Physical renormalization condition for the quark-mixing matrix*

Markus Roth
Max-Planck-Institut, Munich

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in collaboration with A. Denner and E. Kraus

- Introduction
- Restrictions on counterterms from symmetries
- Physical renormalization condition for quark-mixing matrix (QMM)
- Summary and conclusions

*More details can be found in Phys. Rev. D **70** (2004) 033002 (hep-ph/0402130)

Introduction

Motivation

- Precise measurement at the B factories Babar and Belle
  Renormalization effects of QMM in SM are small
  W decay into quark pair: $\frac{\delta\sigma}{\sigma} = \mathcal{O}\left(\frac{\alpha}{\pi}\frac{m_{\mathrm{b}}^2}{M_{\mathrm{W}}^2}\right) \approx 10^{-5}$
- Renormalization of QMM conceptually interesting
  Possible guideline for other mixing matrices in extensions of the SM

Desirable properties of a renormalization condition for the QMM

- Gauge independence of the renormalized QMM and physical matrix elements
- Unitarity of the renormalized QMM
- Physically motivated renormalization condition,
  where all quark generations should be treated on equal footing.
- Smooth limit for physical matrix element in the limit degenerated quark masses

QMM in the Standard Model (SM)

Appearance of QMM in SM:

$$\underbrace{\bar{Q}_i'^{\mathrm{L}} G_{ij}^d \Phi d_j'^{\mathrm{R}}}_{\text{Yukawa interaction}} \quad \to \quad -m_{\mathrm{d}}^2 \left(\bar{d}_i^{\mathrm{L}} d_i^{\mathrm{R}} + \bar{d}_i^{\mathrm{R}} d_i^{\mathrm{L}}\right) - \frac{e}{\sqrt{2} M_{\mathrm{W}} s_{\mathrm{W}}} \left(\bar{u}_i^{\mathrm{L}} V_{ij} \phi^+ m_{\mathrm{d},j} d_j^{\mathrm{R}}\right),$$

$$\underbrace{\mathrm{i}\bar{Q}_i'^{\mathrm{L}} \not{D}_{ij} Q_j'^{\mathrm{L}}}_{\text{Kinetic term}} \quad \to \quad -\mathrm{i}\frac{e}{\sqrt{2} s_{\mathrm{W}}} \left(\bar{u}_i^{\mathrm{L}} V_{ij} W^+ d_j^{\mathrm{L}} + \bar{d}_i^{\mathrm{L}} V_{ij}^\dagger W^- u_j^{\mathrm{L}}\right).$$

Natural choice for renormalization condition:

$$\mathrm{W}^+ \to \mathrm{u}_i \bar{\mathrm{d}}_j \qquad \text{or} \qquad \bar{\mathrm{u}}_i \to \mathrm{W}^- \bar{\mathrm{d}}_j \quad \text{(for top quark)}$$

Unstable particles: Unsolved problem, will be ignored in the following.

Unitarity of the QMM: $VV^\dagger = \mathbf{1}$

Physical parameters: 3 angles and 1 CP-violating phase

W-boson decay $\mathrm{W}^+ \to \mathrm{u}_i\bar{\mathrm{d}}_j$

Matrix element to $\mathrm{W}^+ \to \mathrm{u}_i\bar{\mathrm{d}}_j$:

$$\mathcal{M}_{ij} = \sum_{a=1}^{2}\sum_{\sigma=\pm} F^{\sigma}_{a,ij}\mathcal{M}^{\sigma}_{a,ij}, \qquad \mathcal{M}^{-}_{1,ij} = -\frac{e}{\sqrt{2}s_{\mathrm{W}}}\bar{u}(p_{\mathrm{u},i})\not{\varepsilon}(p_{\mathrm{W}})\omega_{-}v(p_{\mathrm{d},j}),$$

$$F^{-}_{1} = V + \delta F^{-}_{\mathrm{loop},1} + \delta F_{\mathrm{ct}} + \delta V.$$

Explicit 1-loop calculation in dimensional regularization:

$\Rightarrow$ Divergences in $\delta F^{-}_{\mathrm{loop},1}V^{\dagger}$ = hermitian

