QCD Phenomenology

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YETI ’05
January 6th 2005
Tools of the Trade

- Fixed-order perturbation theory
- All-order perturbation theory
- ‘Best of both’
- Non-perturbative fits and models
- Analysis tools
QCD Phenomenology – Why?

- Testing QCD
  - Again???

![Graph showing QCD Phenomenology](image-url)
QCD Phenomenology – Why?

- Testing QCD
  - Again???
- Measuring $\alpha_s$
  - Again???
QCD Phenomenology – Why?

• Testing QCD
  – Again???

• Measuring $\alpha_s$
  – Again???

• Testing our understanding of QCD
  – Perturbative calculations
  – Non-perturbative models
  – Perturbative/non-perturbative interface

• QCD as a tool
  – Electroweak measurements
  – Searches for new physics
Tools of the Trade

- **Fixed-order perturbation theory**
  - LO – state of the art for high parton multiplicity – highly automated
  - NLO – up to five partons – partly automated
  - NNLO – two partons ($\sigma(e^+e^-)$, DIS, Drell–Yan)
  - Valid for infrared safe observables

- **All-order perturbation theory**
- ‘Best of both’
- Non-perturbative fits and models
- Analysis tools
Higher Order Perturbation Theory – Why?

• $\alpha_s$ not so small – need several orders for $\sim%$ accuracy
• Renormalization/Factorization scales $\Rightarrow$ LO perturbation theory not predictive

$$\sigma(s) = \int dx_1 f_i(x_1, \mu_F) \, dx_2 \, f_j(x_2, \mu_F) \, \sigma_{ij}(x_1 x_2 s, \mu_F, \mu_R; \alpha_s(\mu_R))$$

• Higher orders can be enhanced by large coefficients
• 1 parton = 1 jet $\Leftrightarrow$ no dependence on jet definition
Higher Order Calculations – Why So Hard?

1) Loop integrals for virtual matrix elements
   • Get harder with increasing number of legs
   • State of the art: one-loop pentagon, two-loop box

• Some progress with automation?
Higher Order Calculations – Why So Hard?

2) Numerical cancellation of infrared poles
   • Real and virtual emission each divergent in infrared
     → Divergences must be extracted analytically
   • Must be integrated over different phase spaces
   • Observable arbitrarily complicated
     → Must be integrated numerically
Monte Carlo Calculations of NLO QCD

Two separate divergent integrals:

$$\sigma_{NLO} = \int_{m+1} d\sigma^R + \int_m d\sigma^V$$

Must combine before numerical integration.

Jet definition could be arbitrarily complicated.

$$d\sigma^R = d\prod_{m+1} |\mathcal{M}_{m+1}|^2 F^J_{m+1}(p_1, \ldots, p_{m+1})$$

How to combine without knowing $F^J$?
Subtraction Method

• Seek to define an approximate cross section that matches all the real singularities

\[ \sigma^{NLO} = \int_{m+1} \left[ d\sigma^R - d\sigma^A \right] + \int_{m+1} d\sigma^A + \int_m d\sigma^V \]

• but is feasible to integrate analytically

\[ \sigma^{NLO} = \int_{m+1} \left[ (d\sigma^R)_{\epsilon=0} - (d\sigma^A)_{\epsilon=0} \right] + \int_m \left[ d\sigma^V + \int_1 d\sigma^A \right]_{\epsilon=0} \]

• To avoid dependence on unknown \( F^J \), approximate cross section must project event kinematics onto an \( m \)-parton configuration and calculate \( F^J \) from that.

\[ d\sigma^A \stackrel{\text{kin}}{\rightarrow} m+1 \]  
\[ d\prod_{m+1} |\mathcal{M}_{m+1}^{\text{approx}}|^2 F^J_m (\tilde{p}_1, \ldots, \tilde{p}_m). \]

\[ \tilde{p}_i = \tilde{p}_i (p_1, \ldots, p_{m+1}) \]
Implementations

• **NLOJET++** (Zoltan Nagy) for pure jet processes
  – 1-, 2- and 3-jet production

