LUND UNIVERSITY

Parton Showers

- Introduction
- DGLAP-based PS
- $k_\perp$-factorization
- Matching with Matrix Elements
- Summary

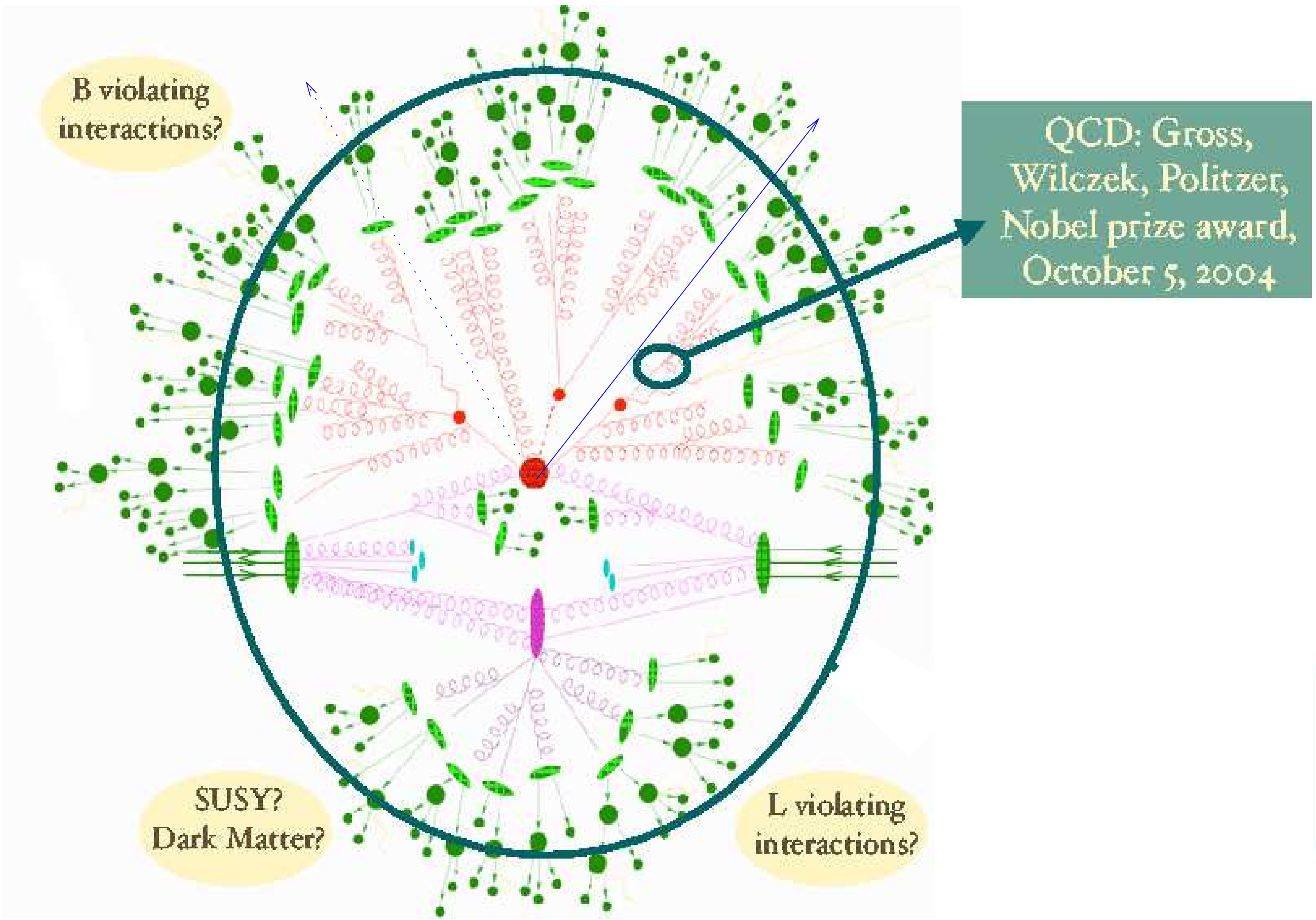
Durham
2005.01.06
Leif Lönnblad

We need Event Generators to understand our detectors and to understand the physics.

In Electro-weak and many Beyond Standard Models things are easy. With a few particle final-states we can calculate distributions analytically to a high precision, and generating corresponding events is trivial.

If there are a handful or more particles, there are automatic Matrix Element Generators which will do it for you.

If QCD is involved we always get lots of final-state particles and matrix element generators are typically not enough.


B violating interactions?
QCD: Gross, Wilczek, Politzer, Nobel prize award, October 5, 2004
SUSY? Dark Matter?
L violating interactions?

At LHC everything is QCD

Every signal will have a QCD background

Every observable will have QCD corrections.

$\mathcal{O} = \sigma_0(1 + C_1\alpha_\mathrm{s} + C_2\alpha_\mathrm{s}^2 + \ldots)$

For QCD we need not only describe how partons are produced, we also need to model how they evolve and form hadrons. And also to take into account how these hadrons decay.

QCD is difficult since a parton is not a jet.

Leading order is never enough

$$\mathcal{O} = \sigma_0(1 + C_1\alpha_\text{s} + C_2\alpha_\text{s}^2 + \ldots)$$

If we have a large scale μ^2, $\alpha_\text{s} \propto 1/\log(\mu^2)$ is small. But the coefficients contains integrals over gluon emissions giving $C_n \propto \log(\mu^2)^n$.

This means we need to resum the series — DGLAP.

Collinear factorization: Convolute ME with PDFs using the approximation that the incoming partons are collinear. Sum over all emissions with a scale below μ.

If μ^2 is not so large we in addition get $C_n \propto \log(S/\mu^2)$, this is not taken into account in DGLAP.

