Extra dimensions

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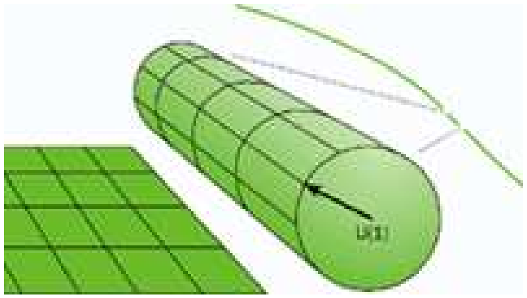
1. The Kaluza-Klein zoo
2. 'Postmodern' extra dimensions (flat)
3. Power law running
4. Detecting (flat) extra dimensions - 1...6
5. Randall Sundrum models
6. Summary

1. The Kaluza Klein zoo (1923/1926)

Geometric idea for unifying electromagnetism and gravity; Compactified 5D space:



- The noncompact theory has full 5D Lorentz invariance - graviton g^{MN} where $M, N = 0...4$
- Compact theory has 4D Lorentz invariance plus U(1)

$g^{\mu\nu}$	$g^{4\nu} = A^{\nu}$
$g^{\mu 4} = A^{\mu}$	$g^{44} = \phi$

1. Kaluza-Klein modes (higher dimensional fields)
2. non-topological solitons - e.g. Q-balls etc
3. topological solitons - domain walls, D-branes

1. KK-Modes

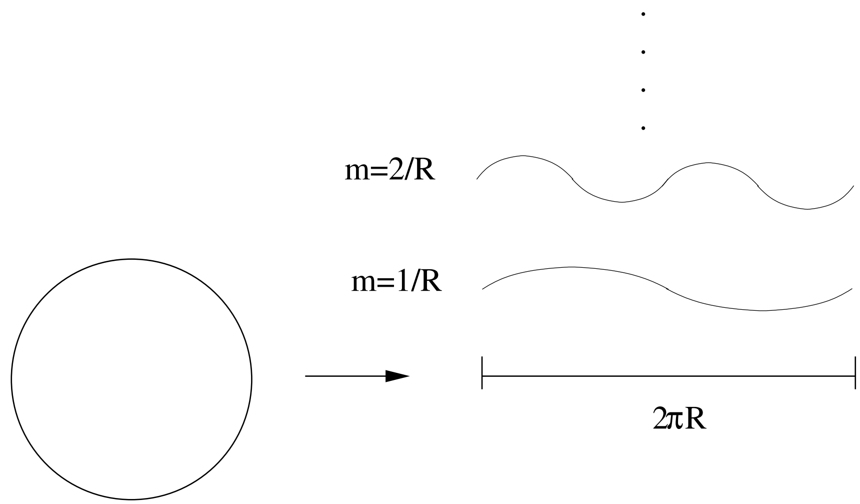
Get 5D action from Einstein-Hilbert action:

$$\mathcal{L}_5 \supset -\frac{1}{4g_5^2} F^{\mu\nu} F_{\mu\nu}$$

- Kaluza-Klein modes from (factor of $1/\sqrt{2\pi R}$ to normalize KE term)

$$A_\mu(x,y) = \frac{A_\mu^{(0)}}{\sqrt{2\pi R}} + \sqrt{\frac{2}{2\pi R}} \sum_{n=1}^{\infty} \left(A_\mu^{(n)}(x) \cos\frac{ny}{R} + A_\mu'^{(n)}(x) \sin\frac{ny}{R} \right)$$

- Equation of motion $\Box_5 A_\mu^{(n)} = (\partial^\mu \partial_\mu - \frac{n^2}{R^2})\, A_\mu^{(n)}$


m=2/R
m=1/R
2πR

- Dilution of couplings: consider non-abelian generalization
  -$g(\partial^\mu \phi) A_\mu \phi \supset \frac{1}{\sqrt{2\pi R}}\, g(\partial \phi^{(0)}) A_\mu^{(0)} \phi^{(0)}$

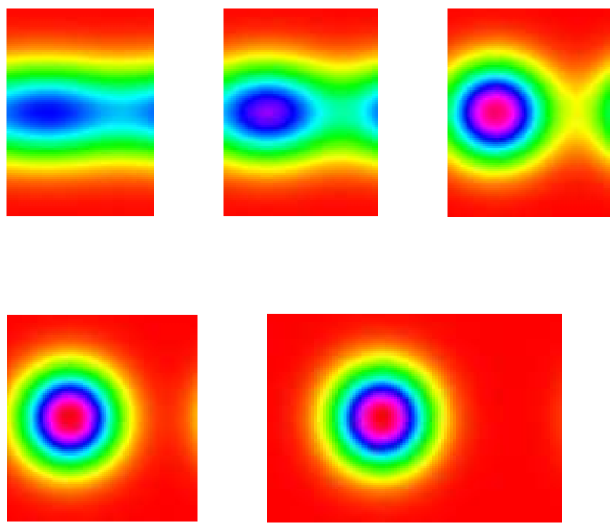
$$\begin{aligned} g_4^2 &= \frac{g_5^2}{2\pi R} \\ &= \frac{g_5^2}{Vol} \end{aligned}$$

  More simply rescale so

$$\mathcal{L}_4 = \int d^{D-4}y\, \frac{F^{\mu\nu}F_{\mu\nu}}{g_5^2} \equiv \frac{1}{g_4^2} F^{\mu\nu}F_{\mu\nu}$$

2. Non-topological objects - e.g. Q-balls (lumps).

Take scalar theory with e.g. global U(1) symmetry and quartic potential with high m^2 and λ_3. $\phi = \varphi(x^M)e^{i\omega t}$ gives lumps that hold a given Q-charge with less energy than Q separate particles. e.g. with one extra dimension

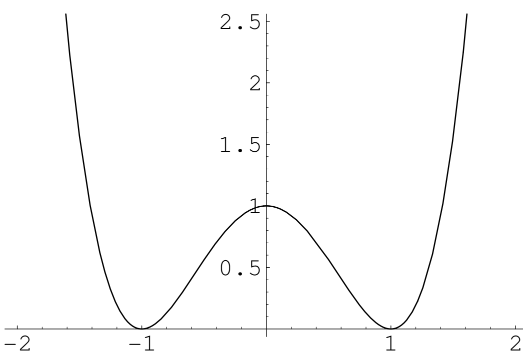


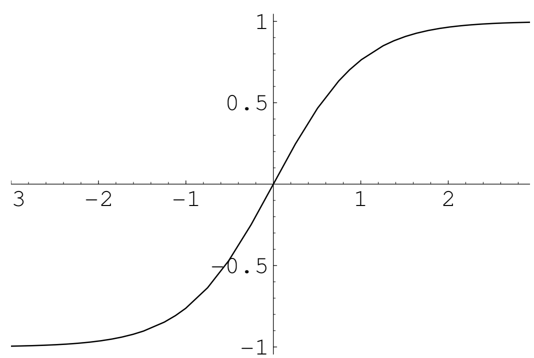
3. Topological solitons e.g. 1: Domain walls.