• **MCFM** (John Campbell and Keith Ellis) for everything else
  – Monte Carlo for FeMtobarn processes
Tools of the Trade

- Fixed-order perturbation theory
- All-order perturbation theory
  - Identification of large logarithmic terms at every order
  - Evolution (Altarelli–Parisi, BFKL)
  - Exponentiation and resummation (Sudakov)
  - Only possible for well-defined observables
  - Numerical resummation (parton showers)
- ‘Best of both’
- Non-perturbative fits and models
- Analysis tools
All Orders Resummation

• For cross sections that force us close to phase space boundary, real–virtual cancellation spoiled $\rightarrow$ large logarithms
• Cross sections can be calculated by recurrence relations or evolution equations $\rightarrow$ multiple emissions factorize
• Must ensure observable does not disturb factorization
  $\rightarrow$ Possible only for limited set of observables
• Rule of thumb: $P(\text{no emission}) = \exp\left\{-P_0(\text{emission})\right\}$
  (Sudakov form factor) (exponentiation)
eg Thrust Distribution

- Thrust $\rightarrow 1 \Rightarrow$ very two-jet-like events
- Suppression of radiation harder than $1 - T$
- Leading order probability of radiation harder than $1 - T$:
  \[ C_F \frac{\alpha_s}{2\pi} 2L^2 \quad (L = \log 1 - T) \]
  $\Rightarrow$ all orders probability for thrust above $T$
  \[ e^{-C_F \frac{\alpha_s}{2\pi} 2L^2} \]
- State of the art:
  - next-to-leading log generalized exponentiation:
    \[ (1 + C\alpha_s)e^{-Lg_1(\alpha_sL)+g_2(\alpha_sL)} \]
  
  Leading log: sums terms like $\alpha_s^n L^{n+1}$
  Next-to-leading log: sums terms like $\alpha_s^n L^n$
- Effect of resummation

![Graph showing the effect of resummation on 1/σ dσ/d(1-T)](image)

• Resummation alone not enough to fit data also need non-perturbative power corrections (shape functions)

Event Shapes in hadron–hadron collisions

- In their early days
  - ‘transverse thrust’
  - ‘$K_{\text{out}}$’ (A. Banfi, G. Marchesini, G. Smye and G. Zanderighi, JHEP 0108 (2001) 047)

High-$p_t$ W/Z events:

\[ K_{\text{out}} \equiv \sum_i |p_{t,\text{out},i}| \]
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• ‘Best of both’
  – Parton showers with matrix element matching
• Non-perturbative fits and models
• Analysis tools
Parton Showers

• Numerical attempt to calculate all exclusive final states
  → all orders calculation for any observable
• Must introduce resolution criterion: large logs
  → evolution driven by Sudakov form factor
Matrix element matching

- Parton shower accurate in soft and collinear limits
  - Model dependent extrapolation to hard emission
- Fixed order accurate for hard well separated partons
  - Divergent in soft and collinear regions

→ Need to combine
Tools of the Trade

• Fixed-order perturbation theory
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• ‘Best of both’
• Non-perturbative fits and models
  – Parton distribution functions
  – Fragmentation functions
  – Hadronization models
  – Underlying event models
• Analysis tools
Parton Distribution Functions

- Cross section for deep inelastic scattering:
  \[ \frac{d^2\sigma}{dx\,dQ^2} = \frac{2\pi\alpha^2}{xQ^4}[(1 + (1 - y)^2)F_2(x, Q^2) - y^2F_L(x, Q^2)] \]

- Parton model:
  \[ F_2(x, Q^2) = \sum_q e_q^2 x f_q(x) \]

- Collinear factorization: \( Q^2 \)-dep.