To take these into account we need $k_\perp$-factorization.

If your observable includes jets, it's not enough to resum and forget about additional parton emission – these need to be generated, and hadronized.

For hadronization we need models (string or cluster) which only work if soft and collinear partons are included consistently.

To reconstruct these resummed emissions we use Parton Shower Generators.

To generate *real* exclusive events we want to generate according to

$$
\begin{aligned}
\mathcal{O}_{+0} &= \sigma_0(1 + C_{01}\alpha_s + C_{02}\alpha_s^2 + C_{03}\alpha_s^3 + \ldots) \\
\mathcal{O}_{+1} &= \sigma_0(C_{11}\alpha_s + C_{12}\alpha_s^2 + C_{13}\alpha_s^3 + \ldots) \\
\mathcal{O}_{+2} &= \sigma_0(C_{22}\alpha_s^2 + C_{23}\alpha_s^3 + C_{24}\alpha_s^4 + \ldots) \\
&\vdots
\end{aligned}
$$

To generate *real* exclusive events we want to generate according to

$$
\begin{aligned}
\mathcal{O}_{+0} &= \sigma_0(1 + C_{01}\alpha_s + C_{02}\alpha_s^2 + C_{03}\alpha_s^3 + \ldots) \\
\mathcal{O}_{+1} &= \sigma_0(C_{11}\alpha_s + C_{12}\alpha_s^2 + C_{13}\alpha_s^3 + \ldots) \\
\mathcal{O}_{+2} &= \sigma_0(C_{22}\alpha_s^2 + C_{23}\alpha_s^3 + C_{24}\alpha_s^4 + \ldots) \\
&\vdots
\end{aligned}
$$

Matrix Element Generators only gives tree-level diagrams and we get inclusive events.

To generate *real* exclusive events we want to generate according to

$$\begin{aligned}
\mathcal{O}_{+0} &= \sigma_0(1 + C_{01}\alpha_\mathrm{s} + C_{02}\alpha_\mathrm{s}^2 + C_{03}\alpha_\mathrm{s}^3 + \ldots) \\
\mathcal{O}_{+1} &= \sigma_0(C_{11}\alpha_\mathrm{s} + C_{12}\alpha_\mathrm{s}^2 + C_{13}\alpha_\mathrm{s}^3 + \ldots) \\
\mathcal{O}_{+2} &= \sigma_0(C_{22}\alpha_\mathrm{s}^2 + C_{23}\alpha_\mathrm{s}^3 + C_{24}\alpha_\mathrm{s}^4 + \ldots) \\
&\vdots
\end{aligned}$$

Matrix Element Generators only gives tree-level diagrams and we get inclusive events.

Next-to-leading order generators only gives us one extra parton.

$$
\begin{aligned}
\mathcal{O}_{+0} &= \sigma_0(1 + C_{01}\alpha_s + C_{02}\alpha_s^2 + C_{03}\alpha_s^3 + \ldots) \\
\mathcal{O}_{+1} &= \sigma_0(C_{11}\alpha_s + C_{12}\alpha_s^2 + C_{13}\alpha_s^3 + \ldots) \\
\mathcal{O}_{+2} &= \sigma_0(C_{22}\alpha_s^2 + C_{23}\alpha_s^3 + C_{24}\alpha_s^4 + \ldots) \\
&\vdots
\end{aligned}
$$

All coefficients in the expansion are divergent due to soft and collinear poles. We need to introduce a cutoff. If we use a cutoff in a jet-clustering variable and get well separated hard partons, the result can be compared to real jets clustered with the corresponding jet algorithm.

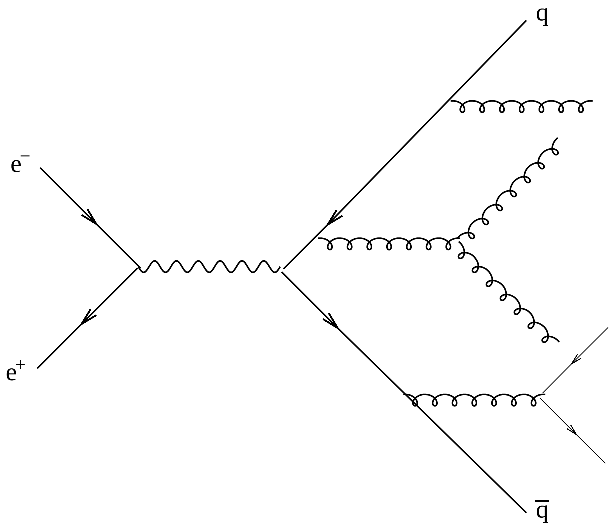
But a jet is not a parton. To understand corrections we need to model also the emission of soft and collinear partons, and the transition into hadrons.

Hadronization models (PYTHIA string, HERWIG cluster) work fairly well, but they are non-perturbative models and need exclusive partonic states with soft and collinear partons resolved down to scales of around a GeV.

In a Parton Shower we do this by including all orders in α_{s}, but approximated to leading-logarithmic accuracy.

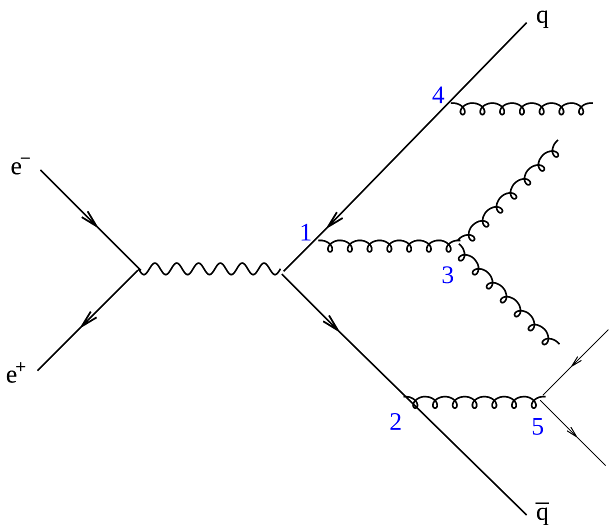
All coefficients are still divergent, but summed up to all orders, the result is finite.