Consider a (higgs) scalar and fermion in 5D

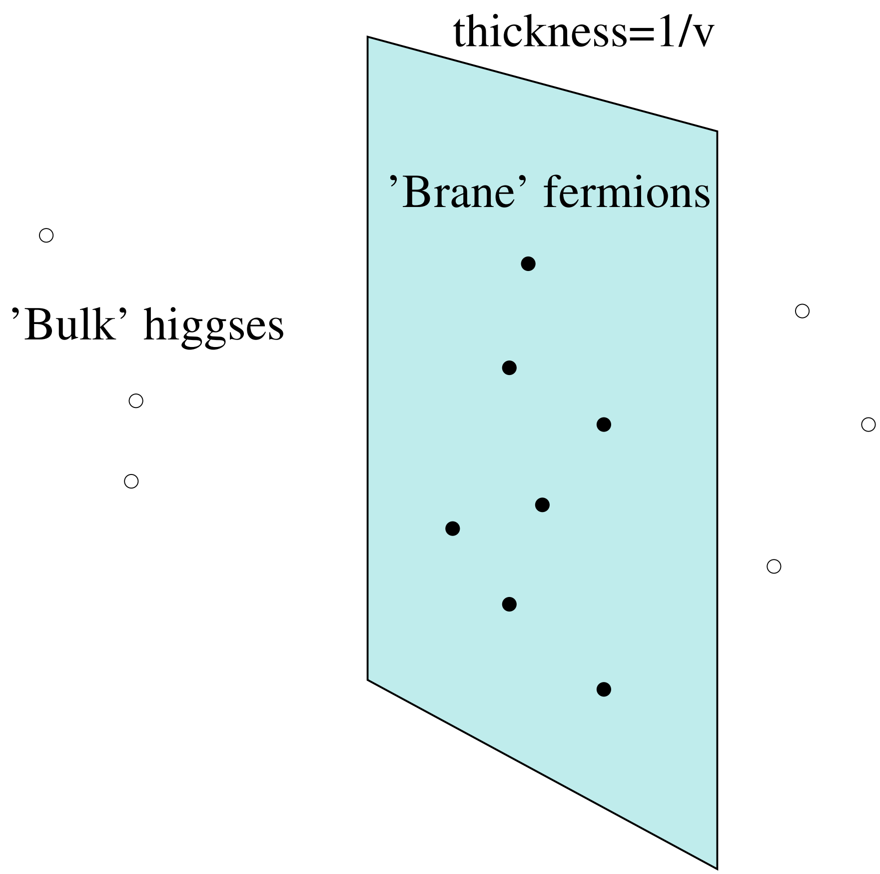
$$\mathcal{L} = \frac{1}{2}(\partial_M \phi)^2 + \bar{\psi}(i\gamma^M \partial_M - \phi\psi) - U(\phi)$$

$$U = \lambda(\phi^2 - v^2)^2$$

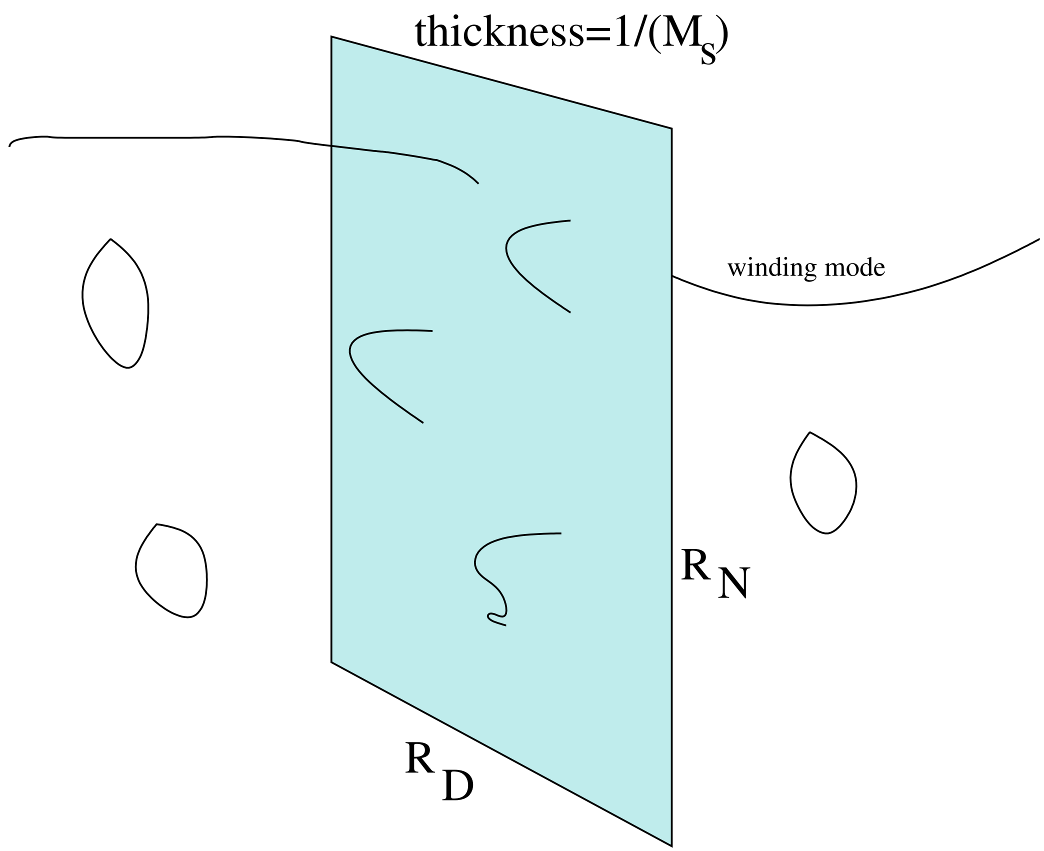




A static solution to eqns of motion is the domain wall of thickness $\sim 1/v$. The fermions can be trapped on the wall where their mass vanishes!

