
Electroweak Precision Physics

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IPPP Durham

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1. Introduction
2. Electroweak precision observables
3. Precision tests: Standard Model and Supersymmetry
4. Conclusions

1. Introduction

Electroweak precision measurements:

M_W [GeV]	$= 80.425 \pm 0.034$	0.04%
$\sin^2 \theta_{\text{eff}}^{\text{lept}}$	$= 0.23150 \pm 0.00016$	0.07%
Γ_Z [GeV]	$= 2.4952 \pm 0.0023$	0.09%
M_Z [GeV]	$= 91.1875 \pm 0.0021$	0.002%
G_μ [GeV $^{-2}$]	$= 1.16637(1) 10^{-5}$	0.0009%
m_t [GeV]	$= 178.0 \pm 4.3$	2.4%
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Quantum effects of the theory:

Loop corr. to “pseudo-observables” $M_W, \sin^2 \theta_{\text{eff}}, \dots : \sim \mathcal{O}(1\%)$

⇒ EW prec. measurements test quantum effects of the theory

Comparison of electroweak precision data with theory predictions:

EW precision data:

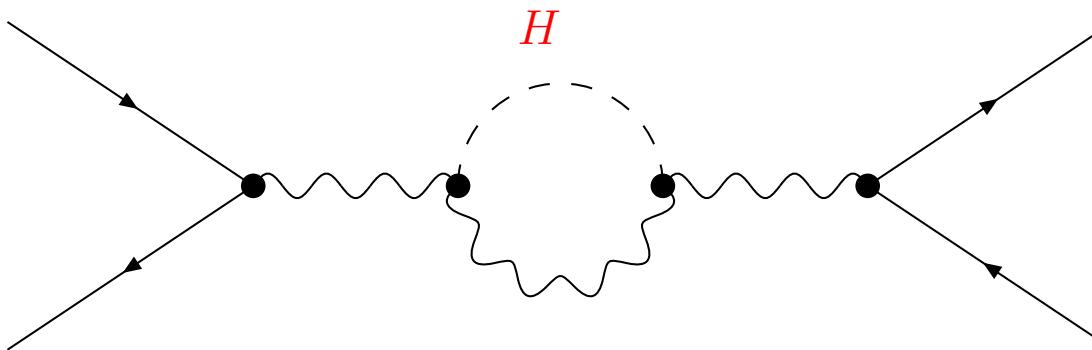
M_Z , M_W , $\sin^2 \theta_{\text{eff}}^{\text{lept}}$, ...

Theory:

SM, MSSM, ...



Test of theory at quantum level: sensitivity to loop corrections



Indirect constraints on unknown parameters: M_H , ...

Effects of “new physics”?

Example: indirect determination of m_t from precision data vs. direct measurement

Indirect det. of m_t from precision data: $m_t = 180.3^{+11.7}_{-9.2}$ GeV

Direct measurement: $m_t = 178.0 \pm 4.3$ GeV

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Theoretical uncertainties:

- unknown higher-order corrections
- experimental error of input parameters: m_t , $\Delta\alpha_{\text{had}}$, ...

Observables vs. “pseudo-observables”:

Couplings, masses, mixing angles, etc. are not (directly) physical observables \Rightarrow “pseudo-observables”

Actual observables: σ , BRs, asymmetries, ...

Need deconvolution procedure (unfolding) to determine masses, partial widths, etc. from measured cross sections

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What is a suitable definition of model parameters?

need some compromise between

- simple interpretation within given model
- model independence

Experimental determination of model parameters (masses, couplings, . . .):

Particle masses: relatively small model dependence

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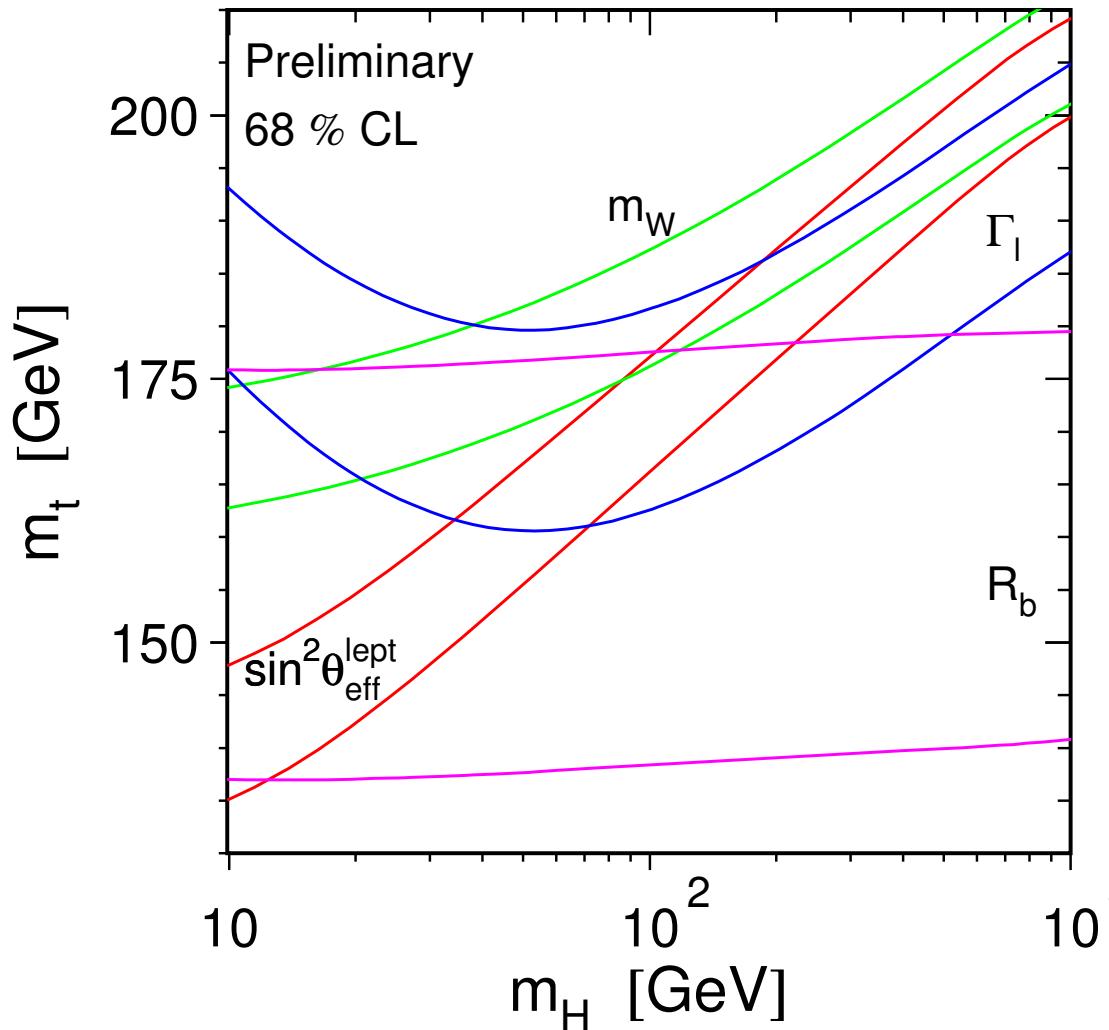
Particle masses: relatively small model dependence

⇒ The experimental value of M_Z (slightly) depends on the value of the SM Higgs mass

$(\delta M_Z = \pm 0.2 \text{ MeV} \text{ for } 100 \text{ GeV} \leq M_H \leq 1 \text{ TeV})$

Couplings, mixing angles, etc.: relatively large model dependence

Sensitivity of different (pseudo-)observables to the Higgs mass in the SM



[LEPEWWG '04]

⇒ highest sensitivity from $\sin^2 \theta_{\text{eff}}$ and M_W

Comparison of current and anticipated future experimental errors

Present errors vs. Run II of Tevatron, LHC, and ILC with and without low-energy running mode (GigaZ):

	now	Tevatron	LHC	ILC	GigaZ
$\delta \sin^2 \theta_{\text{eff}} (\times 10^5)$	16	—	14–20	—	1.3
$\delta M_W [\text{MeV}]$	34	20	15	10	7
$\delta m_t [\text{GeV}]$	4.3	2.5	1–2	0.1	0.1
$\delta m_h [\text{MeV}]$	—	—	200	50	50

⇒ Large improvement at next generation of colliders

2. *Electroweak precision observables*

Sensitivity to quantum effects (loop corrections) of new physics:

- Precision measurements resolve %-level loop effects:
 M_W , $\sin^2 \theta_{\text{eff}}$, Γ_Z , ...

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- Loop-induced processes \Leftrightarrow new physics contribution doesn't compete with large SM lowest-order prediction:
 $(g - 2)_\mu$, $b \rightarrow s\gamma$, $B_s \rightarrow \mu^+ \mu^-$, EDMs, ...

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 M_W , $\sin^2 \theta_{\text{eff}}$, Γ_Z , ...
- Loop-induced processes \Leftrightarrow new physics contribution doesn't compete with large SM lowest-order prediction:
 $(g - 2)_\mu$, $b \rightarrow s\gamma$, $B_s \rightarrow \mu^+ \mu^-$, EDMs, ...
- Future precision measurements, possibly very large loop effects: m_h , other Higgs-sector observables

Theoretical predictions for M_W , $\sin^2 \theta_{\text{eff}}$:

Comparison of prediction for muon decay with experiment (Fermi constant G_μ)

$$\Rightarrow M_W^2 \left(1 - \frac{M_W^2}{M_Z^2} \right) = \frac{\pi \alpha}{\sqrt{2} G_\mu} (1 + \Delta r),$$

↔
loop corrections

\Rightarrow Theo. prediction for M_W in terms of M_Z , α , G_μ , $\Delta r(m_t, M_H, \dots)$

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Effective couplings at the Z resonance:

$$\Rightarrow \sin^2 \theta_{\text{eff}} = \frac{1}{4} \left(1 - \text{Re} \frac{g_V}{g_A} \right) = \left(1 - \frac{M_W^2}{M_Z^2} \right) \text{Re} \kappa_l(s = M_Z^2)$$

Theoretical predictions for M_W , $\sin^2 \theta_{\text{eff}}$:

Current status in the SM: complete two-loop result known for M_W , fermionic two-loop corrections known for $\sin^2 \theta_{\text{eff}}$

Necessary diagrams for evaluation of 2-loop corrections to $\sin^2 \theta_{\text{eff}}$:

- Renormalisation: $\delta M_W^2 = \text{Re}\Sigma_{T(2)}^W(M_W^2) + \dots$
⇒ 2-loop self-energies with arbitrary momentum and masses

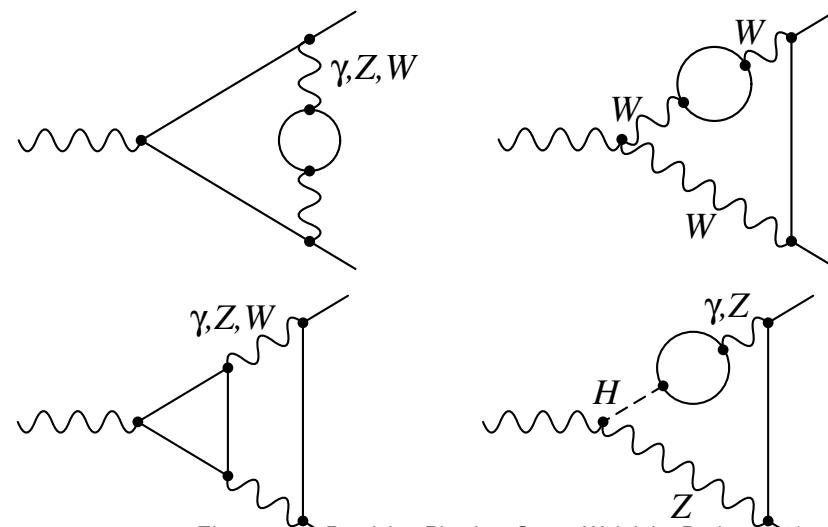
No analytical results available for general case