Unitarity of QMM yields $\delta V V^{\dagger}$ = anti-hermitian

Decompose counterterms in a hermitian and an anti-hermitian part:

$$\delta F_{\mathrm{ct}}V^{\dagger} = \underbrace{\left(\delta Z_{\mathrm{W}} + \frac{\delta e}{e} - \frac{\delta s_{\mathrm{W}}}{s_{\mathrm{W}}}\right) + \frac{1}{2}\left\{\left[\delta Z^{\mathrm{u,L}\dagger} + \delta Z^{\mathrm{u,L}}\right] + V\left[\delta Z^{\mathrm{d,L}} + \delta Z^{\mathrm{d,L}\dagger}\right]V^{\dagger}\right\}}_{\text{hermitian}}$$

$$\underbrace{+\frac{1}{2}\left\{\left[\delta Z^{\mathrm{u,L}\dagger} - \delta Z^{\mathrm{u,L}})\right] + V\left[\delta Z^{\mathrm{d,L}} - \delta Z^{\mathrm{d,L}\dagger}\right]V^{\dagger}\right\}}_{\text{anti-hermitian}}.$$

Proposals for renormalization conditions

- Denner,Sack'90:
  Counterterm for QMM guided by UV divergences:

$$\delta V^{\mathrm{DS}} \stackrel{!}{=} -\frac{1}{2}\left\{\left[(\delta Z^{\mathrm{u,L}})^\dagger - \delta Z^{\mathrm{u,L}}\right] V + V\left[\delta Z^{\mathrm{d,L}} - (\delta Z^{\mathrm{d,L}})^\dagger\right]\right\}.$$

  Problem: δV and matrix element for $\mathrm{W}^+ \to \mathrm{u}_i \bar{\mathrm{d}}_j$ is gauge dependent at 1 loop!
  Gambino,Grassi,Madricardo'98

- Gambino,Grassi,Madricardo'98:
  Same δV as in Denner-Sack scheme but with field renormalization constants $\delta Z^{\mathrm{u/d,L}}$ fixed at zero momenta via

$$\Gamma_{\bar{\mathrm{u}}_i \mathrm{u}_j}(0) = 0, \qquad \frac{\partial}{\partial \not{p}} \Gamma^{\mathrm{L}}_{\bar{\mathrm{u}}_i \mathrm{u}_j}(0) = 0.$$

  - Gauge independence of QMM shown only at 1-loop level.

  Remark: Appearance of terms $\propto 1/(m_{q,i}^2 - m_{q,j}^2)$.

- $\overline{\mathrm{MS}}$ scheme: Balzereit,Mannel,Plümper'99;Pilaftsis'02

$$\delta V^{\overline{\mathrm{MS}}} \stackrel{!}{=} -\frac{1}{2}\left\{\left[(\delta Z^{\mathrm{u,L},\overline{\mathrm{MS}}})^\dagger - \delta Z^{\mathrm{u,L},\overline{\mathrm{MS}}}\right] V + V\left[\delta Z^{\mathrm{d,L},\overline{\mathrm{MS}}} - (\delta Z^{\mathrm{d,L},\overline{\mathrm{MS}}})^\dagger\right]\right\}.$$

  - Fully consistent, yields gauge-independent results.
  - Results depend on unphysical renormalization scale.

  Remark: Physical matrix elements include terms $\propto 1/(m_{q,i}^2 - m_{q,j}^2)$.

- Diener,Kniehl'01:
  QMM counterterms from difference to reference theories with no mixing; restore unitarity of QMM in a second step
  - Renormalization procedure worked out only at 1 loop.

- Zhou'03:
  Renormalization condition for QMM:

$$\delta V = -\delta F_{\mathrm{ct}} - \text{corresponding loop contributions.}$$

  Restore unitarity in a second step.
  - Only proposal, leaves many questions open

- Denner,Roth,Kraus'04:
  Physical renormalization condition for QMM
  - Fully consistent and fulfills all desired properties
  - Renormalization condition equivalent to the one of Zhou
  - Detail discussion of restrictions of symmetries on counterterms

Gauge-parameter dependence

Kluberg-Stern,Zuber'75
Piguet,Sibold'85

Extending BRS symmetry by BRS-varying gauge parameter ξ:

$$s_{\mathsf{BRS}}\xi = \chi, \qquad s_{\mathsf{BRS}}\chi = 0, \qquad \chi = \text{constant ghost field.}$$

