

Monte Carlo Event Generators

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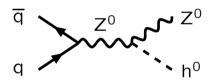
- 1. (yesterday) Introduction and Overview; Monte Carlo Techniques
 - 2. (yesterday) Matrix Elements; Parton Showers I
 - 3. (today) Parton Showers II; Matching Issues
 - 4. (today) Multiple Interactions and Beam Remnants
- 5. (tomorrow) Hadronization and Decays; Summary and Outlook

Event Physics Overview

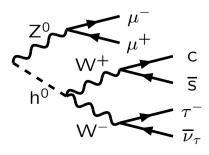
Repetition: from the "simple" to the "complex", or from "calculable" at large virtualities to "modelled" at small

Matrix elements (ME):

1) Hard subprocess: $|\mathcal{M}|^2$, Breit-Wigners, parton densities.

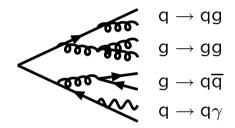


2) Resonance decays: includes correlations.

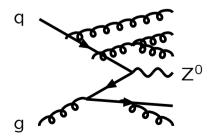


Parton Showers (PS):

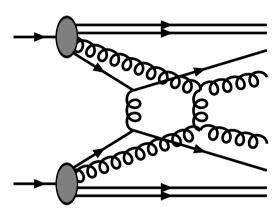
3) Final-state parton showers.



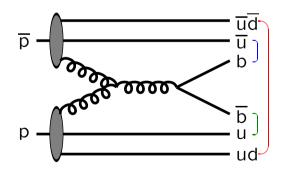
4) Initial-state parton showers.



5) Multiple parton–parton interactions.

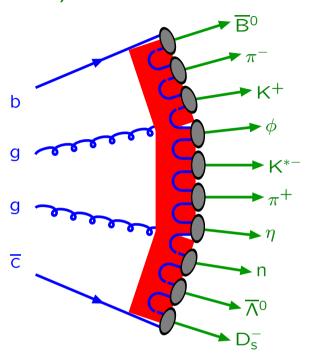


6) Beam remnants, with colour connections.

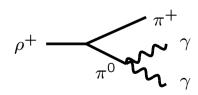


5) + 6) = Underlying Event

7) Hadronization

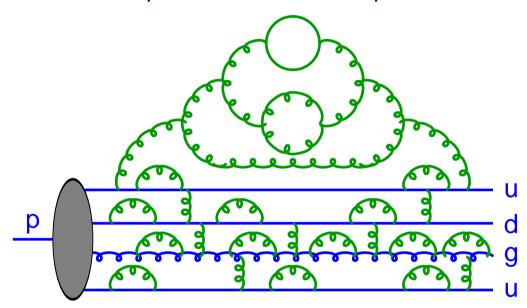


8) Ordinary decays: hadronic, τ , charm, ...



Parton Distribution Functions

Hadrons are composite, with time-dependent structure:



 $f_i(x,Q^2)$ = number density of partons iat momentum fraction x and probing scale Q^2 .

Linguistics (example):

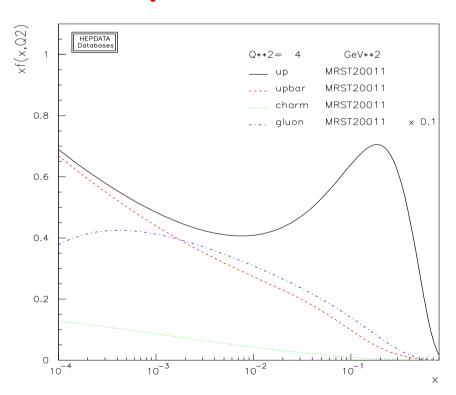
$$F_2(x,Q^2) = \sum_i e_i^2 x f_i(x,Q^2)$$

structure function parton distributions

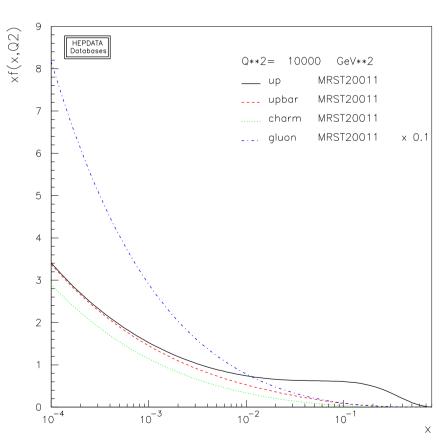
Absolute normalization at small Q_0^2 unknown. Resolution dependence by DGLAP:

$$\frac{\mathrm{d}f_b(x,Q^2)}{\mathrm{d}(\ln Q^2)} = \sum_{a} \int_{x}^{1} \frac{\mathrm{d}z}{z} f_a(x',Q^2) \frac{\alpha_{\rm S}}{2\pi} P_{a \to bc} \left(z = \frac{x}{x'} \right)$$

 $Q^2 = 4 \text{ GeV}^2$

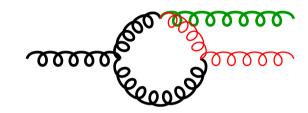


 $Q^2 = 10000 \text{ GeV}^2$

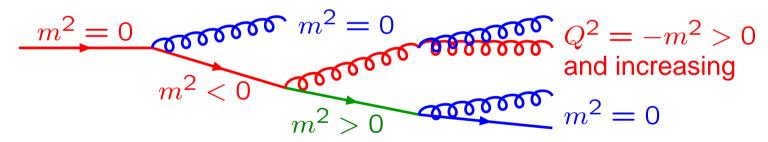


Initial-State Shower Basics

- Parton cascades in p are continuously born and recombined.
- ullet Structure at Q is resolved at a time $t \sim 1/Q$ before collision.
- A hard scattering at Q^2 probes fluctuations up to that scale.
- A hard scattering inhibits full recombination of the cascade.



• Convenient reinterpretation:



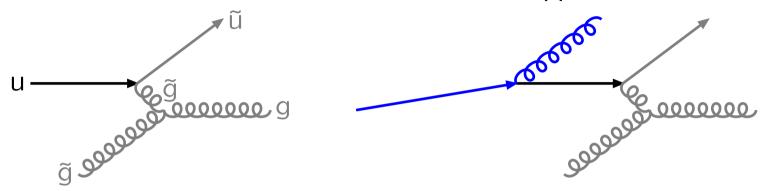
Event generation could be addressed by **forwards evolution**: pick a complete partonic set at low Q_0 and evolve, see what happens.