⇒ numerical integration

- Two-loop vertex diagrams:

two classes:

top, light fermions



Evaluation with different methods

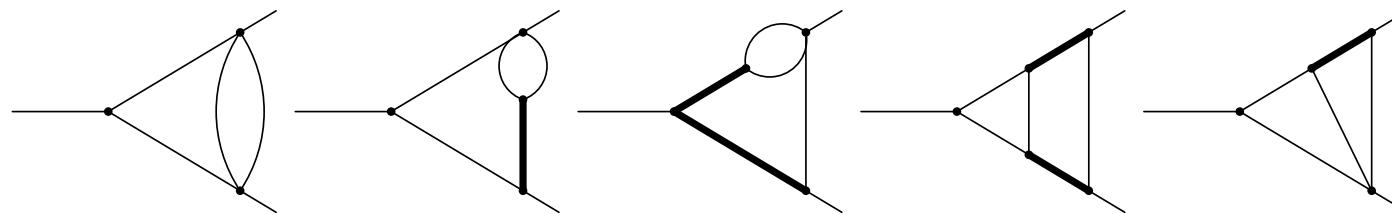
Top-quark contributions: expansion in M_Z^2/m_t^2

⇒ expansion up to $(M_Z^2/m_t^2)^5$ yields intrinsic precision of 10^{-7}

Light fermion contributions:

depend on only one variable ⇒ reduction to master integrals using integration by parts and Lorentz invariance identities

[*Chetyrkin, Tkachov '81*] [*Gehrmann, Remiddi '00*] [*Laporta '00*]

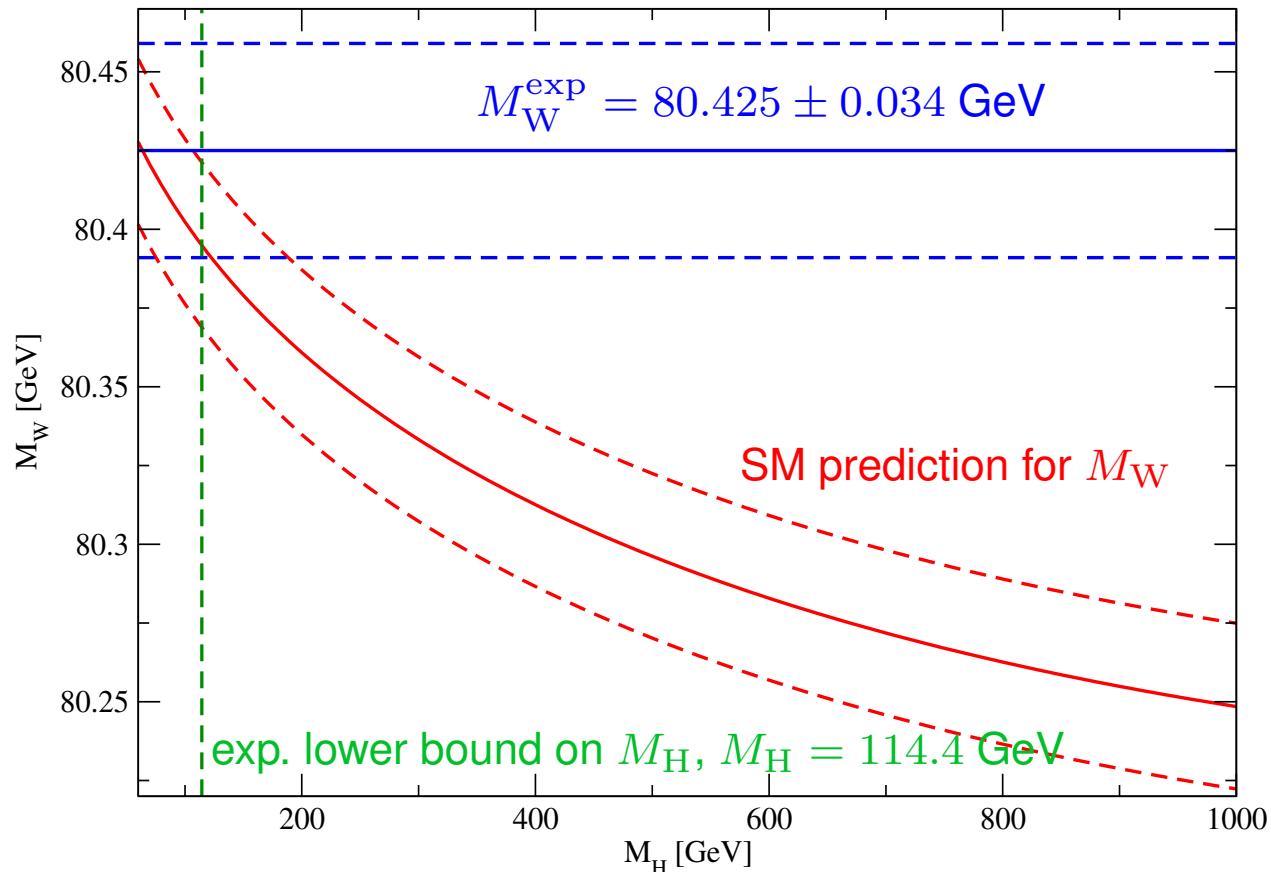


Analytical results for master integrals via differential equations

SM prediction for M_W (complete 2-loop result)

vs. experimental result

[M. Awramik, M. Czakon, A. Freitas, G.W. '04]

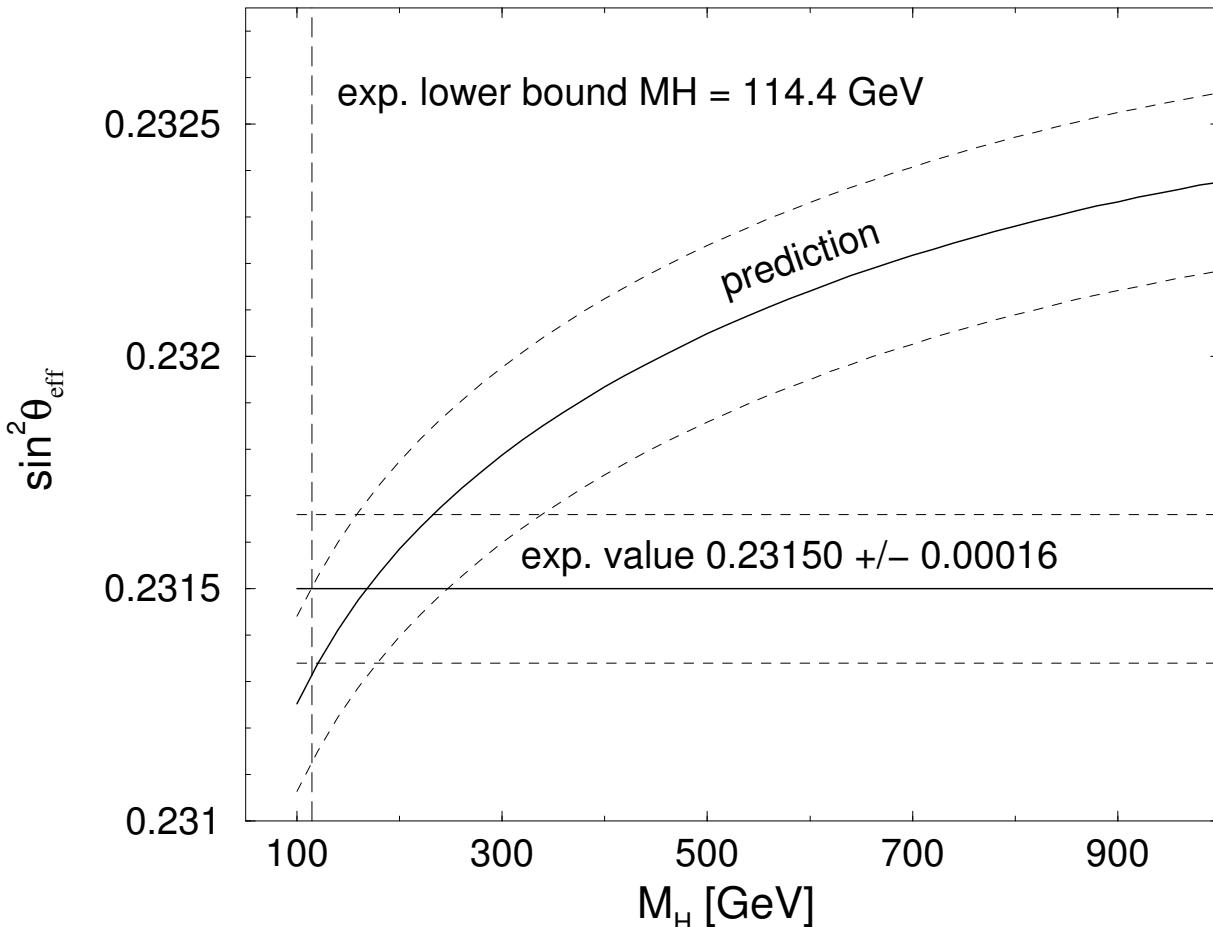


Main source of
theo. uncertainty:
 $\delta m_t = 4.3$ GeV
 $\Rightarrow \Delta M_W^{\text{para}} \approx 26$ MeV,
 $\Delta \sin^2 \theta_{\text{eff}}^{\text{para}} \approx 14 \times 10^{-5}$

⇒ Preference for light Higgs

SM prediction for $\sin^2 \theta_{\text{eff}}$ (fermionic 2-loop result)