χ is a constant ghost

$\Rightarrow$ Gauge independence of physical matrix elements is proven. Kummer'01

Requirement for BRS-invariant counterterms:

$$s_{\mathsf{BRS}}\Gamma_{\mathrm{ct}} = 0$$

Parameters of QMM have to be gauge independent:

$$0 \stackrel{!}{=} s_{\mathsf{BRS}}\left(\delta\theta_n \frac{\partial}{\partial\theta_n}\Gamma_{\mathrm{cl}}\right) = (s_{\mathsf{BRS}}\delta\theta_n)\frac{\partial}{\partial\theta_n}\Gamma_{\mathrm{cl}} + \delta\theta_n \frac{\partial}{\partial\theta_n}\underbrace{(s_{\mathsf{BRS}}\Gamma_{\mathrm{cl}})}_{=0} = \chi(\partial_\xi\delta\theta_n)\frac{\partial}{\partial\theta_n}\Gamma_{\mathrm{cl}}.$$

$\Rightarrow$ Parameters of QMM are gauge independent ($\partial_\xi\delta\theta_n = 0$).

Restrictions from $\mathrm{SU(2)_L}$ gauge symmetry

Global (spontaneously broken) classical $\mathrm{SU(2)_L}$ invariance:

$$\delta_+^{\mathrm{rig}} u_i^{\mathrm{L}} = \frac{\mathrm{i}}{\sqrt{2}} V_{ij} d_j^{\mathrm{L}}, \qquad \delta_+^{\mathrm{rig}} \bar{u}_i^{\mathrm{L}} = 0,$$
$$\delta_+^{\mathrm{rig}} d_i^{\mathrm{L}} = 0, \qquad \delta_+^{\mathrm{rig}} \bar{d}_i^{\mathrm{L}} = -\frac{\mathrm{i}}{\sqrt{2}} \bar{u}_j^{\mathrm{L}} V_{ji}, \qquad \text{etc.}$$

Further restrictions on counterterms: $\delta_a^{\mathrm{rig}} \Gamma_{\mathrm{ct}} = 0 \qquad (a = +, -, 3).$

1. Relates field renormalization of left-handed quarks:

$$u_i^{\mathrm{L}} \to \left(\mathbf{1} + \delta z_{\mathrm{inv}}^{\mathrm{u,L}}\right)_{ij} u_j^{\mathrm{L}}, \qquad d_i^{\mathrm{L}} \to \left(\mathbf{1} + V^\dagger \delta z_{\mathrm{inv}}^{\mathrm{u,L}} V\right)_{ij} d_j^{\mathrm{L}}, \qquad \text{etc.}$$

2. Requires extension of $\delta\theta_n$ by field renormalizations: $\left[\delta_a^{\mathrm{rig}}, \delta\theta_n \frac{\partial}{\partial\theta_n}\right] \neq 0$

$$\Gamma_{\mathrm{ct}} = \delta\theta_n \left[\frac{\partial}{\partial\theta_n} + \frac{1}{2}\int \mathrm{d}^4x \left(u_i^{\mathrm{L}} \frac{\delta}{\delta u_j^{\mathrm{L}}} \frac{\partial V_{ik}}{\partial\theta_n} V_{kj}^\dagger + \text{etc}\right)\right] \Gamma_{\mathrm{cl}}.$$

Available counterterms:

$$\begin{aligned}
V_{ij} &\to V_{ij} + \delta\theta_n \partial_{\theta_n} V_{ij}, \\
u_i^{\mathrm{L}} &\to Z_{ij}^{\mathrm{u,L}}(\xi) u_j^{\mathrm{L}} = \left[\mathbf{1} + \delta z_{\mathrm{inv}}^{\mathrm{u,L}}(\xi) + \tfrac{1}{2}\delta\theta_n (\partial_{\theta_n} V) V^\dagger\right]_{ij} u_j^{\mathrm{L}}, \\
d_i^{\mathrm{L}} &\to Z_{ij}^{\mathrm{d,L}}(\xi) d_j^{\mathrm{L}} = \left[\mathbf{1} + V^\dagger \delta z_{\mathrm{inv}}^{\mathrm{u,L}}(\xi) V - \tfrac{1}{2}\delta\theta_n V^\dagger (\partial_{\theta_n} V)\right]_{ij} d_j^{\mathrm{L}},
\end{aligned}$$

etc.

