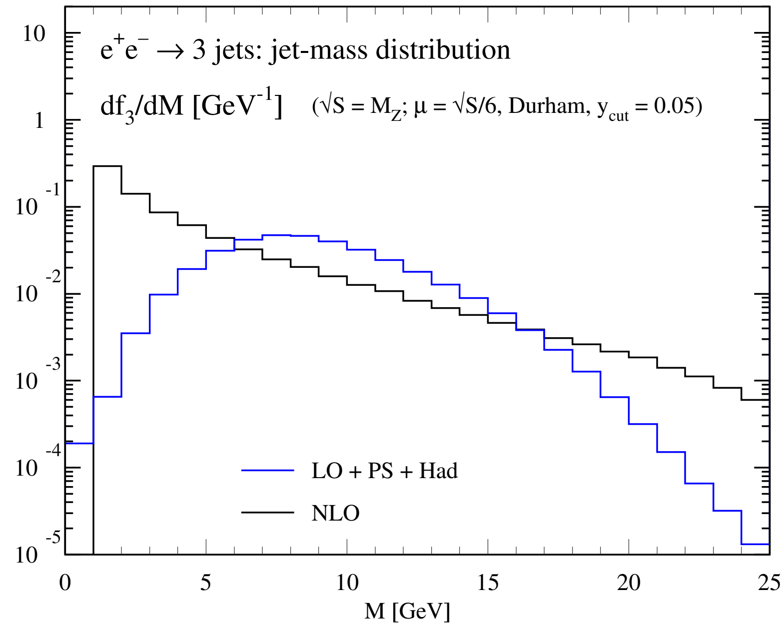
| NLO calculation & first splitting $\longrightarrow$ parton events with NLO weights | $\longrightarrow$ | Pythia: further parton showers and/or hadronization $\longrightarrow$ complete hadronic events with NLO weights |
|---|---|---|

— Program works as event generator with NLO (positive or negative) weights and produces a multi-parton or multi-hadron final state

— The Les Houches interface had to be extended to include information about the splitting history of a parton

# Adding parton showers to a NLO calculation

- 3-jet event shape variable $df_3/dM$: compare NLO, LO $\oplus$ PS and NLO $\oplus$ PS



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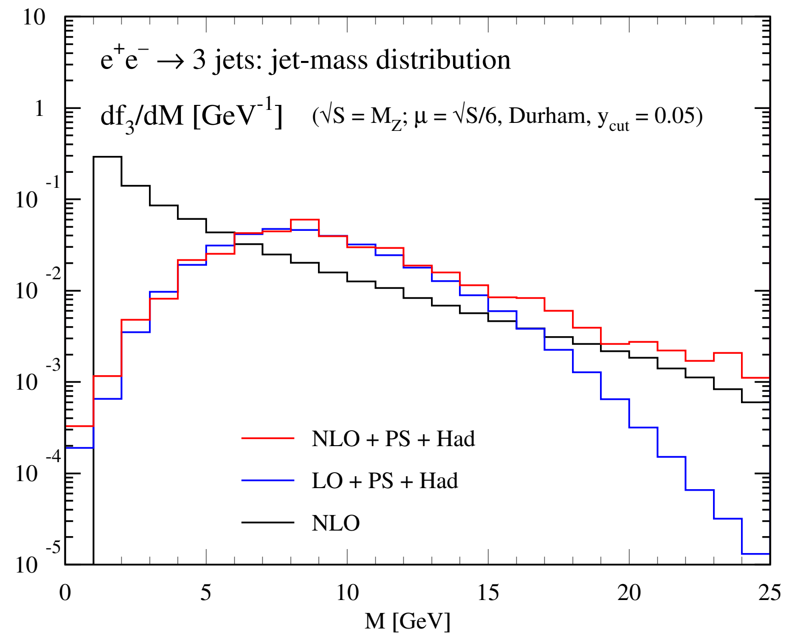

