



LUND UNIVERSITY

YETI'06-SM
IPPP, Durham, UK
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Monte Carlo Event Generators

Torbjörn Sjöstrand

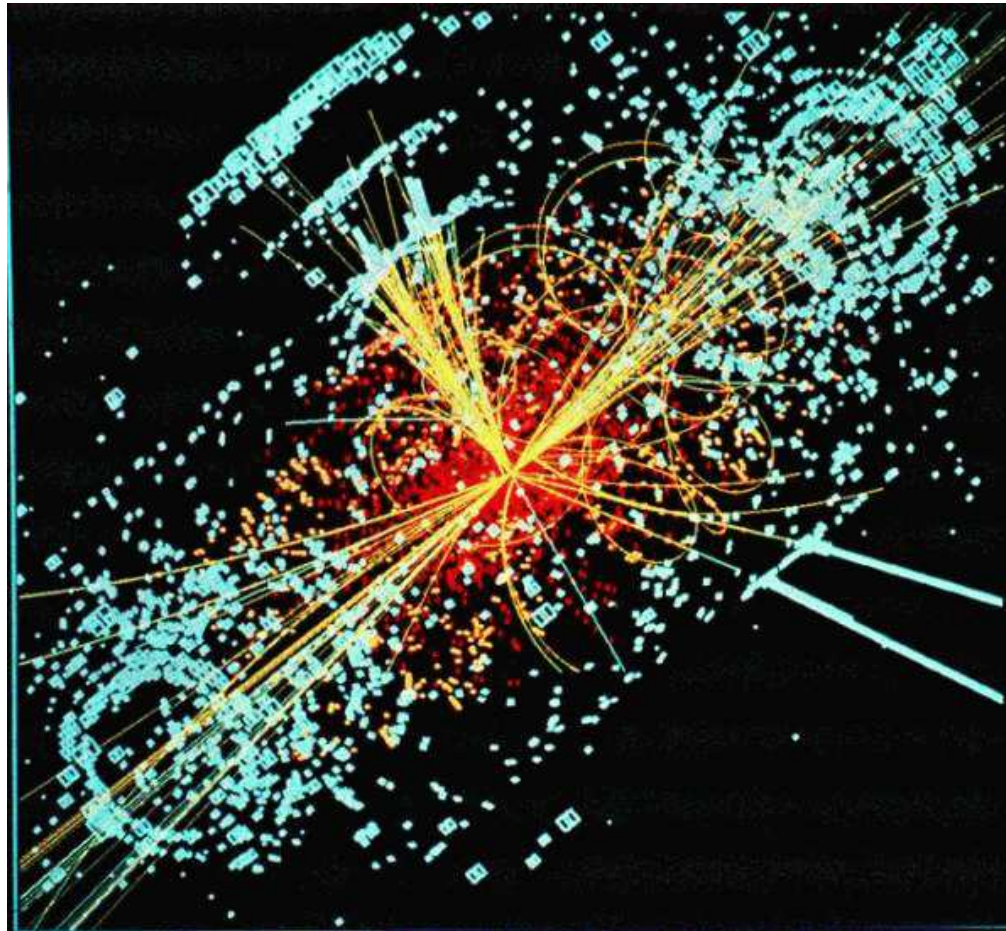
Lund University

1. (today) Introduction and Overview; Monte Carlo Techniques
2. (today) **Matrix Elements; Parton Showers I**
3. (tomorrow) Parton Showers II; Matching Issues
4. (tomorrow) Multiple Interactions and Beam Remnants
5. (Wednesday) Hadronization and Decays; Summary and Outlook

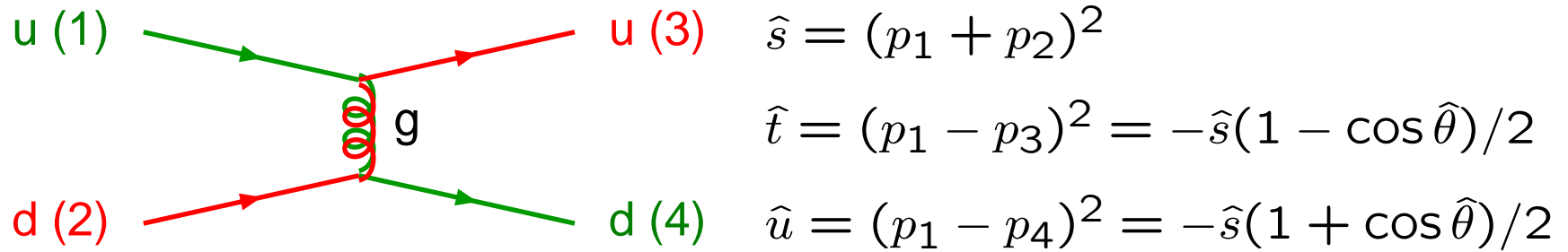
Matrix Elements and Their Usage

\mathcal{L}

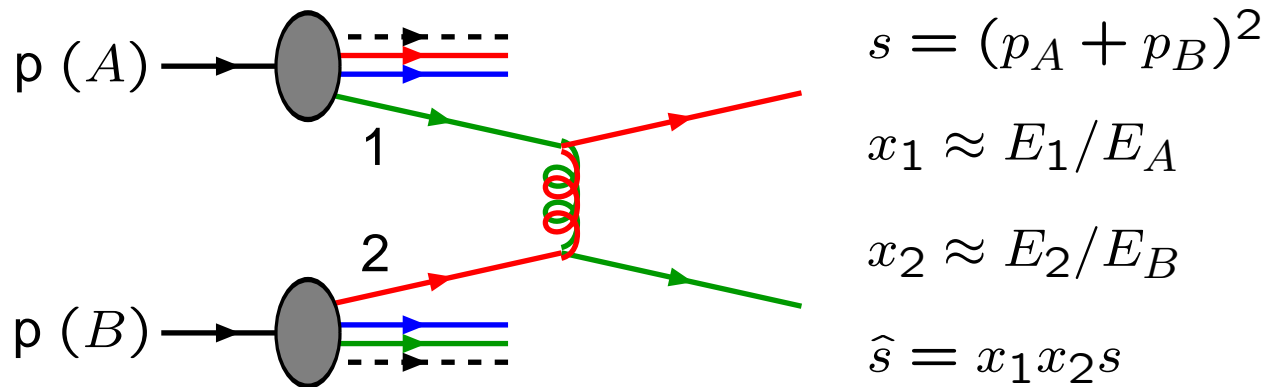
- ⇒ Feynman rules
- ⇒ **Matrix Elements**
- ⇒ **Cross Sections**
- + **Kinematics**
- ⇒ **Processes**
- ⇒ ... ⇒



Cross sections and kinematics

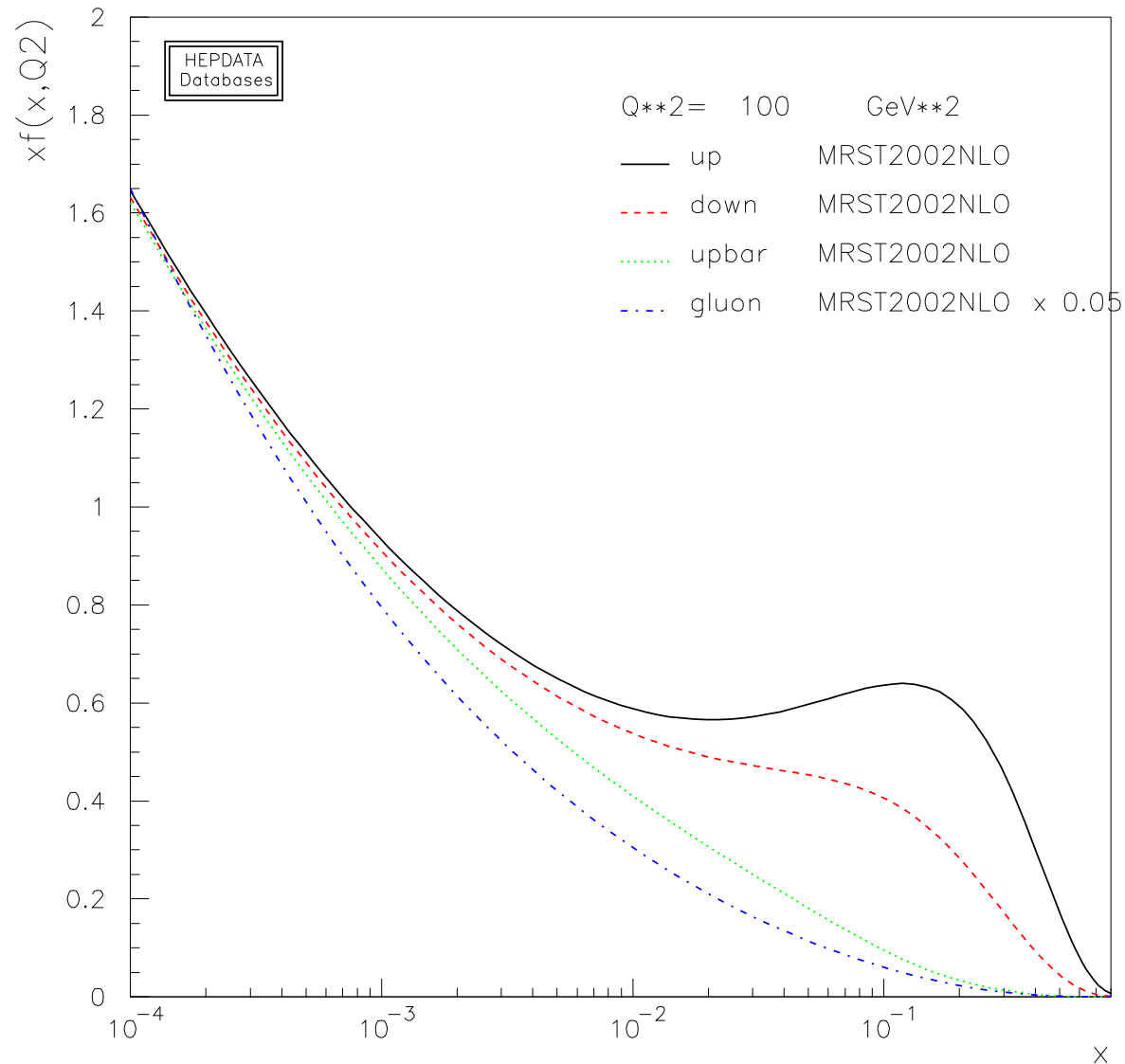


$$qq' \rightarrow qq' : \frac{d\hat{\sigma}}{d\hat{t}} = \frac{\pi}{\hat{s}^2} \frac{4}{9} \alpha_s^2 \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} \quad (\sim \text{Rutherford})$$



$$\sigma = \sum_{i,j} \iiint dx_1 dx_2 d\hat{t} f_i^{(A)}(x_1, Q^2) f_j^{(B)}(x_2, Q^2) \frac{d\hat{\sigma}_{ij}}{d\hat{t}}$$

Parton Distribution/Density Functions (PDF)