Classical SU(2) invariance yields the Denner-Sack prescription:

$$\delta V \equiv \delta\theta_n \partial_{\theta_n} V = -\frac{1}{2}\left\{\left[(\delta Z^{\mathrm{u,L}}(\xi))^\dagger - \delta Z^{\mathrm{u,L}}(\xi)\right] V + V \left[\delta Z^{\mathrm{d,L}}(\xi) - (\delta Z^{\mathrm{d,L}}(\xi))^\dagger\right]\right\}.$$

Consequences:

- Physical parameters of QMM $\delta\theta_n$ replaced by $\delta Z^{\mathrm{d,L}}(\xi)$.
- Gauge dependence of $\delta Z^{\mathrm{d,L}}(\xi)$ restricted by gauge independence of $\delta\theta_n$.
- Need full freedom in $\delta Z^{\mathrm{u,L}}(\xi)$, $\delta Z^{\mathrm{d,L}}(\xi)$ for on-shell conditions:

$$\Gamma_{\bar{q}_i q_j}(m_{q_j}^2) = 0, \qquad \frac{\partial}{\partial \not{p}} \Gamma_{\bar{q}_i q_j}(m_{q_j}^2) = \delta_{ij}.$$

$\Rightarrow$ Not enough freedom in gauge-dependent part of $\delta Z^{\mathrm{u,L}}(\xi)$, $\delta Z^{\mathrm{d,L}}(\xi)$

The Denner-Sack prescription together with a complete on-shell scheme induces an artificial gauge-parameter dependence in $\delta\theta_n$.

$\Rightarrow$ Violation of the extended BRS invariance

$\Rightarrow$ Artificial gauge dependence of physical matrix elements

Extended BRS symmetry & $\mathrm{SU(2)_L}$ symmetry in its classical form & complete on-shell conditions are in general not possible at the same time!

Finite field redefinitions

Introduction of finite field redefinitions:

$$d_i^{\mathrm{L}} \longrightarrow R_{ij}^{\mathrm{fin}} d_j^{\mathrm{L}} = (\delta_{ij} + \delta R_{ij}^{\mathrm{fin}}) d_j^{\mathrm{L}}.$$

done everywhere, in effective action, BRS and $\mathrm{SU}(2)_{\mathrm{L}}$ symmetry.

- Does not spoil symmetries.
- R^{fin} appear explicitly in $\mathrm{SU}(2)_{\mathrm{L}}$ transformations:

$$\begin{aligned} \delta_+^{\mathrm{rig}} u_i^{\mathrm{L}} &= \tfrac{\mathrm{i}}{\sqrt{2}} V_{ij} R_{jk}^{\mathrm{fin}} d_k^{\mathrm{L}}, & \delta_+^{\mathrm{rig}} \bar{u}_i^{\mathrm{L}} &= 0, \\ \delta_+^{\mathrm{rig}} d_i^{\mathrm{L}} &= 0, & \delta_+^{\mathrm{rig}} \bar{d}_i^{\mathrm{L}} &= -\tfrac{\mathrm{i}}{\sqrt{2}} \bar{u}_j^{\mathrm{L}} V_{jk} (R_{ki}^{\mathrm{fin}})^{*-1}, \qquad \text{etc.} \end{aligned}$$

$\Rightarrow \mathrm{SU}(2)_{\mathrm{L}}$ symmetry is renormalized!

Available parameters for on-shell conditions:

$$V_{ij} \to V_{ij} + \delta\theta_n \partial_{\theta_n} V_{ij},$$
$$u_i^{\mathrm{L}} \to Z_{ij}^{\mathrm{u,L}}(\xi) u_j^{\mathrm{L}} = \left[\mathbf{1} + \delta z_{\mathrm{inv}}^{\mathrm{u,L}}(\xi) + \tfrac{1}{2}\delta\theta_n (\partial_{\theta_n} V) V^\dagger\right]_{ij} u_j^{\mathrm{L}},$$
$$d_i^{\mathrm{L}} \to Z_{ij}^{\mathrm{d,L}}(\xi) d_j^{\mathrm{L}} = \left[\mathbf{1} + V^\dagger \delta z_{\mathrm{inv}}^{\mathrm{u,L}}(\xi) V - \tfrac{1}{2}\delta\theta_n V^\dagger (\partial_{\theta_n} V) + \delta R^{\mathrm{fin}}(\xi)\right]_{ij} d_j^{\mathrm{L}},$$

etc.