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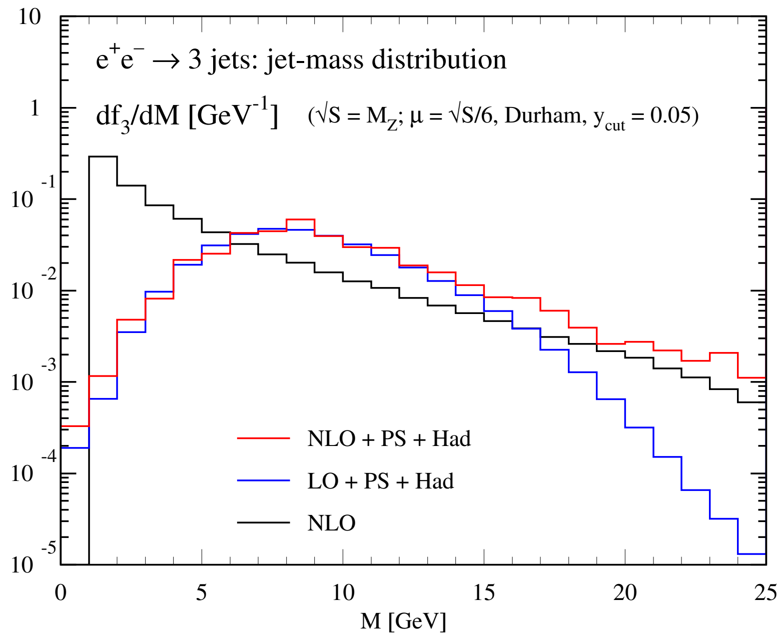


- NLO plus parton shower

+ provides a NLO estimate of $\int dM\, d\sigma/dM$
+ includes the correct hard scattering kinematics $\longrightarrow$ realistic jet structure at large $M$
+ includes the summation of collinear logs $\longrightarrow$ realistic jet structure at small $M$
+ provides a realistic final state (complete hadronic event)

# Adding parton showers to a NLO calculation

- Work in progress

— numerical tests

— add $Z$-boson exchange ($\gamma^*$ only at the moment)

— study dependence on choices for parameters and schemes ($\longrightarrow$ effects of $\mathcal{O}(\alpha_s^2)$)

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## Summary & Outlook

— NLO calculations can include parton showers

— Primary splittings must be matched to NLO calculations

— Exclusive set of partonic events can be generated and interfaced with standard event generators (Pythia, Herwig, Ariadne,. . . ) for further showering and hadronization

— Users can mix and match

# Backup slides

— Proof of the “Subtraction to multiplication theorem”

— Alternative matching schemes

— Results on the thrust distribution

# Proof of the "Subtraction to multiplication theorem"

- Proof step 0: add and subtract

$$\mathcal{I}[\text{shower}] = \mathcal{I}_1[\text{shower}] + \mathcal{I}_2[\text{shower}]$$

with

$$\begin{aligned}
\mathcal{I}_1[\text{shower}] \;&=\; \int \frac{d\vec{q}}{2|\vec{q}\,|}\,\mathrm{Tr}\Bigg\{\int_0^\infty \frac{d\bar{q}^2}{\bar{q}^2}\int_0^1 dx \int_{-\pi}^{\pi}\frac{d\phi}{2\pi}\,\exp\left(-\int_{\bar{q}^2}^\infty \frac{d\bar{l}^2}{\bar{l}^2}\int_0^1 dz\,\frac{\alpha_s}{2\pi}\,\mathcal{P}_{g/q}(\bar{l}^2,z)\right)\\
&\times\left[\frac{\alpha_s}{2\pi}\mathcal{M}_{g/q}(\bar{q}^2,x,\phi)\,R(\bar{q}^2,x,\phi) - \frac{\alpha_s}{2\pi}\,\mathcal{P}_{g/q}(\bar{q}^2,x)\not{q}\,R_0\right]\Bigg\}
\end{aligned}$$