DGLAP-based PS



The tree-level matrix element for an n-parton state can be approximated by a product of splitting functions corresponding to a sequence of one-parton emissions from the zeroth order state.

DGLAP-based PS



The tree-level matrix element for an n-parton state can be approximated by a product of splitting functions corresponding to a sequence of one-parton emissions from the zeroth order state.

These emissions need to be ordered in some resolution scale, ρ, so that every emission is softer and more collinear that the previous.

Then they need to be made exclusive, ensuring there are no additional emissions between ρ_i and ρ_{i+1}.

The probability of a emitting parton b from parton a is then given by

$$d\mathcal{P}_{ab} = \hat{P}_{ab}(\rho_i, \Omega_i) d\rho_i d\Omega_i \times \exp\left(- \sum_d \int_{\rho_i}^{\rho_{i-1}} d\rho \int d\Omega \hat{P}_{ad}(\rho, \Omega) \right)$$

$\exp(-\int_{\rho_i}^{\rho_{i-1}} \ldots) = \Delta_S(\rho_i, \rho_{i-1})$ is the Sudakov form factor.

$\hat{P}_{ab}(\rho_i, \Omega_i)$ are the splitting functions which, when multiplied together, corresponds to the approximate full tree-level ME

$$\hat{P}_{ab} d\rho d\Omega = \frac{\alpha_s}{2\pi} P_{ab}(z) \frac{dp_\perp^2}{p_\perp^2} dz d\phi$$

where $P(z)$ are the standard DGLAP splitting functions.

Integrating we get schematically

$$\begin{aligned} O_{+0} &= \sigma_0 \Delta_{\mathrm{S}0} \\ O_{+1} &= \sigma_0 C_{11}^{\mathrm{PS}} \alpha_{\mathrm{s}} \Delta_{\mathrm{S}1} \\ O_{+2} &= \sigma_0 C_{22}^{\mathrm{PS}} \alpha_{\mathrm{s}}^2 \Delta_{\mathrm{S}2} \\ &\vdots \end{aligned}$$

We still need a cutoff, ρ_{cut}, and the coefficients C_{nn}^{PS} diverges as $\log^n \rho_{\max}/\rho_{\mathrm{cut}}$

Integrating we get schematically

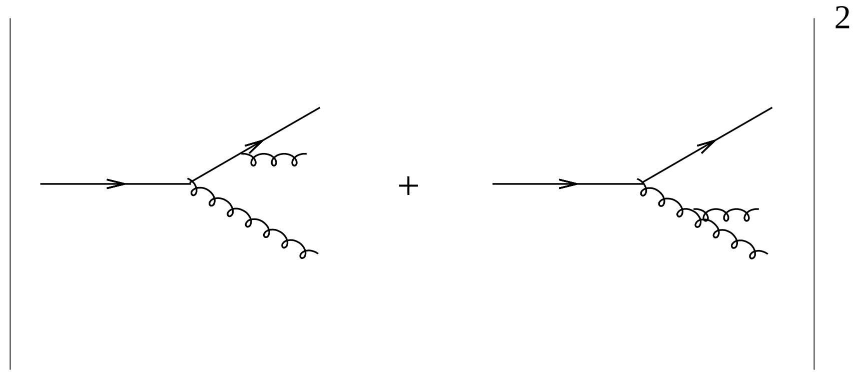
$$
\begin{aligned}
O_{+0} &= \sigma_0 \Delta_{S0} = \sigma_0(1 + C_{01}^{\mathrm{PS}}\alpha_s + C_{02}^{\mathrm{PS}}\alpha_s^2 + \ldots) \\
O_{+1} &= \sigma_0 C_{11}^{\mathrm{PS}}\alpha_s \Delta_{S1} = \sigma_0(C_{11}^{\mathrm{PS}}\alpha_s + C_{12}^{\mathrm{PS}}\alpha_s^2 + C_{13}^{\mathrm{PS}}\alpha_s^3 + \ldots) \\
O_{+2} &= \sigma_0 C_{22}^{\mathrm{PS}}\alpha_s^2 \Delta_{S2} = \sigma_0(C_{22}^{\mathrm{PS}}\alpha_s^2 + C_{23}^{\mathrm{PS}}\alpha_s^3 + C_{24}^{\mathrm{PS}}\alpha_s^4 + \ldots) \\
&\vdots
\end{aligned}
$$

We still need a cutoff, ρ_{cut}, and the coefficients C_{nn}^{PS} diverges as $\log^n \rho_{\mathrm{max}}/\rho_{\mathrm{cut}}$

but the Sudakovs corresponds to the leading-log resummation of all virtual terms and makes things finite, and we can use $\rho_{\mathrm{cut}} \sim 1$ GeV.

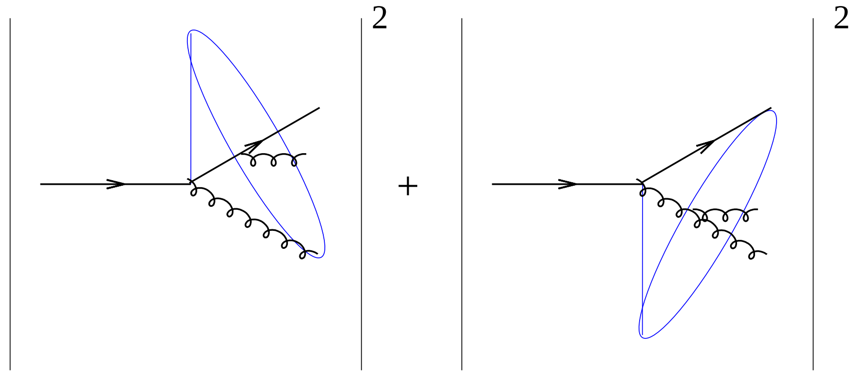
Parton showers cannot model several hard jets very well. Especially the correlations between hard jets are poorly described.

