Small x Physics and the LHC

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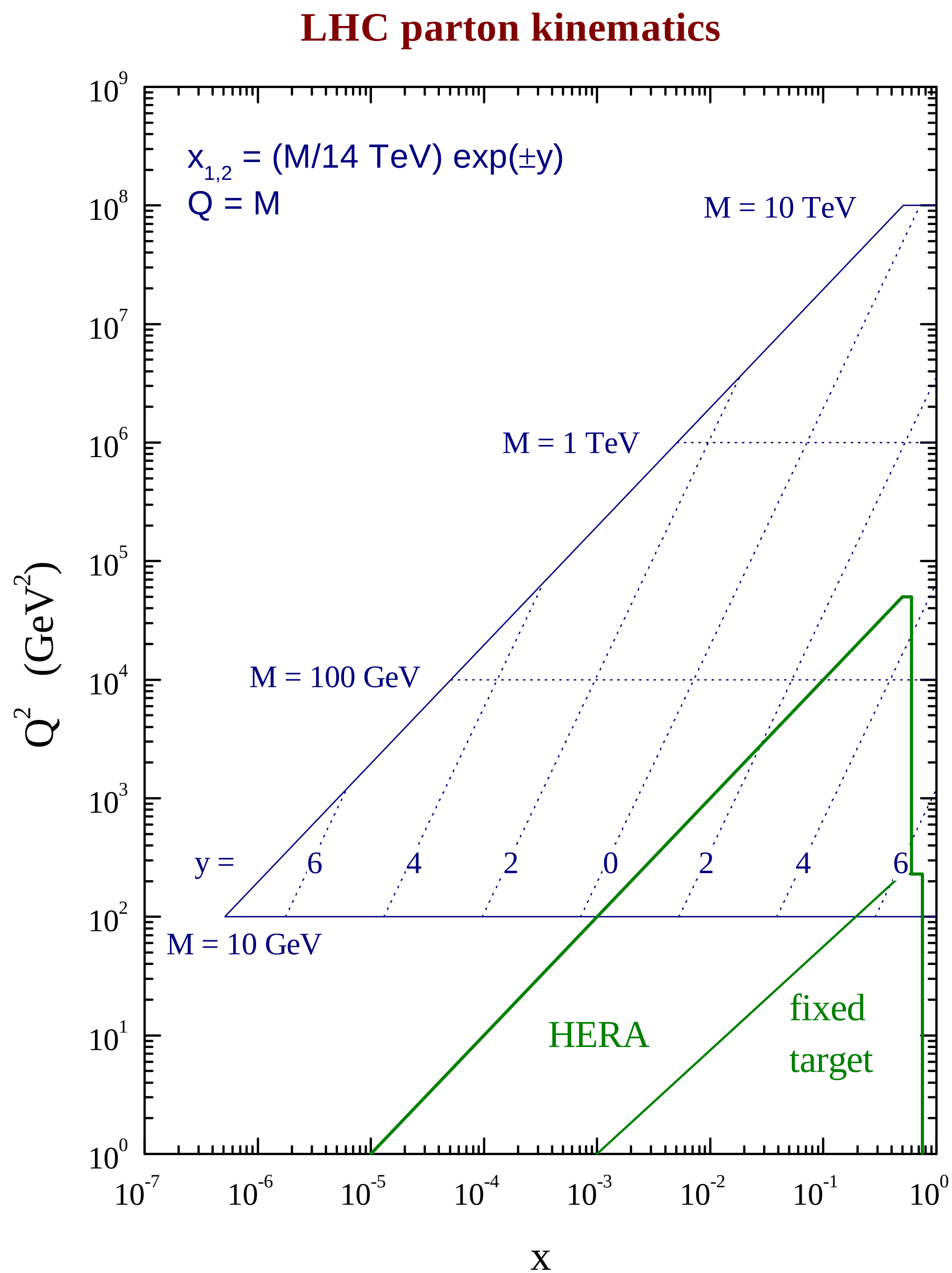
The kinematic range for particle production at the LHC is shown.

Smallish $x \sim 0.001 - 0.01$ parton distributions therefore vital for understanding the standard production processes at the LHC.

However, even smaller (and higher x) required when one moves away from zero rapidity, e.g when calculating total cross-section.



Small-x Theory

It is known that at each order in α_S each splitting function and coefficient function obtains an extra power of $\ln(1/x)$ (some accidental zeros in P_{gg}), i.e.

$$P_{ij}(x, \alpha_s(Q^2)), \quad C_i^P(x, \alpha_s(Q^2)) \sim \alpha_s^m(Q^2) \ln^{m-1}(1/x).$$

$\rightarrow$ no guarantee of convergence at small x!

$$x < 0.01, \qquad \ln(1/x) > 5, \qquad \rightarrow \alpha_S \ln(1/x) > 1.$$

The global fits usually assume that this turns out to be unimportant in practice, and proceed regardless.

I will do this for the moment. It is not necessarily a good approach.

Parton Uncertainties from experiment – currently an issue attracting a lot of work. Often assumed to be dominant source of uncertainty. Number of approaches.

Hessian (Error Matrix) approach first used by H1 and ZEUS, recently extended by CTEQ.

$$\chi^2 - \chi^2_{min} \equiv \Delta\chi^2 = \sum_{i,j} H_{ij}(a_i - a_i^{(0)})(a_j - a_j^{(0)})$$

We can then use the standard formula for linear error propagation.