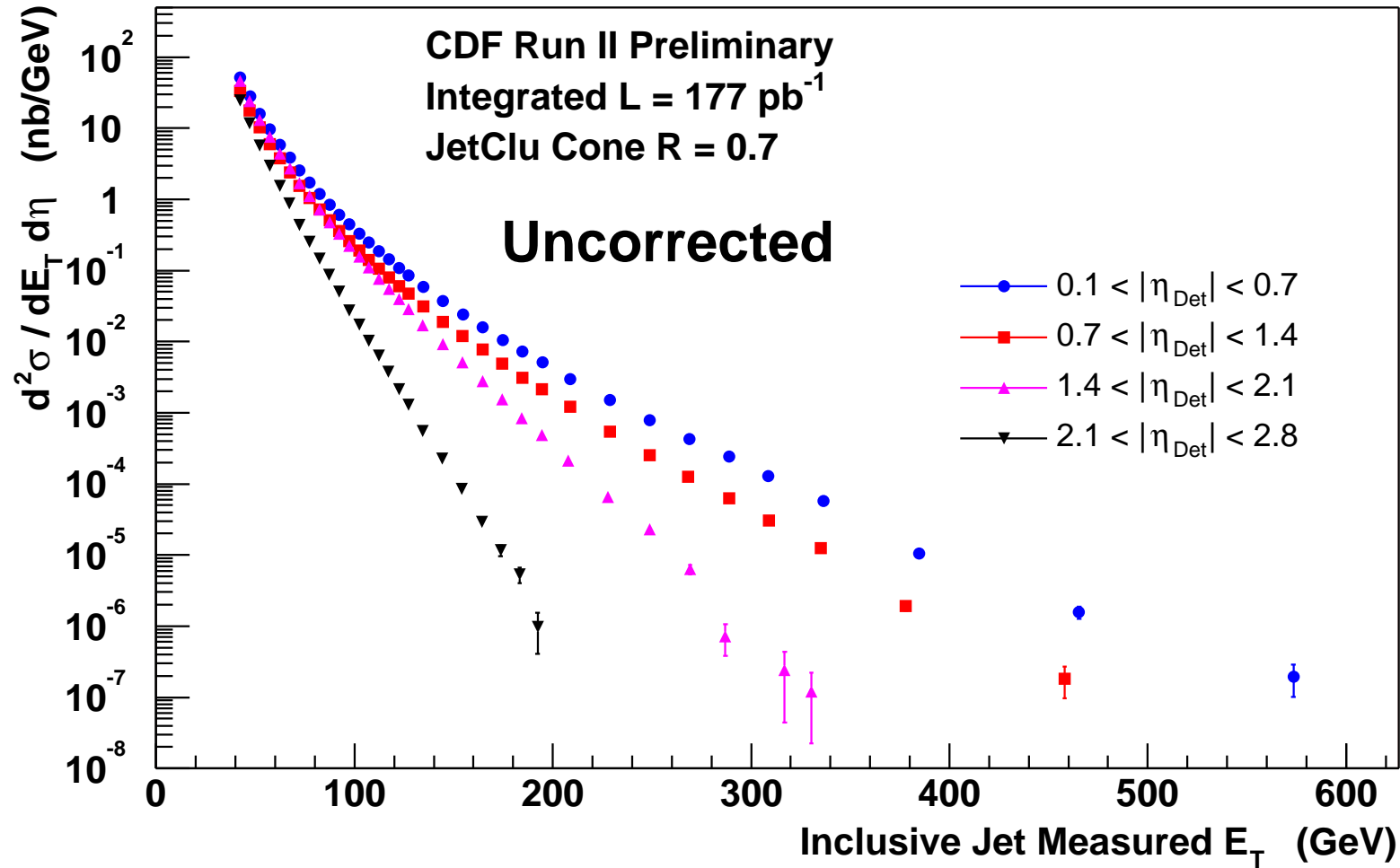


initial
conditions
nonperturbative

evolution
perturbative
(DGLAP)

<http://durpdg.dur.ac.uk/hepdata/pdf.html>

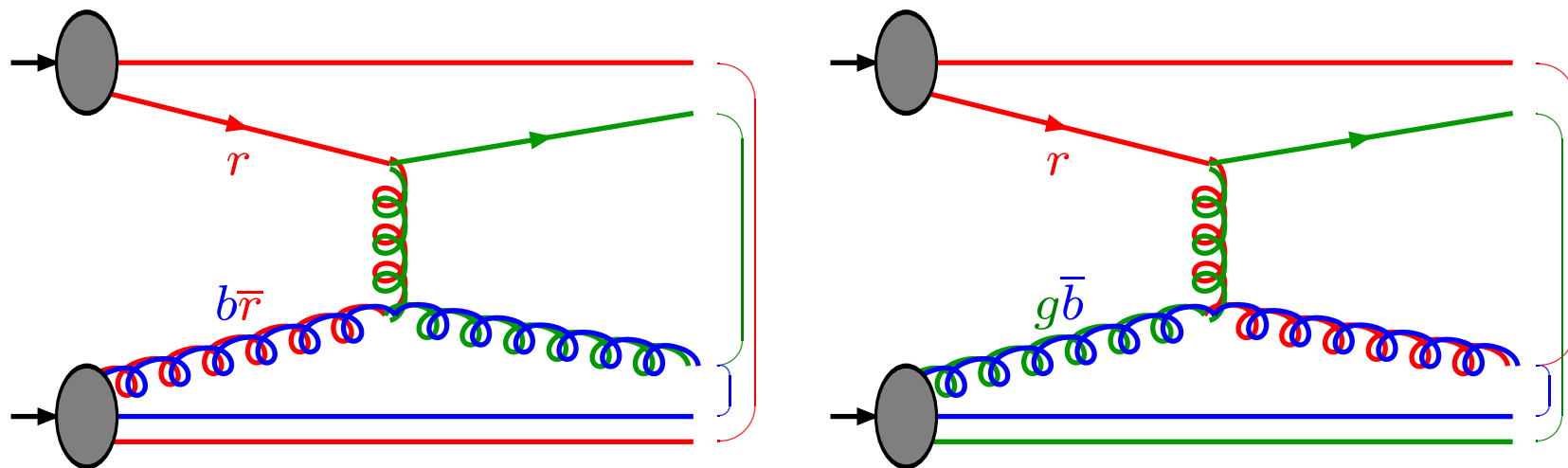
Peaking of PDF's at small x and of QCD ME's at low p_{\perp}
 \implies most of the physics is at low transverse momenta ...



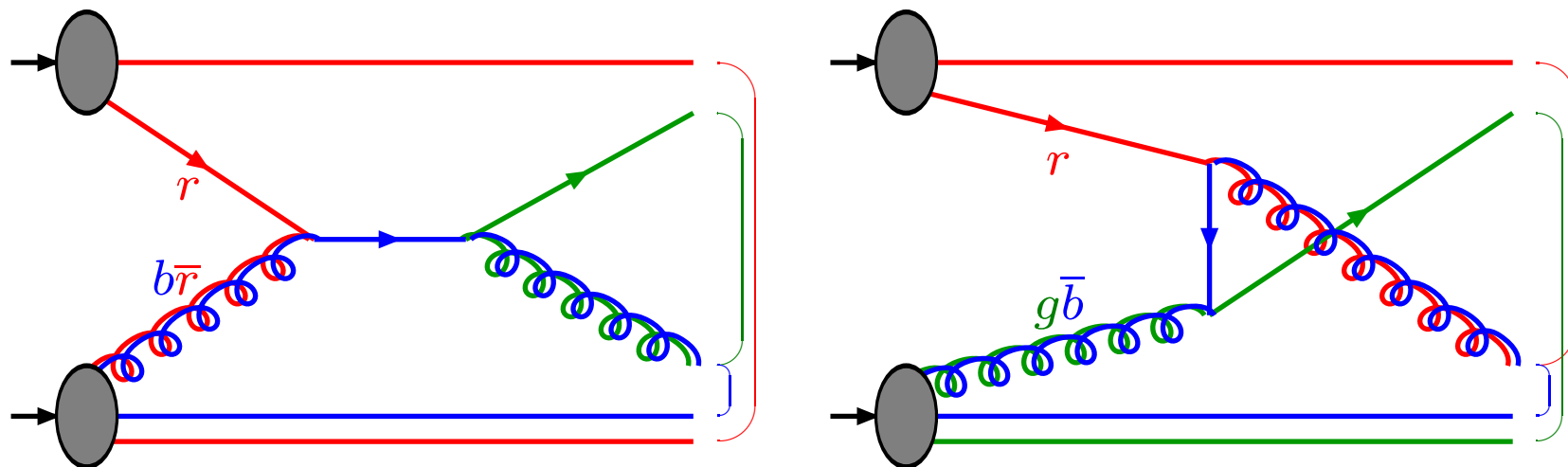
... but New Physics likely to show up at large masses/ p_{\perp} 's

Colour flow in hard processes

One Feynman graph can correspond to several possible colour flows, e.g. for $qg \rightarrow qg$:



while other $qg \rightarrow qg$ graphs only admit one colour flow:



so nontrivial mix of kinematics variables (\hat{s}, \hat{t})
and colour flow topologies I, II:

$$\begin{aligned} |\mathcal{A}(\hat{s}, \hat{t})|^2 &= |\mathcal{A}_I(\hat{s}, \hat{t}) + \mathcal{A}_{II}(\hat{s}, \hat{t})|^2 \\ &= |\mathcal{A}_I(\hat{s}, \hat{t})|^2 + |\mathcal{A}_{II}(\hat{s}, \hat{t})|^2 + 2 \operatorname{Re} (\mathcal{A}_I(\hat{s}, \hat{t}) \mathcal{A}_{II}^*(\hat{s}, \hat{t})) \end{aligned}$$

with $\operatorname{Re} (\mathcal{A}_I(\hat{s}, \hat{t}) \mathcal{A}_{II}^*(\hat{s}, \hat{t})) \neq 0$

\Rightarrow indeterminate colour flow, while

- showers *should* know it (coherence),
- hadronization *must* know it (hadrons singlets).

Normal solution:

$$\frac{\text{interference}}{\text{total}} \propto \frac{1}{N_C^2 - 1}$$

so split I : II according to proportions in the $N_C \rightarrow \infty$ limit, i.e.

$$\begin{aligned} |\mathcal{A}(\hat{s}, \hat{t})|^2 &= |\mathcal{A}_I(\hat{s}, \hat{t})|_{\text{mod}}^2 + |\mathcal{A}_{II}(\hat{s}, \hat{t})|_{\text{mod}}^2 \\ |\mathcal{A}_I(\hat{s}, \hat{t})|_{\text{mod}}^2 &= |\mathcal{A}_I(\hat{s}, \hat{t}) + \mathcal{A}_{II}(\hat{s}, \hat{t})|^2 \left(\frac{|\mathcal{A}_I(\hat{s}, \hat{t})|^2}{|\mathcal{A}_I(\hat{s}, \hat{t})|^2 + |\mathcal{A}_{II}(\hat{s}, \hat{t})|^2} \right)_{N_C \rightarrow \infty} \\ |\mathcal{A}_{II}(\hat{s}, \hat{t})|_{\text{mod}}^2 &= \dots \end{aligned}$$

The Smaller Picture: Subprocess Survey

Kind	Process	PYT	HER	ISA
QCD & related	Soft QCD	★	★	★
	Hard QCD	★	★	★
	Heavy flavour	★	★	★
Electroweak SM	Single $\gamma^*/Z^0/W^\pm$	★	★	★
	$(\gamma/\gamma^*/Z^0/W^\pm/f/g)^2$	★	★	★
	Light SM Higgs	★	★	★
	Heavy SM Higgs	★	★	★
SUSY BSM	$h^0/H^0/A^0/H^\pm$	★	★	★
	SUSY	★	★	★
	\mathbb{R} SUSY	★	★	—
Other BSM	Technicolor	★	—	(★)
	New gauge bosons	★	—	—
	Compositeness	★	—	—
	Leptoquarks	★	—	—
	$H^{\pm\pm}$ (from LR-sym.)	★	—	—
	Extra dimensions	(★)	(★)	(★)

A Giant on Clay Feet

Subprocess lists *look* impressive, and have involved a lot of hard work, but:

- ★ Processes usually only in lowest nontrivial order

⇒ need programs that include HO loop corrections to cross sections, alternatively do (p_{\perp}, y) -dependent rescaling by hand?