Inefficient:

- 1) have to evolve and check for all potential collisions, but 99.9...% inert
- 2) impossible to steer the production e.g. of a narrow resonance (Higgs)

Backwards evolution

Backwards evolution is viable and ~equivalent alternative: start at hard interaction and trace what happened "before"



Monte Carlo approach, based on conditional probability: recast

$$\frac{\mathrm{d}f_b(x,Q^2)}{\mathrm{d}t} = \sum_a \int_x^1 \frac{\mathrm{d}z}{z} f_a(x',Q^2) \frac{\alpha_s}{2\pi} P_{a \to bc}(z)$$

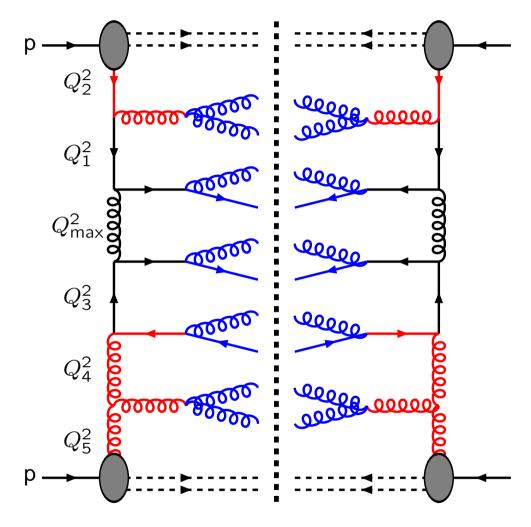
with $t = \ln(Q^2/\Lambda^2)$ and z = x/x' to

$$\mathrm{d}\mathcal{P}_b = \frac{\mathrm{df}_b}{f_b} = |\mathrm{d}t| \sum_a \int \mathrm{d}z \, \frac{x' f_a(x',t)}{x f_b(x,t)} \frac{\alpha_\mathrm{S}}{2\pi} \, P_{a \to bc}(z)$$

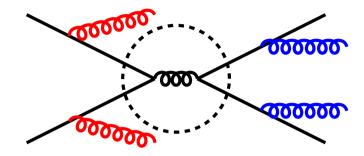
then solve for decreasing t, i.e. backwards in time, starting at high Q^2 and moving towards lower, with Sudakov form factor $\exp(-\int d\mathcal{P}_b)$

Ladder representation combines whole event:

cf. previously:



DGLAP:
$$Q^2_{\max} > Q^2_1 > Q^2_2 \sim Q^2_0$$
 $Q^2_{\max} > Q^2_3 > Q^2_4 > Q^2_5 \sim Q^2_0$

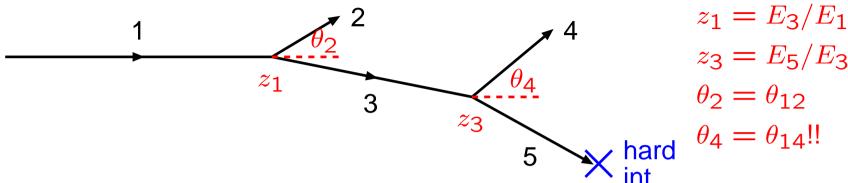


One possible

Monte Carlo order:

- 1) Hard scattering
- Initial-state shower from center outwards
- 3) Final-state showers

Coherence in spacelike showers



with $Q^2 = -m^2 =$ spacelike virtuality

• kinematics only:

$$Q_3^2 > z_1 Q_1^2, Q_5^2 > z_3 Q_3^2, \dots$$

i.e. Q_i^2 need not even be ordered

• coherence of leading collinear singularities:

$$Q_5^2 > Q_3^2 > Q_1^2$$
, i.e. Q^2 ordered

coherence of leading soft singularities (more messy):

$$E_3\theta_4 > E_1\theta_2$$
, i.e. $z_1\theta_4 > \theta_2$

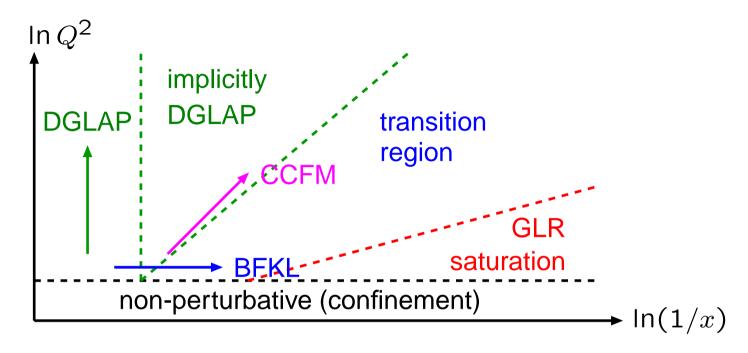
$$z \ll 1$$
: $E_1 \theta_2 \approx p_{\perp 2}^2 \approx Q_3^2$, $E_3 \theta_4 \approx p_{\perp 4}^2 \approx Q_5^2$

i.e. reduces to Q^2 ordering as above

 $z \approx 1$: $\theta_4 > \theta_2$, i.e. angular ordering of soft gluons

⇒ reduced phase space

Evolution procedures



DGLAP: Dokshitzer–Gribov–Lipatov–Altarelli–Parisi evolution towards larger Q^2 and (implicitly) towards smaller x

BFKL: Balitsky–Fadin–Kuraev–Lipatov evolution towards smaller x (with small, unordered Q^2)

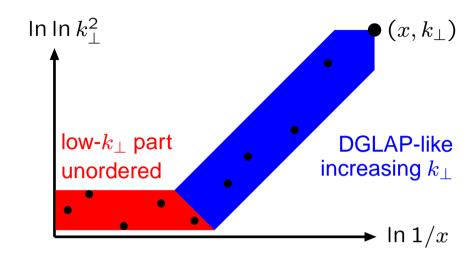
CCFM: Ciafaloni–Catani–Fiorani–Marchesini interpolation of DGLAP and BFKL