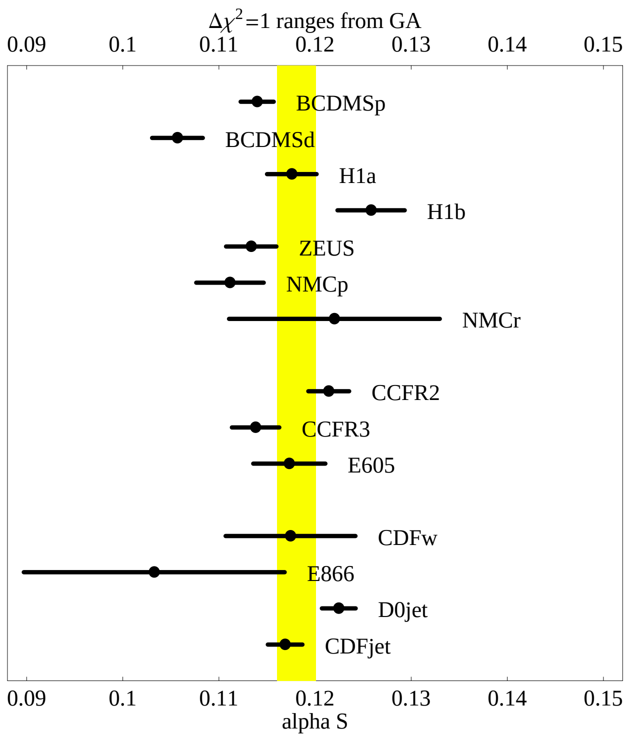
$$(\Delta F)^2 = \Delta\chi^2 \sum_{i,j} \frac{\partial F}{\partial a_i}(H)^{-1}_{ij}\frac{\partial F}{\partial a_j},$$

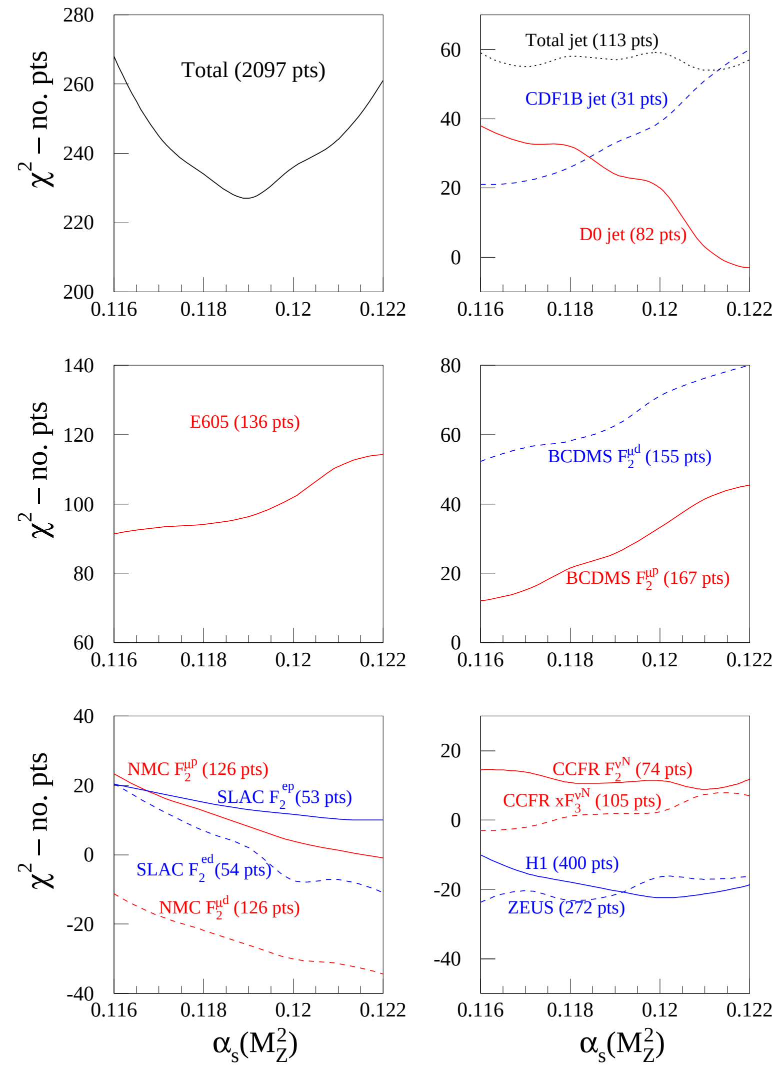
This has been used to find partons with errors by Alekhin and H1, each with restricted data sets.

Simple method problematic due to extreme variations in $\Delta\chi^2$ in different directions in parameter space - particularly with more parameters (more data). $\longrightarrow$ numerical instability.

Solved (helped) by finding and rescaling eigenvectors of H leading to diagonal form $\Delta\chi^2 = \sum_i z_i^2$. First used by CTEQ. Now used in slightly weaker form by MRST and ZEUS.

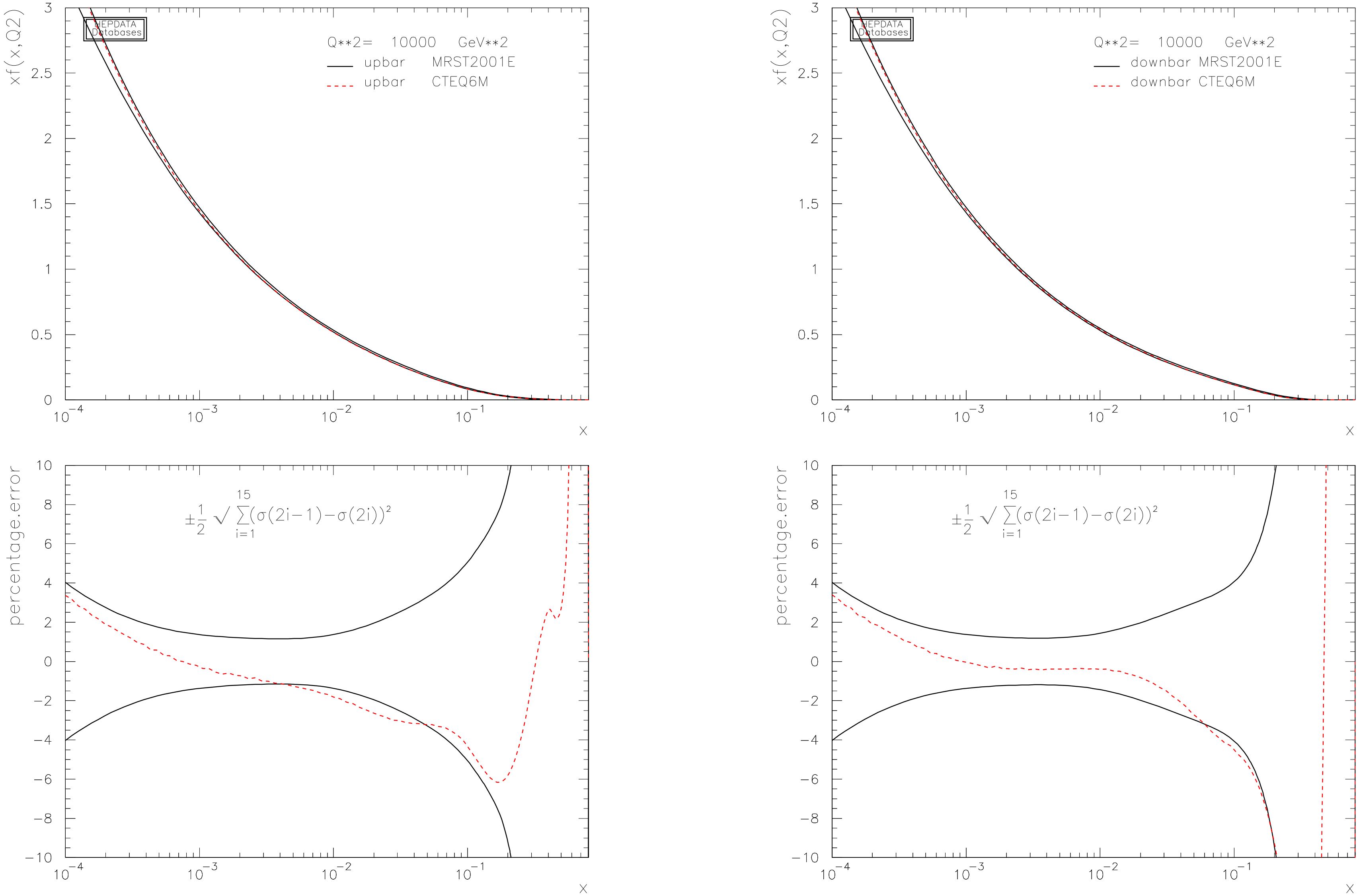
In full **global** fit art in choosing "correct" $\Delta\chi^2$ given complication of errors. Ideally $\Delta\chi^2 = 1$, but unrealistic.