- ★ No multijet topologies

⇒ have to trust shower to get it right, alternatively match to HO (non-loop) ME generators

- ★ Spin correlations often absent or incomplete

e.g. top produced unpolarized, while $t \rightarrow bW^+ \rightarrow b\ell^+\nu_{\ell}$ decay correct

⇒ have to use external programs when important

- ★ New physics scenarios appear at rapid pace

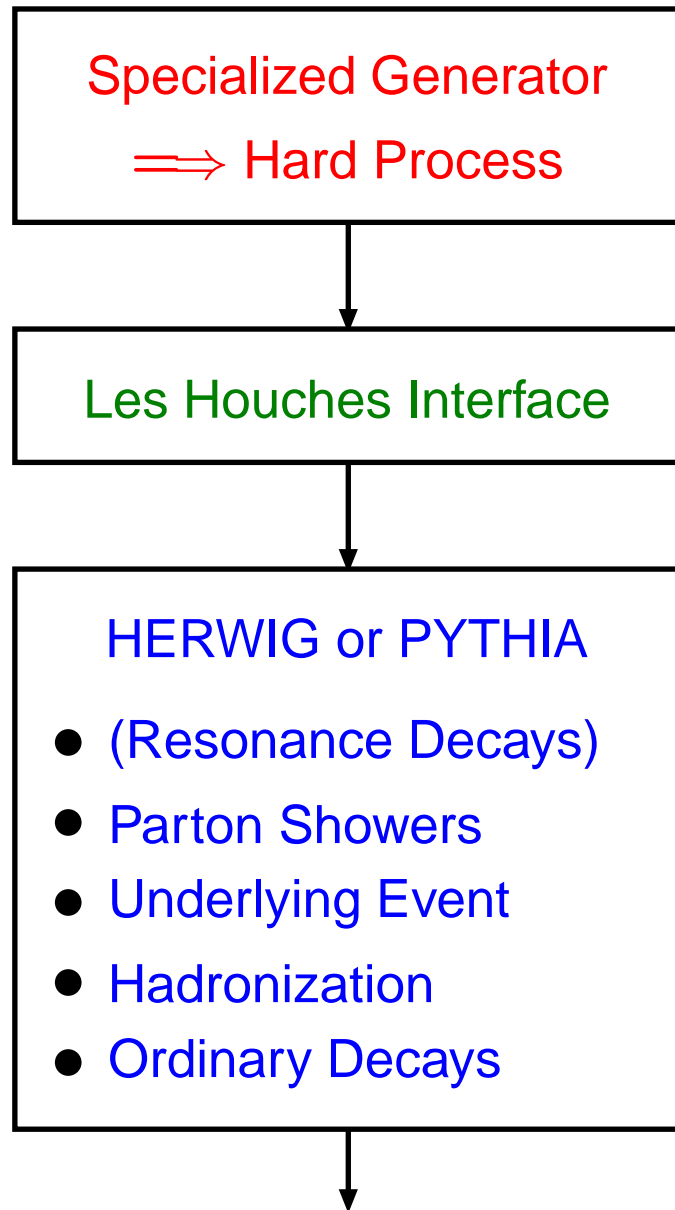
⇒ need to have a bigger class of “one-issue experts” contributing code

⇒ **The Les Houches Accord**

(Q: So why were the process libraries ever built?)

A: Automatic code generation only maturing in recent years!)

The Les Houches Accord



Some Specialized Generators:

- AcerMC: $t\bar{t}b\bar{b}$, ...
- ALPGEN: $W/Z + \leq 6j$,
 $nW + mZ + kH + \leq 3j$, ...
- AMEGIC++: generic LO
- CompHEP: generic LO
- GRACE+Bases/Spring:
generic LO+ some NLO loops
- GR@PPA: $b\bar{b}b\bar{b}$
- MadCUP: $W/Z + \leq 3j$, $t\bar{t}b\bar{b}$
- MadGraph+HELAS: generic LO
- MCFM: NLO $W/Z + \leq 2j$,
 $WZ, WH, H + \leq 1j$
- O'Mega+WHIZARD: generic LO
- VECBOS: $W/Z + \leq 4j$

Apologies for all unlisted programs

Initialization

```
INTEGER MAXPUP
PARAMETER (MAXPUP=100)
INTEGER IDBMUP,PDFGUP,PDFSUP, IDWTUP, NPRUP, LPRUP
DOUBLE PRECISION EBMUP, XSECUP, XERRUP, XMAXUP
COMMON/HEPRUP/IDBMUP(2), EBMUP(2), PDFGUP(2), PDFSUP(2), IDWTUP,
&NPRUP, XSECUP(MAXPUP), XERRUP(MAXPUP), XMAXUP(MAXPUP), LPRUP(MAXPUP)
```

IDBMUP: incoming beam particles (PDG codes, $p = 2212$, $\bar{p} = -2212$)

EBMUP: incoming beam energies (GeV)

PDFGUP, PDFSUP: PDFLIB parton distributions (not used by PYTHIA)

IDWTUP: weighting strategy

- = 1: PYTHIA mixes and unweights events, according to known $d\sigma_{\max}$
- = 2: PYTHIA mixes and unweights events, according to known σ_{tot}
- = 3: unit-weight events, given by user, always to be kept
- = 4: weighted events, given by user, always to be kept
- = -1, -2, -3, -4: also allow negative $d\sigma$

NPRUP: number of separate user processes

XSECUP(i): σ_{tot} for each user process

XERRUP(i): error on σ_{tot} for each user process

XMAXUP(i): $d\sigma_{\max}$ for each user process

LPRUP(i): integer identifier for each user process

The event

```
INTEGER MAXNUP
PARAMETER (MAXNUP=500)
INTEGER NUP, IDPRUP, IDUP, ISTUP, MOTHUP, ICOLUP
DOUBLE PRECISION XWGTUP, SCALUP, AQEDUP, AQCDUP, PUP, VTIMUP, SPINUP
COMMON/HEPEUP/NUP, IDPRUP, XWGTUP, SCALUP, AQEDUP, AQCDUP,
&IDUP(MAXNUP), ISTUP(MAXNUP), MOTHUP(2, MAXNUP), ICOLUP(2, MAXNUP),
&PUP(5, MAXNUP), VTIMUP(MAXNUP), SPINUP(MAXNUP)
```

IDPRUP: identity of current process

XWGTUP: event weight (meaning depends on IDWTUP weighting strategy)

SCALUP: scale Q of parton distributions etc.

AQEDUP: α_{em} used in event

AQCDUP: α_S used in event

NUP: number of particles in event

IDUP(i): PDG identity code for particle i

ISTUP(i): status code

MOTHUP(j, i): position of one or two mothers

ICOLUP(j, i): colour and anticolour indices

PUP(j, i): (p_x, p_y, p_z, E, m)

VTIMUP(i): invariant lifetime $c\tau$

SPINUP(i): spin (helicity) information

Do it yourself

CompHEP and MadGraph can easily be run interactively:

- user specifies process, e.g. $gg \rightarrow W^+ \bar{u}d$,
- program finds all contributing lowest-order Feynman graphs,
- the required amplitudes/cross sections are calculated,
- phase-space is sampled (with tricks) and unweighted to give a set of parton-level events,
- parton-level properties can be histogrammed,
- Les Houches Accord \implies complete events.

CompHEP (matrix-elements-based, good for $\sim \leq 4$ outgoing partons):

<http://theory.sinp.msu.ru/comphep/>

MadGraph (amplitude-based, can handle $\sim \leq 7$ outgoing partons):

<http://madgraph.physics.uiuc.edu/>

...but

- stiff price to pay for each additional parton \implies LO libraries,
- confined to lowest-order processes \implies NLO libraries.

Ready-made libraries

Many leading-order (LO) ones, e.g.:

- ALPGEN: $W/Z+ \leq 6j$, $nW + mZ + kH+ \leq 3j$, $Q\bar{Q}+ \leq 6j$, ...

<http://mlm.home.cern.ch/mlm/alpgen/>

- AcerMC: $t\bar{t}b\bar{b}$, $WWb\bar{b}$, ...

<http://borut.home.cern.ch/borut/>

- VECBOS: $W/Z+ \leq 4j$
- GR@PPA: $b\bar{b}b\bar{b}$, ...
- TopReX: $t\bar{t}$, ...

Not as many NLO, but still quite a few, e.g.

- MCFM: NLO $W/Z+ \leq 2j$, WZ , WH , $H+ \leq 1j$

<http://mcfm.fnal.gov/>

- PHOX family: photons + jets

http://wwwlapp.in2p3.fr/lapth/PHOX_FAMILY/main.html

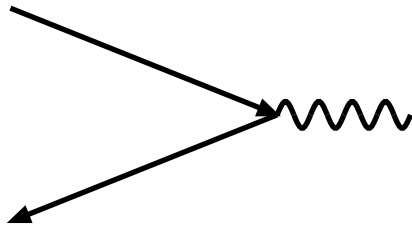
- MNR: $c\bar{c}$, $b\bar{b}$
- AYLEN/EMILIA: WW , WZ , ZZ , $W\gamma$, $Z\gamma$
- EKS: $2j$
- PROSPINO: $\tilde{q}\tilde{q}$, $\tilde{q}\tilde{g}$, $\tilde{g}\tilde{g}$
- HIGLU: $gg \rightarrow H$

Next-to-leading order (NLO) calculations

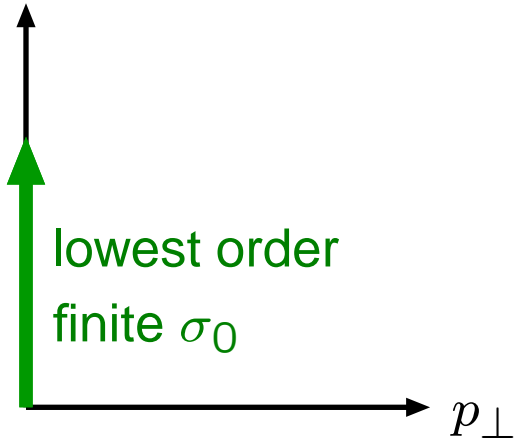
I. Lowest order,

$\mathcal{O}(\alpha_{em})$:

$q\bar{q} \rightarrow Z^0$



$d\sigma/dp_{\perp}$

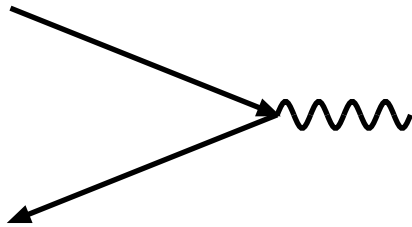


Next-to-leading order (NLO) calculations

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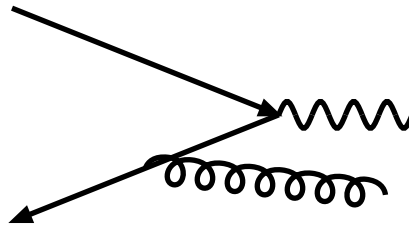
lowest order
finite σ_0

p_{\perp}

II. First-order real,

$\mathcal{O}(\alpha_{em}\alpha_s)$:

$q\bar{q} \rightarrow Z^0 g$ etc.