GLR: Gribov–Levin–Ryskin nonlinear equation in dense-packing (saturation) region, where partons recombine, not only branch

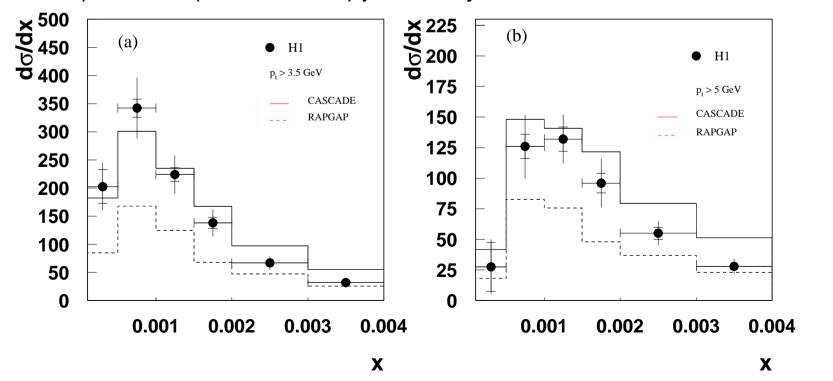
Initial-State Shower Comparison

Two(?) CCFM Generators:
(SMALLX (Marchesini, Webber))
CASCADE (Jung, Salam)
LDC (Gustafson, Lönnblad):
reformulated initial/final rad.

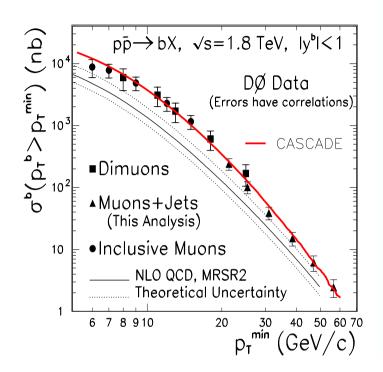
⇒ eliminate non-Sudakov

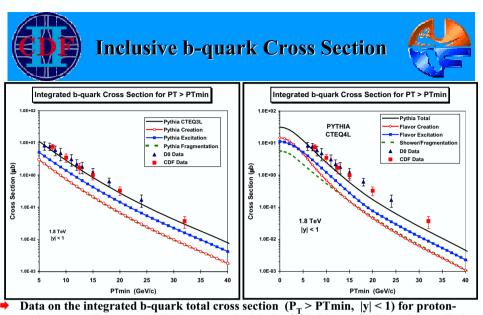


Test 1) forward (= p direction) jet activity at HERA



2) Heavy flavour production





Data on the integrated b-quark total cross section (P_T > PTmin, |y| < 1) for proton-antiproton collisions at 1.8 TeV compared with the QCD Monte-Carlo model predictions of PYTHIA 6.115 (CTEQ3L) and PYTHIA 6.158 (CTEQ4L). The four curves correspond to the contribution from flavor creation, flavor excitation, shower/fragmentation, and the resulting total.</p>

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but also explained by DGLAP with leading order pair creation

- + flavour excitation (\approx unordered chains)
 - + gluon splitting (final-state radiation)

CCFM requires off-shell ME's + unintegrated parton densities

$$F(x,Q^2) = \int^{Q^2} \frac{\mathrm{d}k_{\perp}^2}{k_{\perp}^2} \mathcal{F}(x,k_{\perp}^2) + (\text{suppressed with } k_{\perp}^2 > Q^2)$$

so not ready for prime time in pp

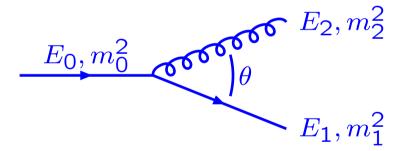
Initial- vs. final-state showers

Both controlled by same evolution equations

$$\mathrm{d}\mathcal{P}_{a\to bc} = \frac{\alpha_\mathrm{S}}{2\pi} \frac{\mathrm{d}Q^2}{Q^2} P_{a\to bc}(z) \, \mathrm{d}z \, \cdot \, (\mathrm{Sudakov})$$
 but

Final-state showers:

 Q^2 timelike ($\sim m^2$)



decreasing E, m^2, θ both daughters $m^2 \ge 0$ physics relatively simple \Rightarrow "minor" variations: Q^2 , shower vs. dipole, . . . Initial-state showers:

 Q^2 spacelike ($\approx -m^2$)

$$E_{0}, Q_{0}^{2}$$
 E_{1}, Q_{1}^{2}

decreasing E, increasing Q^2 , θ one daughter $m^2 \geq 0$, one $m^2 < 0$ physics more complicated \Rightarrow more formalisms: DGLAP, BFKL, CCFM, GLR, ...

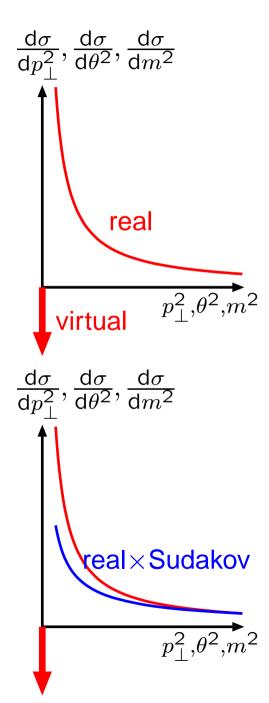
Matrix Elements vs. Parton Showers

ME: Matrix Elements

- + systematic expansion in α_s ('exact')
- powerful for multiparton Born level
- + flexible phase space cuts
- loop calculations very tough
- negative cross section in collinear regions
 unpredictive jet/event structure
- no easy match to hadronization

PS: Parton Showers

- approximate, to LL (or NLL)
- main topology not predetermined
 inefficient for exclusive states
- + process-generic ⇒ simple multiparton
- + Sudakov form factors/resummation⇒ sensible jet/event structure
- + easy to match to hadronization



Matrix Elements and Parton Showers

Recall complementary strengths:

- ME's good for well separated jets
- PS's good for structure inside jets

Marriage desirable! But how?

- Problems: gaps in coverage?
 - doublecounting of radiation?
 - Sudakov?
 - NLO consistency?