Many approaches use $\Delta\chi^2 \sim 1$. CTEQ choose $\Delta\chi^2 \sim 100$ (conservative?). MRST choose $\Delta\chi^2 \sim 20$ for $1-\sigma$ error.

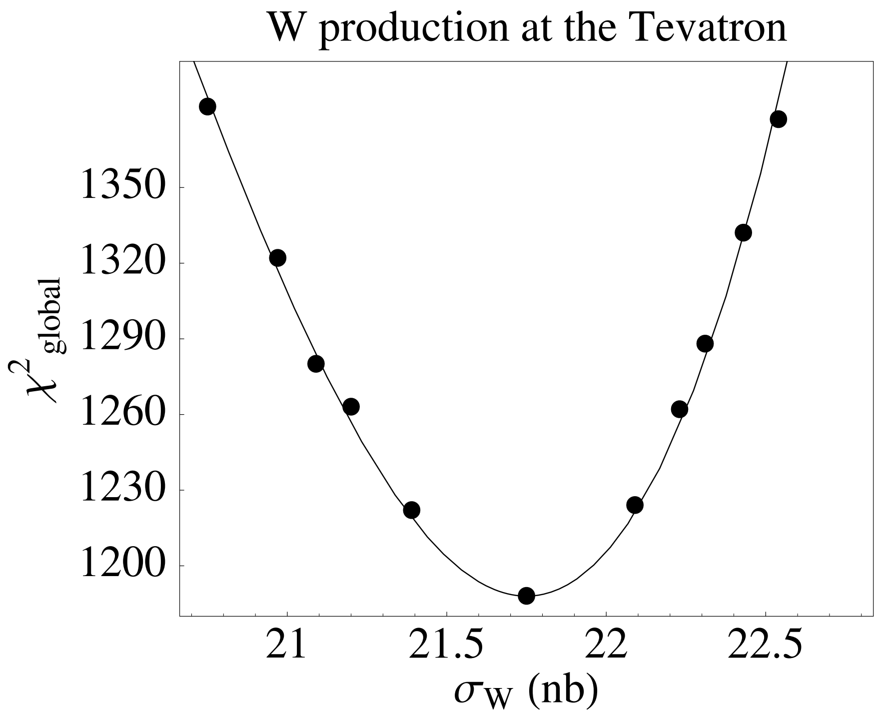
Uncertainty on MRST $\bar{u}$ and $\bar{d}$ distributions, along with CTEQ6. Central rapidity $x = 0.006$ is ideal for MRST uncertainty. CTEQ values very similar at this x.



Can also look at uncertainty on a given physical quantity using Lagrange Multiplier method, first suggested by CTEQ and concentrated on by MRST. Minimize

$$\Psi(\lambda, a) = \chi^2_{global}(a) + \lambda F(a).$$

Gives best fits for particular values of quantity $F(a)$ without relying on Gaussian approx for χ^2. Uncertainty then determined by deciding allowed range of $\Delta\chi^2$.



CTEQ obtain for $\alpha_S = 0.118$

$$\Delta\sigma_W(\mathrm{LHC}) \approx \pm 4\% \quad \Delta\sigma_{\mathrm{W}}(\mathrm{Tev}) \approx \pm 4$$