- DGLAP evolution:
  \[ f_q(x, Q^2) = \frac{\alpha_s}{2\pi} \log \frac{Q^2}{Q_0^2} \int_x^1 \frac{dy}{y} f_q(y, Q_0^2) P_{qq}(x/y) + \alpha_s^2 \log^2 \int \int + \ldots \]

- pdf at some \( x \) and \( Q^2 \) point depends on pdf at all lower \( Q^2 \) and higher \( x \) values
Parton Distribution Functions

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\[
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\[
f_q(x, Q^2) = \frac{\alpha_s}{2\pi} \log \frac{Q^2}{Q_0^2} \int_1^x \frac{dy}{y} f_q(y, Q_0^2) P_{qq} \left( \frac{x}{y} \right) + \alpha_s^2 \log^2 \ldots
\]

- pdf at some \(x\) and \(Q^2\) point depends on pdf at all lower \(Q^2\) and higher \(x\) values
Global Fits

- **State of the art: NLO cross sections with NLO evolution**
  - Deep Inelastic Scattering (NNLO starting to be used)
    - electron/positron/neutrino/antineutrino
    - neutral current/charged current
    - proton/deuterium/nuclear targets
    - charm-tagged final states
  - Drell–Yan production (hh→ll+X)
    - fixed target (virtual photon)
    - W/Z production and forward/backward asymmetry
  - Tevatron high–E_T jet data
    - rapidity–dependence crucial
    - cf previous high–E_T excess
Global Fits

• State of the art: NLO cross sections with NLO evolution
• Global fits are reperformed ~ once every two years
  – Martin, Roberts, Stirling and Thorne (MRST)
  – a Coordinated Theoretical and Experimental project for QCD (CTEQ)
• pdfs now come with uncertainties
  – propagated from experimental errors
  – uncertainties due to theoretical assumptions?
• Valence quarks very well known at intermediate $x$
• Gluons and sea quarks less so
• All poor at large $x$
Drell–Yan Production in p–p

- Global fits dominated by HERA data, but don't forget fixed target DIS and Drell–Yan
- Best way to measure sea quark distributions
- Separate $d$ and $u$ shapes
- Fermilab E866/NuSea experiment
Isospin Violation in Valence Distributions

- Isospin \( \Rightarrow f_{u/p}(x,Q^2) = f_{d/n}(x,Q^2) \) etc
- Models of explicit isospin violation at low \( Q^2 \rightarrow \) small
- Global fit (MRST) \( \Rightarrow \sim 0.5\% \) violation in momentum sum

Isospin Violation in Valence Distributions

- Isospin $\Rightarrow f_{u/p}(x,Q^2) = f_{d/n}(x,Q^2)$ etc
- Models of explicit isospin violation at low $Q^2 \rightarrow$ small
- Global fit (MRST) $\Rightarrow \sim 0.5\%$ violation in momentum sum
- QED effects in evolution equation $\Rightarrow u$ quarks evolve faster than $d$ quarks
- Next MRST set will include $f_{\gamma/p}$ and hence this source of isospin violation…

$\Rightarrow$ Important to have data on deuterium (isoscalar) targets (see later…)
Strange–Antistrange Asymmetry

- Strange quark distribution best measured from CC charm production, e.g. $\nu_\mu + s \rightarrow \mu^- + c$ (CCFR+NuTeV)
- CTEQ fits (Kretzer et al hep-ph/0312322) allow $s(x) \neq \bar{s}(x)$

![Strangeness Asymmetry](image1)

![Momentum Asymmetry](image2)

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Strange–Antistrange Asymmetry

• Strange quark distribution best measured from CC charm production, eg $\nu_\mu + s \rightarrow \mu^- + c$ (CCFR+NuTeV)
• CTEQ fits (Kretzer et al hep-ph/0312322) allow $s(x) \neq \bar{s}(x)$
• Results on $x[s(x) - \bar{s}(x)]$ stable $> 0$
  $\Rightarrow$ important to have sign-selected data
- NuTeV $\sin^2 \theta_w$ measurement (from charged/neutral current $\nu$ DIS) relies on these effects being small
- $\sin^2 \theta_w = (1-m_w^2/m_z^2) \Rightarrow m_w$ measurement