Much work ongoing ⇒ no established orthodoxy

Three main areas, in ascending order of complication:

- 1) Match to lowest-order nontrivial process merging
- 2) Combine leading-order multiparton process vetoed parton showers
 - 3) Match to next-to-leading order process MC@NLO

Merging

= cover full phase space with smooth transition ME/PS

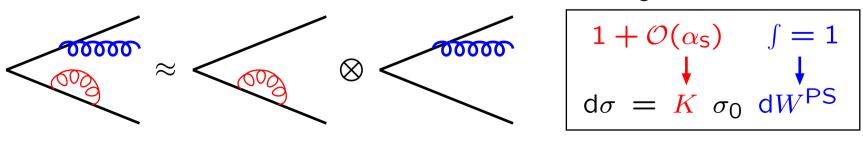
Want to reproduce
$$W^{\text{ME}} = \frac{1}{\sigma(\text{LO})} \frac{d\sigma(\text{LO} + g)}{d(\text{phasespace})}$$

by shower generation + correction procedure

Exponentiate ME correction by shower Sudakov form factor:

$$W_{\text{actual}}^{\text{PS}}(Q^2) = W^{\text{ME}}(Q^2) \exp\left(-\int_{Q^2}^{Q_{\text{max}}^2} W^{\text{ME}}(Q'^2) dQ'^2\right)$$

• Do not normalize W^{ME} to $\sigma(\text{NLO})$ (error $\mathcal{O}(\alpha_s^2)$ either way)



Normally several shower histories ⇒ ~equivalent approaches

Final-State Shower Merging

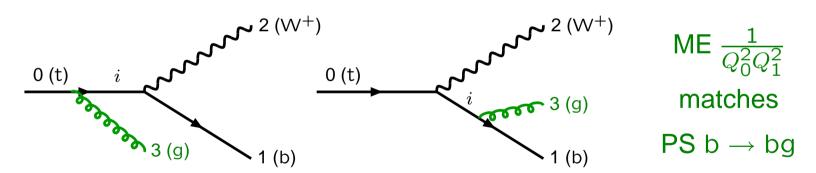
Merging with $\gamma^*/Z^0 \rightarrow q\overline{q}g$ for $m_q = 0$ since long

(M. Bengtsson & TS, PLB185 (1987) 435, NPB289 (1987) 810)

For $m_{\rm q}>0$ pick $Q_i^2=m_i^2-m_{i,\rm onshell}^2$ as evolution variable since

$$W^{\text{ME}} = \frac{(\dots)}{Q_1^2 Q_2^2} - \frac{(\dots)}{Q_1^4} - \frac{(\dots)}{Q_2^4}$$

Coloured decaying particle also radiates:



 \Rightarrow can merge PS with generic $a \rightarrow bcg$ ME

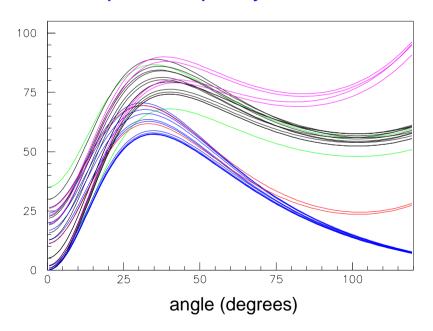
(E. Norrbin & TS, NPB603 (2001) 297)

Subsequent branchings q → qg: also matched to ME, with reduced energy of system

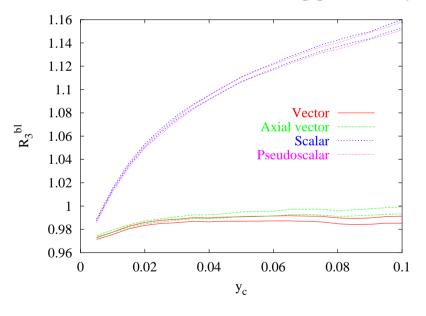
PYTHIA performs merging with generic FSR $a \rightarrow bcg$ ME,

in SM: $\gamma^*/Z^0/W^{\pm} \to q\overline{q}$, $t \to bW^+$, $H^0 \to q\overline{q}$, and MSSM: $t \to bH^+$, $Z^0 \to \tilde{q}\overline{\tilde{q}}$, $\tilde{q} \to \tilde{q}'W^+$, $H^0 \to \tilde{q}\overline{\tilde{q}}$, $\tilde{q} \to \tilde{q}'H^+$, $\chi \to q\overline{\tilde{q}}$, $\chi \to q\overline{\tilde{q}}$, $\tilde{q} \to q\chi$, $t \to \tilde{t}\chi$, $\tilde{g} \to q\overline{\tilde{q}}$, $\tilde{q} \to q\tilde{g}$, $t \to \tilde{t}\tilde{g}$

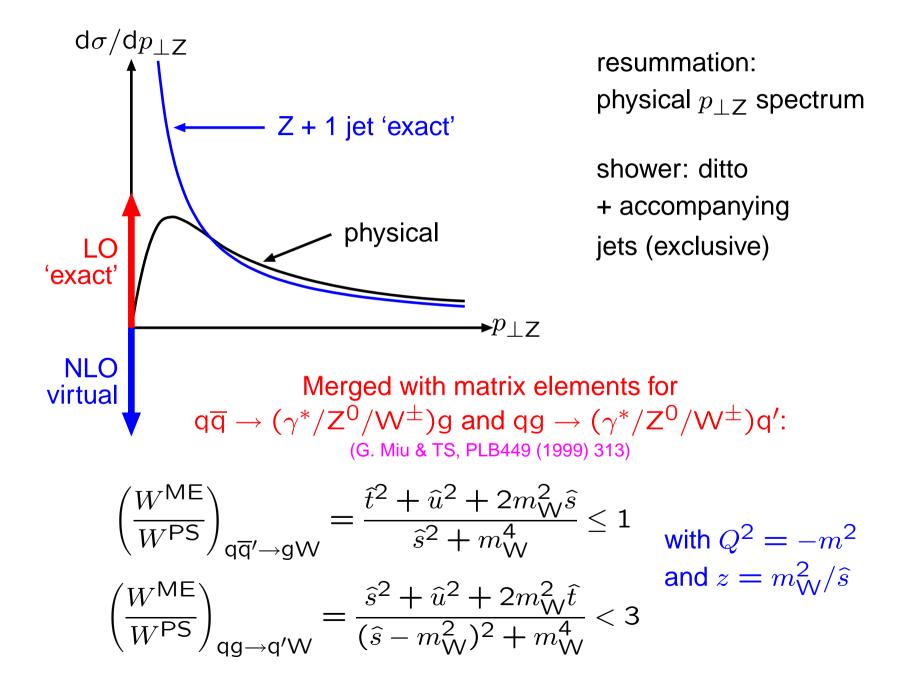
g emission for different colour, spin and parity:



 $R_3^{\mathsf{bl}}(y_c)$: mass effects in Higgs decay:



Initial-State Shower Merging



Merging in HERWIG

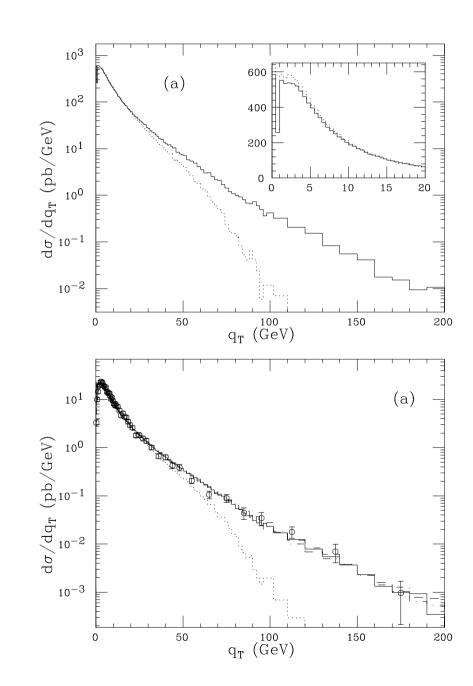
HERWIG also contains merging, for

- $\bullet \ Z^0 \to q \overline{q}$
- \bullet t \rightarrow bW⁺
- $\bullet \ q\overline{q} \to Z^0$

and some more

Special problem:
angular ordering does not
cover full phase space; so
(1) fill in "dead zone" with ME
(2) apply ME correction
in allowed region

Important for agreement with data:



Vetoed Parton Showers

S. Catani, F. Krauss, R. Kuhn, B.R. Webber, JHEP 0111 (2001) 063; L. Lönnblad, JHEP0205 (2002) 046; F. Krauss, JHEP 0208 (2002) 015; S. Mrenna, P. Richardson, JHEP0405 (2004) 040; M.L. Mangano, in preparation

Generic method to combine ME's of several different orders to NLL accuracy; will be a 'standard tool' in the future

Basic idea:

- consider (differential) cross sections $\sigma_0, \sigma_1, \sigma_2, \sigma_3, \ldots$, corresponding to a lowest-order process (e.g. W or H production), with more jets added to describe more complicated topologies, in each case to the respective leading order
- σ_i , $i \geq 1$, are divergent in soft/collinear limits
- absent virtual corrections would have ensured "detailed balance", i.e. an emission that adds to σ_{i+1} subtracts from σ_i
- such virtual corrections correspond (approximately)
 to the Sudakov form factors of parton showers
- so use shower routines to provide missing virtual corrections
 rejection of events (especially) in soft/collinear regions

Veto scheme:

- 1) Pick hard process, mixing according to σ_0 : σ_1 : σ_2 : ..., above some ME cutoff (e.g. all $p_{\perp i} > p_{\perp 0}$, all $R_{ij} > R_0$), with large fixed $\alpha_{\rm S0}$
- 2) Reconstruct imagined shower history (in different ways)
- 3) Weight $W_{\alpha} = \prod_{\text{branchings}} (\alpha_{\text{S}}(k_{\perp i}^2)/\alpha_{\text{S0}}) \Rightarrow \text{accept/reject}$

CKKW-L:

4) Sudakov factor for non-emission on all lines above ME cutoff

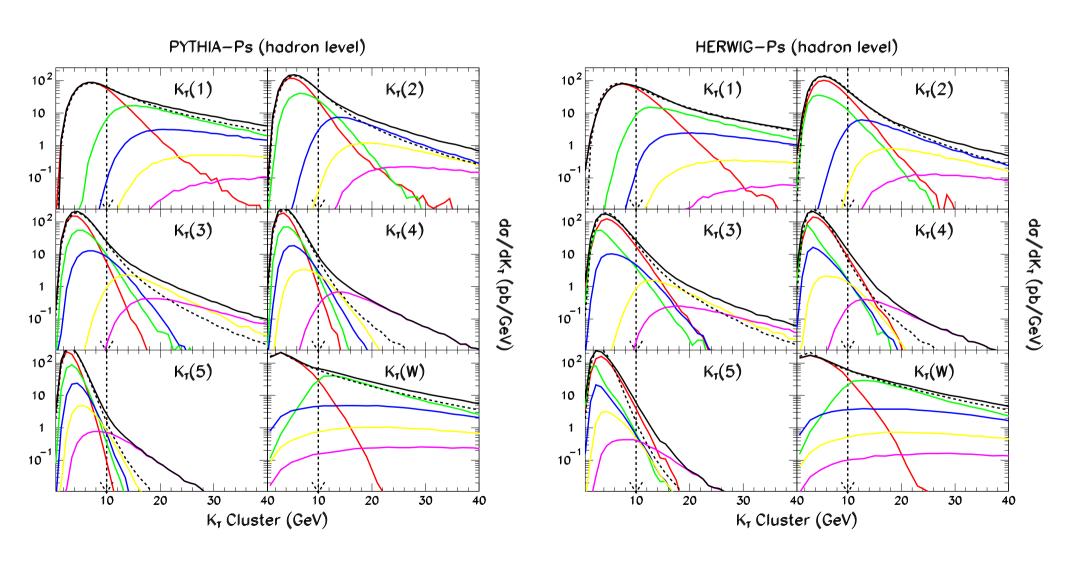
$$W_{\mathrm{Sud}} = \prod \text{"propagators"}$$
 Sudakov $(k_{\perp \mathrm{beg}}^2, k_{\perp \mathrm{end}}^2)$