$d\sigma/dp_{\perp}$



real, $+\infty$

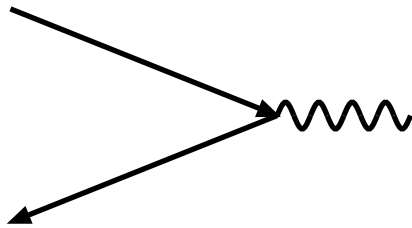
p_{\perp}

Next-to-leading order (NLO) calculations

I. Lowest order,

$\mathcal{O}(\alpha_{em})$:

$q\bar{q} \rightarrow Z^0$



$d\sigma/dp_{\perp}$



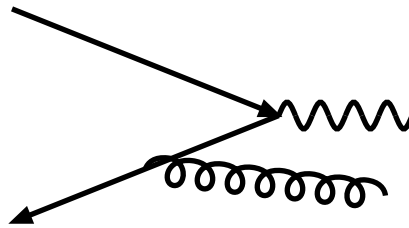
lowest order
finite σ_0

p_{\perp}

II. First-order real,

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$q\bar{q} \rightarrow Z^0 g$ etc.



$d\sigma/dp_{\perp}$

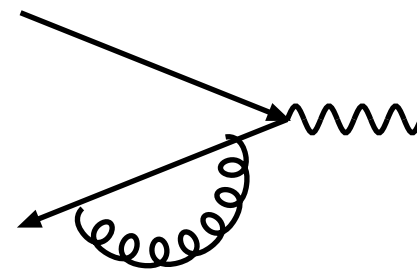
real, $+\infty$

p_{\perp}

III. First-order virtual,

$\mathcal{O}(\alpha_{em}\alpha_s)$:

$q\bar{q} \rightarrow Z^0$ with loops



$d\sigma/dp_{\perp}$

virtual, $-\infty$

p_{\perp}

$$\sigma_{\text{NLO}} = \int_n d\sigma_{\text{LO}} + \int_{n+1} d\sigma_{\text{Real}} + \int_n d\sigma_{\text{Virt}}$$

Simple one-dimensional example: $x \sim p_{\perp}/p_{\perp\text{max}}, 0 \leq x \leq 1$

Divergences regularized by $d = 4 - 2\epsilon$ dimensions, $\epsilon < 0$

$$\sigma_{\text{R+V}} = \int_0^1 \frac{dx}{x^{1+\epsilon}} M(x) + \frac{1}{\epsilon} M_0$$

KLN cancellation theorem: $M(0) = M_0$

Phase Space Slicing:

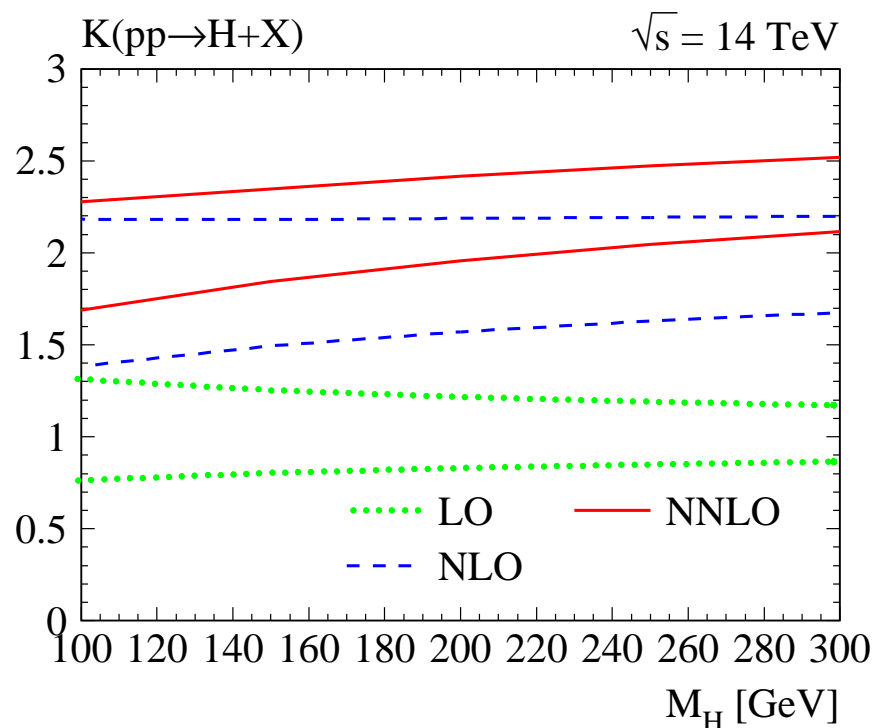
Introduce arbitrary *finite* cutoff $\delta \ll 1$ (so $\delta \gg |\epsilon|$)

$$\begin{aligned} \sigma_{\text{R+V}} &= \int_{\delta}^1 \frac{dx}{x^{1+\epsilon}} M(x) + \int_0^{\delta} \frac{dx}{x^{1+\epsilon}} M(x) + \frac{1}{\epsilon} M_0 \\ &\approx \int_{\delta}^1 \frac{dx}{x} M(x) + \int_0^{\delta} \frac{dx}{x^{1+\epsilon}} M_0 + \frac{1}{\epsilon} M_0 \\ &= \int_{\delta}^1 \frac{dx}{x} M(x) + \frac{1}{\epsilon} (1 - \delta^{-\epsilon}) M_0 \\ &\approx \int_{\delta}^1 \frac{dx}{x} M(x) + \ln \delta M_0 \end{aligned}$$

Alternatively **Subtraction**:

$$\begin{aligned}
 \sigma_{R+V} &= \int_0^1 \frac{dx}{x^{1+\epsilon}} M(x) - \int_0^1 \frac{dx}{x^{1+\epsilon}} M_0 + \int_0^1 \frac{dx}{x^{1+\epsilon}} M_0 + \frac{1}{\epsilon} M_0 \\
 &= \int_0^1 \frac{M(x) - M_0}{x^{1+\epsilon}} dx + \left(-\frac{1}{\epsilon} + \frac{1}{\epsilon} \right) M_0 \\
 &\approx \int_0^1 \frac{M(x) - M_0}{x} dx + \mathcal{O}(1) M_0
 \end{aligned}$$

NLO provides a more accurate answer for an integrated cross section:



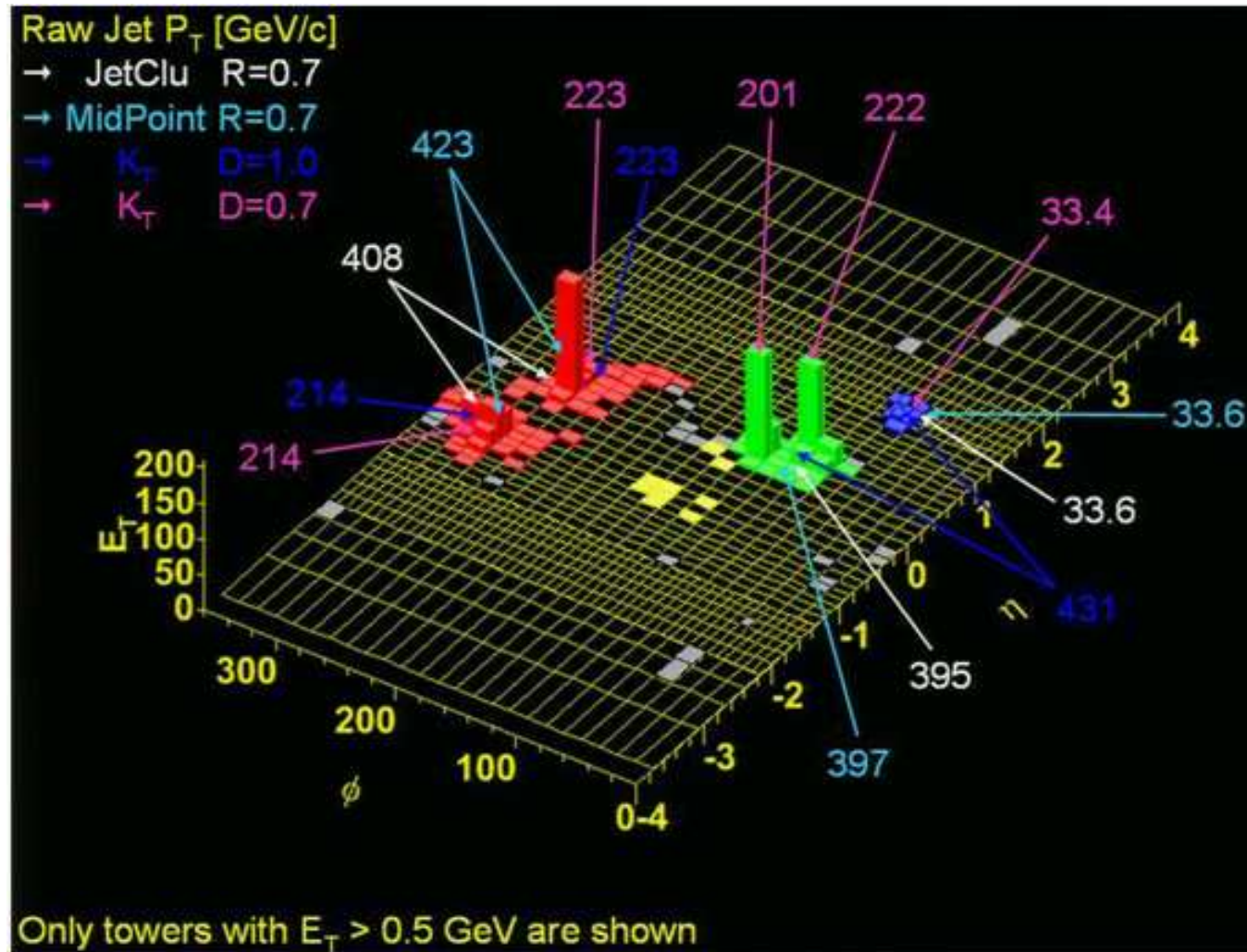
Warning!