⇒ Enough parameters in $\delta Z^{\mathrm{u,L}}(\xi)$, $\delta Z^{\mathrm{d,L}}(\xi)$ to fulfill on-shell conditions!

Denner-Sack prescription replaced by

$$\begin{aligned}\delta V &= -\frac{1}{2}\left\{\left[(\delta Z^{\mathrm{u,L}}(\xi))^\dagger - \delta Z^{\mathrm{u,L}}(\xi)\right] V + V\left[\delta Z^{\mathrm{d,L}}(\xi) - (\delta Z^{\mathrm{d,L}}(\xi))^\dagger\right]\right\} \\ &\quad + \frac{1}{2} V\left[\delta R^{\mathrm{fin}}(\xi) - (\delta R^{\mathrm{fin}}(\xi))^\dagger\right].\end{aligned}$$

⇒ $\delta\theta_n$ have to be fixed by an additional renormalization condition.

Physical renormalization condition

Use matrix element of W decay $\mathrm{W}^+ \longrightarrow \mathrm{u}_i\bar{\mathrm{d}}_j$:

$$\mathcal{M}_{ij} = \sum_{a=1}^{2}\sum_{\sigma=\pm} F^{\sigma}_{a,ij}\mathcal{M}^{\sigma}_{a,ij}, \qquad \mathcal{M}^{-}_{1,ij} = -\frac{e}{\sqrt{2}s_{\mathrm{W}}}\bar{u}(p_{\mathrm{u},i})\not{\varepsilon}(p_{\mathrm{W}})\omega_{-}v(p_{\mathrm{d},j}),$$

$$F^{-}_{1} = V + \delta V + \delta F_{\mathrm{ct}} + \delta F^{-}_{\mathrm{loop},1}.$$

Decompose F^{-}_{1} into a hermitian and an unitary part:

$$F^{-}_{1} = HY, \qquad H^{\dagger} = H, \qquad Y^{\dagger}Y = \mathbf{1}.$$

Physical renormalization condition:

$$Y \overset{!}{=} V \qquad \text{or} \qquad F^{-}_{1} \overset{!}{=} HV$$

The unitary part of the form factor F^{-}_{1} does not receive quantum corrections!

This renormalization condition leads to

$$\delta V \overset{!}{=} -\frac{1}{2}\left\{\left[(\delta Z^{\mathrm{u,L}})^\dagger - \delta Z^{\mathrm{u,L}}\right] V + V\left[\delta Z^{\mathrm{d,L}} - (\delta Z^{\mathrm{d,L}})^\dagger\right]\right\}$$
$$-\frac{1}{2}\left[\delta F^{-}_{\mathrm{loop},1} - V(\delta F^{-}_{\mathrm{loop},1})^\dagger V\right].$$

Renormalization condition fixes $N^2 = 9$ parameters of a general unitary matrix, but QMM has only $(N-1)^2 = 4$ physical parameters.

Need in addition the $2N-1=5$ unphysical phases for field renormalization of the quark fields which are not fixed by on-shell conditions.

Summary and conclusions

- Extended BRS symmetry rules gauge-parameter dependence of counterterms and physical matrix elements.

  ⇒ Counterterms to QMM must be gauge-parameter independent.

- Extended BRS symmetry & $\mathrm{SU(2)_L}$ symmetry in its classical form & on-shell scheme are in general at the same time not possible!

  ⇒ Using an on-shell scheme, the mass eigenstates of the quark fields are not the original "$\mathrm{SU(2)_L}$-symmetric" quark fields.

  ⇒ New mixing parameter (finite field redefinitions) appear in the $\mathrm{SU(2)_L}$-gauge transformations.