The splitting probabilities means that coherence effects are not taken into account



Parton showers cannot model several hard jets very well. Especially the correlations between hard jets are poorly described.

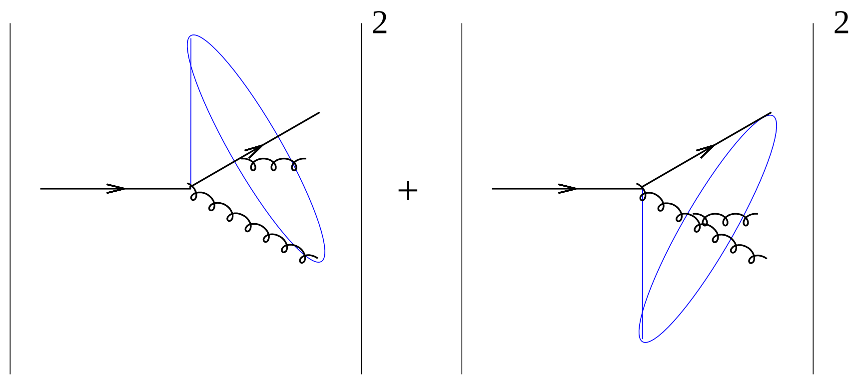
The splitting probabilities means that coherence effects are not taken into account



Most coherence effects can be taken into account by angular ordering.

Parton showers cannot model several hard jets very well. Especially the correlations between hard jets are poorly described.

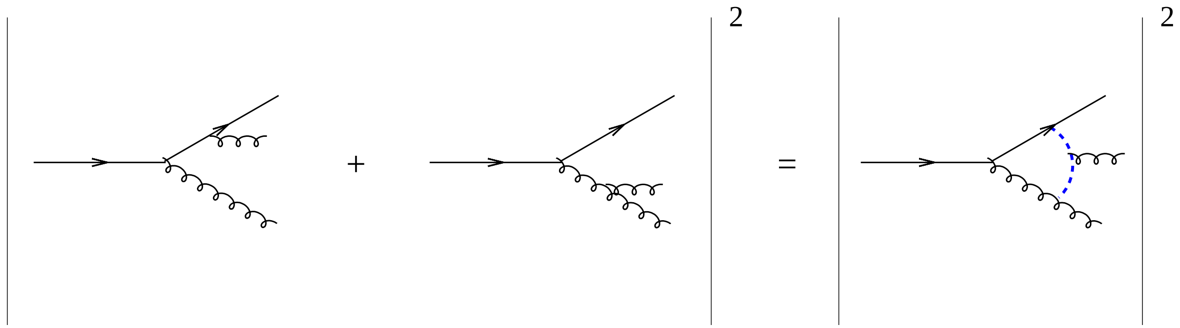
The splitting probabilities means that coherence effects are not taken into account



Most coherence effects can be taken into account by angular ordering.

Some angular correlations can also be taken into account by adjusting the azimuthal angles after a shower is generated.

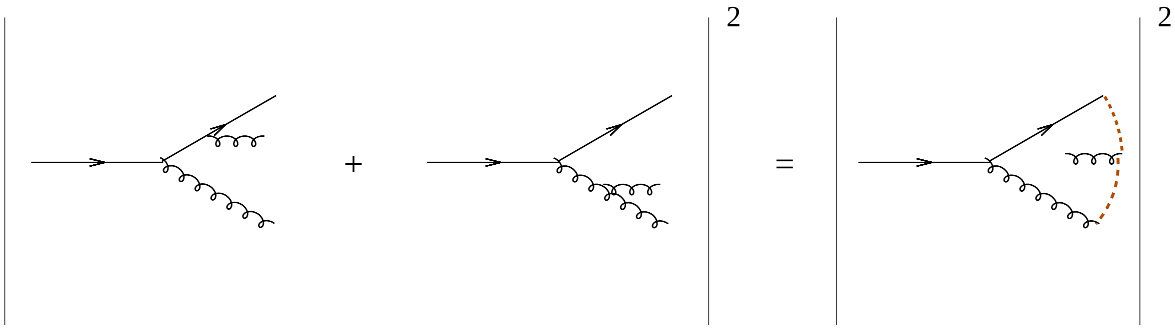
Coherence effects can be included directly, by considering gluon radiation from colour dipoles between colour-connected partons.



Rather than iterating $1 \to 2$ parton splitting we iterate $2 \to 3$ splittings. Each emission from a dipole will create two new dipoles which can continue radiating.

This is implemented in the ARIADNE generator.

Coherence effects can be included directly, by considering gluon radiation from colour dipoles between colour-connected partons.



Rather than iterating $1 \rightarrow 2$ parton splitting we iterate $2 \rightarrow 3$ splittings. Each emission from a dipole will create two new dipoles which can continue radiating.

This is implemented in the ARIADNE generator.

Initial-state shower

For incoming hadrons, we need to consider the evolution of the parton densities. Using collinear factorization and DGLAP evolution we have (with $t = \log k_\perp^2/\Lambda^2$)

$$\frac{df_b(x,t)}{dt} = \sum_a \int \frac{dx'}{x'} f_a(x',t) \frac{\alpha_s}{2\pi} P_{ab}\left(\frac{x}{x'}\right)$$

We can interpret this as during a small increase ṭ there is a probability for parton a with momentum fraction x' to become resolved into parton b at $x = zx'$ and another parton c at $x' - x = (1-z)x'$.