Neither approach operates with positive definite quantities

No obvious event-generator implementation

No trivial connection to physical events

Parton Showers



- Final-State (Timelike) Showers
- Initial-State (Spacelike) Showers
 - Matching to Matrix Elements

Divergences

Emission rate $q \rightarrow qg$ diverges when

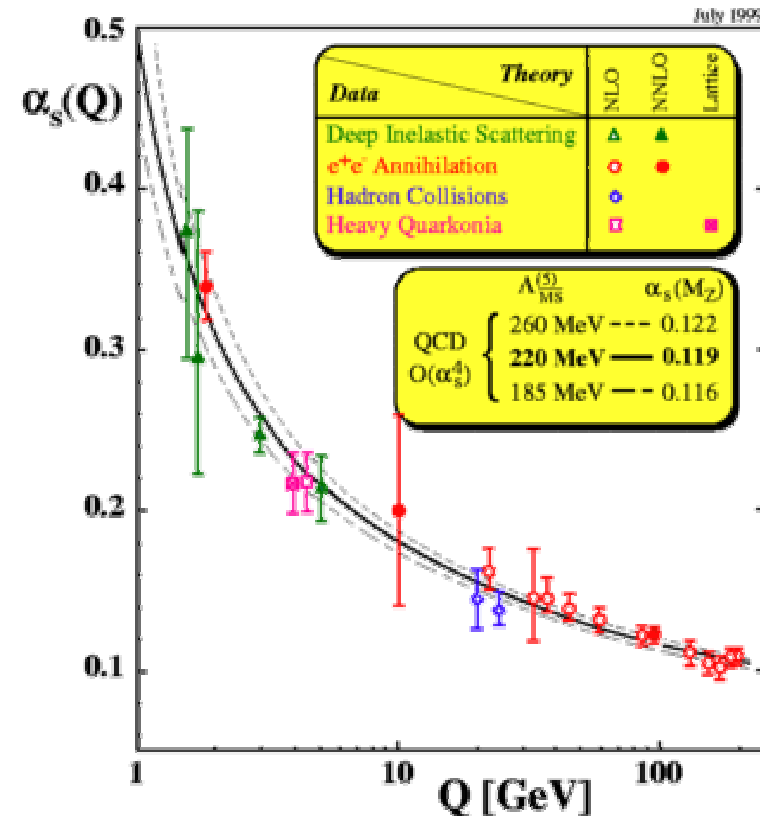
- collinear: opening angle $\theta_{qg} \rightarrow 0$
- soft: gluon energy $E_g \rightarrow 0$

Almost identical to $e \rightarrow e\gamma$

(“bremsstrahlung”),

but QCD is non-Abelian so additionally

- $g \rightarrow gg$ similarly divergent
- $\alpha_s(Q^2)$ diverges for $Q^2 \rightarrow 0$
(actually for $Q^2 \rightarrow \Lambda_{\text{QCD}}^2$)

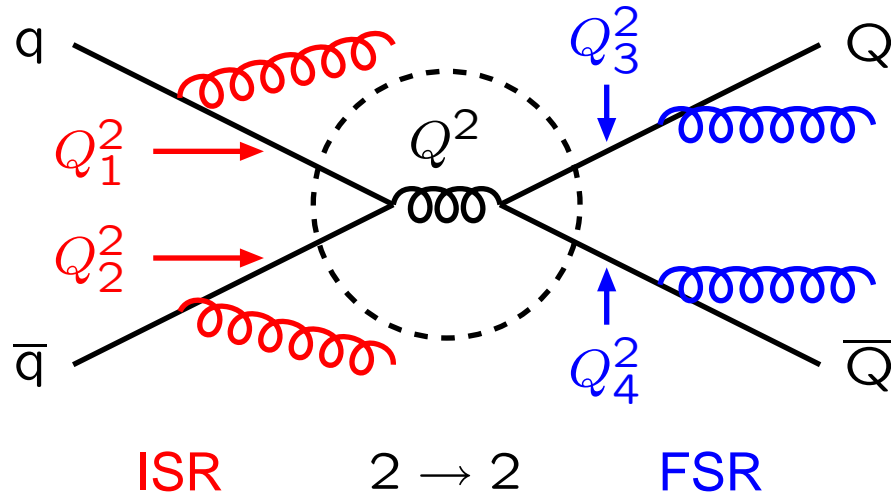


Big probability for one emission \implies also big for several
 \implies with ME's need to calculate to high order **and** with many loops
 \implies extremely demanding technically (not solved!), and
involving big cancellations between positive and negative contributions.

Alternative approach: **parton showers**

The Parton-Shower Approach

$$2 \rightarrow n = (2 \rightarrow 2) \oplus \text{ISR} \oplus \text{FSR}$$



FSR = Final-State Rad.;
timelike shower

$Q_i^2 \sim m^2 > 0$ decreasing

ISR = Initial-State Rad.;
spacelike shower

$Q_i^2 \sim -m^2 > 0$ increasing

$2 \rightarrow 2 =$ hard scattering (on-shell):

$$\sigma = \iiint dx_1 dx_2 d\hat{t} f_i(x_1, Q^2) f_j(x_2, Q^2) \frac{d\hat{\sigma}_{ij}}{d\hat{t}}$$

Shower evolution is viewed as a probabilistic process,

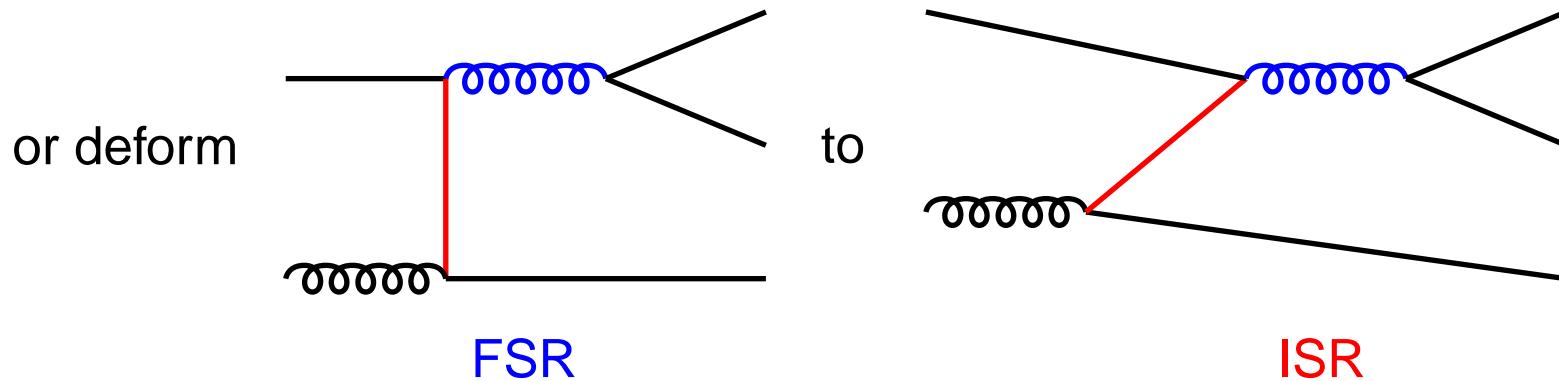
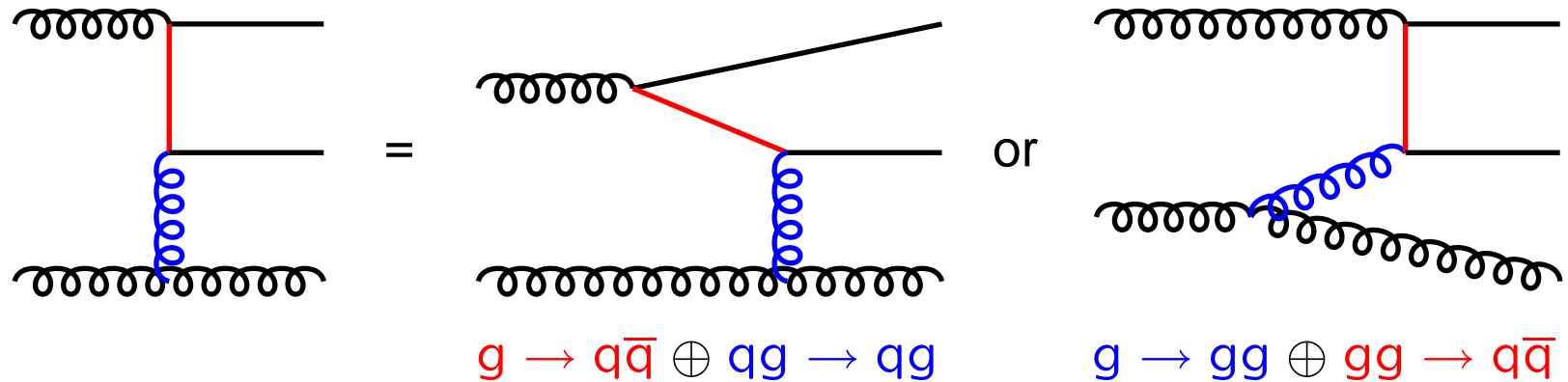
which occurs with unit total probability:

the cross section is not directly affected,

but indirectly it is, via the changed event shape

Doublecounting

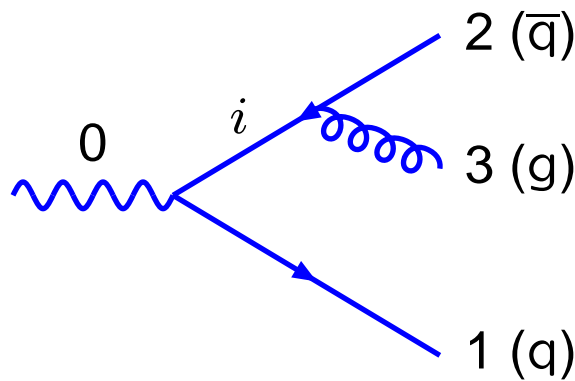
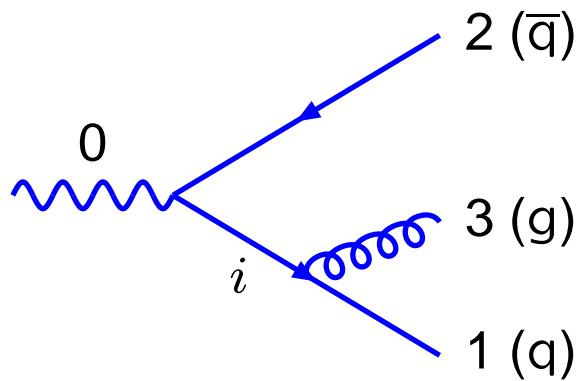
A $2 \rightarrow n$ graph can be “simplified” to $2 \rightarrow 2$ in different ways:



Do not doublecount: $2 \rightarrow 2 = \text{most virtual} = \text{shortest distance}$

Conflict: theory derivations often assume virtualities strongly ordered;
interesting physics often in regions where this is not true!

From Matrix Elements to Parton Showers



$$e^+e^- \rightarrow q\bar{q}g$$

$$x_j = 2E_j/E_{\text{cm}} \Rightarrow x_1 + x_2 + x_3 = 2$$

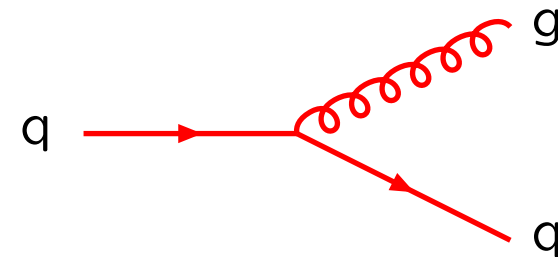
$$m_q = 0 : \frac{d\sigma_{\text{ME}}}{\sigma_0} = \frac{\alpha_s}{2\pi} \frac{4}{3} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} dx_1 dx_2$$

Rewrite for $x_2 \rightarrow 1$, i.e. q-g collinear limit:

$$1 - x_2 = \frac{m_{13}^2}{E_{\text{cm}}^2} = \frac{Q^2}{E_{\text{cm}}^2} \Rightarrow dx_2 = \frac{dQ^2}{E_{\text{cm}}^2}$$

$$x_1 \approx z \Rightarrow dx_1 \approx dz$$

$$x_3 \approx 1 - z$$



$$\Rightarrow d\mathcal{P} = \frac{d\sigma}{\sigma_0} = \frac{\alpha_s}{2\pi} \frac{dx_2}{(1-x_2)} \frac{4}{3} \frac{x_2^2 + x_1^2}{(1-x_1)} dx_1 \approx \frac{\alpha_s}{2\pi} \frac{dQ^2}{Q^2} \frac{4}{3} \frac{1+z^2}{1-z} dz$$

Generalizes to DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi)