and

$$\begin{aligned}
\mathcal{I}_2[\text{shower}] \;&=\; \int \frac{d\vec{q}}{2|\vec{q}\,|}\,\mathrm{Tr}\Bigg\{\not{q}\,R_0\int_0^\infty d\bar{q}^2\\
&\times\frac{1}{\bar{q}^2}\int_0^1 dx\,\frac{\alpha_s}{2\pi}\,\mathcal{P}_{g/q}(\bar{q}^2,x)\,\exp\left(-\int_{\bar{q}^2}^\infty \frac{d\bar{l}^2}{\bar{l}^2}\int_0^1 dz\,\frac{\alpha_s}{2\pi}\,\mathcal{P}_{g/q}(\bar{l}^2,z)\right)\Bigg\}
\end{aligned}$$

# Proof of the "Subtraction to multiplication theorem"

- Proof step 1: expand $\mathcal{I}_1[\text{shower}]$

$$
\begin{aligned}
\mathcal{I}_1[\text{shower}] &= \int \frac{d\vec{q}}{2|\vec{q}|}\,\mathrm{Tr}\Bigg\{\int_0^\infty \frac{d\bar{q}^2}{\bar{q}^2}\int_0^1 dx \int_{-\pi}^{\pi}\frac{d\phi}{2\pi}\,\exp\left(-\int_{\bar{q}^2}^\infty \frac{d\bar{l}^2}{\bar{l}^2}\int_0^1 dz\,\frac{\alpha_s}{2\pi}\,\mathcal{P}_{g/q}(\bar{l}^2,z)\right) \\
&\quad \times\left[\frac{\alpha_s}{2\pi}\mathcal{M}_{g/q}(\bar{q}^2,x,\phi)\,R(\bar{q}^2,x,\phi) - \frac{\alpha_s}{2\pi}\,\mathcal{P}_{g/q}(\bar{q}^2,x)\not{q}\,R_0\right]\Bigg\} \\
&= \int \frac{d\vec{q}}{2|\vec{q}|}\,\mathrm{Tr}\Bigg\{\int_0^\infty \frac{d\bar{q}^2}{\bar{q}^2}\int_0^1 dx \int_{-\pi}^{\pi}\frac{d\phi}{2\pi}\times 1 \\
&\quad \times\left[\frac{\alpha_s}{2\pi}\mathcal{M}_{g/q}(\bar{q}^2,x,\phi)\,R(\bar{q}^2,x,\phi) - \frac{\alpha_s}{2\pi}\,\mathcal{P}_{g/q}(\bar{q}^2,x)\not{q}\,R_0\right]\Bigg\} + \mathcal{O}(\alpha_s^2\times R) \\
&= \mathcal{I}[\text{real}] + \mathcal{I}[\text{virtual}] + \mathcal{O}(\alpha_s^2\times R)
\end{aligned}
$$

# Proof of the “Subtraction to multiplication theorem”

- Proof step 2: calculate $\mathcal{I}_2[\text{shower}]$

$$\mathcal{I}_2[\text{shower}] = \int \frac{d\vec{q}}{2|\vec{q}\,|}\,\text{Tr}\Bigg\{\not{q}\,R_0 \int_0^\infty d\bar{q}^2 \times \frac{1}{\bar{q}^2}\int_0^1 dx\,\frac{\alpha_s}{2\pi}\,\mathcal{P}_{g/q}(\bar{q}^2,x)\,\exp\left(-\int_{\bar{q}^2}^\infty \frac{d\bar{l}^2}{\bar{l}^2}\int_0^1 dz\,\frac{\alpha_s}{2\pi}\,\mathcal{P}_{g/q}(\bar{l}^2,z)\right)\Bigg\}$$