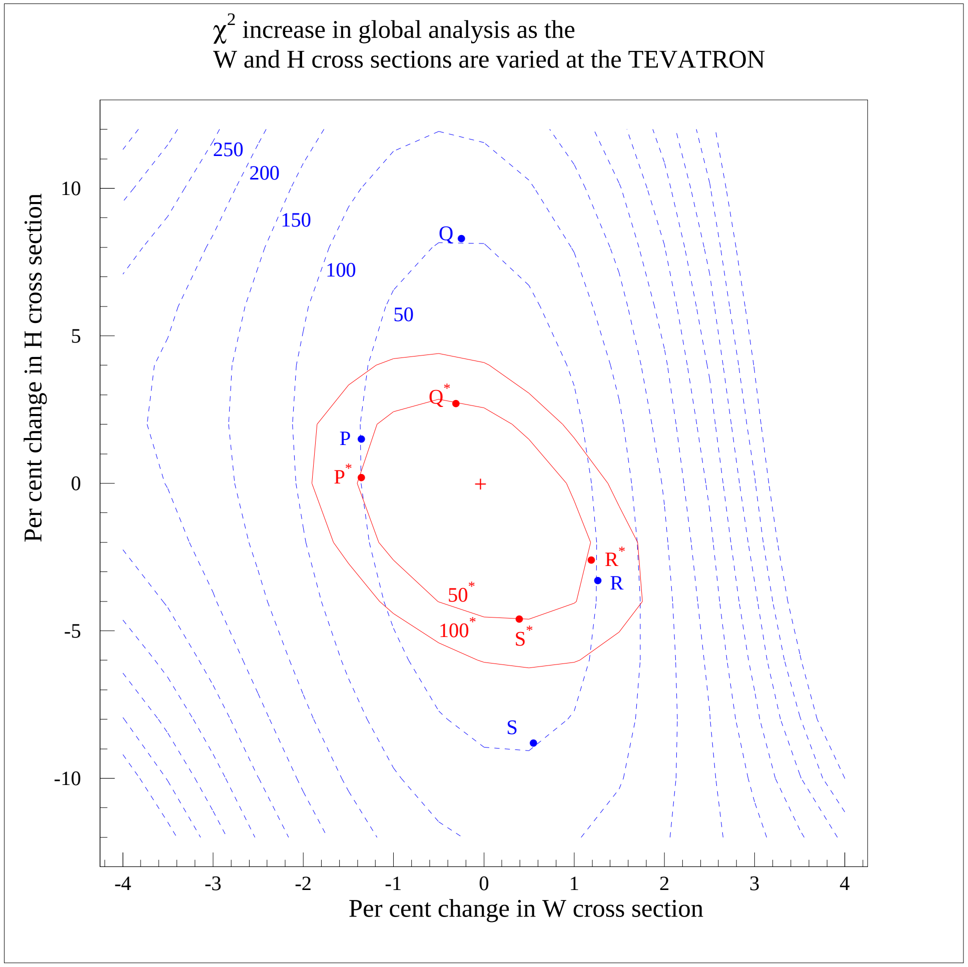
$$\Delta\sigma_H(\mathrm{LHC}) \approx \pm 5\%.$$

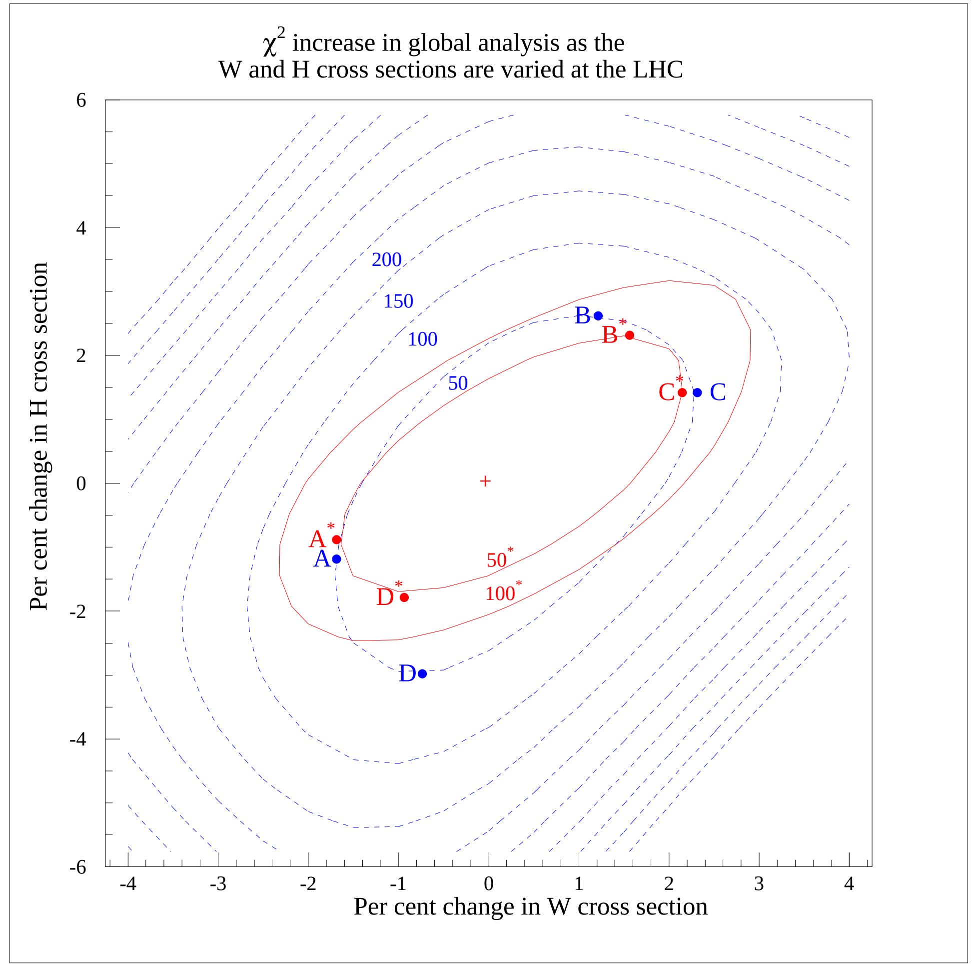
MRST use a wider range of data, and if $\Delta\chi^2 \sim 50$ find for $\alpha_S = 0.119$

$$\Delta\sigma_W(\mathrm{Tev}) \approx \pm 1.2\% \quad \Delta\sigma_{\mathrm{W}}(\mathrm{LHC}) \approx \pm 2\%$$

$$\Delta\sigma_H(\mathrm{Tev}) \approx \pm 4\% \quad \Delta\sigma_{\mathrm{H}}(\mathrm{LHC}) \approx \pm 2\%.$$

MRST also allow α_S to be free.





χ^2-plots for W and Higgs (120GeV) production at the Tevatron and LHC α_S free (blue) and fixed (red) at $\alpha_S = 0.119$.

Current best (MRST) estimate

$$\delta\sigma_{W,Z}^{\rm NLO}(\text{expt pdf}) = \pm 2\%$$

but note that there is a greater uncertainty in the NLO prediction, due to possible problems at small x in the global fit to DIS data.

This is because the large rapidity W and Z total cross-sections sample very small x

$\sigma(W^+)/\sigma(W^-)$ is **gold-plated**

$$R_\pm = \frac{\sigma(W^+)}{\sigma(W^-)} \simeq \frac{u(x_1)\bar{d}(x_2)}{d(x_1)\bar{u}(x_2)} \simeq \frac{u(x_1)}{d(x_1)}$$

since sea is u, d symmetric at small x, and using MRST2001E

$$\delta R_\pm(\text{expt. pdf}) = \pm 1.4\%$$

Assuming all other uncertainties cancel, this is probably the most accurate SM cross-section test at LHC.