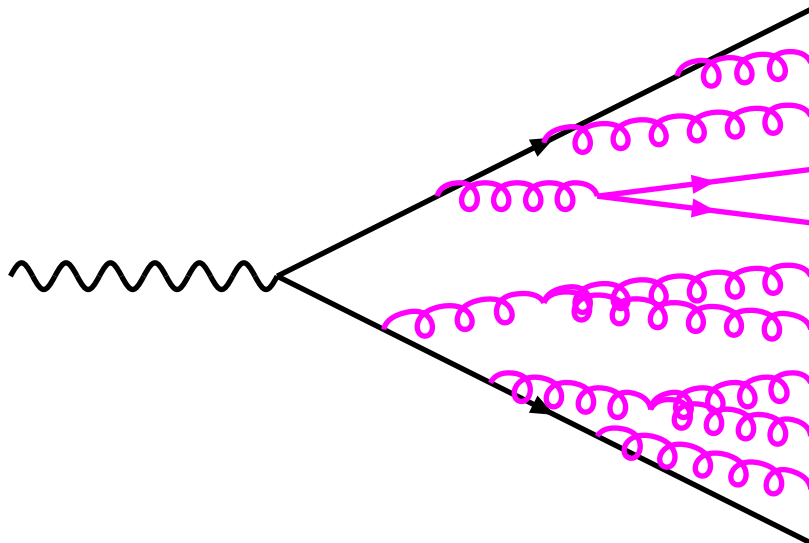
$$d\mathcal{P}_{a \rightarrow bc} = \frac{\alpha_s}{2\pi} \frac{dQ^2}{Q^2} P_{a \rightarrow bc}(z) dz$$

$$P_{q \rightarrow qg} = \frac{4}{3} \frac{1+z^2}{1-z}$$

$$P_{g \rightarrow gg} = 3 \frac{(1-z(1-z))^2}{z(1-z)}$$

$$P_{g \rightarrow q\bar{q}} = \frac{n_f}{2} (z^2 + (1-z)^2) \quad (n_f = \text{no. of quark flavours})$$

Iteration gives final-state parton showers



Need soft/collinear cut-offs
to stay away from
nonperturbative physics.

Details model-dependent, e.g.

$Q > m_0 = \min(m_{ij}) \approx 1 \text{ GeV}$,

$z_{\min}(E, Q) < z < z_{\max}(E, Q)$

or $p_{\perp} > p_{\perp \min} \approx 0.5 \text{ GeV}$

The Sudakov Form Factor

Conservation of total probability:

$$\mathcal{P}(\text{nothing happens}) = 1 - \mathcal{P}(\text{something happens})$$

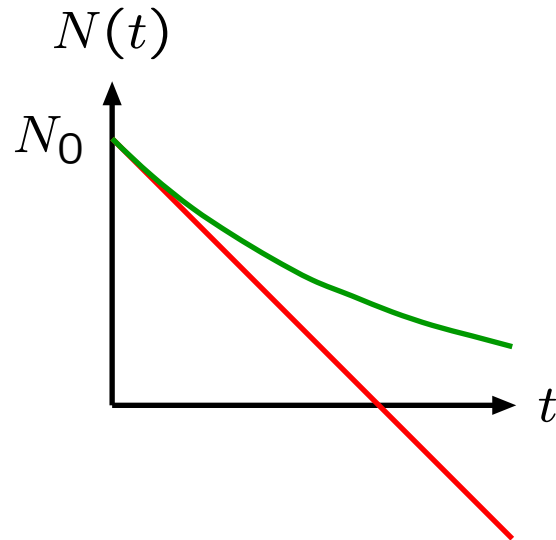
“multiplicativeness” in “time” evolution:

$$\mathcal{P}_{\text{nothing}}(0 < t \leq T) = \mathcal{P}_{\text{nothing}}(0 < t \leq T_1) \mathcal{P}_{\text{nothing}}(T_1 < t \leq T)$$

Subdivide further, with $T_i = (i/n)T$, $0 \leq i \leq n$:

$$\begin{aligned} \mathcal{P}_{\text{nothing}}(0 < t \leq T) &= \lim_{n \rightarrow \infty} \prod_{i=0}^{n-1} \mathcal{P}_{\text{nothing}}(T_i < t \leq T_{i+1}) \\ &= \lim_{n \rightarrow \infty} \prod_{i=0}^{n-1} \left(1 - \mathcal{P}_{\text{something}}(T_i < t \leq T_{i+1}) \right) \\ &= \exp \left(- \lim_{n \rightarrow \infty} \sum_{i=0}^{n-1} \mathcal{P}_{\text{something}}(T_i < t \leq T_{i+1}) \right) \\ &= \exp \left(- \int_0^T \frac{d\mathcal{P}_{\text{something}}(t)}{dt} dt \right) \\ \implies d\mathcal{P}_{\text{first}}(T) &= d\mathcal{P}_{\text{something}}(T) \exp \left(- \int_0^T \frac{d\mathcal{P}_{\text{something}}(t)}{dt} dt \right) \end{aligned}$$

Example: radioactive decay of nucleus



naively: $\frac{dN}{dt} = -cN_0 \Rightarrow N(t) = N_0 (1 - ct)$

depletion: a given nucleus can only decay once

correctly: $\frac{dN}{dt} = -cN(t) \Rightarrow N(t) = N_0 \exp(-ct)$

generalizes to: $N(t) = N_0 \exp\left(-\int_0^t c(t') dt'\right)$

or: $\frac{dN(t)}{dt} = -c(t) N_0 \exp\left(-\int_0^t c(t') dt'\right)$

sequence allowed: nucleus₁ → nucleus₂ → nucleus₃ → ...

Correspondingly, with $Q \sim 1/t$ (Heisenberg)

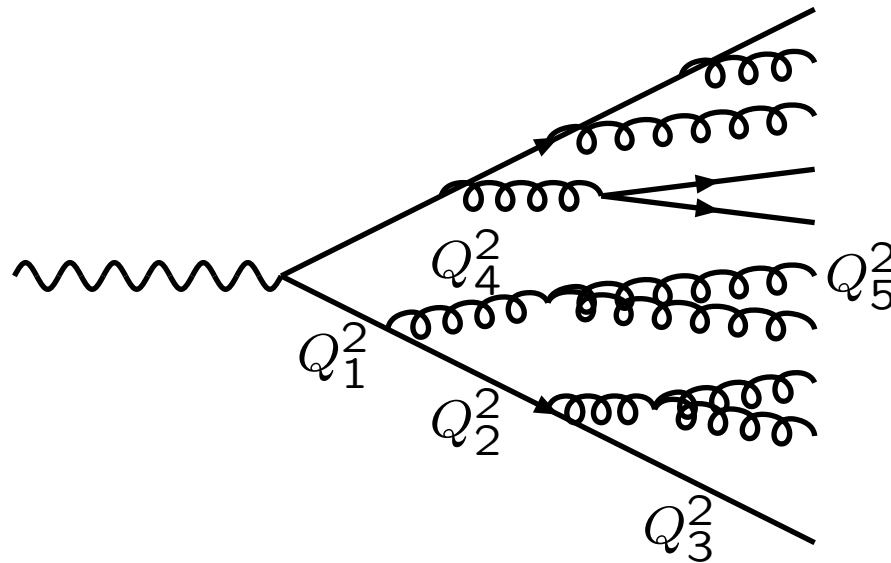
$$d\mathcal{P}_{a \rightarrow bc} = \frac{\alpha_s}{2\pi} \frac{dQ^2}{Q^2} P_{a \rightarrow bc}(z) dz \exp\left(-\sum_{b,c} \int_{Q^2}^{Q_{\max}^2} \frac{dQ'^2}{Q'^2} \int \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z') dz'\right)$$

where the exponent is (one definition of) the Sudakov form factor

A given parton can only branch once, i.e. if it did not already do so

Note that $\sum_{b,c} \int \int d\mathcal{P}_{a \rightarrow bc} \equiv 1 \Rightarrow$ convenient for Monte Carlo

($\equiv 1$ if extended over whole phase space, else possibly nothing happens)



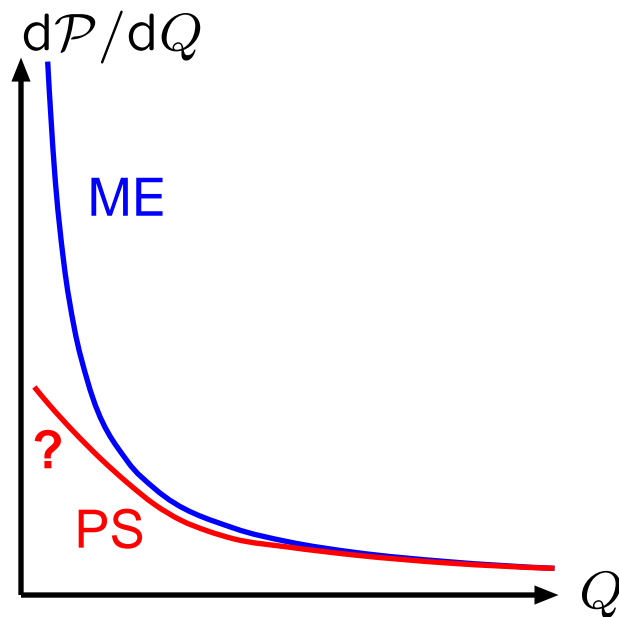
Sudakov form factor provides
 “time” ordering of shower:
 lower $Q^2 \iff$ longer times

$$Q_1^2 > Q_2^2 > Q_3^2$$

$$Q_1^2 > Q_4^2 > Q_5^2$$

etc.

Sudakov regulates singularity for *first* emission ...



... but in limit of *repeated soft*
 emissions $q \rightarrow qg$ (but no $g \rightarrow gg$)
 one obtains the same inclusive
 Q emission spectrum as for ME,

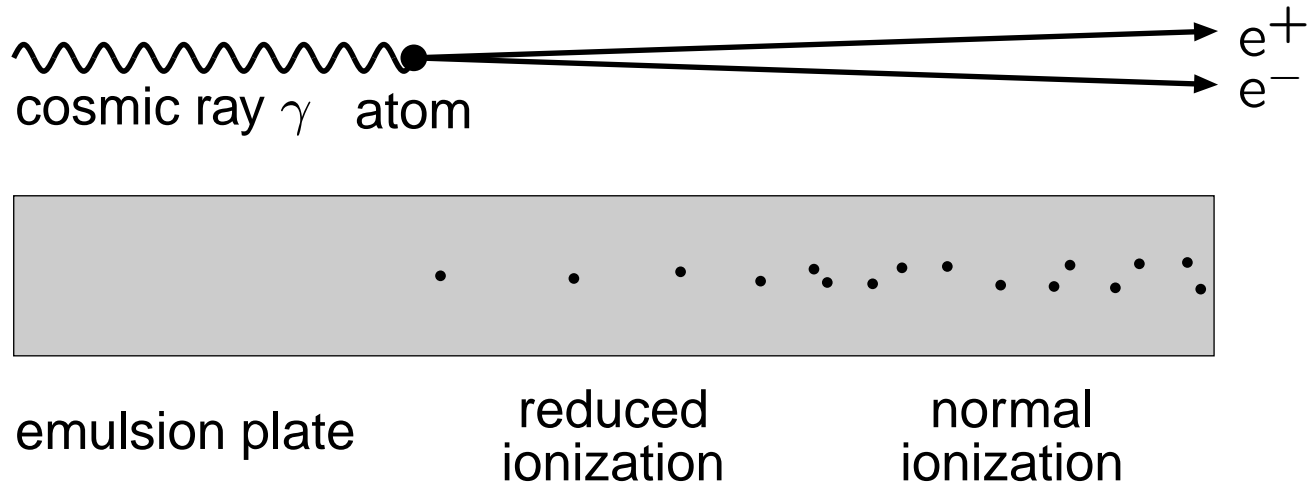
i.e. divergent ME spectrum

\iff **infinite number of PS emissions**

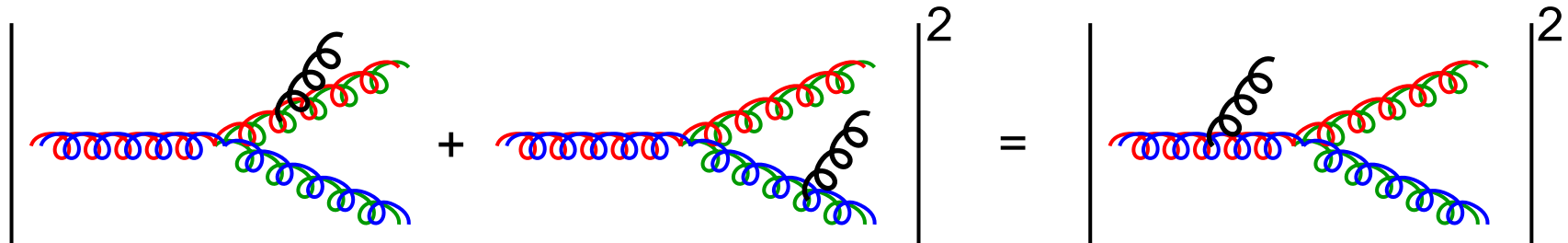
Proof: as for veto algorithm (what is
 probability to have an emission at Q
 after 0, 1, 2, 3, ... previous ones?)