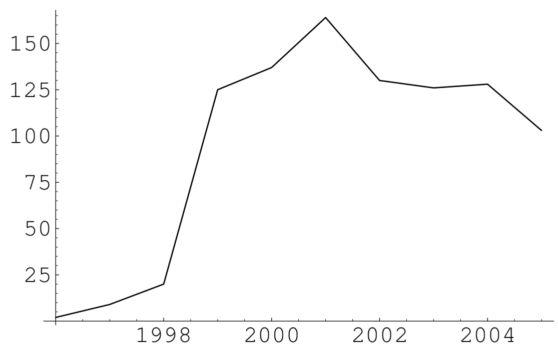


Topological solitons e.g. 2: D(irichlet)-branes



Note that D-branes can be wrapped in the N directions - it then has KK-modes of mass $m = 1/R_N$ and winding modes with $m = R_D$

2. Post-modern EDs (Antoniadis, Arkani-Hamed, Dimopoulos, Dvali 1998)



$y(x) =$"find title extra and title dimension# and date $= x$"

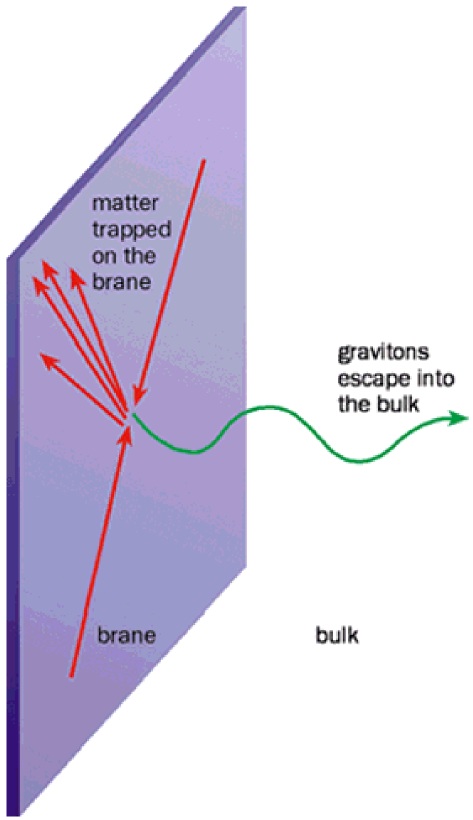
What is the fundamental scale of gravity?

Planck scale i.e.

$$
\begin{aligned}
G_N &= 6.673 \times 10^{-11} m^3 kg^{-1} s^{-2} \\
h &= 1.055 \times 10^{-34} Js \\
c &= 2.997 \times 10^5 ms^{-1} \\
L_P &= \sqrt{\frac{Gh}{c^3}} = 1.61 \times 10^{-33} cm
\end{aligned}
$$

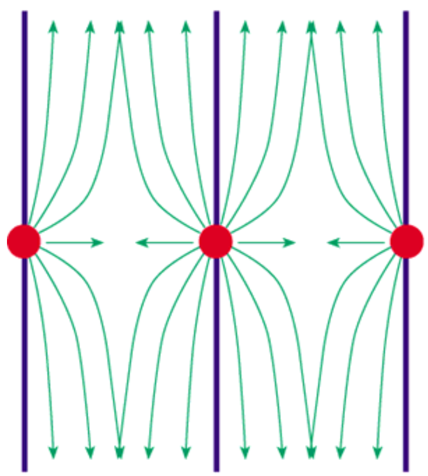
c is the fundamental speed of any radiation. However G_N is measured from the attraction between bodies in 4 dimensions. To infer $M_s \sim m_{Pl}$ must assume no large parameters (e.g. volumes) in getting from strings to 4-D gravity.

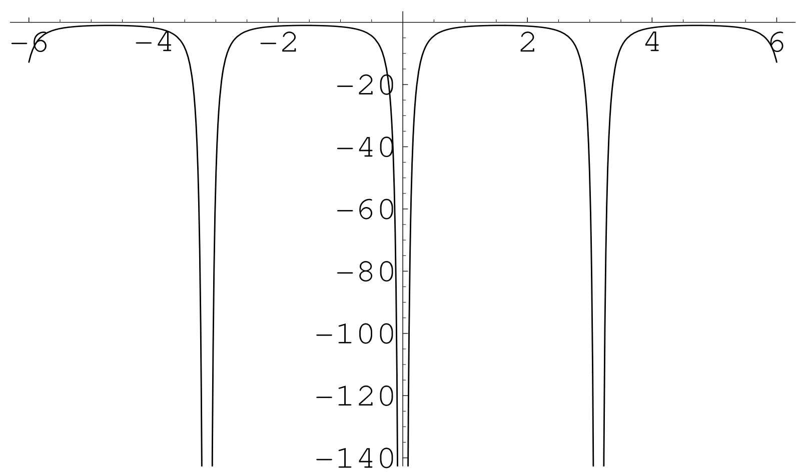
General idea - everything changes if we live (gauge and matter fields trapped) on a hypersurface (brane), and only gravity knows about the full volume (bulk).