In a backward evolution scenario we start out with the hard sub-process at some scale $t_{\max}$

$$\sigma_0 \propto \hat{\sigma}_{ab\to X} f_a(x_a, t_{\max}) f_b(x_b, t_{\max})$$

and we get the relative probability for the parton a to be *unresolved* into parton c during a decrease in resolution scale dt

$$d\mathcal{P}_a = \frac{df_a(x_a,t)}{f_a(x_a,t)} = |dt| \sum_c \int \frac{dx'}{x'} \frac{f_c(x',t)}{f_a(x_a,t)} \frac{\alpha_{\mathrm{s}}}{2\pi} P_{ca}\left(\frac{x_a}{x'}\right)$$

Summing up the cumulative effect of many small changes dt, the probability for no radiation exponentiates and we get a Sudakov

$$\Delta_{S_a}(x_a, t_{\max}, t) = \exp\left\{-\int_t^{t_{\max}} dt' \sum_c \int \frac{dx'}{x'} \frac{f_c(x',t')}{f_a(x_a,t')} \frac{\alpha_{\mathrm{s}}(t')}{2\pi} P_{ca}\left(\frac{x_a}{x'}\right)\right\}$$

This now gives us the probability for a backwards initial-state splitting

$$d\mathcal{P}_{ca} = \frac{\alpha_\mathrm{s}}{2\pi} P_{ac}(z) \frac{f_c(x_a/z,t)}{f_a(x_a,t)} dt \frac{dz}{z} d\phi \times \Delta_{S_a}(x_a, t_{\max}, t)$$

In a hadronic collision we first generate the hard scattering, then evolve the incoming partons backward to lower scales, and then alow for a final-state shower from all partons from the hard scattering and the initial-state shower.

This works nicely as long as the hard scale is large and DGLAP evolution is applicable with decreasing virtualities in each backward initial-state splitting.

$k_\perp$-factorization

DGLAP evolution is not applicable if the hard scale is much smaller than the total energy and the virtuality of the incoming partons are not much smaller than the hard scale.

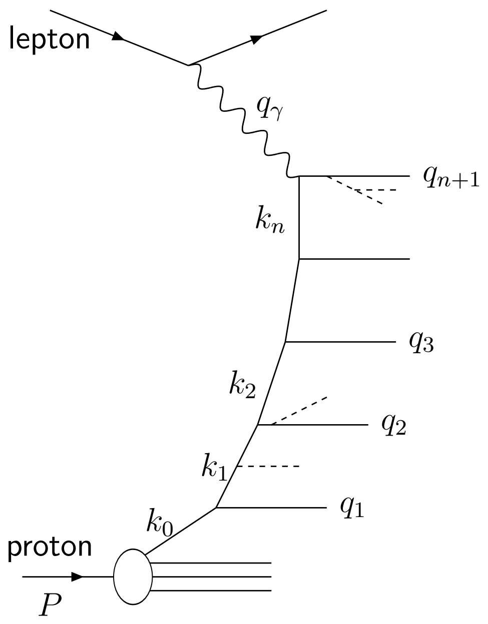
Collinear factorization $\Longrightarrow$ $k_\perp$-factorization

$$\int dx_a dx_b \hat{\sigma}_{ab\to X} f_a(x_a, Q^2) f_b(x_b, Q^2) \Longrightarrow$$

$$\int dx_a dx_b dk_{\perp a} dk_{\perp b} \hat{\sigma}^{\star}_{ab\to X} \mathcal{F}_a(x_a, k_{\perp a}, Q^2) \mathcal{F}_b(x_b, k_{\perp b}, Q^2)$$

$\mathcal{F}$ an unintegrated parton density.

$\hat{\sigma}^{\star}$ is the off-shell matrix element

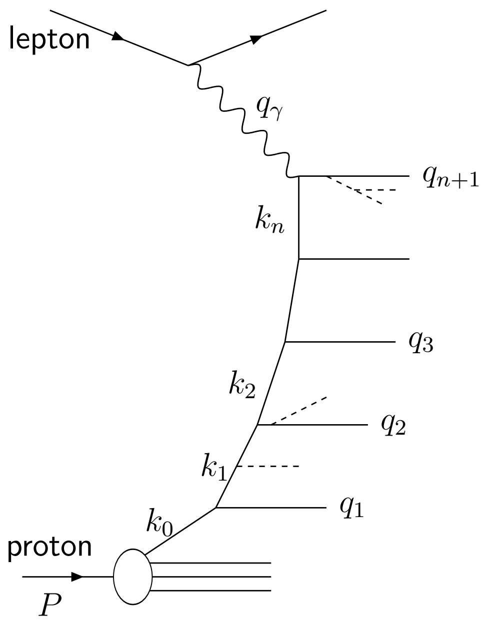


In DIS, the cross section is dominated by events with small $Q^2 = -q_\gamma^2$ and small x.

The available phase space for emitting partons is not limited by Q^2, but rather by the total hadronic energy, $W^2 \approx Q^2/x$.

The $1/z$ pole in the gluon splitting function makes it possible to emit many initial-state gluons even for small Q^2.

We need to take into account unordered evolution.



In DIS, the cross section is dominated by events with small $Q^2 = -q_\gamma^2$ and small x.

The available phase space for emitting partons is not limited by Q^2, but rather by the total hadronic energy, $W^2 \approx Q^2/x$.

The $1/z$ pole in the gluon splitting function makes it possible to emit many initial-state gluons even for small Q^2.

We need to take into account unordered evolution.

Forward jets at HERA cannot be reproduced by DGLAP based initial-state parton showers.