vs. experimental result

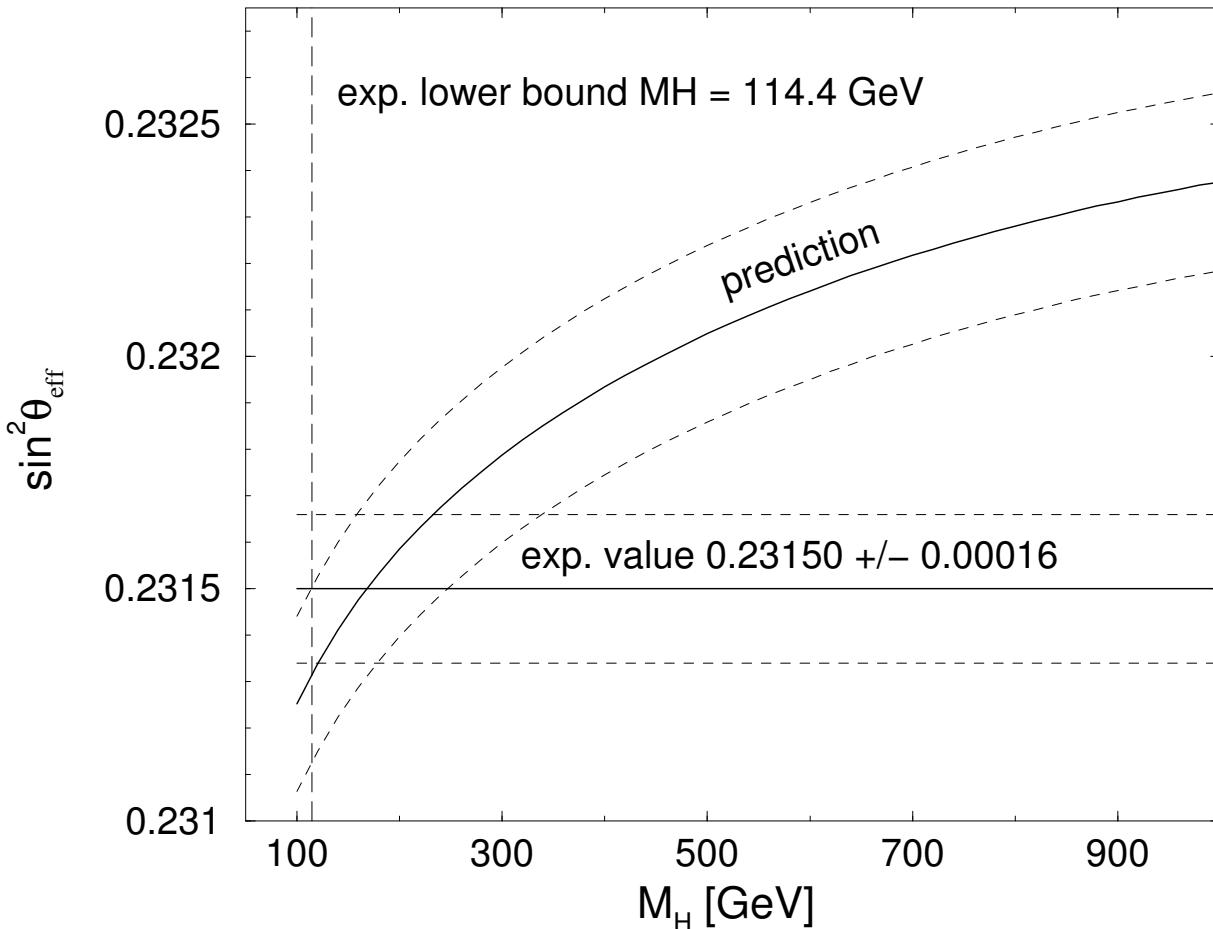


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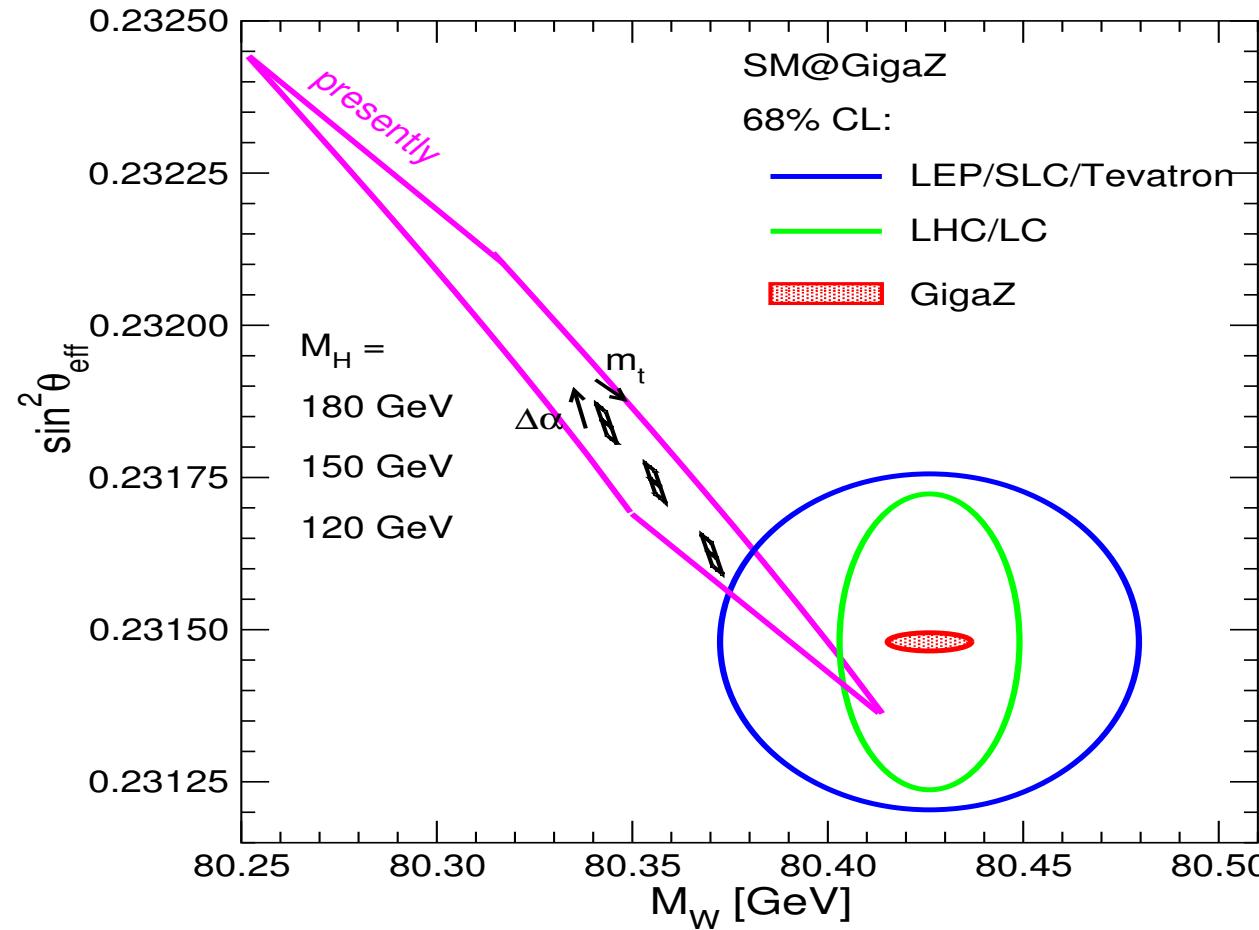
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⇒ Preference for light Higgs

However: experimental value for $\sin^2 \theta_{\text{eff}}$ contains average over
 $A_l = 0.23098 \pm 0.00026$ (SLD) and $A_{\text{fb}}^{0,b} = 0.23210 \pm 0.00030$ (LEP)

SM prediction for M_W and $\sin^2 \theta_{\text{eff}}$ vs. current experimental result (LEP2/Tevatron) and prospective accuracies at the LHC and a LC with low-energy option (GigaZ):

[J. Erler, S. Heinemeyer, W. Hollik, G.W., P. Zerwas '00]

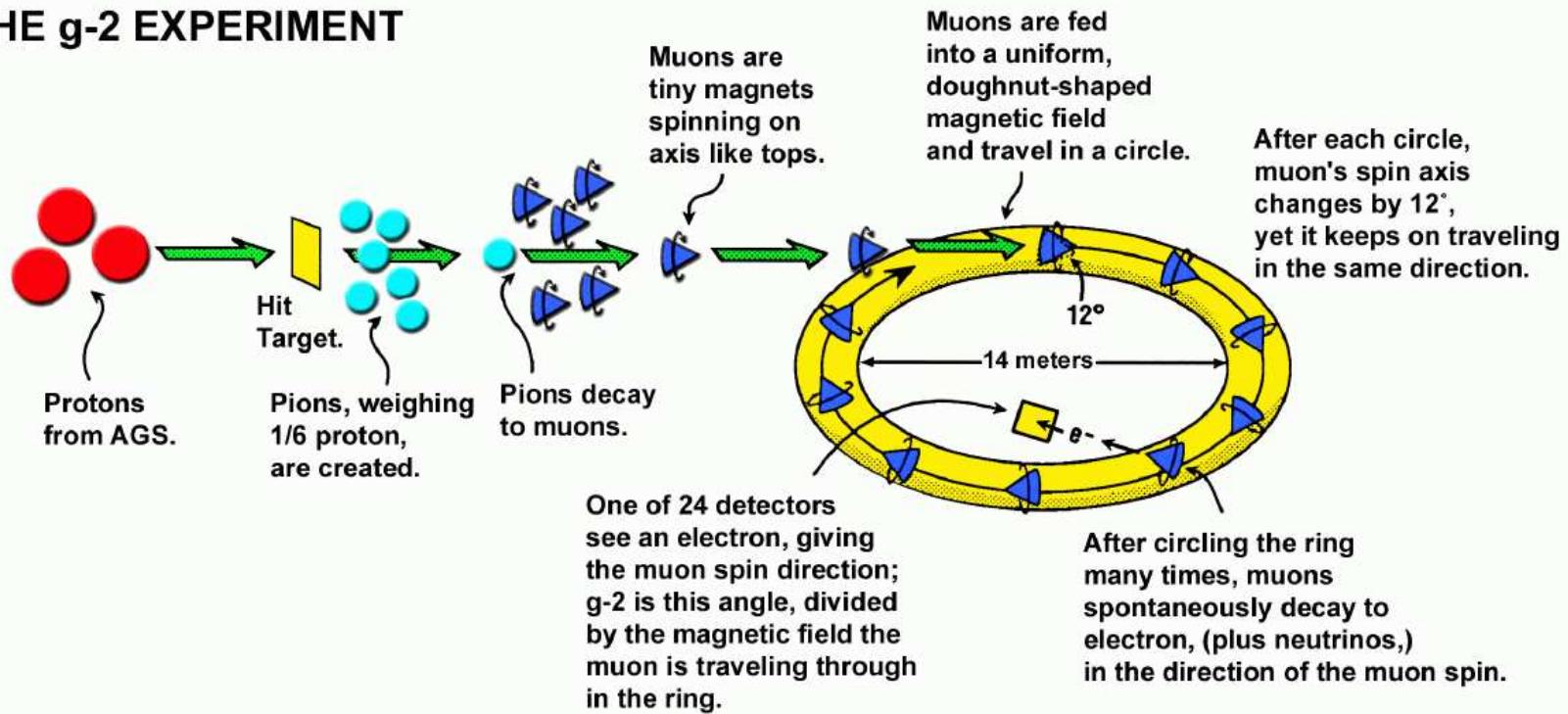


⇒ Highly sensitive test of electroweak theory:
improved accuracy of observables and input parameters

The anomalous magnetic moment of the muon:

$$(g - 2)_\mu \equiv 2a_\mu$$

LIFE OF A MUON: THE g-2 EXPERIMENT



Coupling of muon to magnetic field: $\mu - \bar{\mu} - \gamma$ coupling

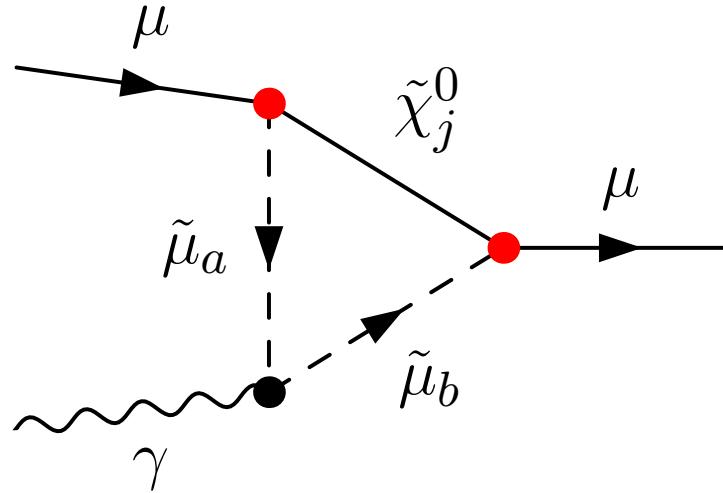
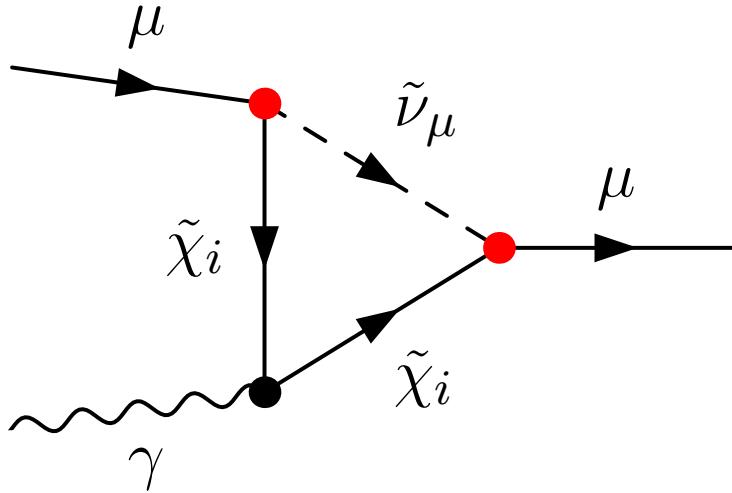
$$\bar{u}(p') \left[\gamma^\mu F_1(q^2) + \frac{i}{2m_\mu} \sigma^{\mu\nu} q_\nu F_2(q^2) \right] u(p) A_\mu \quad F_2(0) = (g-2)_\mu \equiv 2a_\mu$$

a_μ : experimental result vs. SM prediction

$$a_\mu^{\text{exp}} - a_\mu^{\text{theo}} = (25.2 \pm 9.2) \times 10^{-10} : 2.7\sigma .$$

Better agreement between theory and experiment possible in models of physics beyond the SM

