Yeti 2005/6: Effective Field Theories

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If you chose (b) — seems you are in the right talk.

EFTs – the tool of choice for smart YEATs

In a nutshell:

> An effective field theory is a field theory with a restricted range of application: at energy $E \ll \Lambda$, it contains a series of contributions of type $(E/\Lambda)^n$, n positive integer. Truncating the series at some $n_{\rm max}$ implies that predictions are only accurate up to terms of $O((E/\Lambda)^{n_{\rm max}+1})$.

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Why would anyone want to do such a crazy thing and expand a theory in E/Λ?

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Effective Lagrangian (Fermi's 4-fermion Lagrangian):

$$\mathcal{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} J^\mu J_\mu + O\left(\frac{1}{m_W^4}\right)$$

with

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8m_W^2}, \qquad G_F|_{\text{exp}} = 1.166 \cdot 10^{-5}\,\text{GeV}^{-2}$$

from the partial decay rate $\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu)$

(g_W: weak interaction coupling).

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- Lessons to be learned:
- if a theory contains largely different scales, e.g. $m_\mu \ll m_W$, one can expand low-energy amplitudes with "light" external states in inverse powers of the heavy scale Λ

So where does new physics enter the game?

Let's pretend we don't know the SM, don't know that charged weak interactions are mediated by W and in particular don't know the formula

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$$\Lambda \equiv \left(\frac{\sqrt{2}}{G_F}\right)^{1/2} \approx 350\,\mathrm{GeV}$$

is that new scale and characterices the "new physics" of weak interactions.

Some formalities

- For energies $E \ll \Lambda$, an EFT Lagrangian can be written as

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{d \leq 4} + \sum_{d>4} \frac{1}{\Lambda^{d-4}} \sum_{i_d} g_{i_d} O_{i_d}$$

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- as long as g_{i_d} not known, can't measure Λ!
- a measurement of Λ in the EFT term O/Λ^n is only indicative for the fundamental scale of the truly fundamental theory, because the couplings g_{i_d} have to be set to 1

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  Example: Fermi's four-fermion interaction is matched to the SM.
- If the fundamental theory is unknown, one can still write down $\mathcal{L}_{\text{eff}}$ where the allowed operators O_{i_d} are constrained by the symmetries of $\mathcal{L}_{d\leq 4}$. The associated couplings g_{i_d} serve as bookmarks for new physics:

  Neither Λ nor g_{i_d} can be determined directly from experiment, only the ratio of couplings.

  Example: the SM.

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- That is: whatever the new physics beyond the SM, it has to naturally explain the suppression of these processes, for example in form of a symmetry that forbids them

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- Chiral Perturbation Theory: describe low-energy QCD in terms of meson fields
- other currently popular theory: soft collinear effective theory: interactions between b quarks and energetic light quarks, e.g. $b \to u$ and $b \to s$ transitions

Summary: Principles of EFT

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- The EFT has the same infra-red behaviour/particle content as the underlying fundamental theory, but a different one in the ultra-violet.
- The only remnants of the high-energy dynamics are in the low-energy couplings and in the symmetries of the theory.

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- EFT tells you that new physics just longs to be discovered at 1 TeV!
  If only it would! But there is no strong hint at $\Lambda = 1$ TeV from any analysis I am aware of.
- EFTs are very suspicious because they don’t allow precise predictions.
  Who says that Fermi’s weak interaction Lagrangian is not precise enough? Theories in physics always come with a certain region of applicability. EFT are very precise as long as you don’t want to reach energies of $O(\Lambda)$.

- EFTs are not renormalisable.
  That is true – and irrelevant. Renormalisability means that the infra-red behaviour of a theory does not depend at all on the ultra-violet regime. A renormalisable theory is the limit of an EFT for $\Lambda \to \infty$. It's a nice thing to have, for practical calculations, but also rather unrealistic, because it is an illusion to believe that a renormalisable theory like QED (or the SM) would be valid at all energy scales right up to m_{Planck}.

Lectures & Reviews

1. H. Georgi, Ann. Rev. Nucl. Part. Sci. **43** (1993) 209.
2. A. V. Manohar, arXiv:hep-ph/9606222.
3. A. Pich, arXiv:hep-ph/9806303.
4. I. Z. Rothstein, arXiv:hep-ph/0308266.
5. G. Ecker, arXiv:hep-ph/0507056.
6. D. B. Kaplan, arXiv:nucl-th/0510023.

All these papers require basic knowledge of quantum field theory.