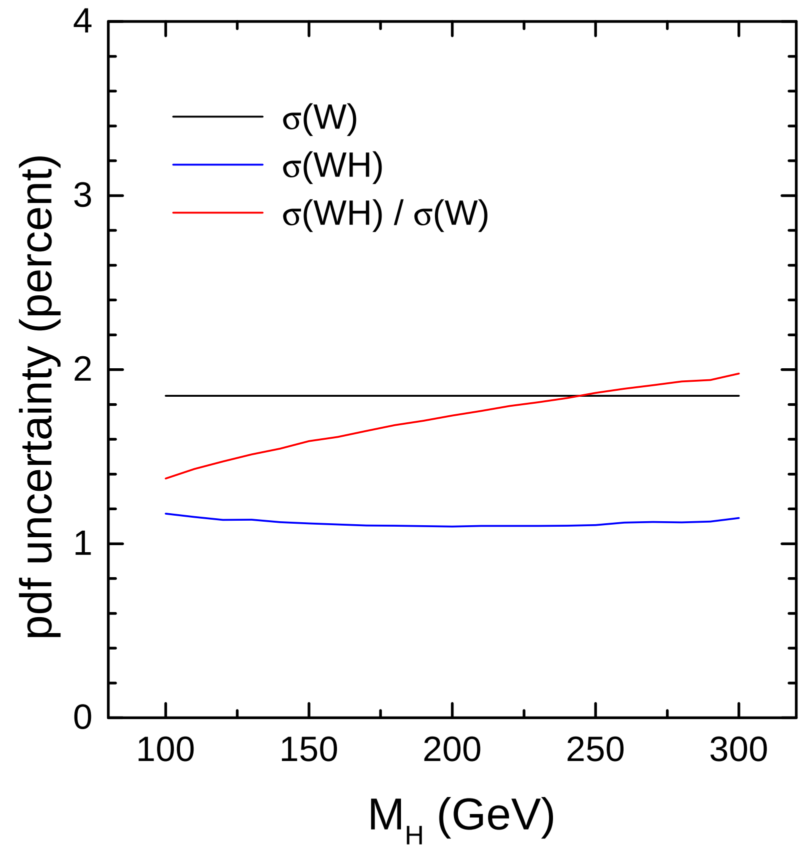
Could $\sigma(W)$ or $\sigma(Z)$ be used to calibrate other cross-sections, e.g. $\sigma(WH)$, $\sigma(Z')$?

$\sigma(WH)$ more precisely predicted because it samples quark pdfs at higher x, and scale, than $\sigma(W)$.

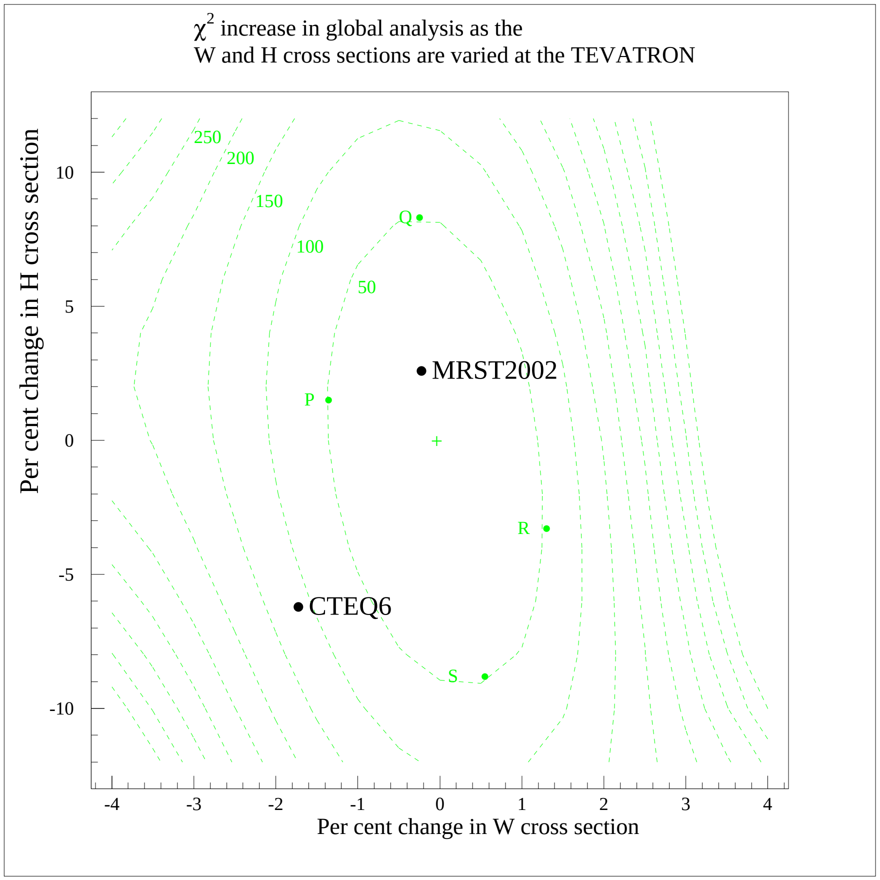
However, ratio shows no improvement in uncertainty, and can be worse.