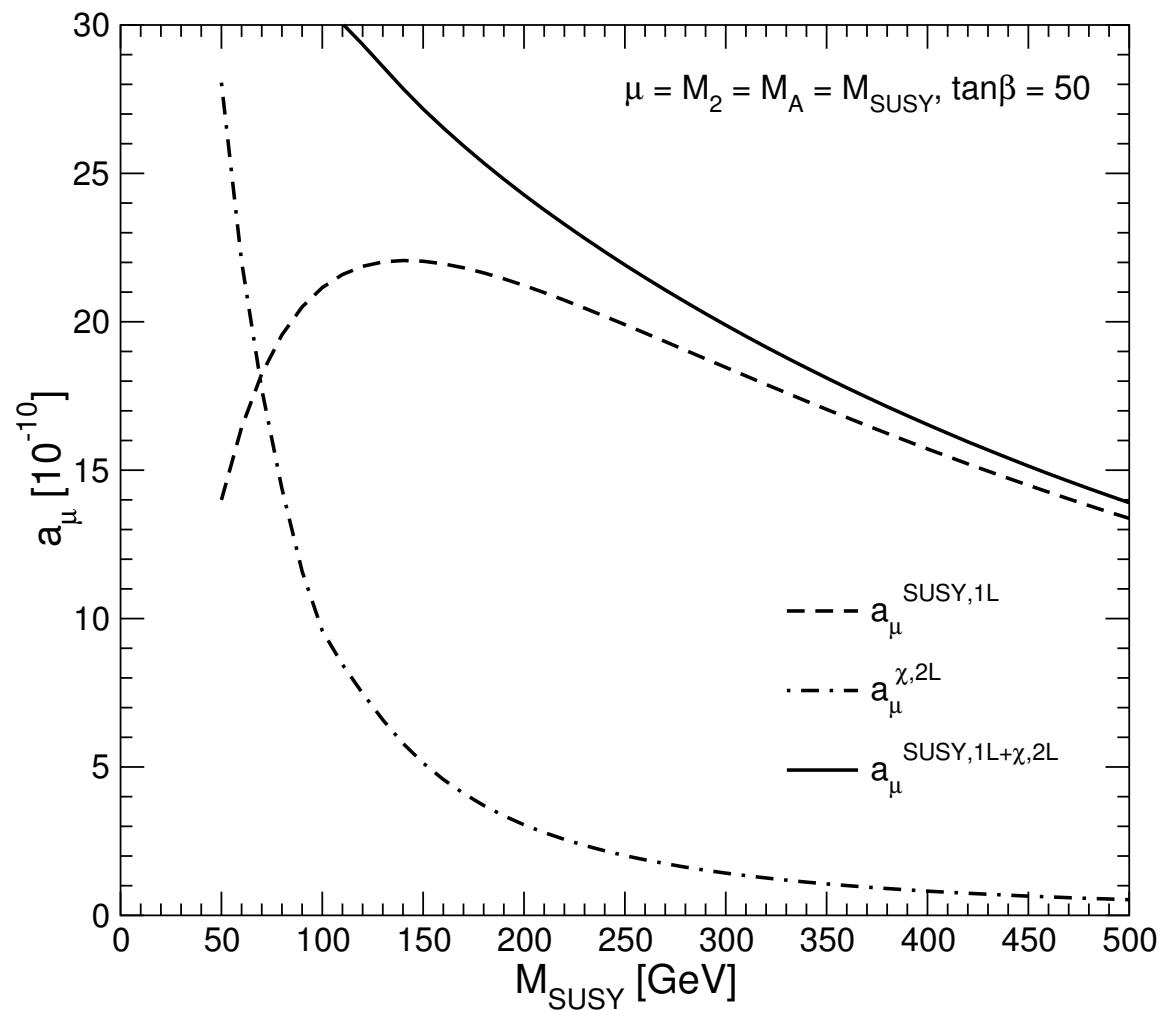
Example: one-loop contributions of superpartners of fermions and gauge bosons



SUSY contributions to a_μ

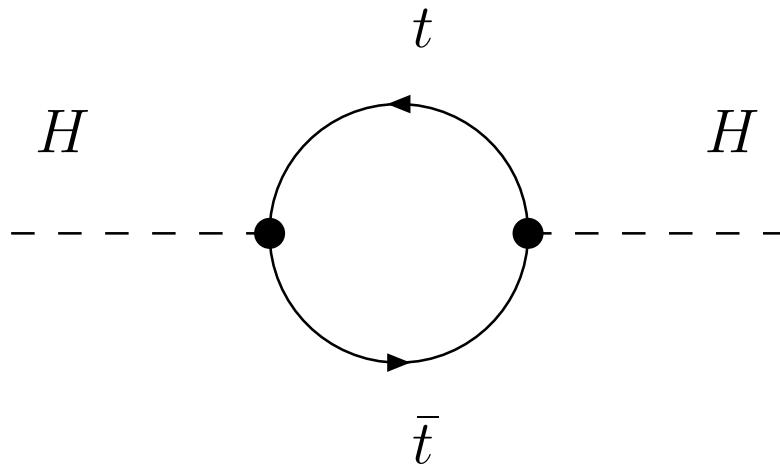
One-loop SUSY contribution (dashed),
two-loop chargino/neutralino contributions (dash-dotted)
and the sum (full line)
for
 $\mu = M_2 = M_A \equiv M_{\text{SUSY}}$,
 $m_{\tilde{f}} = 1 \text{ TeV}$, $\tan \beta = 50$:

[S. Heinemeyer, D. Stöckinger, G. W. '04]



Precision Higgs physics

Large coupling of Higgs to top quark

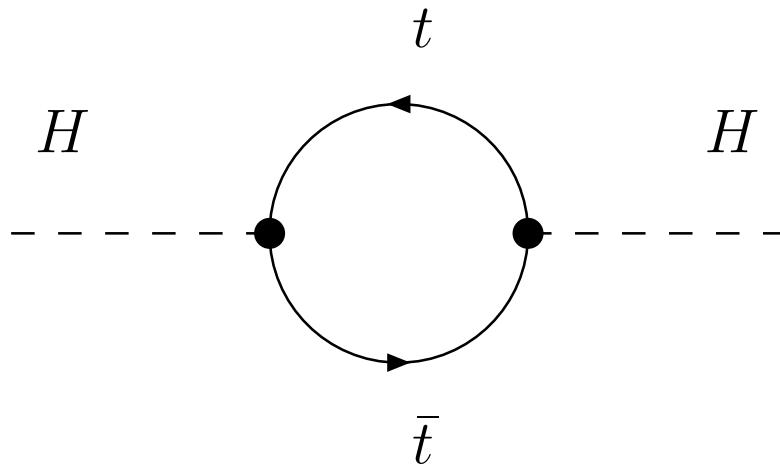


One-loop correction $\sim G_\mu m_t^4$

$\Rightarrow M_H$ depends sensitively on m_t in all models where M_H can be predicted (SM: M_H is free parameter)

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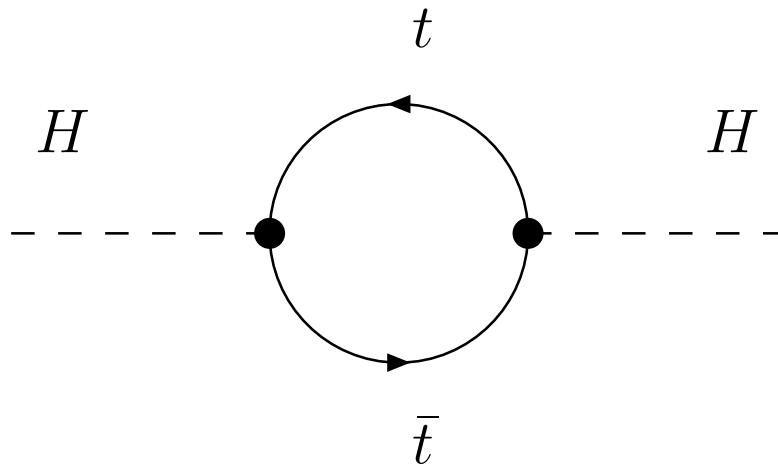
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\Rightarrow Precision Higgs physics needs precision top physics
(ILC: $\Delta m_t \lesssim 0.1 \text{ GeV}$)

Higgs mass bound in the MSSM:

⇒ Prediction for m_h , m_H , ...

Tree-level result for m_h , m_H :

$$m_{H,h}^2 = \frac{1}{2} \left[M_A^2 + M_Z^2 \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4M_Z^2 M_A^2 \cos^2 2\beta} \right]$$

⇒ $m_h \leq M_Z$ at tree level

MSSM tree-level bound (gauge sector): excluded by LEP!

Large radiative corrections (Yukawa sector, ...):

Yukawa couplings: $\frac{e m_t}{2 M_W s_W}$, $\frac{e m_t^2}{M_W s_W}$, ...

⇒ Dominant one-loop corrections: $G_\mu m_t^4 \ln \left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} \right)$, $\mathcal{O}(100\%)$!

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Upper bound on m_h :

‘Unconstrained’ MSSM: M_A , $\tan \beta$, 5 param. in \tilde{t} - \tilde{b} sector, μ , $m_{\tilde{g}}$, M_2

$$m_h \lesssim 136 \text{ GeV}$$

[S. Heinemeyer, W. Hollik, G. W. '99], [M. Frank, S. Heinemeyer, W. Hollik, G. W. '02] [G. Degrassi, S. Heinemeyer, W. Hollik, P. Slavich, G. W. '02]

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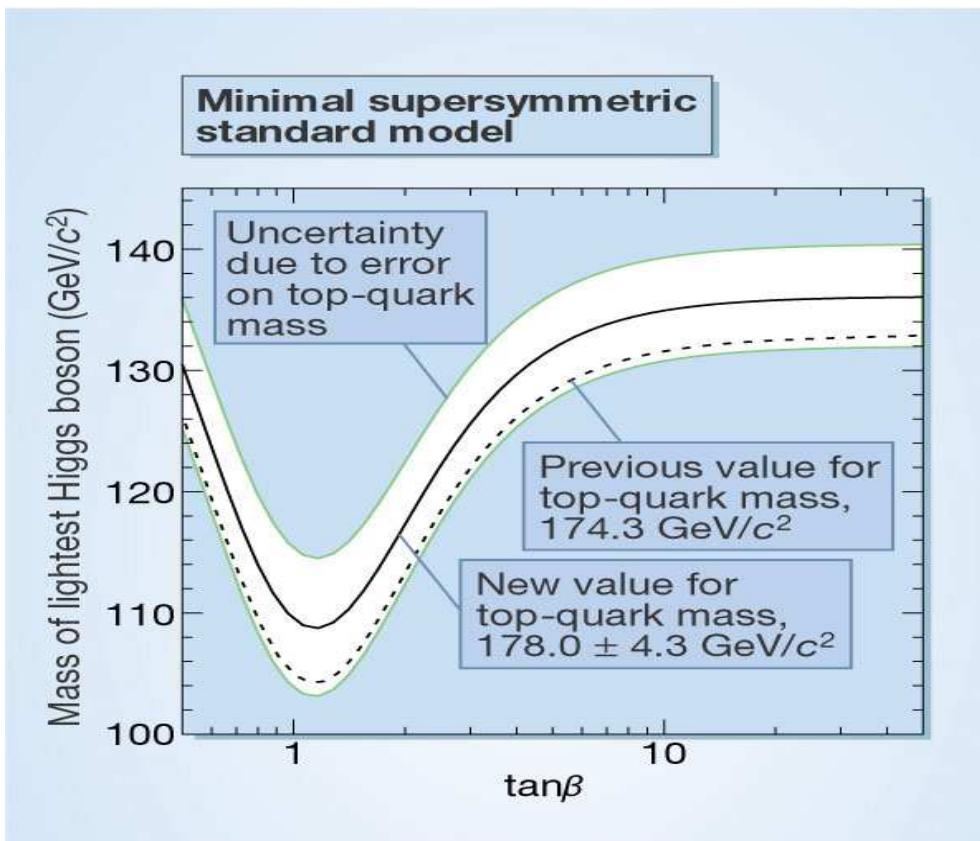
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for $m_t = 178 \text{ GeV}$, no theoretical uncertainties included

Remaining theoretical uncertainties in prediction for m_h :

[G. Degrassi, S. Heinemeyer, W. Hollik, P. Slavich, G. W. '02]