Let's look at the unintegrated gluon density, which should be dominating. Starting from a (non-perturbative) gluon at some x_0 we get the contribution

$$\mathcal{G}(x,k_\perp^2) = \sum_n \prod_i^n \left\{ \int \frac{dq_{\perp i}^2}{q_{\perp i}^2} dz_i \bar{\alpha} \tilde{P}(z_i, q_{\perp i}^2) \Theta(z_i, q_{\perp i}^2) \right\} \delta(x - x_0 \Pi z_i) \delta(k_\perp^2 - k_{\perp n}^2)$$

$\bar{\alpha}$ is a suitably scaled α_s

$\tilde{P}(z_i, q_{\perp i}^2)$ is the splitting function

$\Theta(z_i, q_{\perp i}^2)$ is some phase space limitation defining which emissions we want to include in the evolution.

For large $k_\perp$ and small x we can use the double leading logarithmic approximation with $\tilde{P}(z) \approx 1/z$ and $\Theta = \theta(q_{\perp i} - q_{\perp i-1})$

$$\mathcal{G}(x, k_\perp^2) = \sum_n \prod_i^n \left\{ \int \frac{dq_{\perp i}^2}{q_{\perp i}^2} \frac{dx_i}{x_i} \theta(q_{\perp i} - q_{\perp i-1}) \theta(x_{i-1} - x_i) \right\} \delta(x - x_n) \delta(k_\perp^2 - k_{\perp n}^2)$$

which can be easily integrated to get the well known DLL result

$$\mathcal{G} \propto \exp(2\sqrt{\bar{\alpha} \ln k_\perp^2 \ln 1/x})$$

Using running coupling $\bar{\alpha} = \alpha_0 / \log(q_\perp^2 / \Lambda^2)$ we would instead get

$$\mathcal{G} \propto \exp(2\sqrt{\alpha_0 \ln \ln k_\perp^2 \ln 1/x})$$

In the limit of asymptotically small x and moderate $k_\perp$ we may use BFKL evolution. Here there is no upper limit on the $q_\perp$ of the emitted gluons and the splitting function

$$\tilde{P}(z, k_\perp^2) = \Delta_R(z, k_\perp^2)/z$$

corresponds to real gluon emissions from *Reggeized* gluons, where the Regge form factor corresponding to a sum over virtual diagrams:

$$\Delta_R(z, k_\perp^2) = \exp\left(-\bar{\alpha}\int_{z_i}^{1}\frac{dz}{z}\int_{\mu^2}^{k_{\perp i}^2}\frac{dk_\perp^2}{k_\perp^2}\right)$$

The integration is a bit more tricky, but is doable and the result is the well-known strong rise of the gluon

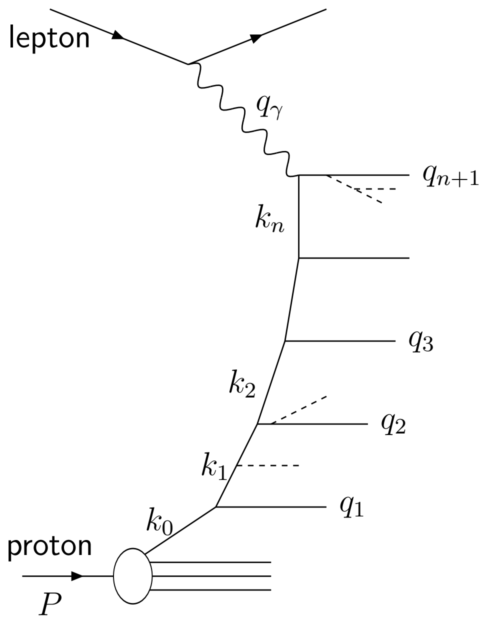
$$\mathcal{G} \propto x^{-\lambda} = x^{-4\ln 2\bar{\alpha}}$$

The next-to-leading logarithmic corrections to BFKL turns out to be massive. The main reason for this seems to be the lack of (transverse) momentum conservation when allowing for unlimited $q_\perp$ in the emissions.

The next-to-leading logarithmic corrections to BFKL turns out to be massive. The main reason for this seems to be the lack of (transverse) momentum conservation when allowing for unlimited $q_\perp$ in the emissions.

The first commandment of Event Generation

Thou Shalt always respect Energy and Momentum conservation



In a parton shower scenario we typically want to separate between initial-state emissions which corresponds to the evolution of the parton densities, and final-state emissions which do not.

In CCFM evolution this done by defining all emissions not corresponding to a angular ordered final-state shower to be initial-state emissions.

CCFM

CCFM limits the initial-state emissions to have increasing opening angles (rapidity). In terms of the rescaled transverse momentum $\bar{q} = q_\perp/(1-z)$ we then get the phase space restriction

$$\Theta = \theta(\bar{q}_i - z_{i-1}\bar{q}_{i-1})$$

Starting from BFKL and resumming all emissions now treated as final-state will cancel parts of the Regge form factor giving

$$\frac{\Delta_R}{z_i} \longrightarrow \frac{\Delta_{ne}}{z_i} = \frac{1}{z_i}\exp\left(-\bar{\alpha}\int_{z_i}^{1}\frac{dz}{z}\int^{k_{\perp i}^2}\frac{d\bar{q}^2}{\bar{q}^2}\theta(\bar{q}-z\bar{q}_i)\right)$$

The angular ordering properly takes into account gluon coherence and also results in less infrared sensitivity.

Here we may also include the soft pole in the splitting function with a corresponding Sudakov form factor to conserve energy

$$\tilde{P} = \frac{\Delta_{ne}}{z} + \frac{\Delta_S}{1-z}$$

which means that for not so small x we recover the main features of DGLAP evolution.