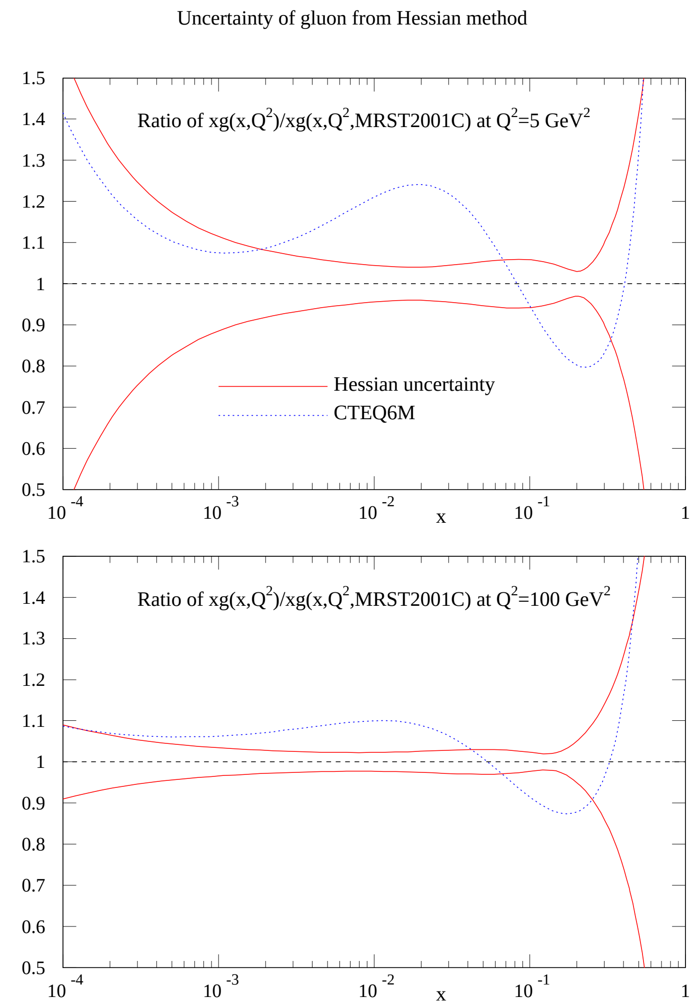
Partons in different regions of x are often anti-correlated rather than correlated, partially due to sum rules.

pdf uncertainties on W, WH
cross sections at LHC (MRST2001E)



Different approaches lead to similar accuracy of measured quantities, but can lead to different central values. Must consider effect of assumptions made during fit.





Cuts made on data, data sets fit, parameterization for input sets, form of strange sea, heavy flavour prescription, assumption of no isospin violation, strong coupling ......

Many can be as important as experimental errors on data used (or more so).

Results from LHC/LP Study Working Group (Bourilkov).

Table 1: Cross sections for Drell-Yan pairs (e^+e^-) with PYTHIA 6.206, rapidity < 2.5. The errors shown are the PDF uncertainties.

PDF set	Comment	xsec [pb]	PDF uncertainty %
$81 < M < 101$ GeV			
CTEQ6	LHAPDF	1065 $\pm$ 46	4.4
MRST2001	LHAPDF	1091 $\pm$ ...	3
Fermi2002	LHAPDF	853 $\pm$ 18	2.2

Comparison of $\sigma_W \cdot B_{l\nu}$ for MRST2002 and Alekhin partons.

PDF set	Comment	xsec [nb]	PDF uncertainty
Alekhin	Tevatron	2.73	$\pm$ 0.05 (tot)
MRST2002	Tevatron	2.59	$\pm$ 0.03 (expt)
CTEQ6	Tevatron	2.54	$\pm$ 0.10 (expt)
Alekhin	LHC	215	$\pm$ 6 (tot)
MRST2002	LHC	204	$\pm$ 4 (expt)
CTEQ6	LHC	205	$\pm$ 8 (expt)

In both cases differences (mainly) due to detailed constraint (by data) on quark decomposition.

Problems in the fit.

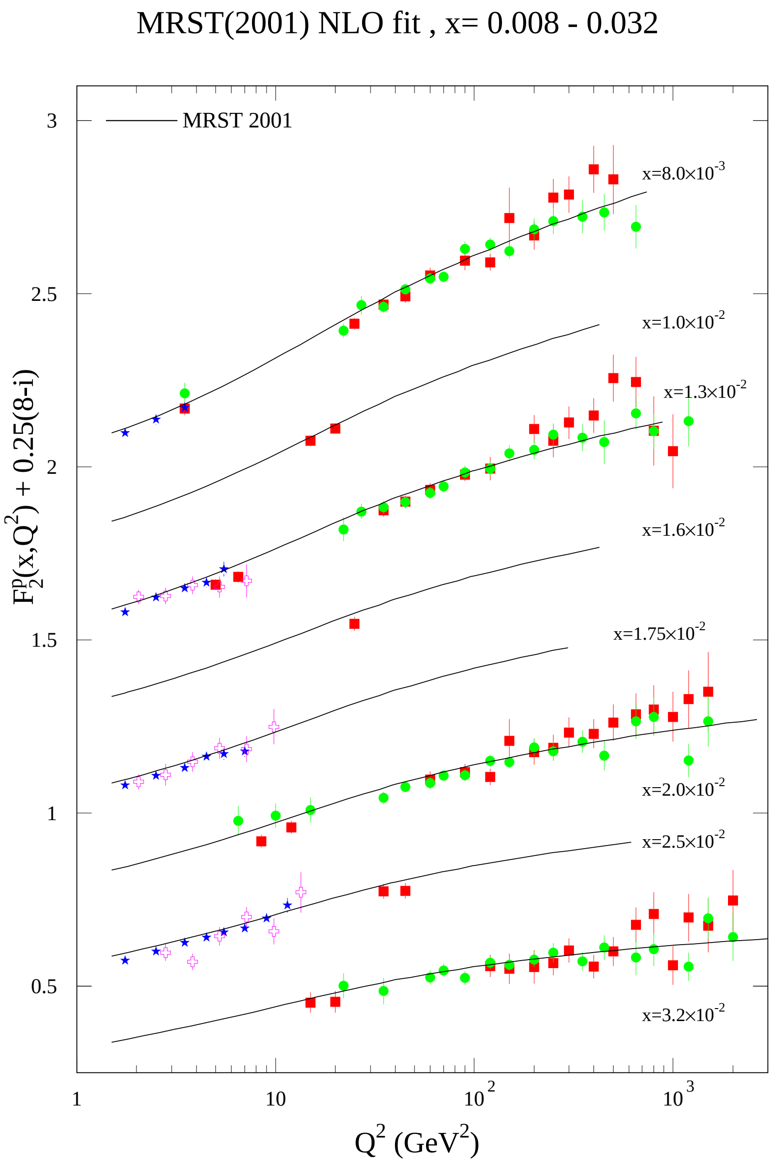
Variations from different approaches partially due to inadequacy of theory .