- 4a) CKKW: use NLL Sudakovs
- 4b) L: use trial showers
- 5) $W_{\text{Sud}} \Rightarrow \text{accept/reject}$
- 6) do shower, vetoing emissions above cutoff

MLM:

- 4) do parton showers
- 5) (cone-)cluster showered event
- 6) match partons and jets
- 7) if all partons are matched, and $n_{\rm jet}=n_{\rm parton},$ keep the event, else discard it

CKKW mix of W + (0, 1, 2, 3, 4) partons, hadronized and clustered to jets:



(S.Mrenna, P. Richardson)

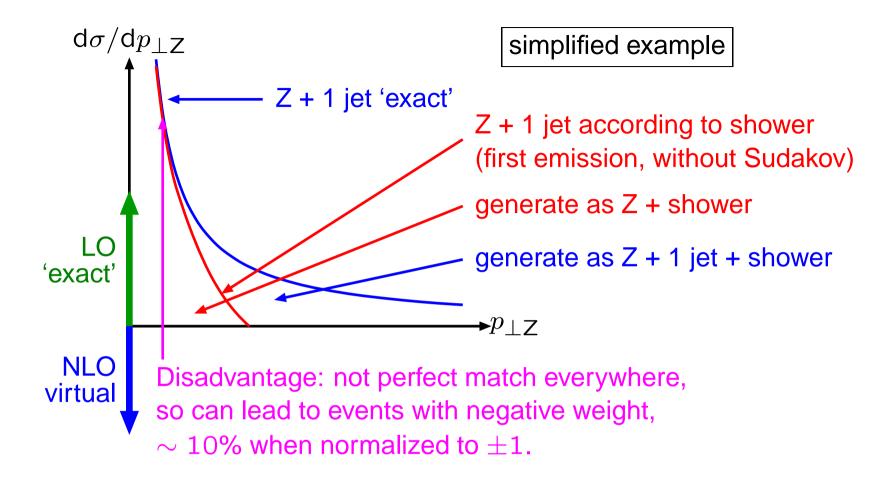
MC@NLO

Objectives:

- Total rate should be accurate to NLO.
- NLO results are obtained for all observables when (formally) expanded in powers of α_s .
- Hard emissions are treated as in the NLO computations.
- Soft/collinear emissions are treated as in shower MC.
- The matching between hard and soft emissions is smooth.
- The outcome is a set of "normal" events, that can be processed further.

Basic scheme (simplified!):

- 1) Calculate the NLO matrix element corrections to an n-body process (using the subtraction approach).
- 2) Calculate analytically (no Sudakov!) how the first shower emission off an n-body topology populates (n + 1)-body phase space.
- 3) Subtract the shower expression from the (n + 1) ME to get the "true" (n + 1) events, and consider the rest of σ_{NLO} as n-body.
- 4) Add showers to both kinds of events.



MC@NLO in comparison:

- Superior with respect to "total" cross sections.
- Equivalent to merging for event shapes (differences higher order).
- Inferior to CKKW–L for multijet topologies.
- ⇒ pick according to current task and availability.

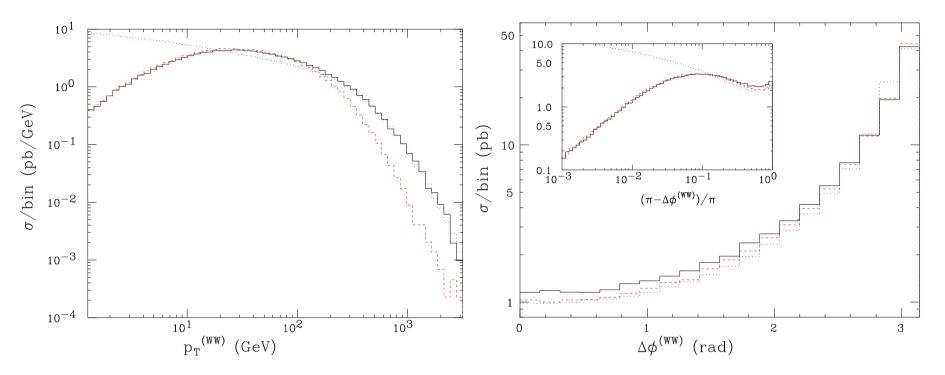
MC@NLO 2.31 [hep-ph/0402116]

TDDOG	D
IPROC	Process
-1350-IL	$H_1H_2 \to (Z/\gamma^* \to) l_{\rm IL}\bar{l}_{\rm IL} + X$
-1360-IL	$H_1H_2 \to (Z \to) l_{\mathrm{IL}} \overline{l}_{\mathrm{IL}} + X$
-1370-IL	$H_1H_2 \to (\gamma^* \to) l_{\rm IL} \bar{l}_{\rm IL} + X$
-1460-IL	$H_1H_2 \rightarrow (W^+ \rightarrow) l_{\rm IL}^+ \nu_{\rm IL} + X$
-1470-IL	$H_1H_2 \to (W^- \to) l_{\rm IL}^- \bar{\nu}_{\rm IL} + X$
-1396	$H_1H_2 \to \gamma^* (\to \sum_i f_i \overline{f_i}) + X$
-1397	$H_1H_2 \rightarrow Z^0 + X$
-1497	$H_1H_2 \to W^+ + X$
-1498	$H_1H_2 \to W^- + X$
-1600-ID	$H_1H_2 \rightarrow H^0 + X$
-1705	$H_1H_2 o bar{b} + X$
-1706	$H_1H_2 \to t\bar{t} + X$
-2850	$H_1H_2 \to W^+W^- + X$
-2860	$H_1H_2 \rightarrow Z^0Z^0 + X$
-2870	$H_1H_2 \to W^+Z^0 + X$
-2880	$H_1H_2 \to W^-Z^0 + X$

(Frixione, Webber)

- Works identically to HERWIG: the very same analysis routines can be used
- Reads shower initial conditions from an event file (as in ME corrections)
- Exploits Les Houches accord for process information and common blocks
- Features a self contained library of PDFs with old and new sets alike
- LHAPDF will also be implemented

W^+W^- Observables



These correlations are problematic: the soft and hard emissions are both relevant. MC@NLO does well, resumming large logarithms, and yet handling the large-scale physics correctly