CCFM has been implemented in two event generators, SMALLX (forward evolution) and CASCADE (backward evolution).

Linked Dipole Chains

The division between initial- and final-state emissions can be made in many ways. However it is reasonable to require that the final-state emissions do not change the basic propagators in the ladder too much.

In the Linked Dipole Chain (LDC) model the final-state emissions are coming from the dipoles between the gluons emitted in the initial-state. A suitable constraint on the initial state emissions turns out to be

$$\Theta = \theta(q_{\perp i} - \min(k_{\perp i-1}, k_{\perp i}))$$

This is a stronger restriction than in CCFM and summing up the contributions from final-state emissions will give us simply

$$\Delta_{ne}/z \longrightarrow 1/z$$

In this way, LDC becomes even less infrared sensitive, and the absence of a form factor makes it easy to include full DGLAP splitting functions (not only the singular parts) and even include the evolution of quarks.

Also LDC has been implemented in an event generator, LDCMC.

But we can also learn some qualitative lessons from the LDC formulation.

Looking at the limit of strongly ordered $k_\perp$, not only increasing but also decreasing, we find that the phase space restriction in LDC means that $q_{\perp i} \approx \max(k_{\perp i-1}, k_{\perp i})$. Also considering strongly ordered x we get for each emission

$$\bar{\alpha}\frac{dz_i}{z_i}\frac{dq^2_{\perp i}}{q^2_{\perp i}} \approx \bar{\alpha}\frac{dz_i}{z_i}\frac{dk^2_{\perp i}}{\max(k_{\perp i-1}, k_{\perp i})} = \bar{\alpha}\frac{dz_i}{z_i}\frac{dk^2_{\perp i}}{k^2_{\perp i}}\min\left(\frac{k_{\perp i}}{k_{\perp i-1}}, 1\right)$$

Comparing with the DLL approximation above which we can rewrite in terms of $\kappa = \log k_{\perp i}^2/\Lambda^2$ and $l_i = \log(1/x_i)$:

$$\mathcal{G}(l,\kappa) \propto \sum_n \prod_i^n \left\{ \bar{\alpha} \int^{\kappa} d\kappa_i \theta(\kappa_i - \kappa_{i-1}) \int^{l} dl_i \theta(l_i - l_{i-1}) \right\} = \sum_n \bar{\alpha}^n \frac{\kappa^n}{n!} \frac{l^n}{n!}$$

we now want to allow also for unordered κ, but we note that taking a step down in κ is punished exponentially by $k_{\perp i}^2/k_{\perp i-1}^2 = \exp(\kappa_{i-1} - \kappa_i)$.

Approximating the exponential suppression with a step function we get an approximate ordering in κ, $\theta(\kappa_i - \kappa_{i-1} + 1)$ and we have

$$\int^{\kappa} \prod_i^n \theta(\kappa_i - \kappa_{i-1} + 1) \approx \frac{(\kappa + n)^n}{n!}$$

For large κ we recover the DLL result

On the other hand if κ is small we get from Sterlings formula

$$\frac{(\kappa+n)^n}{n!} \approx \frac{n^n}{n!} \approx e^n$$

and

$$\mathcal{G}(l,\kappa) \propto \sum_n \bar{\alpha}^n e^n \frac{l^n}{n!} \approx e^{\bar{\alpha}el} = x^{-\lambda}$$

with $\lambda = e\bar{\alpha} \approx 2.72\bar{\alpha}$ which is remarkably close to the BFKL result $\lambda = 4\log 2\bar{\alpha} \approx 2.77\bar{\alpha}$.

We can also get an estimate of where the transition between DGLAP and BFKL should occur, and obtain $\kappa \approx \lambda l$

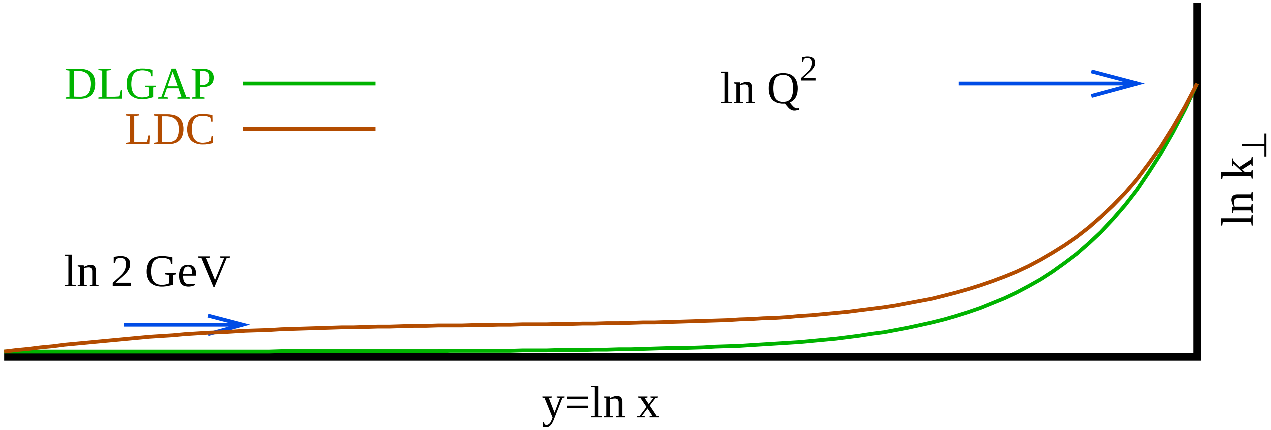
We can now also try to include a running coupling which means

$$\bar{\alpha} d\kappa \rightarrow \alpha_0 \frac{d\kappa}{\kappa} = \alpha_0 du$$

with $u = \log \kappa$.