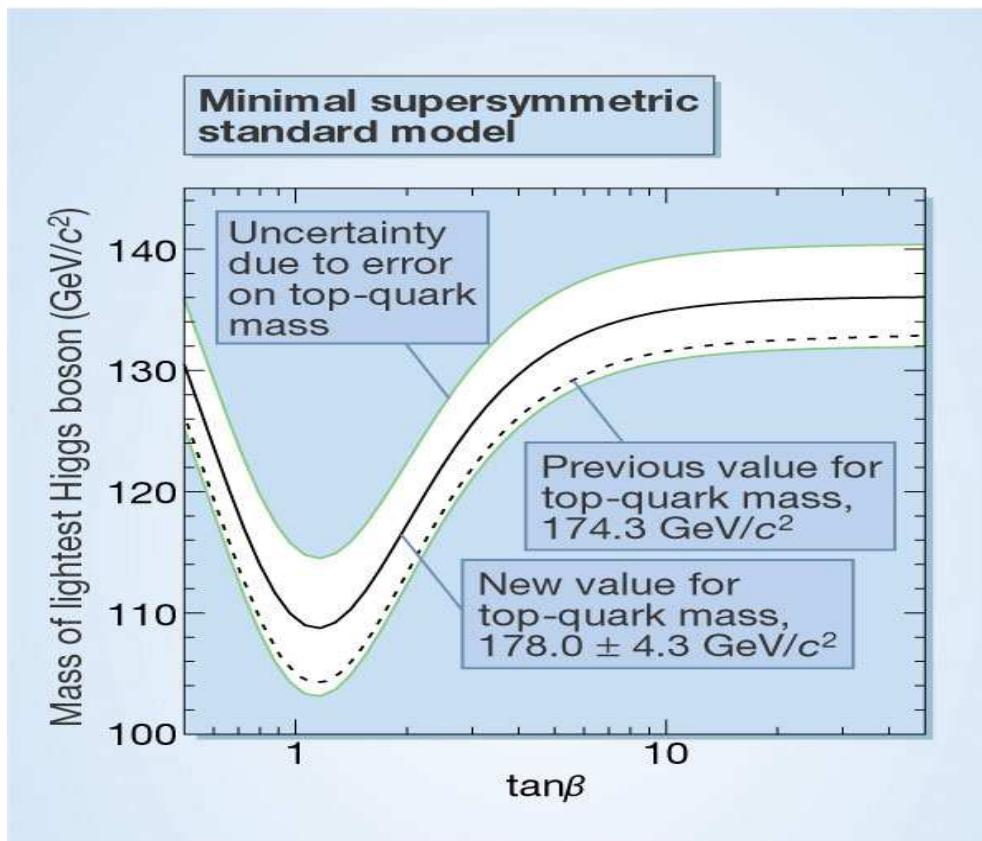
- From unknown higher-order corrections: $\Rightarrow \Delta m_h \approx \pm 3 \text{ GeV}$
- From input parameters: $\Delta m_t \approx \pm 4 \text{ GeV} \Rightarrow \Delta m_h \approx \pm 4 \text{ GeV}$



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For $1-\sigma$ range of m_t :

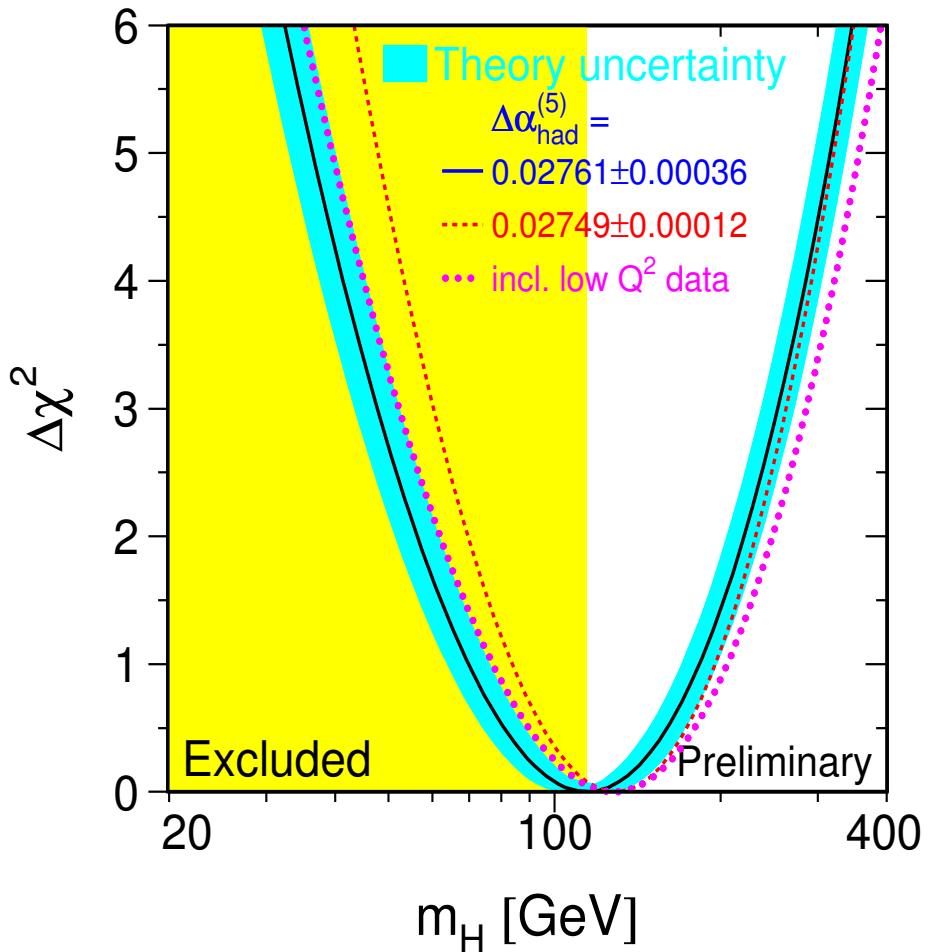
\Rightarrow upper bound beyond
 $m_h = 140 \text{ GeV}$

LEP does not exclude
any $\tan\beta$ value!

3. Precision tests: Standard Model and Supersymmetry

- Global fit in the SM
- SM vs. MSSM
- Fit to precision observables in the constrained MSSM (mSUGRA) with dark matter constraints

Global fit to all data in the SM

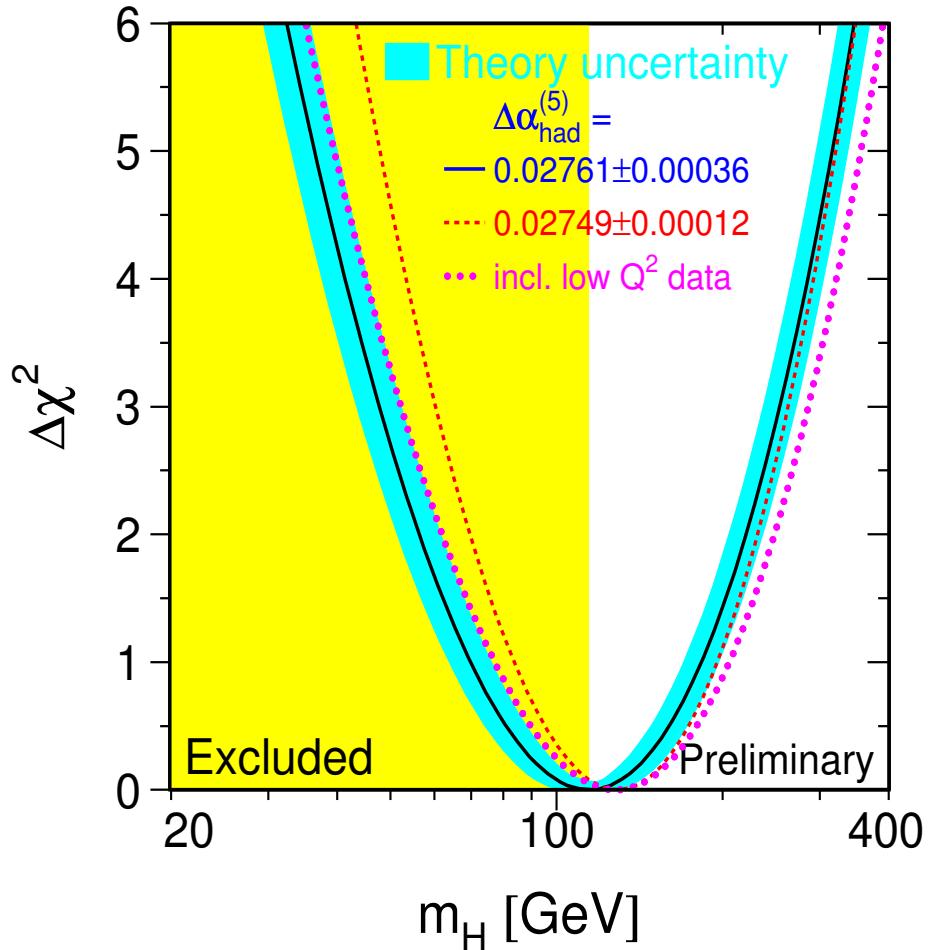


[LEPEEWG '04]

Theoretical uncertainties:

- exp. error of input parameters:
 m_t , $\Delta\alpha_{\text{had}}$, ...
⇒ large $m_t - M_H$ correlation
- unknown higher-order corrections
⇒ “blue band”

Global fit to all data in the SM



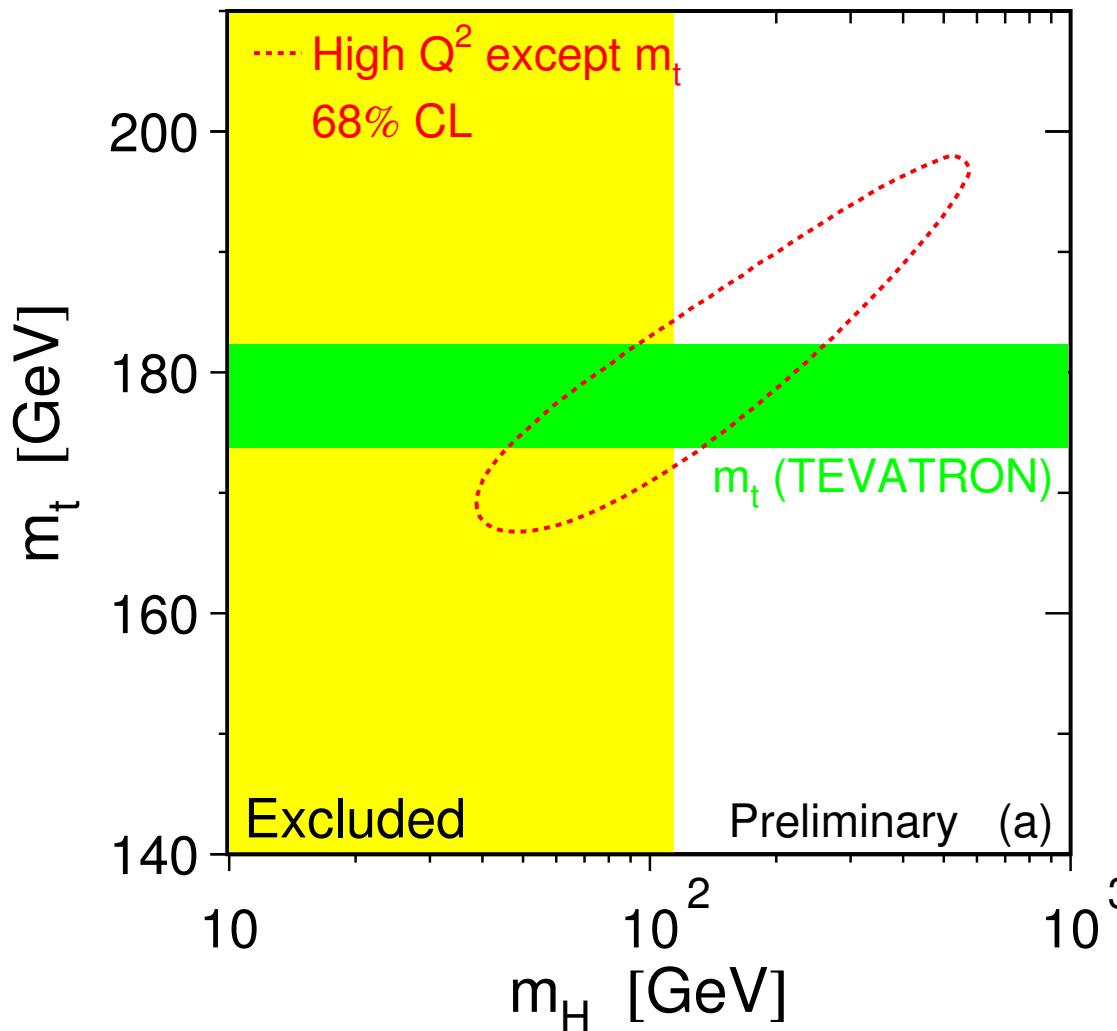
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New m_t value, $m_t = 178.0 \pm 4.3$ GeV, new result for $\sin^2 \theta_{\text{eff}}$
⇒ $M_H = 114^{+69}_{-45}$ GeV, $M_H < 260$ GeV, 95% C.L.