Coherence

QED: Chudakov effect (mid-fifties)



QCD: colour coherence for **soft** gluon emission

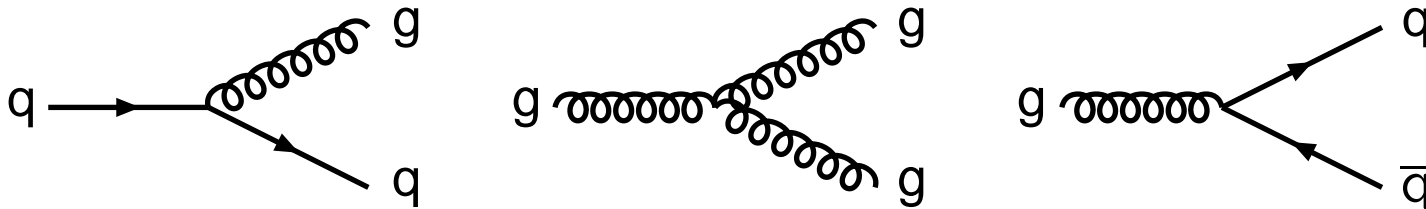


- solved by
- requiring emission angles to be decreasing
 - or
 - requiring transverse momenta to be decreasing

The Common Showering Algorithms

Three main approaches to showering in common use:

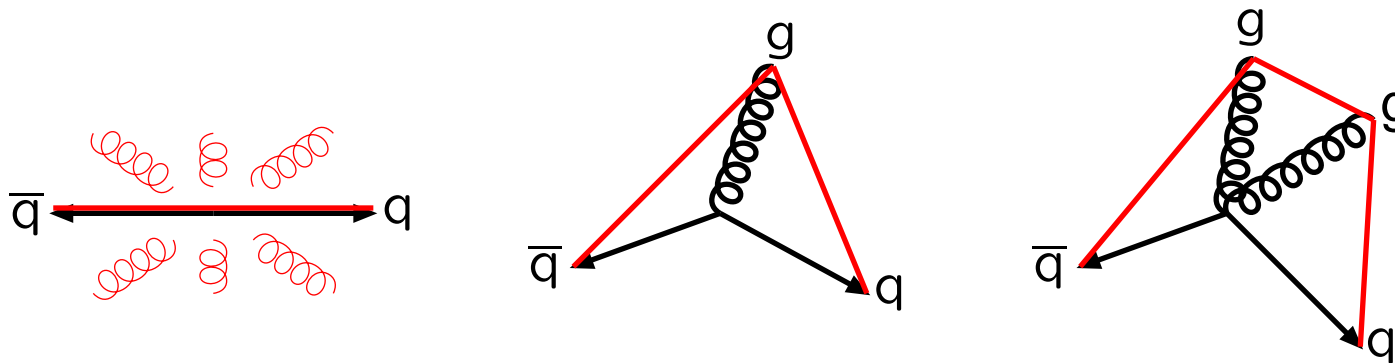
Two are based on the standard shower language
of $a \rightarrow bc$ successive branchings:



HERWIG: $Q^2 \approx E^2(1 - \cos \theta) \approx E^2\theta^2/2$

PYTHIA: $Q^2 = m^2$ (timelike) or $= -m^2$ (spacelike)

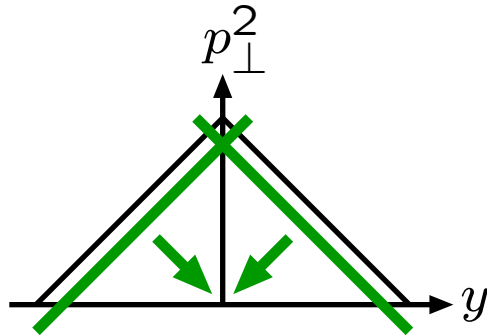
One is based on a picture of dipole emission $ab \rightarrow cde$:



ARIADNE: $Q^2 = p_{\perp}^2$; FSR mainly, ISR is primitive;
there instead LDCMC: sophisticated but complicated

Ordering variables in final-state radiation

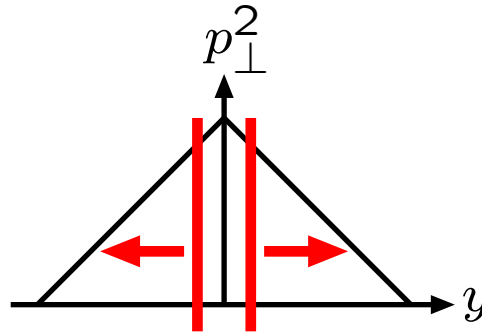
PYTHIA: $Q^2 = m^2$



large mass first
 \Rightarrow “hardness” ordered
coherence brute force

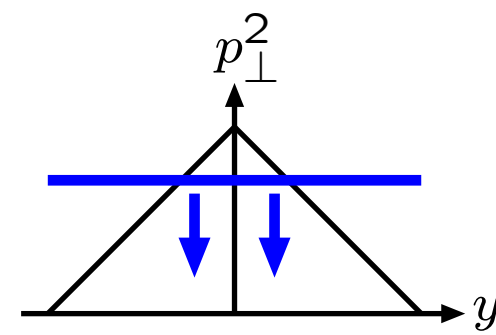
covers phase space
 ME merging simple
 $g \rightarrow q\bar{q}$ simple
not Lorentz invariant
 no stop/restart
 ISR: $m^2 \rightarrow -m^2$

HERWIG: $Q^2 \sim E^2\theta^2$



large angle first
 \Rightarrow **hardness not ordered**
 coherence inherent
gaps in coverage
ME merging messy
 $g \rightarrow q\bar{q}$ simple
not Lorentz invariant
 no stop/restart
 ISR: $\theta \rightarrow \theta$

ARIADNE: $Q^2 = p_{\perp}^2$

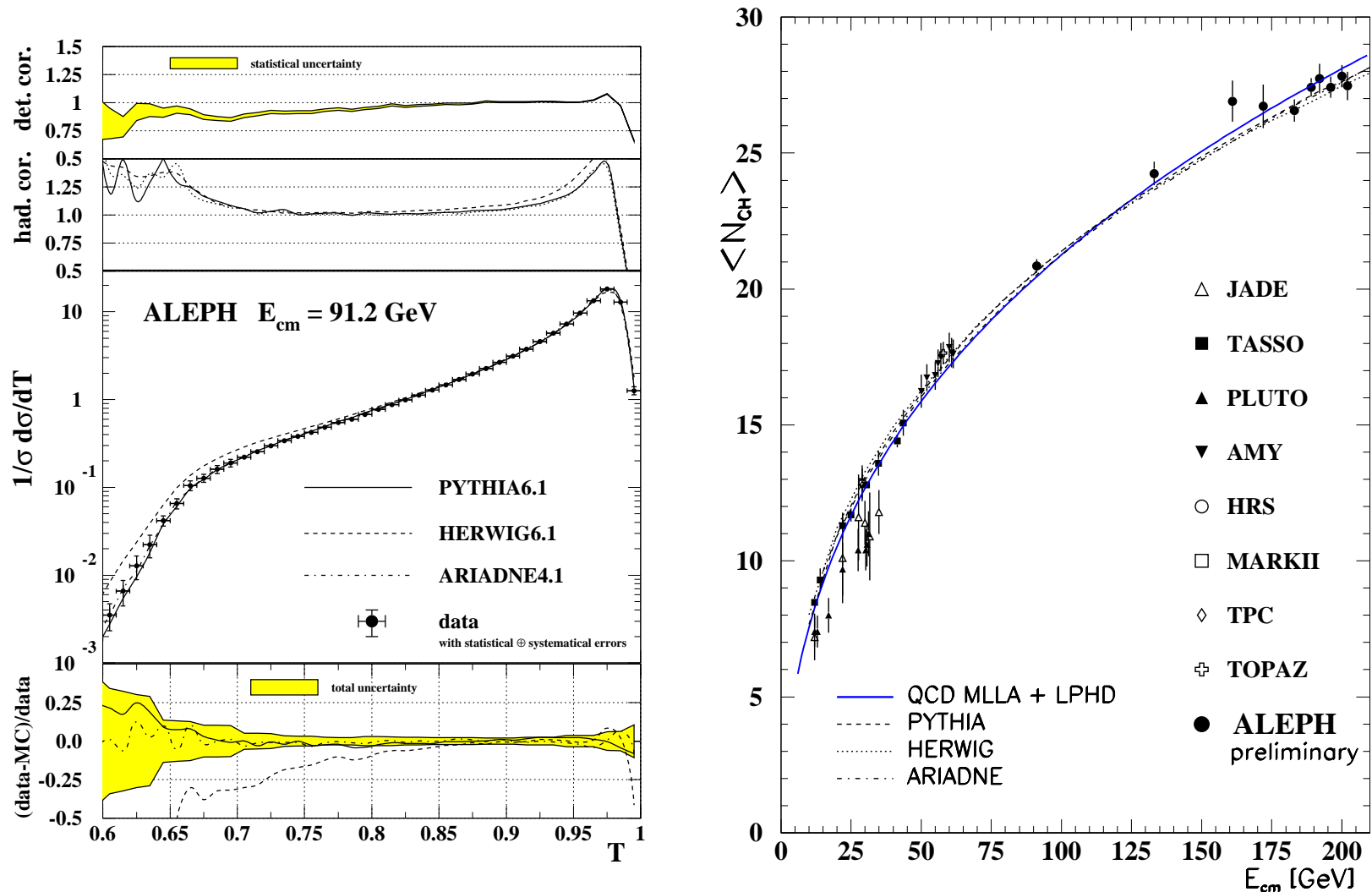


large p_{\perp} first
 \Rightarrow “hardness” ordered
 coherence inherent

covers phase space
 ME merging simple
 $g \rightarrow q\bar{q}$ **messy**
 Lorentz invariant
 can stop/restart
ISR: more messy

Data comparisons

All three algorithms do a reasonable job of describing LEP data, but typically $\text{ARIADNE } (p_{\perp}^2) > \text{PYTHIA } (m^2) > \text{HERWIG } (\theta)$



... and programs evolve to do even better ...