- Physical renormalization condition based on physical matrix elements to

$$\mathrm{W}^+ \to \mathrm{u}_i\,\bar{\mathrm{d}}_j, \qquad \bar{\mathrm{u}}_i \to \mathrm{W}^-\,\bar{\mathrm{d}}_j \quad \text{(for top quark).}$$

Available parameters to fix δV

Not all parameters of field renormalization fixed by on-shell conditions:

$$q_i^{\mathrm{L/R}} \to Z_{ij}^{q,\mathrm{L/R}} q_j^{\mathrm{L/R}}, \qquad Z^{q,\mathrm{L/R}} = \underbrace{U^{q,\mathrm{L/R}}}_{\text{unitary}} \underbrace{H^{q,\mathrm{L/R}}}_{\text{hermitian}}.$$

Quark mass diagonalization:

$$\Gamma_{\mathrm{bil}} = \int \mathrm{d}^4x \Big\{ \underbrace{\mathrm{i}\bar{u}^{\mathrm{L}} (H^{\mathrm{u,L}})^\dagger H^{\mathrm{u,L}} \partial\!\!\!/ u^{\mathrm{L}}}_{\text{fixes hermitian part}} + \underbrace{\bar{u}^{\mathrm{L}} (H^{\mathrm{u,L}})^\dagger (U^{\mathrm{u,L}})^\dagger M^{\mathrm{u}}_{\mathrm{diag}} U^{\mathrm{u,R}} H^{\mathrm{u,R}} u^{\mathrm{R}}}_{\substack{\text{fixes unitary part} \\ \text{up to unphysical phases}}} + \text{etc} \Big\}.$$

$\Rightarrow$ Unphysical phases from a common complex diagonal matrix in $U^{q,\mathrm{L/R}}$ not fixed by on-shell conditions!

Unphysical phases from common complex diagonal matrix:

$$U_{ij}^{q,\mathrm{L/R}} = \mathrm{e}^{\mathrm{i}\tilde{\varphi}_i^q} \tilde{U}_{ij}^{q,\mathrm{L/R}} = \left(\mathrm{e}^{\mathrm{i}\sum_{n=0}^{N-1} \varphi_n^q T_n^{\mathrm{diag}}}\right)_{ik} \tilde{U}_{kj}^{q,\mathrm{L/R}}.$$

Redefinition of QMM by unphysical phases φ_n^q:

$$V \to \mathrm{e}^{-\mathrm{i}(\varphi_0^{\mathrm{u}}-\varphi_0^{\mathrm{d}})T_0^{\mathrm{diag}}} \mathrm{e}^{-\mathrm{i}\sum_{n=1}^{N-1} \varphi_n^{\mathrm{u}} T_n^{\mathrm{diag}}} V \mathrm{e}^{\mathrm{i}\sum_{n=1}^{N-1} \varphi_n^{\mathrm{d}} T_n^{\mathrm{diag}}}, \qquad T_0^{\mathrm{diag}} = \tfrac{1}{\sqrt{2N}}.$$

$\Rightarrow 2N - 1 = 5$ unphysical phases available to adjust δV

9	$[= N^2]$	free parameters to fulfill renormalization condition for δV.
-5	$[= 2N - 1]$	unphysical phases from field renormalization
$= 4$	$[= (N-1)^2]$	physical parameters of the QMM

$\Rightarrow$ Enough free parameters to fulfill renormalization condition for δV.

Physical renormalization condition of the quark-mixing matrix[1]

- Introducing additional BRS transformations for the gauge parameters:

  ⇒ Physical parameters of the QMM are gauge-parameter independent

  ⇒ Physical matrix elements are gauge-parameter independent

  Kummer'01

- Using the extended BRS symmetry and an on-shell scheme for the quark fields:

  ⇒ $\mathrm{SU}(2)_\mathrm{L}$ symmetry has to be renormalized by gauge-dependent counterterms.

  ⇒ Denner-Sack prescription violates extended BRS symmetry yielding gauge-dependent results

  Denner,Sack'90

- Physical renormalization condition based on physical matrix elements to

$$\mathrm{W}^+ \to \mathrm{u}_i\,\bar{\mathrm{d}}_j, \qquad \bar{\mathrm{u}}_i \to \mathrm{W}^-\,\bar{\mathrm{d}}_j \quad \text{(for top quark).}$$

  with the properties:

  – Gauge independence of the renormalized QMM and physical matrix elements
  – Physically motivated renormalization condition, where all quark generations are treated on equal footing.
  – Smooth limit for physical matrix element in the limit degenerated quark masses

[1] More details can be found in Phys. Rev. D **70** (2004) 033002 (hep-ph/0402130)