- Gauge forces do not see extra dimensions - they remain the same
- Gravitational force is diluted in the extra dimensions until it 'sees' the other side - this could be 0.2 mm wide!
- Dilution relates the $D = 4 + n$ dimensional gravitational scale to the 4 dimensional one;

$$m_{Pl}^2 = Vol_n M_s^{n+2}$$





At large distances have $1/r^2$ force-law, small distances get $1/r^{2+n}$:
Indeed if $n = 1$

$$\sum_{n=-\infty}^{\infty} \frac{-1}{|x|^2 + (y - 2\pi n R)^2} = \frac{-1}{2Rx} \frac{\sinh(x/R)}{\cosh(x/R) - \cos(y/R)} \rightarrow \begin{cases} -\frac{1}{2R|x|} & x \gg R \\ \frac{1}{x^2+y^2} & x \ll R \end{cases}$$

TeV scale gravity to solve hierarchy problem?

One possibility is to take a fundamental scale of 1TeV to explain the weak scale without hierarchy problem and no supersymmetry.

For $D = 4 + n$ space time dimensions with n degenerate extra ones;

$$\begin{aligned} m_{Pl}^2 &= M_s^{2+n} R^n \\ R &\sim 10^{\frac{30}{n-17}} (M_s/m_W)^{\frac{n+2}{n}} \end{aligned}$$

- $n = 1 : R \sim 10^{13} cm$- ruled out by planetary orbits
- $n = 2$: $R \sim 100 \mu m$ just about OK (see later)

Interesting coincidences:

- when n=2 and $M_s = 1TeV$ then $R = 1mm \equiv us^{-1}$. In other words,

$$M_{WEAK} = \sqrt{us \times m_{Pl}}$$

- Scale of dark energy $\Lambda^{\frac{1}{4}} \sim us$

- neutrino mass $m_{\nu_3} \sim us$

What's bad with the Extra Dimension picture?

1. Why R is so much bigger than M_s^{-1}? $\equiv$Moduli problem (virtue).
2. Gauge unification lost - every KK mode contributes to running.

3. Power law running (Taylor Veneziano, Dienes Dudas Gherghetta)

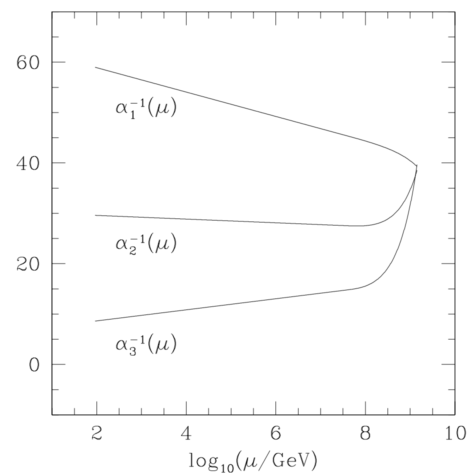
Consider e.g. gauge coupling: in usual 4D case have

$$\alpha = \alpha_{bare} + \beta \log(\Lambda)$$

Now contributions from all the gauge boson KK modes with mass less than Λ

$$\begin{aligned} \frac{\delta\alpha}{\alpha} &= \delta\Lambda\left(\frac{\beta}{\Lambda} + \frac{\beta_{KK}}{\Lambda} + \frac{\partial\beta_{KK}}{\partial\Lambda}\right) \\ \alpha &= \alpha_{bare}(1 + \beta' \log\Lambda + \beta_{N=2}(\Lambda/R)^n). \end{aligned}$$

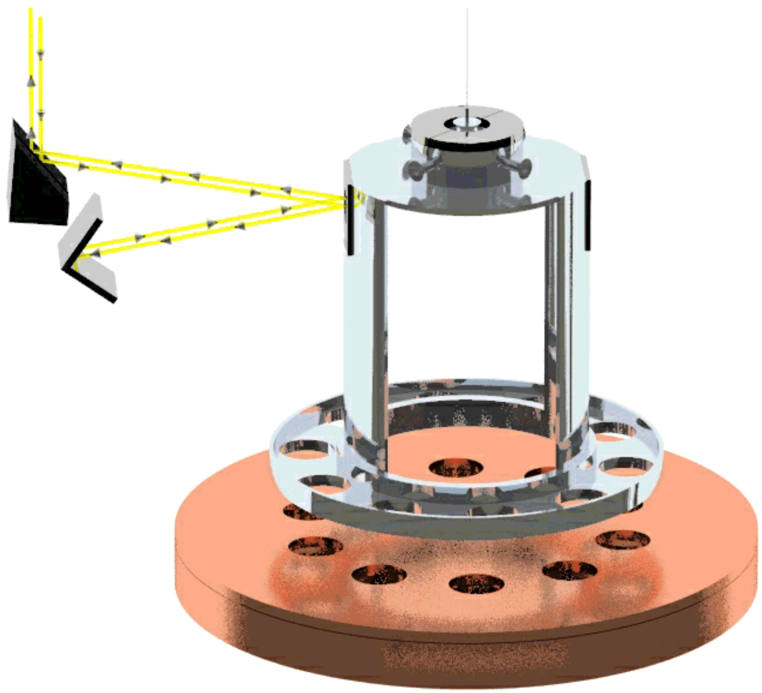
The last term is a power law contribution.



60
40
20
0
$\alpha_1^{-1}(\mu)$
$\alpha_2^{-1}(\mu)$
$\alpha_3^{-1}(\mu)$
2
4
6
8
10
$\log_{10}(\mu/\text{GeV})$

4. Detecting (flat) extra dimensions

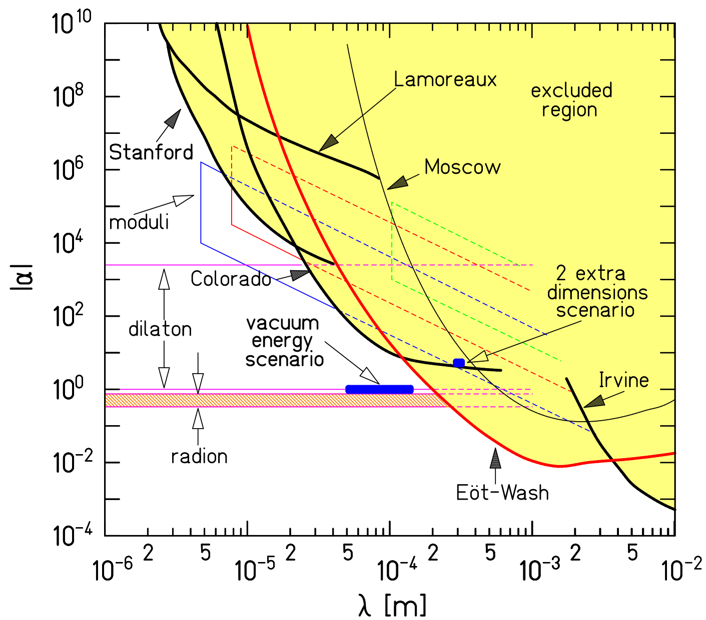
Detection1: Eotvos (torsion balance) experiments - e.g. C.Hoyle et al, hep-ph/0011014, hep-ph/0405262 find $R_{2-extra} < 0.13mm$ at 95% c.l.