$$\int \frac{d\vec{q}}{2|\vec{q}\,|}\,\text{Tr}\Bigg\{\not{q}\,R_0 \int_0^\infty d\bar{q}^2\,\frac{d}{d\bar{q}^2}\exp\left(-\int_{\bar{q}^2}^\infty \frac{d\bar{l}^2}{\bar{l}^2}\int_0^1 dz\,\frac{\alpha_s}{2\pi}\,\mathcal{P}_{g/q}(\bar{l}^2,z)\right)\Bigg\}$$

$$= \int \frac{d\vec{q}}{2|\vec{q}\,|}\,\text{Tr}\left\{\not{q}\,R_0\right\}$$

$$\Rightarrow \quad \mathcal{I}[\text{Born}] + \mathcal{I}[\text{real}] + \mathcal{I}[\text{virtual}] = \mathcal{I}[\text{shower}] \times \left(1 + \mathcal{O}(\alpha_s^2)\right)$$

# Alternative matching schemes

- You could use your favourite $\mathcal{M}'$ & $\mathcal{P}'$

$$\mathcal{I}'[\text{shower}] = \int \frac{d\vec{q}}{2|\vec{q}|}\,\mathrm{Tr}\Bigg\{\int_0^\infty \frac{d\bar{q}^2}{\bar{q}^2}\int_0^1 dx \int_{-\pi}^{\pi}\frac{d\phi}{2\pi}\,\mathcal{M}'_{g/q}(\bar{q}^2,x,\phi)R(\bar{q}^2,x,\phi)$$

$$\times\exp\left(-\int_{\bar{q}^2}^\infty \frac{d\bar{l}^2}{\bar{l}^2}\int_0^1 dz\,\frac{\alpha_s}{2\pi}\,\mathcal{P}'_{g/q}(\bar{l}^2,z)\right)\Bigg\}$$

Then

$$\mathcal{I}'[\text{shower}] = \int \frac{d\vec{q}}{2|\vec{q}|}\,\mathrm{Tr}\Bigg\{\not{q}\,R_0 + \int_0^\infty \frac{d\bar{q}^2}{\bar{q}^2}\int_0^1 dx \int_{-\pi}^{\pi}\frac{d\phi}{2\pi}$$

$$\times\left[\frac{\alpha_s}{2\pi}\,\mathcal{M}'_{g/q}(\bar{q}^2,x,\phi)R(\bar{q}^2,x,\phi) - \frac{\alpha_s}{2\pi}\,\mathcal{P}'_{g/q}(\bar{q}^2,x)\not{q}\,R_0\right]\Bigg\} + \mathcal{O}(\alpha_s^2\times R)$$

$$= \mathcal{I}[\text{Born}] + \mathcal{I}'[\text{real}] + \mathcal{I}'[\text{virtual}]$$

# Alternative matching schemes

- What you get with $\mathcal{M}'$ & $\mathcal{P}'$:

$$\mathcal{I}'[\text{shower}] = \mathcal{I}[\text{Born}] + \mathcal{I}'[\text{real}] + \mathcal{I}'[\text{virtual}]$$

which gives

$$\mathcal{I}[\text{Born}] + \mathcal{I}[\text{real}] + \mathcal{I}[\text{virtual}] =$$

$$\mathcal{I}'[\text{shower}] + (\mathcal{I}[\text{real}] - \mathcal{I}'[\text{real}]) + (\mathcal{I}[\text{virtual}] - \mathcal{I}'[\text{virtual}])$$

$\Rightarrow$ $\mathcal{M}'$ and $\mathcal{P}'$ act as subtractions for $\mathcal{M}$ and $\mathcal{P}$

(as long as they have the right $\bar{q}^2 \to 0$ singularities, the cancel the singularities of $\mathcal{M}$ and $\mathcal{P}$)

# Adding parton showers to a NLO calculation

- Thrust distribution $d\sigma/dt$: compare LO, NLO, LO $\oplus$ PS and NLO $\oplus$ PS