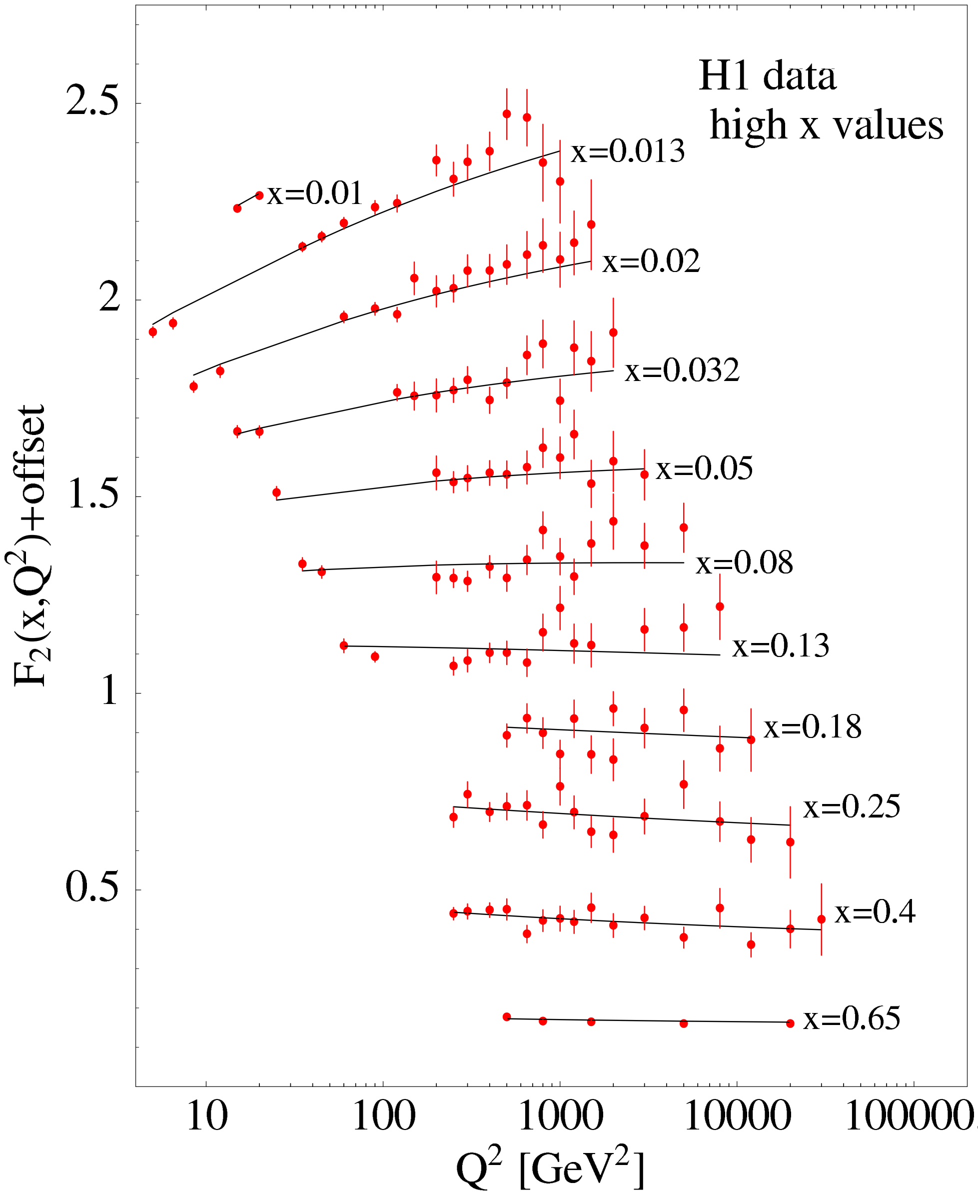
Failings of NLO QCD indicated by some areas where fit quality could be improved.

Good fit to HERA data, but some problems at highest Q^2 at moderate x, i.e. in $dF_2/d\ln Q^2$. $\rightarrow$ possible underestimate of quarks in this region.

Want more gluon in the $x \sim 0.01$ range, and/or larger $\alpha_S(M_Z^2)$.

Possible sign of required $\ln(1/x)$ corrections.