Usually present ISL violation as a Yukawa interaction

$$V(r) = -\frac{Gm_1m_2}{r}(1 + \alpha e^{-r/\lambda})F^{\mu\nu}F_{\mu\nu}$$

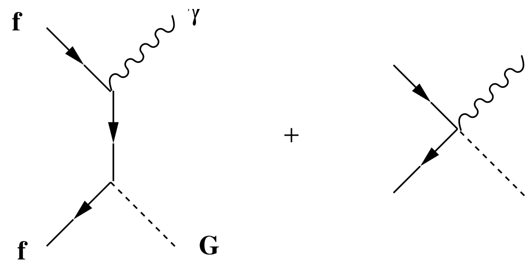
For ADD scenario with n extra dimensions $\lambda = R$ and $\alpha = \frac{8n}{3}$.

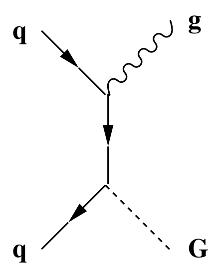


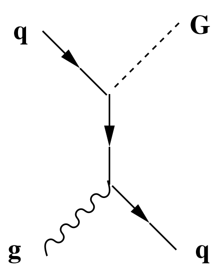
Detection 2: Graviton emmision into the bulk (Guidice, Rattazzi, Wells see e.g. C.Csaki hep-ph/0404096)

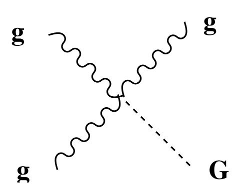
Gravity feels the extra dimensions - so we expect KK modes; each KK mode is emitted with 4-D gravitational strength.

Diagrams are









Hence $\Gamma \sim m^3/m_{Pl}^2$ and the graviton is lost so get e.g.

$$e^+e^- \to \gamma G_{KK}$$

or

$$q\overline{q} \to jet + G_{KK}$$

then $\sigma(E) \sim \frac{\alpha}{m_{Pl}^2} N(E)$. But $N(E) = (ER)^n$ so that get

$$\sigma \sim \frac{\alpha}{E^2} \left(\frac{E}{M_s}\right)^{n+2}.$$

When the energy reached the string scale this missing E_T becomes detectable.

e.g. Acosta et al (CDF) hep-ex/0205057. Run 1b ($87 \pm 4 pb^{-1}$ of $p\bar{p}$ collisions at $\sqrt{s} = 1.8 TeV$). Selection on missing $E_T > 45 GeV$. Backgrounds to $\gamma + missing\ E_T$ include $q\bar{q} \rightarrow \gamma Z \rightarrow \nu\bar{\nu}\gamma$ (3.2), cosmic ray muons (6.3) plus etc. Total is 11.0 ± 2.2, observed 11. At 95% C.L.

$$n = 4 \quad ; \quad M_s > 549 GeV$$

$$n = 6 \quad ; \quad M_s > 581 GeV$$

$$n = 8 \quad ; \quad M_s > 602 GeV$$

Results similar to L3 and DELPHI;

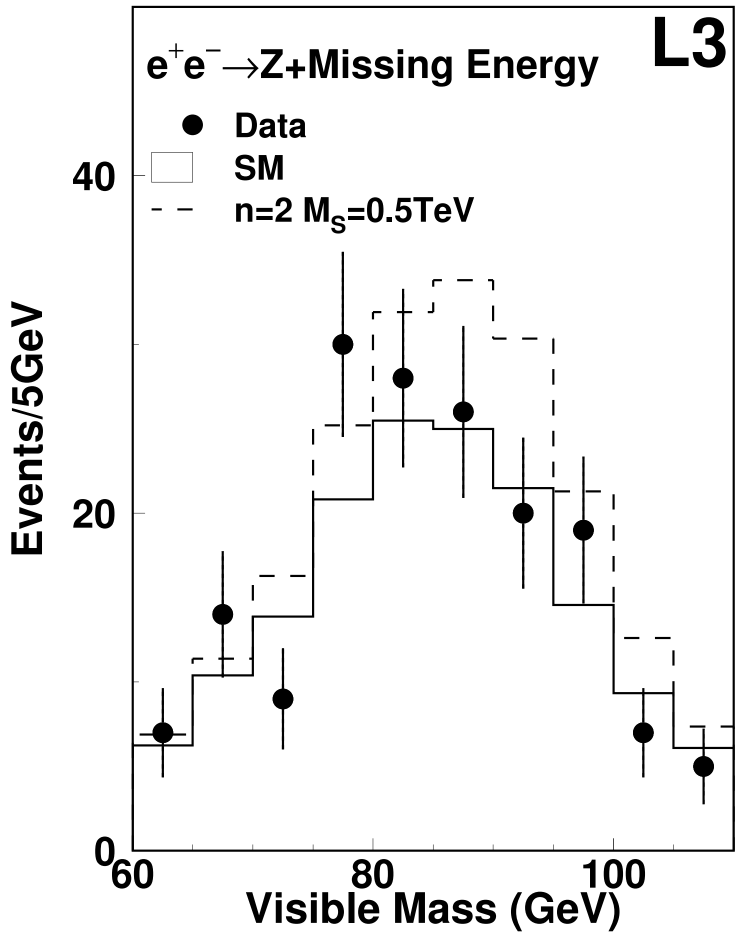


Figure 4: Visible mass for $e^+e^- \to ZG$ candidate events at 188.7 GeV together with SM expectations, dominated by W pair and single W production. The effect of real graviton production with two extra space dimensions and $M_S = 0.5$ TeV is also shown.