**W-Boson Mass [GeV]**

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>TEVATRON</td>
<td>80.452 ± 0.059</td>
</tr>
<tr>
<td>LEP2</td>
<td>80.412 ± 0.042</td>
</tr>
<tr>
<td>Average</td>
<td>80.425 ± 0.034</td>
</tr>
<tr>
<td>NuTeV</td>
<td>80.136 ± 0.084</td>
</tr>
<tr>
<td>LEP1/SLD</td>
<td>80.368 ± 0.032</td>
</tr>
<tr>
<td>LEP1/SLD/$m_t$</td>
<td>80.379 ± 0.023</td>
</tr>
</tbody>
</table>

3σ discrepancy? Could be solved by ~0.5% isospin violation and/or strange asymmetry.
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  – Fragmentation functions
  – Hadronization models
  – Underlying event models
• Analysis tools
What do we mean by the Underlying Event?

“Everything except the hard process”

but...

- initial state radiation
- factorization scale
- parton distribution functions
- parton evolution

→ underlying event model integral part of event model
Why should we be interested?

1. QCD
   Connection with:
   - diffraction
   - saturation
   - confinement
   - total cross section
   Can we predict/understand the properties of hadrons?

2. Experiments
   - Occupancy
   - Pile-up
   - Backgrounds
Why should we be interested?

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   - Occupancy
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   - Backgrounds

3. Physics
   - Jet cross sections
   - Mass reconstruction
   - Rapidity gaps/jet vetoes
   - $E_{\text{miss}}$ reconstruction
   - Photon/lepton isolation

“Don’t worry, we will measure and subtract it”
But… fluctuations and correlations crucial
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   - Photon/lepton isolation
   :

   But... fluctuations and correlations crucial
Fluctuations and correlations

Steep distribution ⇒ small sideways shift = large vertical

Rare fluctuations can have a huge influence

⇒ corrections depend on physics process
How do we Measure Underlying Event?

• “Transverse” region of two-jet events

http://www.phys.ufl.edu/~rfield/cdf/chgjet/chgjet_intro.html
HERWIG’s Soft Underlying Event model


Compare underlying event with ‘minimum bias’ collision

Parameterization of (UA5) data
+ model of energy-dependence
Multiparton Interactions

- Assume p–p collision ≈ local instantaneous sampling of two disks of partons
  - Can have several perturbative scatters within one event
- PYTHIA and Jimmy (add-on for HERWIG)
- Low $p_t$ scatters → underlying event
Proton Radius parameter within Jimmy

I.Borozan, PhD thesis, unpublished

- Increasing $\mu^2$ to 2 GeV$^2$ (i.e. decreasing proton radius by 40%) with $pt_{\text{min}}=3$ GeV gives

~ perfect description of Tevatron data…
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• Analysis tools
  – Event shapes
  – Jet algorithms
  – ‘Event definitions’
Jet Algorithms

- Leading order: a parton is a jet
- But…

one jet or two?

- Need an algorithmic jet definition
Jet Algorithms

• Cone algorithm
  Depends on energy flow relative to cone radius $R$

• Cluster algorithm
  Depends on separation of subjets relative to cutoff $R$
Jet Algorithms

- Cone vs cluster algorithm MHS, Z.Phys.C62(1994)127
- Top mass reconstruction as a test case
- Update in progress (Chris Tevlin, ATLAS student)
Summary

• Every measurement in a hadron collider involves doing QCD phenomenology!

• Most involve a long chain of corrections
  – higher orders
  – parton distribution functions
  – jet algorithm
  – hadronization
  – underlying event

  to connect measurement to physics

• Always ask how well these corrections are known:
  – how much am I relying on QCD phenomenology?
  – how well understood is it?