HERWIG shower improvements

Quasi-Collinear Limit (Heavy Quarks)

Sudakov-basis p, n with $p^2 = M^2$ ('forward'), $n^2 = 0$ ('backward'),

$$p_q = zp + \beta_q n - q_\perp$$

$$p_g = (1-z)p + \beta_g n + q_\perp$$

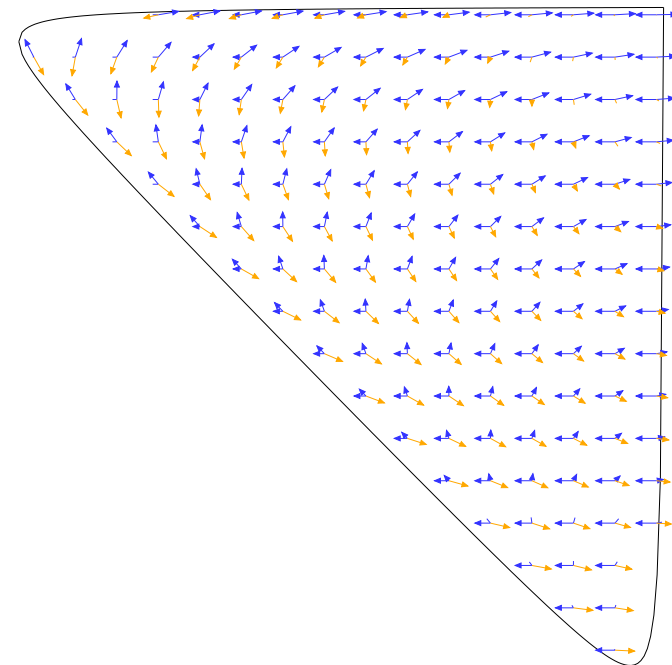
Collinear limit for radiation off heavy quark,

$$P_{gq}(z, \mathbf{q}^2, m^2) = C_F \left[\frac{1+z^2}{1-z} - \frac{2z(1-z)m^2}{\mathbf{q}^2 + (1-z)^2 m^2} \right]$$

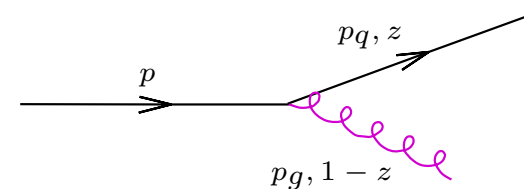
$$= \frac{C_F}{1-z} \left[1 + z^2 - \frac{2m^2}{z\tilde{q}^2} \right]$$

→ $\tilde{q}^2 \sim \mathbf{q}^2$ may be used as evolution variable.

$q\bar{q}g$ -Phase space (x, \bar{x})



Single emission:



New evolution variables

Kinematics to allow better treatment of heavy particles, avoiding overlapping regions in phase space, in particular for soft emissions

We choose \tilde{q}^2 as new evolution variable,

$$\tilde{q}^2 = \frac{\mathbf{q}^2}{z^2(1-z)^2} + \frac{m^2}{z^2} \quad \text{for } q \rightarrow qg$$

and with the argument of running α_S chosen according to

$$\alpha_S(z^2(1-z)^2\tilde{q}^2)$$

angular ordering

$$\tilde{q}_{i+1} < z_i \tilde{q}_i \quad \tilde{k}_{i+1} < (1-z_i) \tilde{q}_i$$

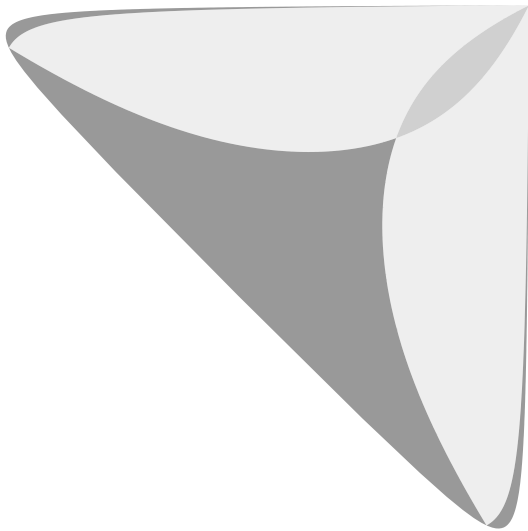
Technically: *reinterpretation* of known evolution variables, i.e. the branching probability for $a \rightarrow bc$ still is

$$dP(a \rightarrow bc) = \frac{d\tilde{q}^2}{\tilde{q}^2} \frac{C_i \alpha_S}{2\pi} P_{bc}(z, \tilde{q}) dz$$

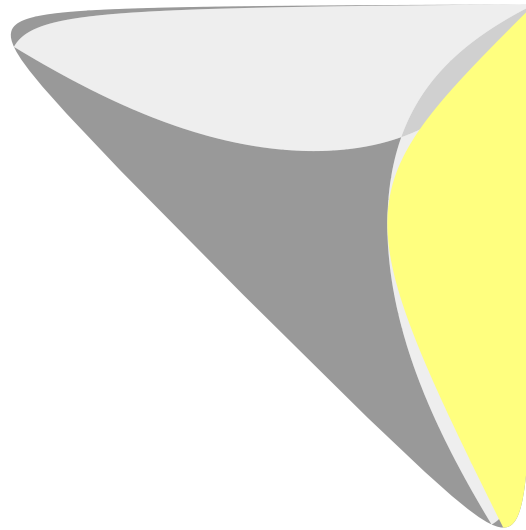
→ Sudakov's etc. technically remain the same!

$q\bar{q}g$ Phase Space old vs new variables

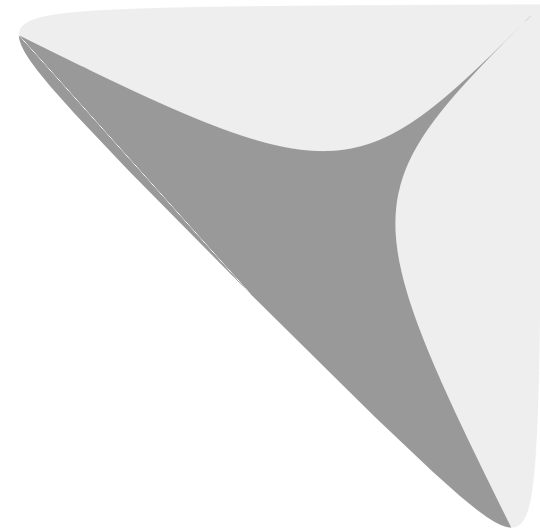
Consider (x, \bar{x}) phase space for $e^+e^- \rightarrow q\bar{q}g$



HERWIG



Comparison

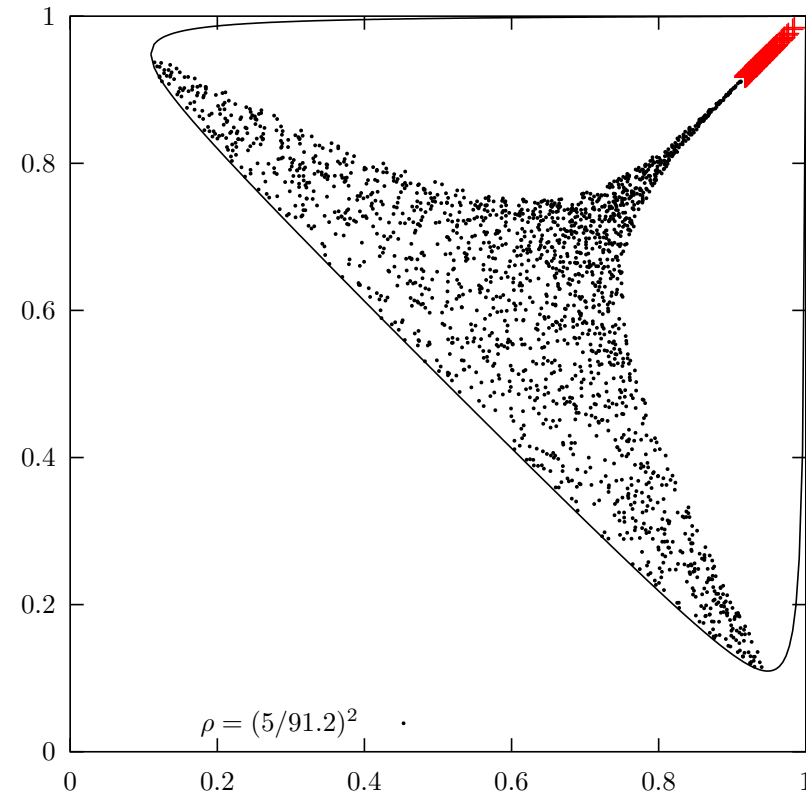


Herwig++

- ✗ Larger dead region with new variables.
- ✓ Smooth coverage of soft gluon region.
- ✓ No overlapping regions in phase space.

Hard Matrix Element Corrections

- Points (x, \bar{x}) in **dead region** chosen acc to LO $e^+e^- \rightarrow q\bar{q}g$ matrix element and accepted acc to ME weight.
- About **3%** of all events are actually hard $q\bar{q}g$ events.
- Red points have **weight** > 1 , practically no error by setting weight to one.
- Event **oriented** according to given $q\bar{q}$ geometry. Quark direction is kept with weight $x^2/(x^2 + \bar{x}^2)$.



Leading Log and Beyond

Neglecting Sudakovs, rate of one emission is:

$$\begin{aligned}\mathcal{P}_{q \rightarrow qg} &\approx \int \frac{dQ^2}{Q^2} \int dz \frac{\alpha_s}{2\pi} \frac{4}{3} \frac{1+z^2}{1-z} \\ &\approx \alpha_s \ln \left(\frac{Q_{\max}^2}{Q_{\min}^2} \right) \frac{8}{3} \ln \left(\frac{1-z_{\min}}{1-z_{\max}} \right) \sim \alpha_s \ln^2\end{aligned}$$

Rate for n emissions is of form:

$$\mathcal{P}_{q \rightarrow qng} \sim (\mathcal{P}_{q \rightarrow qg})^n \sim \alpha_s^n \ln^{2n}$$

Next-to-leading log (NLL): inclusion of *all* corrections of type $\alpha_s^n \ln^{2n-1}$

No existing generator completely NLL (NLLJET?), but

- energy-momentum conservation (and “recoil” effects)
- coherence
- $2/(1-z) \rightarrow (1+z^2)/(1-z)$
- scale choice $\alpha_s(p_{\perp}^2)$ absorbs singular terms $\propto \ln z, \ln(1-z)$ in $\mathcal{O}(\alpha_s^2)$ splitting kernels $P_{q \rightarrow qg}$ and $P_{g \rightarrow gg}$
- ...

⇒ far better than naive, analytical LL

Summary Lecture 2

- Hard processes: ●

- ★ Simple ones: probably built-in in PYTHIA/HERWIG ★
- ★ Multiparton LO: external generator + Les Houches Accord ★
 - ★ NLO: not easily related to physical events ★

- Parton Showers: ●

- ★ 2 kinds: initial-state and final-state ★
- ★ related to and derived from matrix elements ★
- ★ Sudakov form factor ensures sensible physics ★
 - ★ Ordering variable ambiguous: θ , p_{\perp}^2 , m^2 ★
- ★ Constraints from coherence arguments, and from data ★
 - ★ In state of continuous development ★

★ *More to come tomorrow!* ★