Detection 3: Virtual KK-graviton exchange (Guidice, Rattazzi, Wells, hep-ph/9811291)

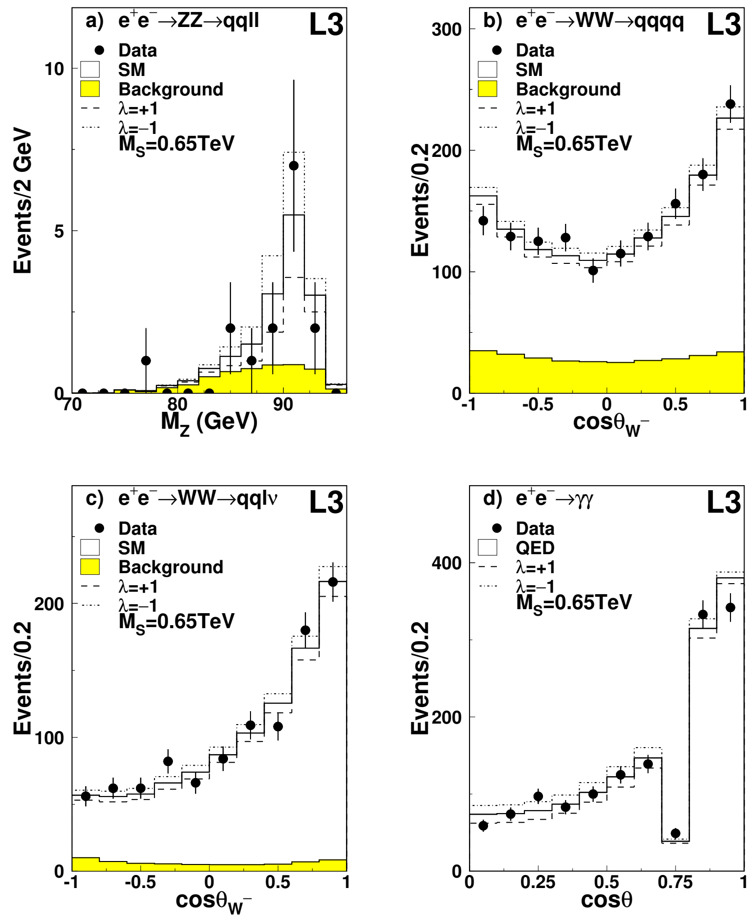
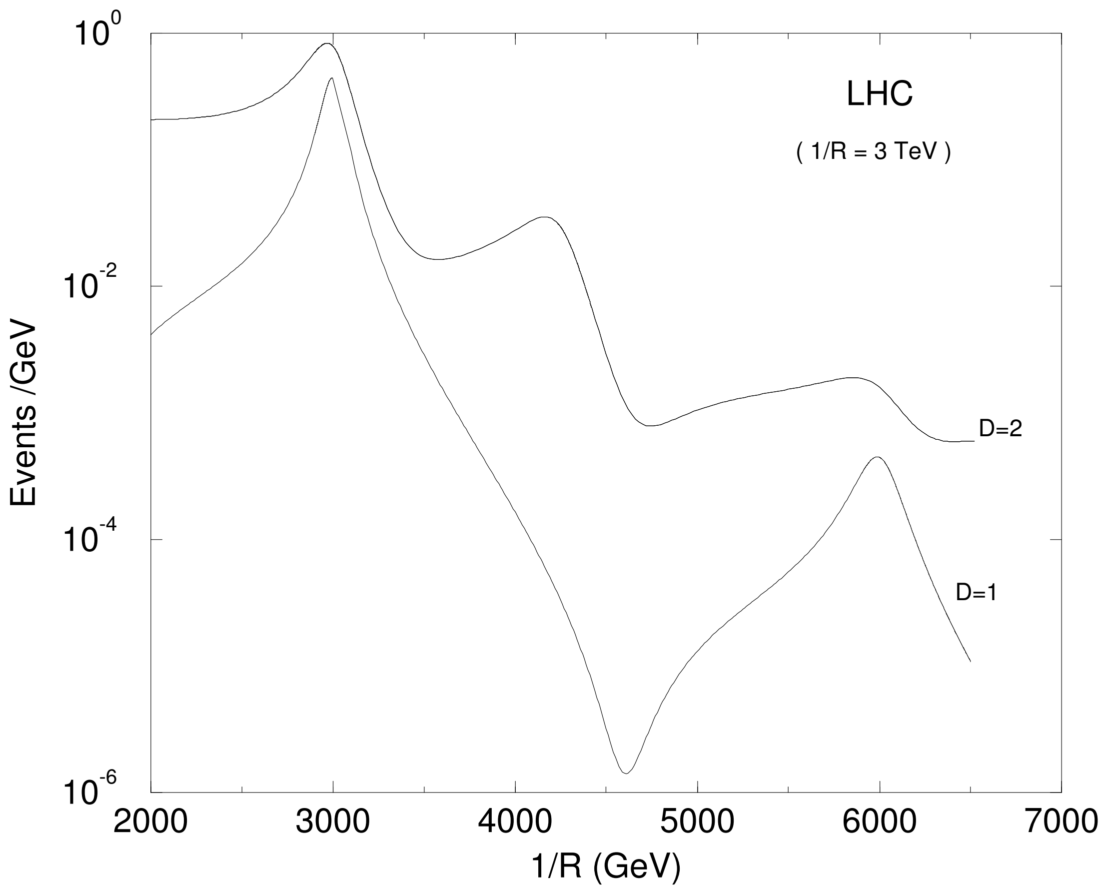


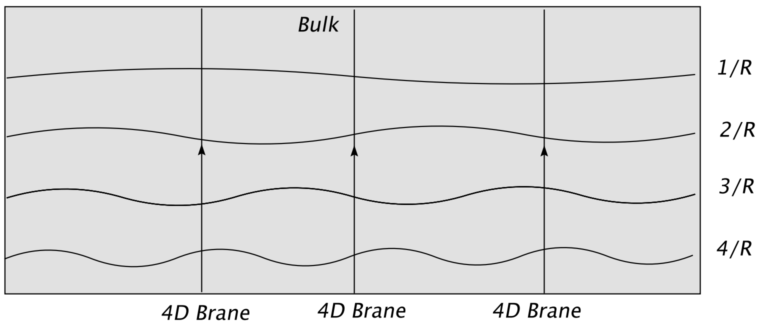
Figure 1: (a) Reconstructed Z mass for $e^+e^- \to ZZ \to q\bar{q}\ell^+\ell^-$ events. Distributions of the polar angle for: (b) hadronic $e^+e^- \to W^+W^-$ events, (c) semileptonic $e^+e^- \to W^+W^-$ events, (d) $e^+e^- \to \gamma\gamma$ events. Data at 188.7 GeV, SM signal and background expectations are presented together with LSG predictions for $M_S = 0.65$ TeV and $\lambda = \pm 1$.