Solid: MC@NLO

Dashed: HERWIG $\times \frac{\sigma_{NLO}}{\sigma_{LO}}$

Dotted: NLO

PYTHIA shower improvements

Objective:

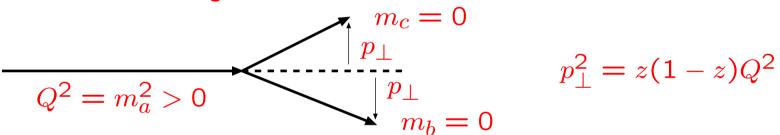
Incorporate several of the good points of the dipole formalism (like ARIADNE) within the shower approach (\Rightarrow hybrid)

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\pm explore alternative p_{\perp} definitions
+ p_{\perp} ordering \Rightarrow coherence inherent
+ ME merging works as before (unique p_{\perp}^2 \leftrightarrow Q_{\parallel}^2 mapping; same z)
+ g \rightarrow q\overline{q} natural
+ kinematics constructed after each branching
  (partons explicitly on-shell until they branch)
+ showers can be stopped and restarted at given p_{\perp} scale
  (not yet worked-out for ISR+FSR)
+ ⇒ well suited for ME/PS matching (L-CKKW, real+fictitious showers)
+ \Rightarrow well suited for simple match with 2 \rightarrow 2 hard processes
++ well suited for interleaved multiple interactions
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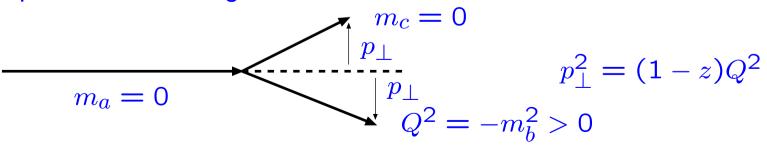
Simple kinematics

Consider branching $a \to bc$ in lightcone coordinates $p^{\pm} = E \pm p_z$

Timelike branching:



Spacelike branching:



Guideline, not final p_{\perp} !

Transverse-momentum-ordered showers

1) Define
$$\begin{array}{l} {\mathsf{p}}_{\perp {\mathsf{evol}}}^2 = z(1-z)Q^2 = z(1-z)M^2 \text{ for FSR} \\ {\mathsf{p}}_{\perp {\mathsf{evol}}}^2 = (1-z)Q^2 = (1-z)(-M^2) \text{ for ISR} \end{array}$$

2) Evolve all partons downwards in $p_{\perp evol}$ from common $p_{\perp max}$

$$d\mathcal{P}_a = \frac{dp_{\perp \text{evol}}^2}{p_{\perp \text{evol}}^2} \frac{\alpha_{\text{S}}(p_{\perp \text{evol}}^2)}{2\pi} P_{a \to bc}(z) dz \exp\left(-\int_{p_{\perp \text{evol}}}^{p_{\perp \text{max}}^2} \cdots\right)$$

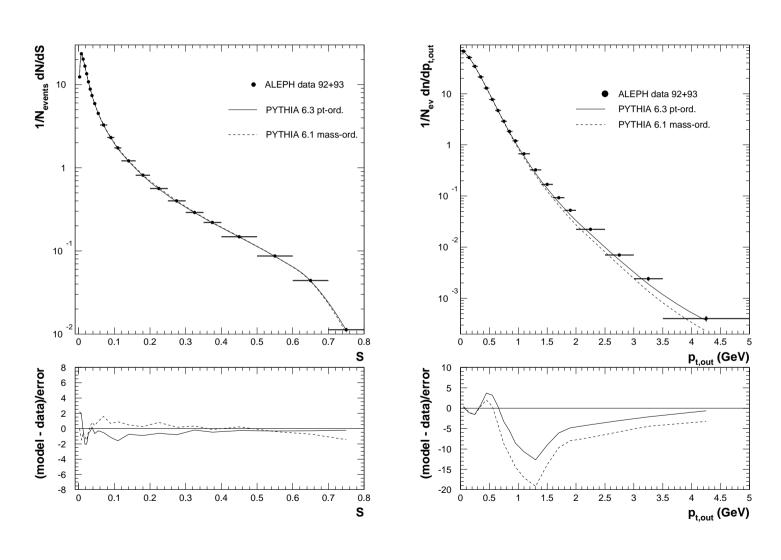
$$d\mathcal{P}_b = \frac{dp_{\perp \text{evol}}^2}{p_{\perp \text{evol}}^2} \frac{\alpha_s(p_{\perp \text{evol}}^2)}{2\pi} \frac{x' f_a(x', p_{\perp \text{evol}}^2)}{x f_b(x, p_{\perp \text{evol}}^2)} P_{a \to bc}(z) dz \exp(-\cdots)$$

Pick the one with *largest* $p_{\perp evol}$ to undergo branching; also gives z.

- 3) Kinematics: $Derive\ Q^2=\pm M^2$ by inversion of 1), but then interpret z as $energy\ fraction$ (not lightcone) in "dipole" rest frame, so that $Lorentz\ invariant$ and matched to matrix elements. Assume yet unbranched partons on-shell and shuffle (E,\mathbf{p}) inside dipole.
- 4) Iterate \Rightarrow combined sequence $p_{\perp max} > p_{\perp 1} > p_{\perp 2} > \ldots > p_{\perp min}$.