Correlation between M_H and m_t in the fit:



[LEPEWWG '04]

⇒ Precise knowledge of m_t crucial for constraining M_H

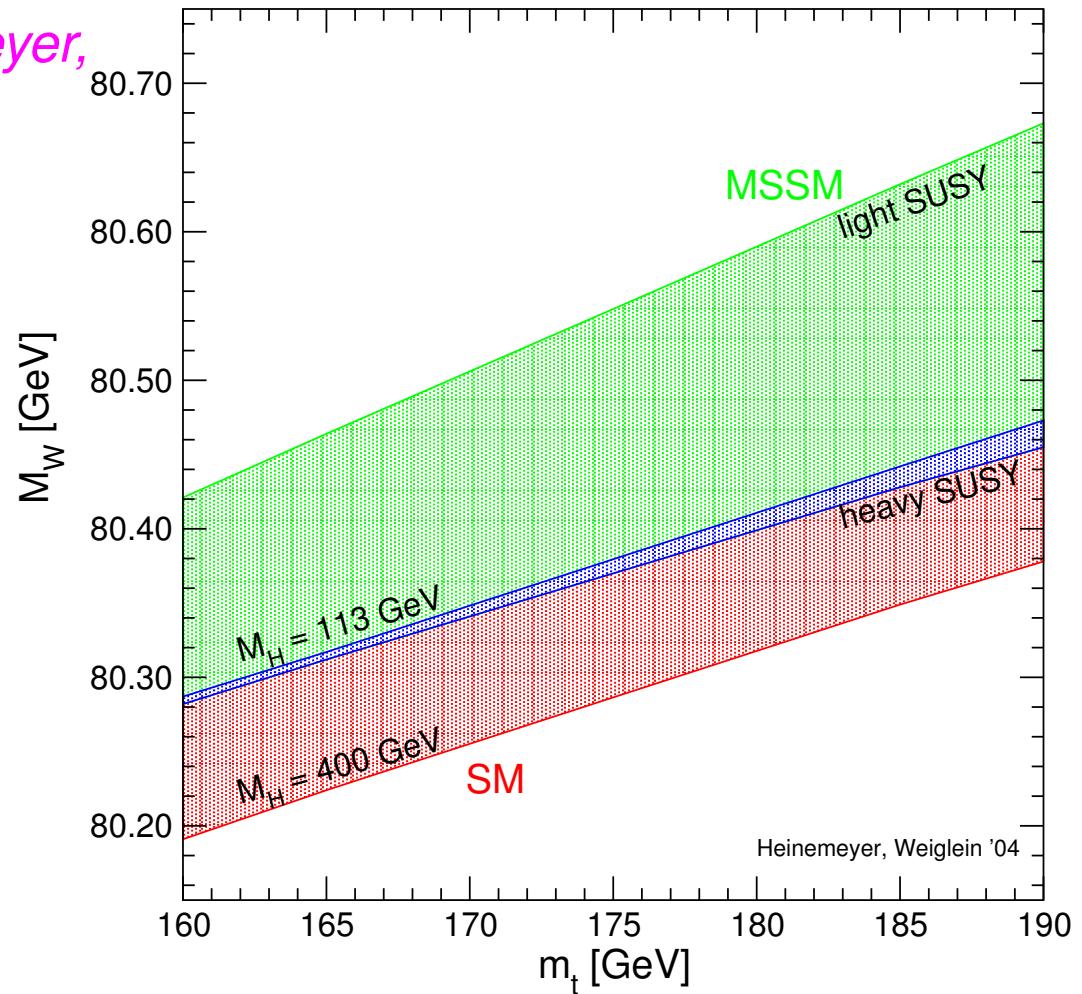
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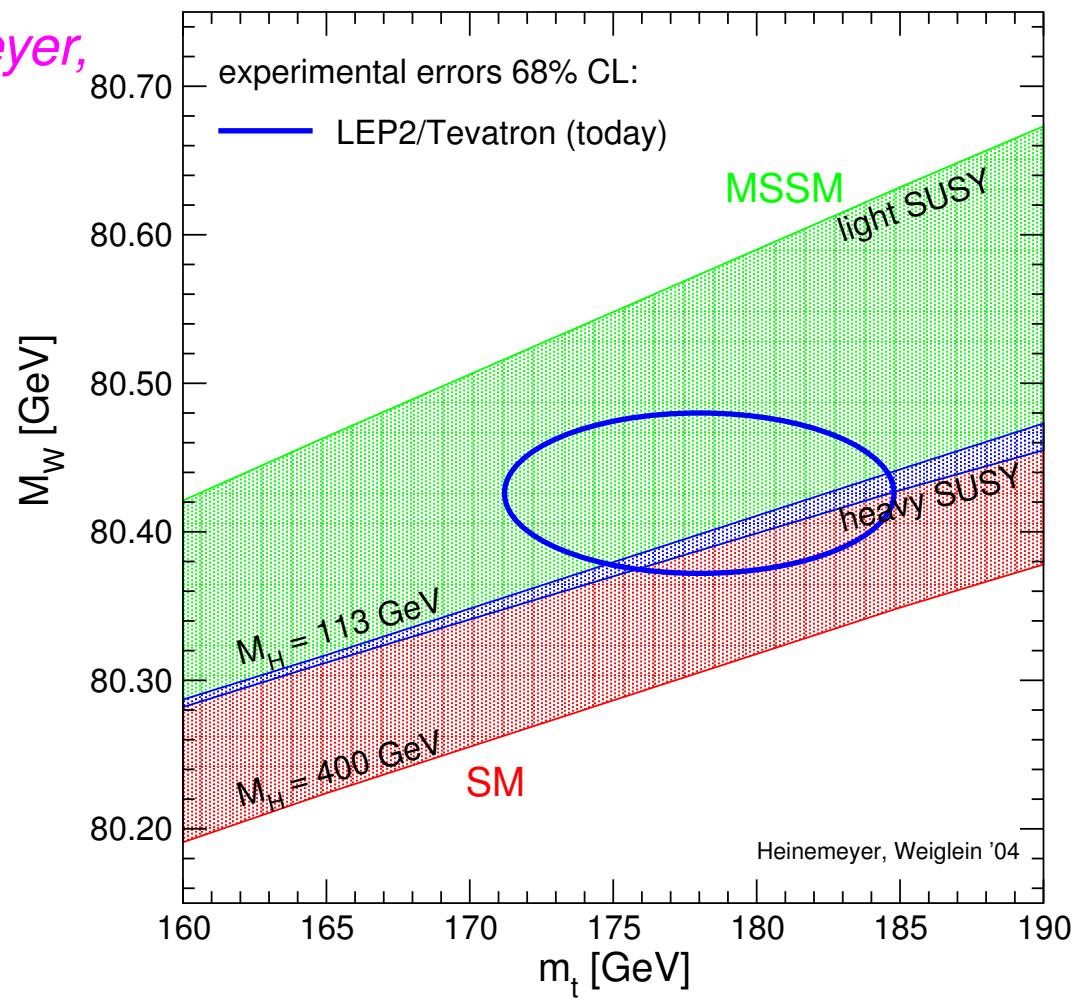
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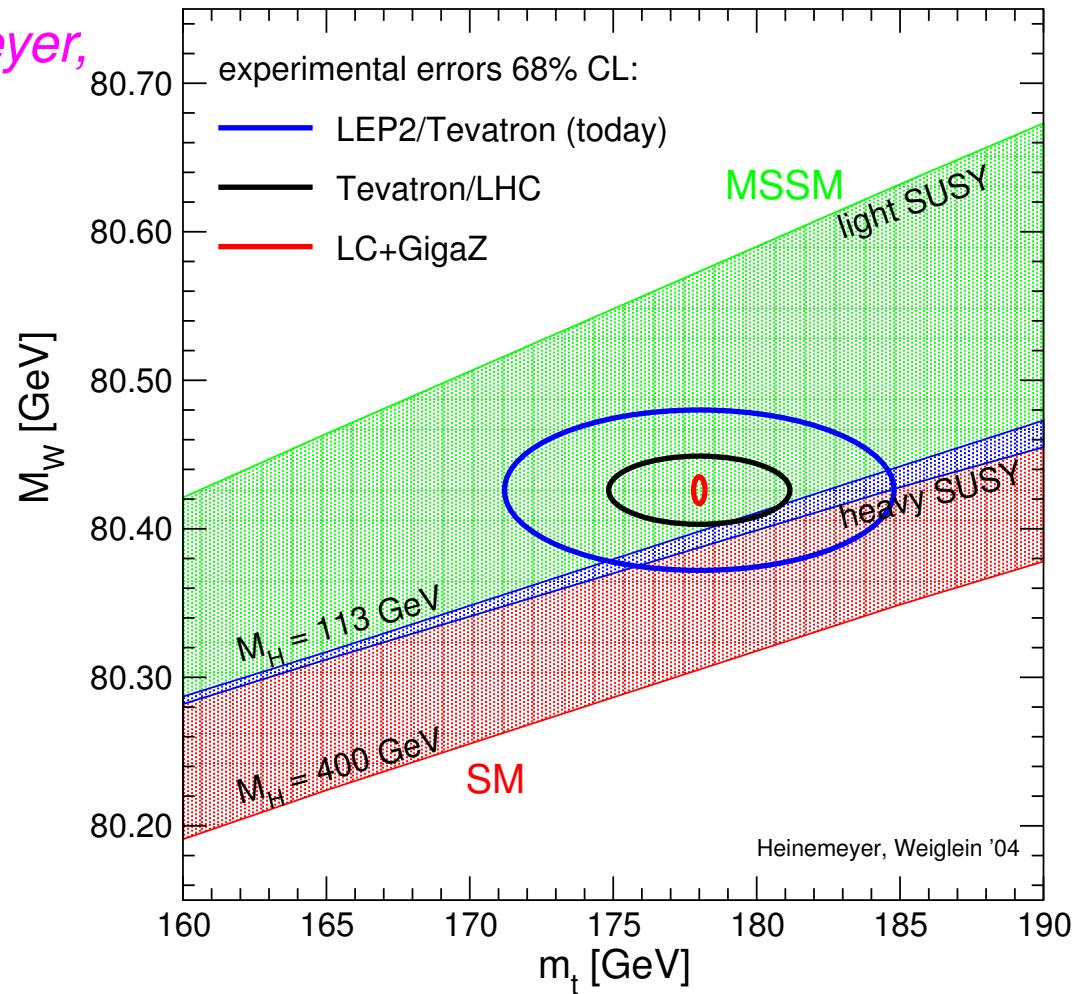
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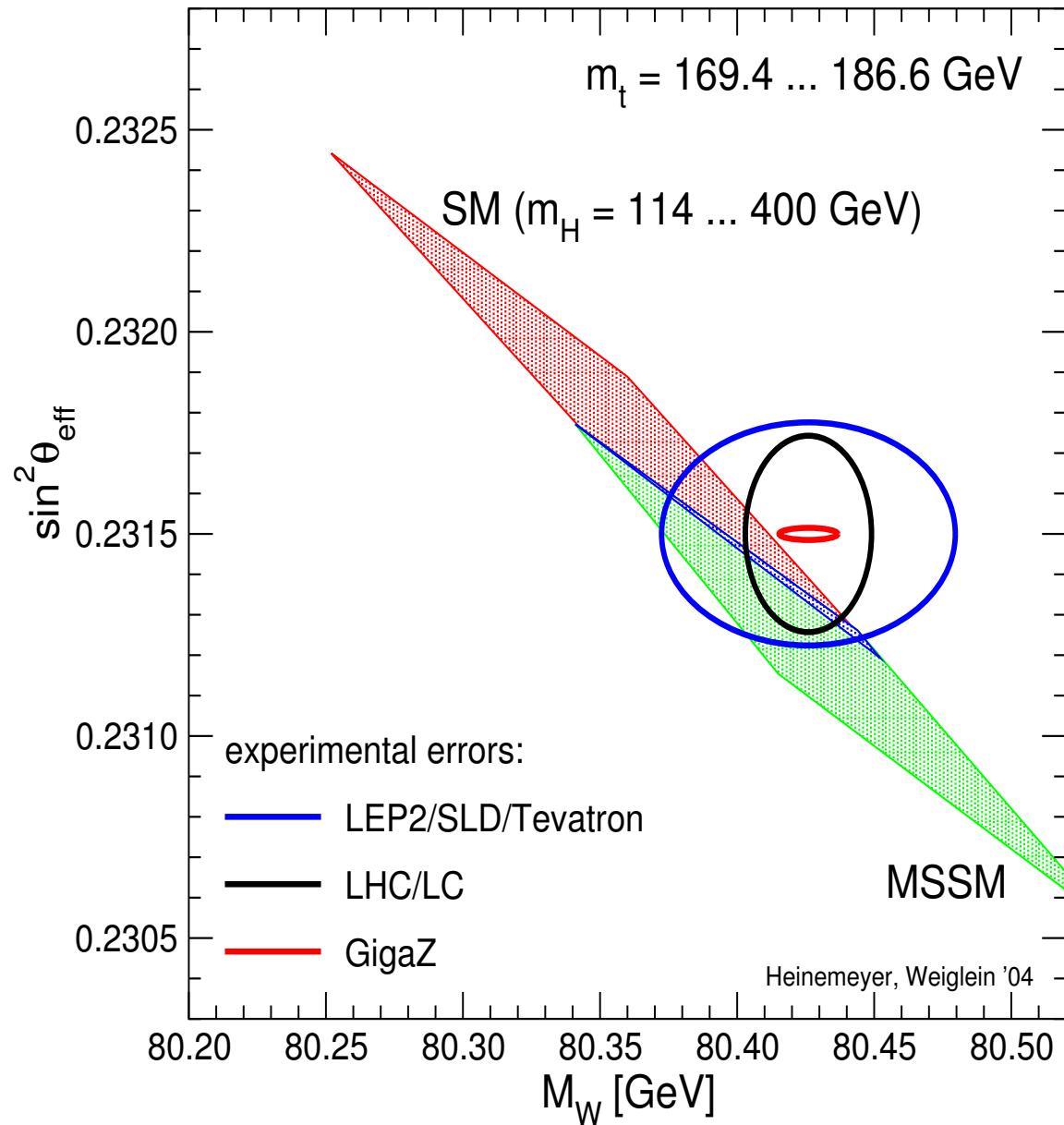
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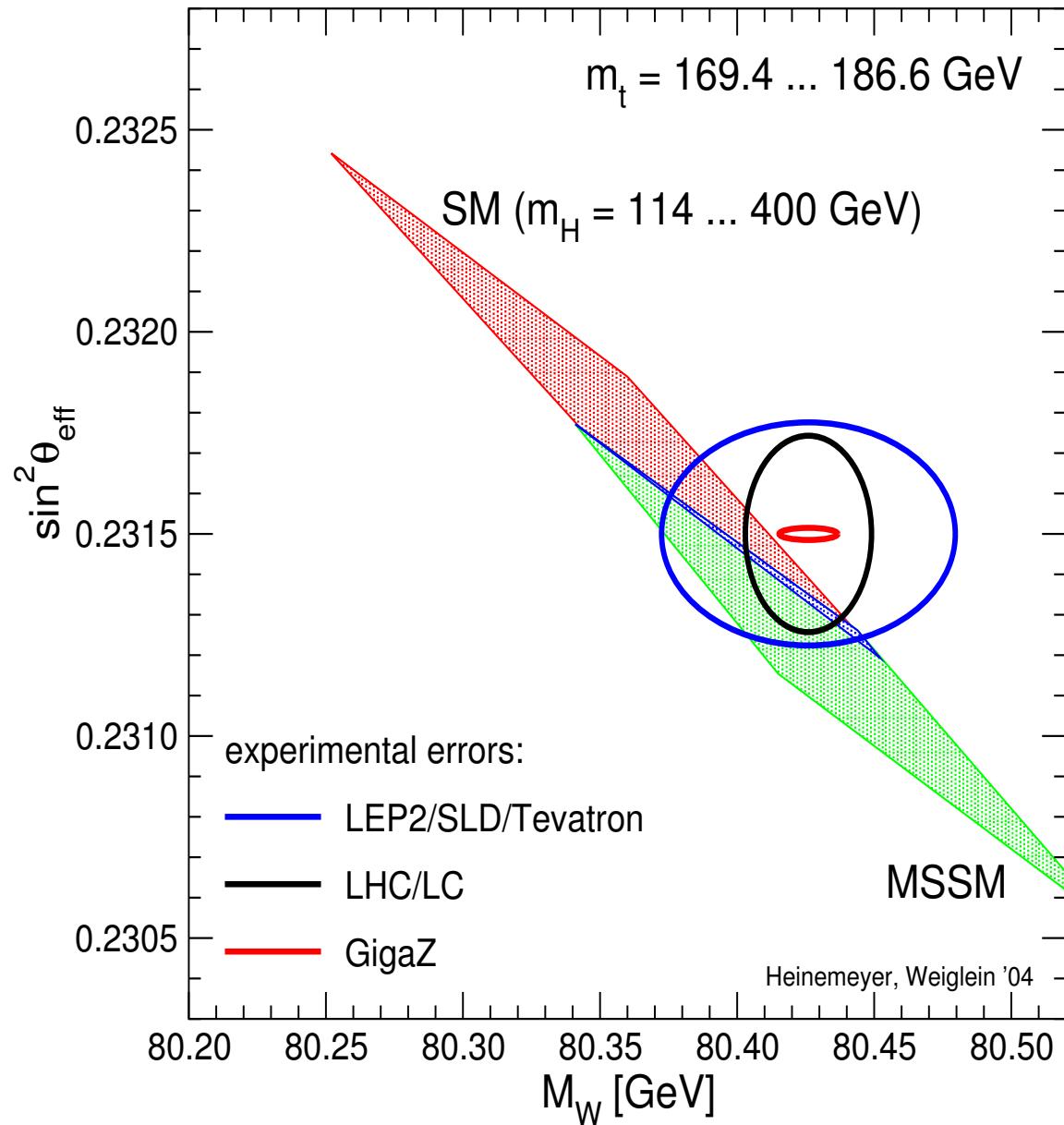