Remembering the approximate phase space constraint $\theta(\kappa_i - \kappa_{i-1} + 1)$ we note that for large κ, one extra unit in κ is negligible in u and we recover the DLL situation, while for small κ the restriction basically vanishes and we get a random walk in κ.

We can therefore expect the typical evolution path, going backwards from the hard scale, to be DGLAP-like until the virtualities reach smaller values where it becomes BFKL-like.



This can be simulated with a DGLAP shower by adding an unnaturally large intrinsic $k_\perp$ (needed to describe $p_\perp$-distributions for prompt photons and W production).

The problem with BFKL/CCFM/LDC

The event generators CASCADE and LDCMC give consistent result.

However, the forward jet rates, a measurement designed to be impossible to reproduce without unordered evolution, is only reproduced by CASCADE and LDCMC if non-singular terms are omitted from the gluon splitting function. Using the full function

$$P_{gg}(z) = \frac{1}{z} + \frac{1}{1-z} + z(1-z) - 2$$

will underestimate forward jet rates by almost a factor 2.

The problem with BFKL/CCFM/LDC

The event generators CASCADE and LDCMC give consistent result.

However, the forward jet rates, a measurement designed to be impossible to reproduce without unordered evolution, is only reproduced by CASCADE and LDCMC if non-singular terms are omitted from the gluon splitting function. Using the full function

$$P_{gg}(z) = \frac{1}{z} + \frac{1}{1-z} + z(1-z) - 2$$

will underestimate forward jet rates by almost a factor 2.

The dipole shower in ARIADNE allows for un-ordered evolution (although not directly related to BFKL/CCFM/LDC) and reproduces forward jets quite nicely.

Matching with Matrix Elements

Tree-level matrix element generators are good for a handful hard, well separated partons, but bad for many soft and collinear partons.

Parton shower generators are good for a handful hard, well separated partons, but bad for many soft and collinear partons.

Why can't we simply combine the two?

Matching with Matrix Elements

Tree-level matrix element generators are good for a handful hard, well separated partons, but bad for many soft and collinear partons.

Parton shower generators are good for a handful hard, well separated partons, but bad for many soft and collinear partons.

Why can't we simply combine the two?

For one extra parton emission we can usually simply modify the splitting functions to reproduce the correct matrix element.

Matching with Matrix Elements

Tree-level matrix element generators are good for a handful hard, well separated partons, but bad for many soft and collinear partons.

Parton shower generators are good for a handful hard, well separated partons, but bad for many soft and collinear partons.

Why can't we simply combine the two?

For one extra parton emission we can usually simply modify the splitting functions to reproduce the correct matrix element.

For more partons we need CKKW.

Parton Shower

$$O_{+0} = \sigma_0 \Delta_{S0}$$
$$O_{+1} = \sigma_0 C_{11}^{PS} \alpha_s \Delta_{S1}$$
$$O_{+2} = \sigma_0 C_{22}^{PS} \alpha_s^2 \Delta_{S2}$$
$$\vdots$$

Matrix Element

$$O_{+0} = \sigma_0$$
$$O_{+1} = \sigma_0 C_{11}^{ME} \alpha_s$$
$$O_{+2} = \sigma_0 C_{22}^{ME} \alpha_s^2$$
$$\vdots$$

Comparing the α_s expansions the strategy should be obvious. Generate events with 1, 2, 3, ..., N extra hard jets according to tree-level matrix elements using some (large) cutoff. Then reweight with Sudakov form factors from the parton shower. Finally add parton shower to get events with more than N partons and with partons below the ME cutoff.

To obtain Sudakov form factors we need to have an ordered set of emission scales. This can be done by applying a jet clustering algorithm to the parton state generated with the Matrix Element.

Alternatively we can make a shower reconstruction (answering the question how would my parton shower have generated this partonic state?)

The Sudakovs can then be calculated analytically or by making trial parton shower emissions from intermediate states in the shower reconstruction, remembering that the Sudakov is a no-emission probability

When adding the parton shower we must make sure we do not double-count and add shower emissions which could also have been generated by the matrix element.

Also we must not under-count and miss phase space regions not covered by the matrix element.

The solution is to do a full parton shower, starting from the highest possible scale, but to veto emissions which are above the matrix element cutoff.

Special care must be taken for the highest parton multiplicity state generated by the matrix element. There we must only veto emissions which are above the lowest reconstructed scale.

Summary

- DGLAP-based parton showers corresponds to an explicit leading-log resummation of QCD corrections
- Essential to understand hadronization effects on jet observables.
- All (both) reasonable hadronization models need exclusive many-parton final-states, well modeled in the soft and collinear region
- Key ingredient to get exclusive states is the Sudakov form factors, corresponding to leading-log resummation of virtual diagrams.

- DGLAP-based parton showers have problems describing small-x final states in hadronic collisions.
  (Higgs production at LHC has $x \lesssim 0.01$)
- Adding an unnatural intrinsic $k_\perp$ will simulate some small-x effects.
- CCFM/LDC based parton showers do better at small-x, but there are still problems.
- With CKKW you can get the best of both the matrix element and parton shower world.

Summary of event generators

- PYTHIA: DGLAP-based PS (virtuality and $k_\perp$-ordered), string fragmentation and . . .
- HERWIG: DGLAP-based PS (angular ordered), cluster fragmentation and . . .
- ARIADNE: Dipole shower (fragmentation and more in PYTHIA).
- CASCADE: CCFM shower (no final-state PS).
- LDCMC: No longer publically available.

References

- PYTHIA manual: hep-ph/0108264
- HERWIG manual: hep-ph/0011363
- ARIADNE manual: *Comput. Phys. Commun.* **71** (1992) 15.
- Small-x: *Eur. Phys. J.* **C25** (2001) 77.
- CKKW: *JHEP* **0111** (2001) 063