Testing the FSR algorithm

Tune performed by Gerald Rudolph (Innsbruck) based on ALEPH 1992+93 data:



Quality of fit

		$\sum \chi^2$ of model		
Distribution	nb.of	PY6.3	PY6.1	
of	interv.	p_\perp -ord.	mass-ord.	
Sphericity	23	25	16	
Aplanarity	16	23	168	
1—Thrust	21	60	8	
Thrust _{minor}	18	26	139	
jet res. $y_3(D)$	20	10	22	
$x = 2p/E_{\rm cm}$	46	207	151	
p_{\perpin}	25	99	170	
$p_{\perp out} < 0.7~GeV$	7	29	24	
$p_{\perp out}$	(19)	(590)	(1560)	
x(B)	19	20	68	
sum $N_{dof} =$	190	497	765	

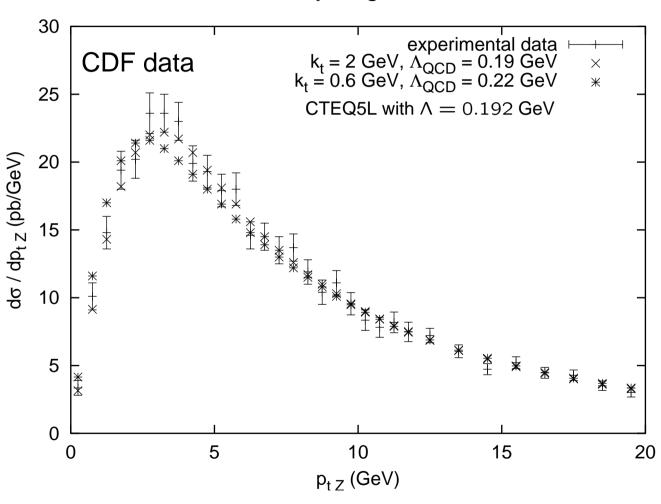
Generator is not assumed to be perfect, so add fraction p of value in quadrature to the definition of the error:

$$p$$
 0% 0.5% 1% $\sum \chi^2$ 523 364 234

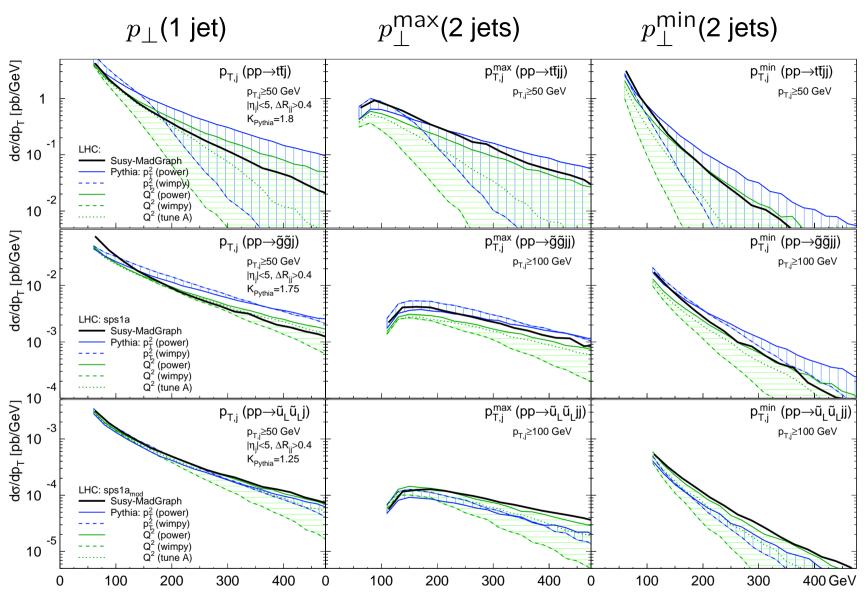
for $N_{
m dof} = 196 \Rightarrow$ generator is 'correct' to \sim 1% except $p_{
m \perp out} > 0.7$ GeV (10%–20% error)

Testing the ISR algorithm

Still only begun...



... but so far no showstoppers

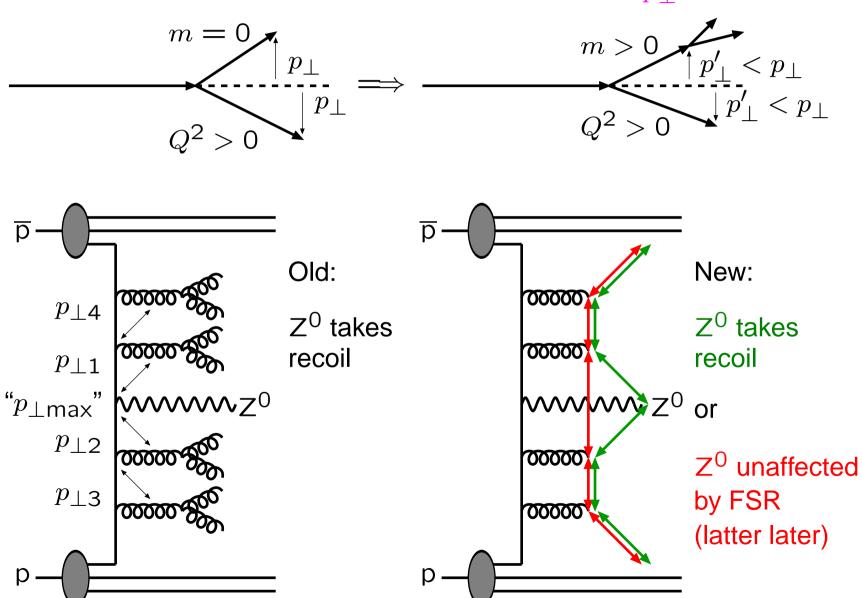


power: $Q^2_{\rm max}=s$; wimpy: $Q^2_{\rm max}=m^2_\perp$; tune A: $Q^2_{\rm max}=4m^2_\perp$ $m_{\rm t}=175$ GeV, $m_{\tilde{\rm g}}=608$ GeV, $m_{\tilde{\rm u}_L}=567$ GeV

(T. Plehn, D. Rainwater, P. Skands)

Combining FSR with ISR

Evolution of timelike sidebranch cascades can reduce p_{\perp} :



Summary Lecture 3

- Showers bring us from few-parton "pencil-jet" topologies
 to multi-broad-jet states.
 - Necessary complement to matrix elements:
- * Do not trust off-the-shelf ME for $R = \sqrt{(\Delta \eta)^2 + (\Delta \phi)^2} \lesssim 1 \star \Delta \phi$ Do not trust unmatched PS for $R \gtrsim 1 \star \Delta \phi$
 - Two main lines of evolution:
- \star (1) Improve algorithm as such: evolution variables, kinematics, NLL, small-x, k_{\perp} factorization, BFKL/CCFM, . . . \star
 - ★ (2) Improve matching ME-PS: merging, vetoed parton showers, MC@NLO ★
 - ⋆ ⇒ active area of development; high profile ⋆