Detection 4: Emission of string and KK-modes - we expect to find excitations of photons, Z's etc. e.g. if 1 or 2 radii are compact or if UED with a size a few TeV (0.1 Fermi) (Antoniadis, Benakli)



Detection 5: Flavour changing neutral currents

Return to extra dimension model and consider localized generations (split fermions). (Why? *a)* exponentially suppressed wavefunction overlaps can explain small Yukawa couplings (Mirabelli, Schmaltz. hep-ph/9912265), *b)* sometimes string models require this sort of set-up for consistency.)



$$L_5 = -\frac{1}{4} F^{MN} F_{MN} + i\overline{f}_i \gamma^\mu D_\mu f_i \delta(y - y_i)$$

- Go to the mass basis from the weak basis;

$$f_i = U_{ia} f_a$$

$$\mathcal{L}_{KE} \supset \sum_n A_\mu^{(n)} i \overline{f}_a \gamma^\mu U_{ai}^\dagger g_i^{(n)} U_{ib} f_b$$

At low energies, 4-fermion interactions are generated

$$\mathcal{L}_{effective} \supset c_{abcd} \left(\overline{f_a} \gamma^\mu f_b \right) \left(\overline{f_c} \gamma^\mu f_d \right)$$

where

$$c_{abcd} = -2g^2 \sum_{ij} U_{ai}^\dagger U_{ib} U_{cj}^\dagger U_{jd} \sum_m \frac{\cos\left(M_m (y_i - y_j)\right)}{M_m^2}$$

and $M_m = m/R$.

- Require $M_m > 100 TeV$ (Delgado Pomarol Quiros)

- But, note divergence problem with $n \geq 2$ extra dimensions;

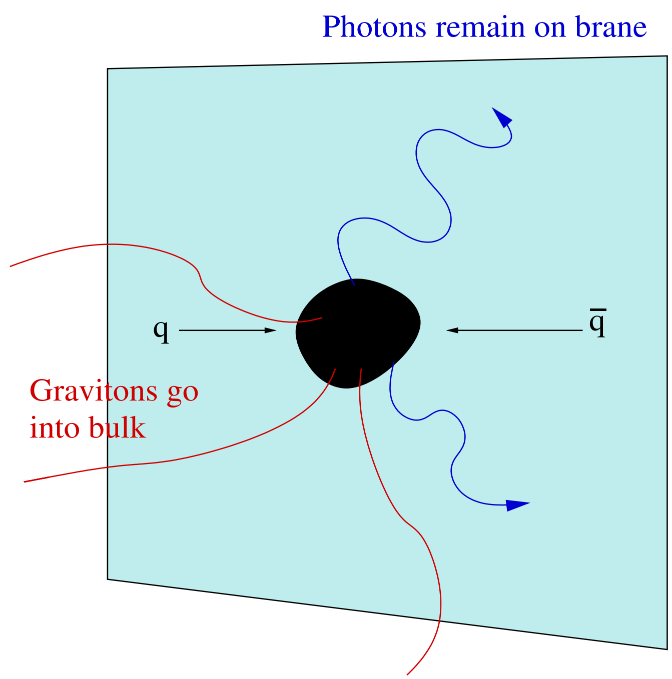
$$c_{abcd} \propto \sum_{\underline{m}} \frac{1}{M_{\underline{m}}^2} \sim R^2 \int d|m|\,|m|^{n-3}$$

physics sensitive to the UV completion

- The UV complete theory really does regulate it! In string theory

$$\mathcal{L} \sim g^2 \left[\bar{f}^i \gamma^\mu f^i \bar{f}^j \gamma_\mu f^j\right] \left(\frac{1}{s} + 2\sum_m \frac{\cos\left(M_m(y_i - y_j)\right) \delta^{-M_m^2/M_s^2}}{s - M_m^2}\right)$$

Detection 6: Detection of micro-Black-holes at e.g. LHC. (See Kanti hep-ph/0402168) (size $10^{-19}m$, lifetime $10^{-27}secs$)



- To first approximation, $\sigma \sim \pi R_s^2 \sim 1\,TeV^{-2} \sim 100\,pb$
- A significant proportion of the collision energy ends up inside the horizon
- Improvements; greybody factors - spectrum $\rightarrow n$.

5. Randall Sundrum models (see C.Csaki hep-ph/0404096)

So far the extra dimensions have been "flat" - i.e. we have ignored back-reaction of brane tension; what happens if strongly curved? Can we still have flat 4-D physics?

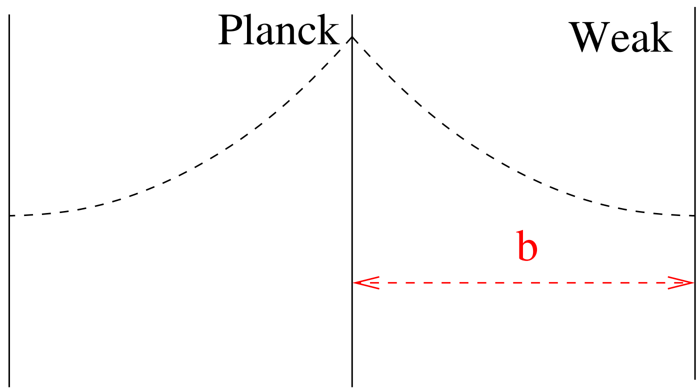
Consistent solutions to 5D einstein equation ($G_{MN} = \kappa^2 T_{MN} = \frac{\Lambda g_{MN}}{2M_s^3}$) with bulk Λ and "warped" metric

$$ds^2 = e^{-2k|y|} dx^\mu dx^\nu \eta_{\mu\nu} - dy^2$$

The $\mu\mu$ components lead to a fine-tuning between 2 *brane tensions* localized at $y = 0$ and $y = b$ and Λ;

$$T_{\mu\nu}^{tension} = \frac{e^{-A/2}}{2}\delta(y)\,\mathrm{diag}(V, -V, -V, -V, 0)$$

$$\Lambda = -\frac{V_0^2}{12M_s^3} \quad ; \quad V_1 = -V_0$$



Solution to the hierarchy problem:

- Consider scalar Higgs fields ϕ trapped on the "weak" brane where $g^{ind}_{\mu\nu}|_{y=b} = e^{-2kb}\eta_{\mu\nu}$.

$$
\begin{aligned}
S^{Higgs} &= \int d^4x \sqrt{g^{ind}}[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi^* - \lambda(|\phi|^2 - v^2)^2] \\
&= \int d^4x\, e^{-4kb}[e^{2kb}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi^* - \lambda(|\phi|^2 - v^2)^2] \\
& \int d^4x\, [\eta^{\mu\nu}\partial_\mu\hat{\phi}\partial_\nu\hat{\phi}^* - \lambda(|\hat{\phi}|^2 - (e^{-kb}v)^2]^2]
\end{aligned}
$$

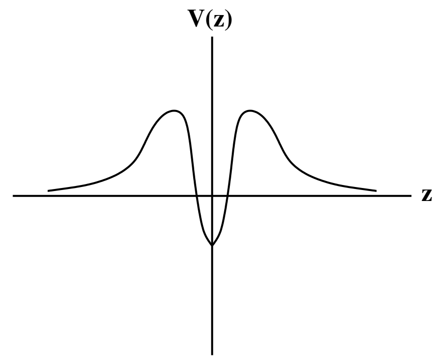
  where $\hat{\phi} = e^{-kb}\phi$.

- A "natural" 5-dimensional VEV of M_s is exponentially warped to $e^{-kb}M_s$ in 4-dimensional physics.

- Note that $m^2_{Pl} = M^3_s \int_0^b e^{-2k|y|}dy = \frac{M^3_s}{k}(1 - e^{-2kb})$ is not changed hierarichally.)

Behaviour of graviton modes in RS1

- zero-mode (our 4D graviton!): sees a volcano potential and is localized near $y = 0$



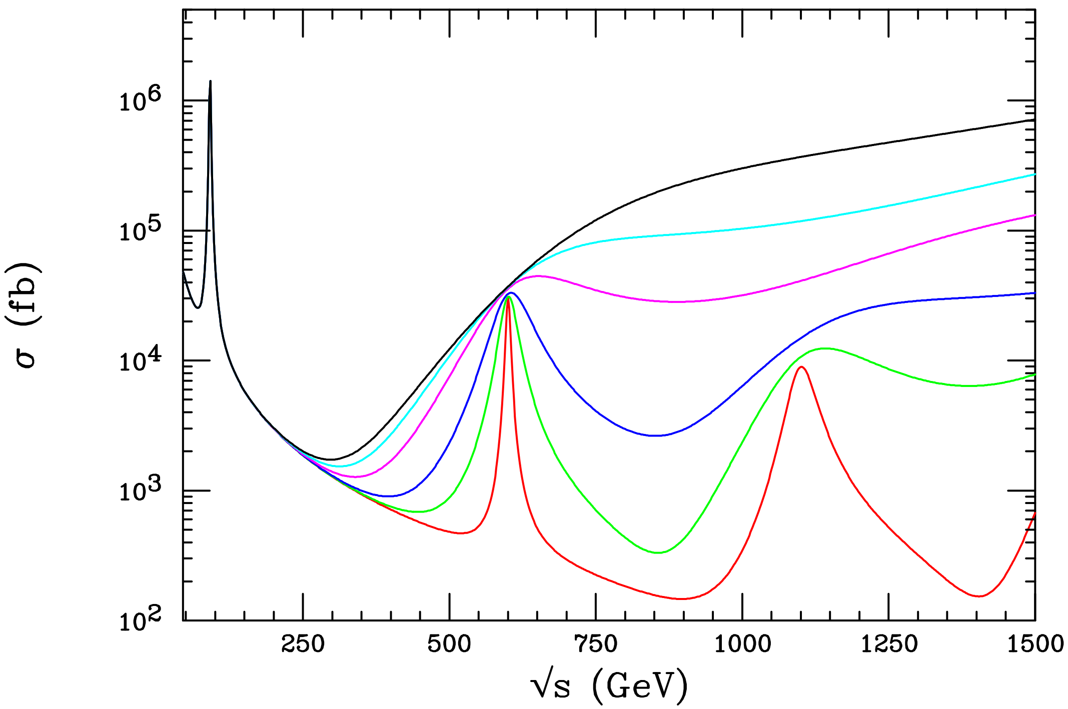
- Correction to Newton potential small for $r \gg M_s^{-1} \sim m_{Pl}^{-1}$;

$$V(r) = G_N \frac{m_1 m_2}{r}(1 + \frac{C}{(kr)^2})$$

- Because of warping KK modes have typical mass splittings of TeV^{-1}!
- But their wave functions are exponentially localized on weak brane - hence coupling suppressed by only TeV^{-1} not m_{Pl}^{-1}!

Detection

e.g. contribution to $e^+e^- \to \mu^+\mu^-$ (Davoudiasl, Hewett, Rizzo, hep-ph/9909255). First mode at 600GeV, $k/m_{Pl} = 1, 0.7, 0.5, 0.3, 0.2, 0.1$ top (black) to bottom (red)



6. Summary

- String theory $\rightarrow$ 6 extra dimensions, but interesting in their own right
- Vacuum degeneracy + D-branes $\rightarrow$ can be 'any' size upto exptl limit (but nucleosynthesis, supernovae constraints etc.)
- Many signatures
- Interesting area for study: do you really believe $M_s \sim TeV$ (why no proton decay?)