Prediction for M_W , $\sin^2 \theta_{\text{eff}}$ in SM and MSSM:



[*S. Heinemeyer, W. Hollik,
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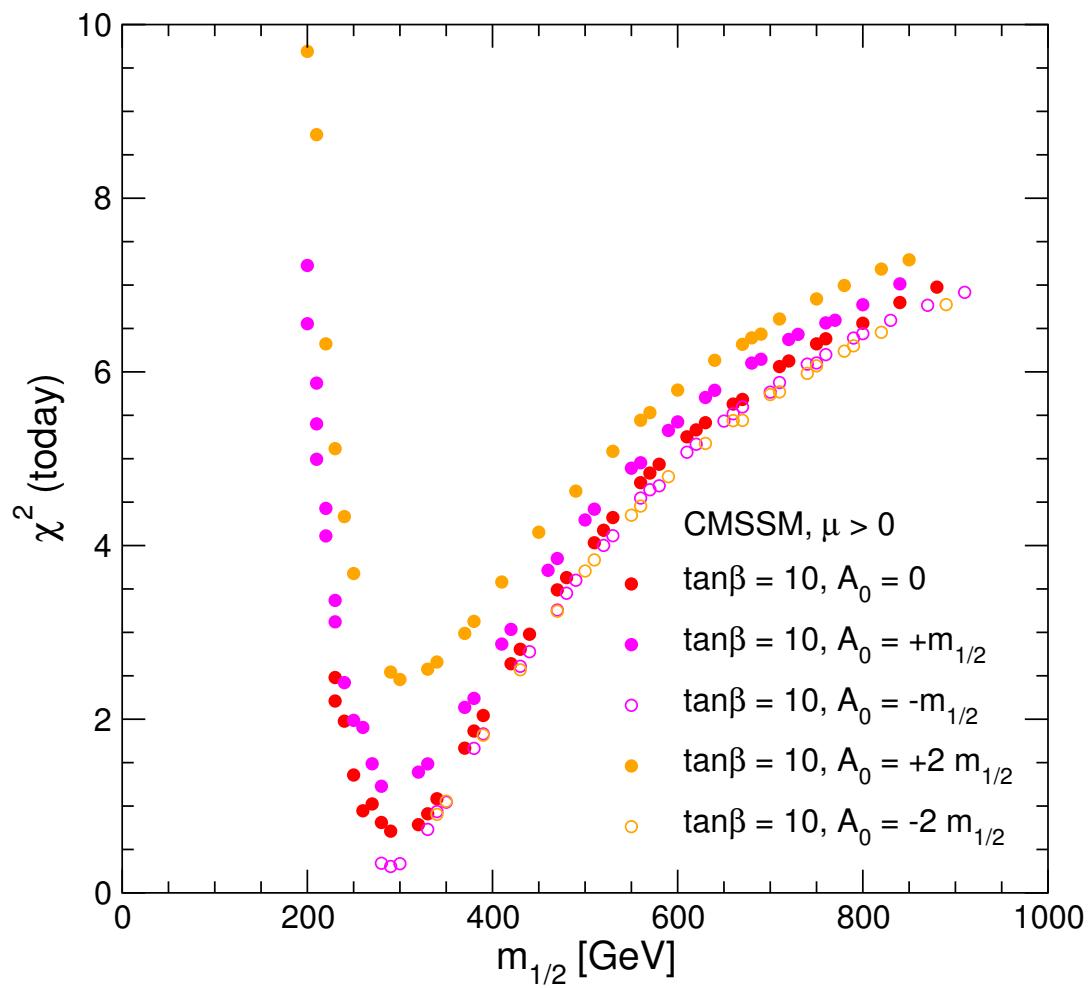
⇒ High sensitivity
to deviations from
both the SM and
the MSSM

χ^2 fit in mSUGRA with dark matter constraints:

$$M_W, \sin^2 \theta_{\text{eff}}, (g - 2)_\mu, \text{BR}(b \rightarrow s\gamma)$$

[J. Ellis, S. Heinemeyer, K. Olive, G. W. '04]

$\tan \beta = 10$:



⇒ very good description
of the data

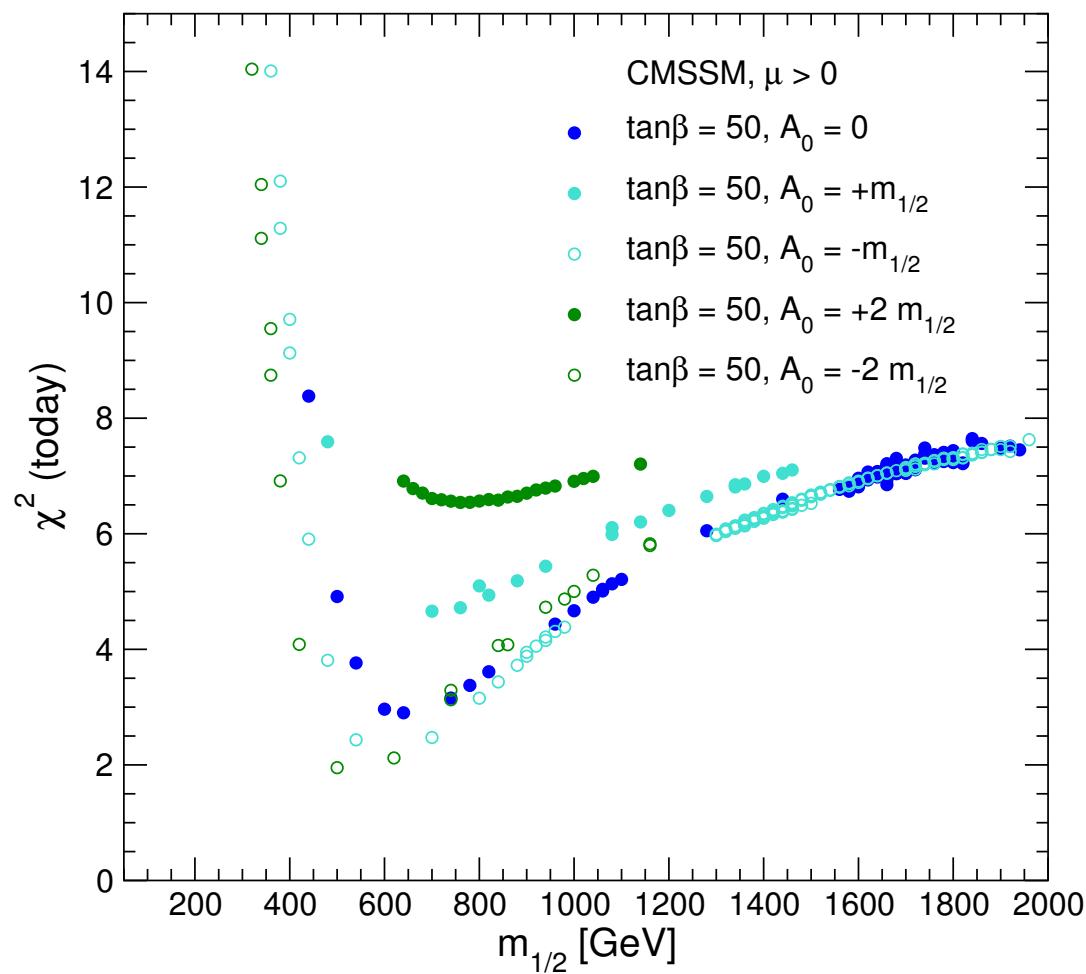
preference for relatively
small mass values

χ^2 fit in mSUGRA with dark matter constraints:

$$M_W, \sin^2 \theta_{\text{eff}}, (g - 2)_\mu, \text{BR}(b \rightarrow s\gamma)$$

[J. Ellis, S. Heinemeyer, K. Olive, G. W. '04]

$\tan \beta = 50$:



⇒ worse fit quality
preferred $m_{1/2}$ values
larger by 200–300 GeV
compared to $\tan \beta = 10$
case

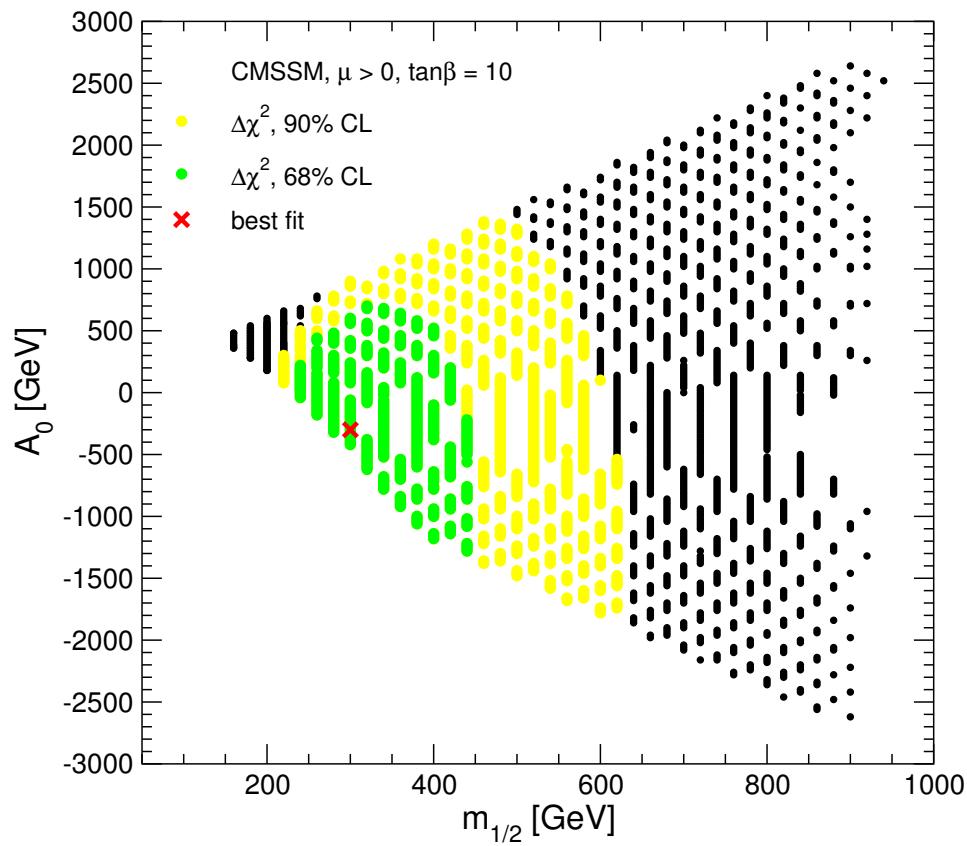
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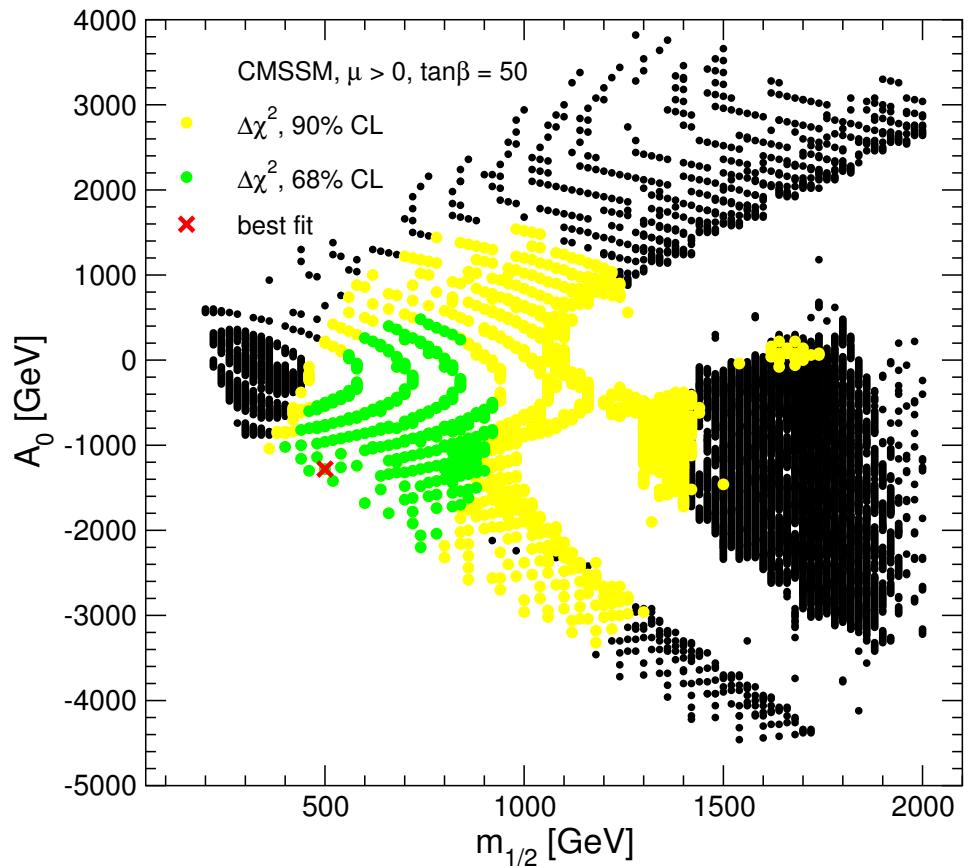
68% and 90% C.L. regions in $m_{1/2}$ – A_0 plane:

[J. Ellis, S. Heinemeyer, K. Olive, G. W. '04]

$\tan \beta = 10$:



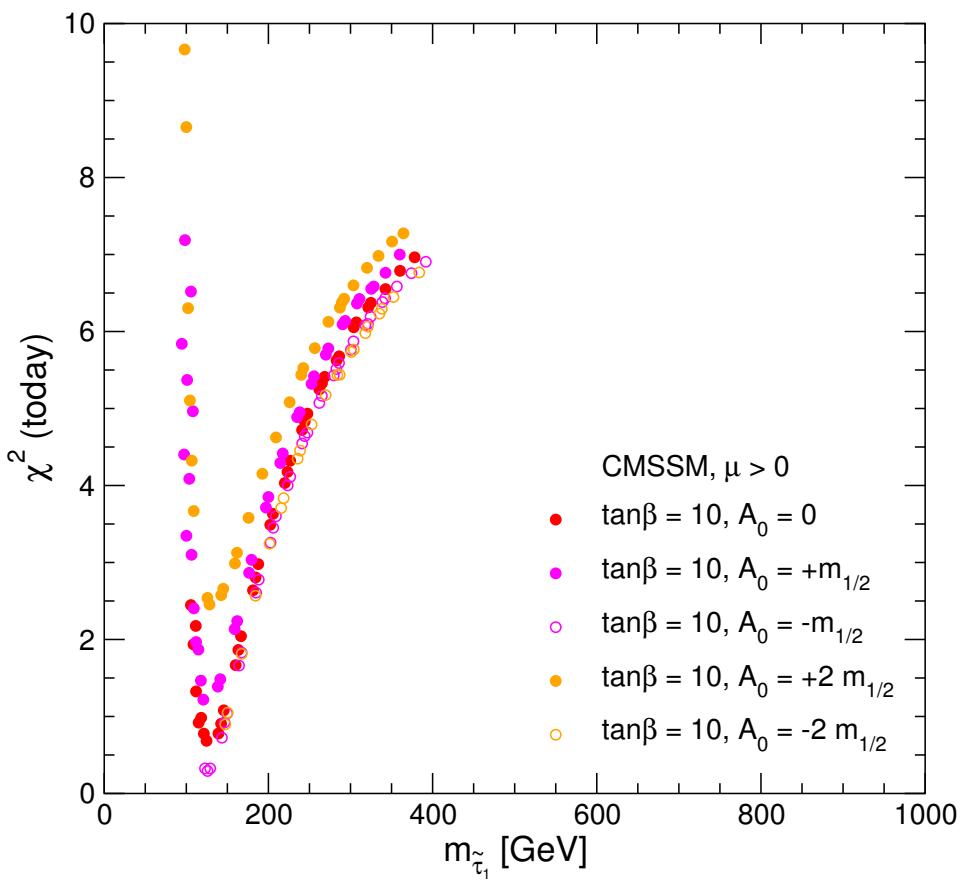
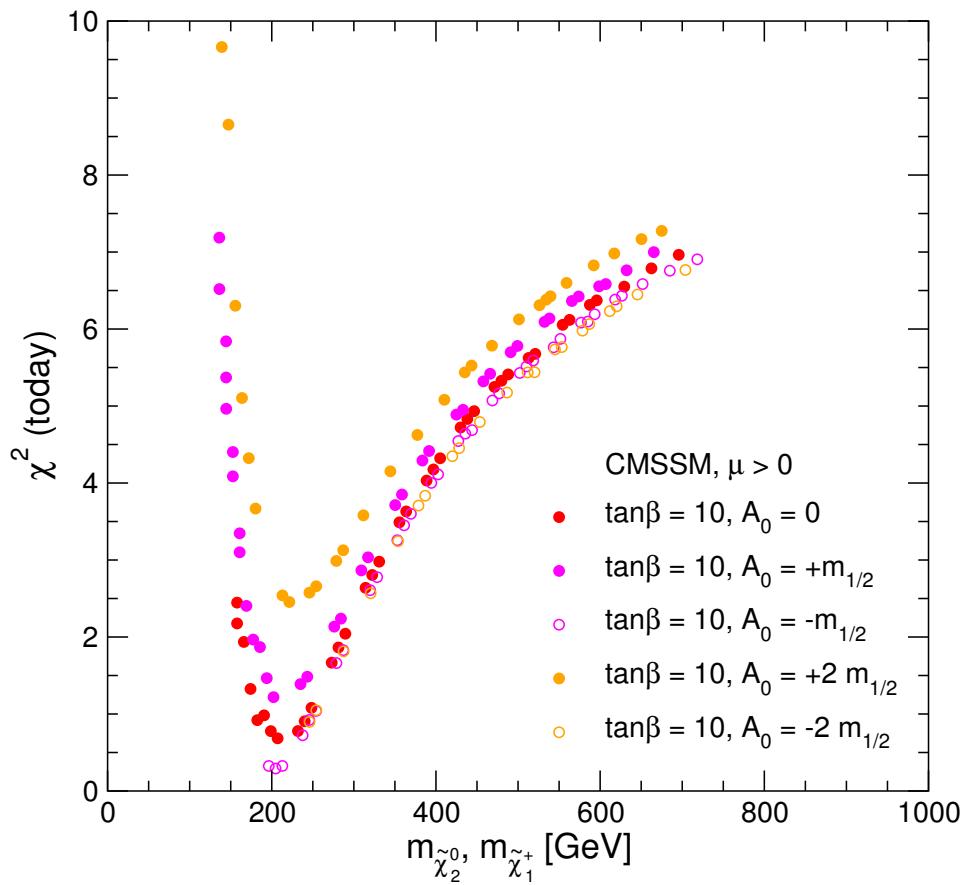
$\tan \beta = 50$:



Fit results for particle masses, $\tan \beta = 10$:

$$m_{\tilde{\chi}_1^+} \approx m_{\tilde{\chi}_2^0}, \quad m_{\tilde{\tau}_1}$$

[J. Ellis, S. Heinemeyer, K. Olive, G. W. '04]



→ Good prospects for the LHC and ILC

4. Conclusions

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⇒ preference for light Higgs, $M_H \lesssim 260$ GeV
strong $m_t - M_H$ correlation

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- At Tevatron, LHC, ILC: improved accuracy of prec. observables M_W , $\sin^2 \theta_{\text{eff}}$, m_h , ... and input parameters m_t , $m_{\tilde{t}}$, ...
 - ⇒ Very sensitive test of electroweak theory