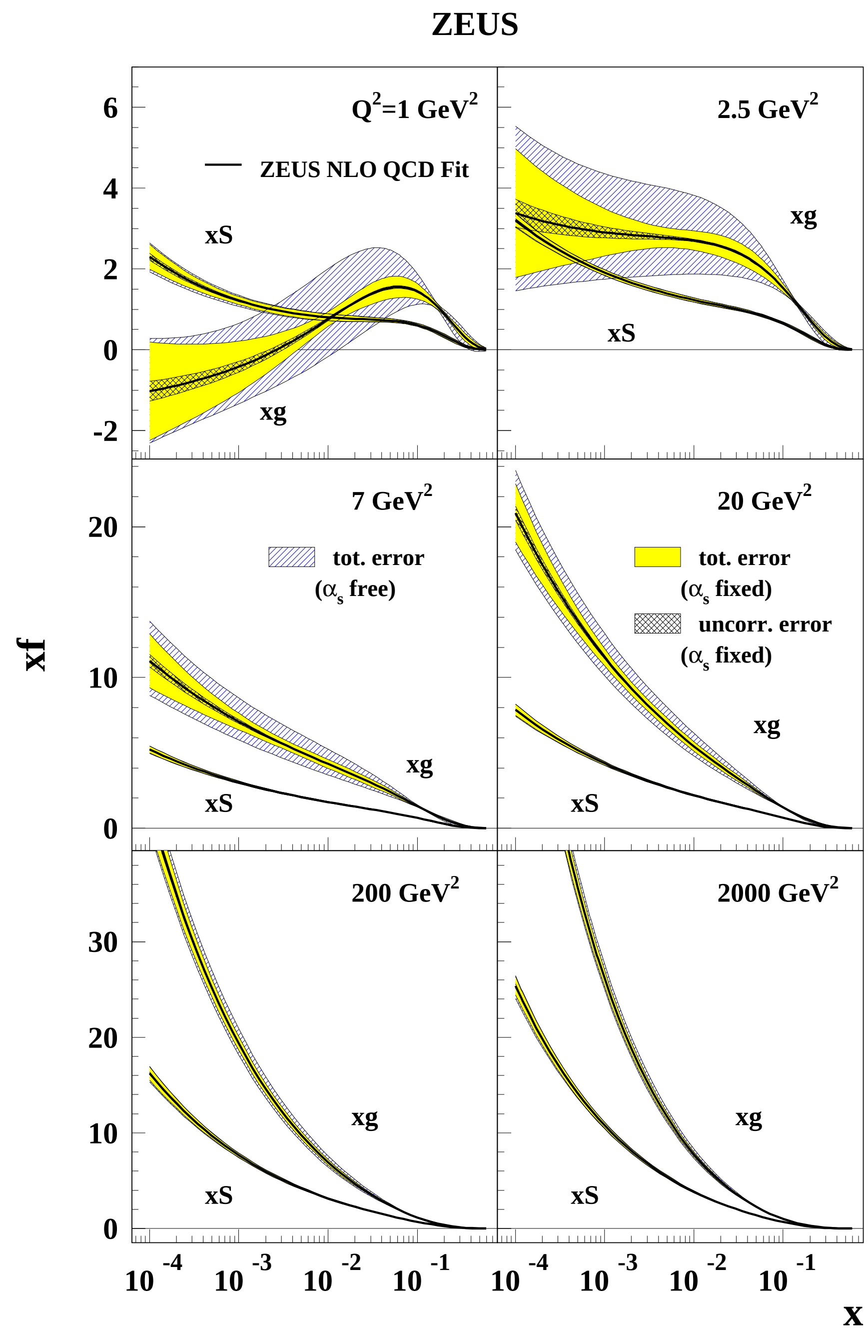


Comparison of MRST(2001) $F_2(x, Q^2)$ with HERA, NMC and E665 data (left) and of CTEQ6 $F_2(x, Q^2)$ and H1 data.

Data require gluon to be negative at low Q^2, e.g. MRST $Q_0^2 = 1\mathrm{GeV}^2$. Needed by all data (e.g Tevatron jets) not just low Q^2 low x data.

$\rightarrow$ $F_L(x, Q^2)$ dangerously small at smallest x, Q^2.

Other groups find similar problems with gluon and/or $F_L(x, Q^2)$ at low x, e.g. ZEUS.

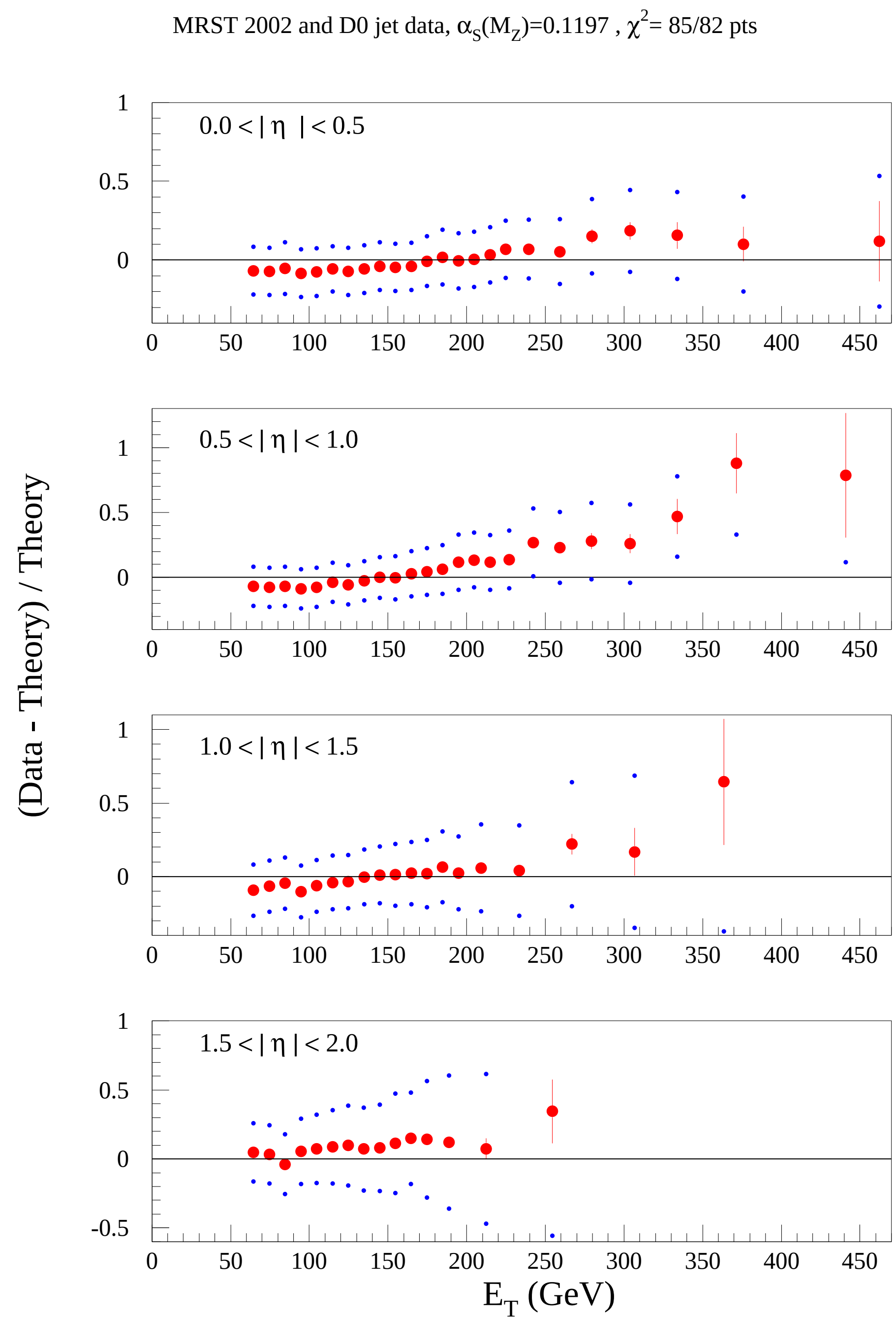


Difficult to reconcile fit to jets and rest of data.

MRST find a reasonable fit to jet data, but need to use the large systematic errors.

Better for CTEQ6 largely due to different cuts on other data. Usually worse for other partons (jets not in fits). General tension between HERA and NMC data and jets.

In general different data compete over the gluon and $\alpha_S(M_Z^2)$.



Theoretical Errors

Hence it is vital to consider theoretical corrections. These include ....

- higher orders (NNLO)
- small x $(\alpha_s^n \ln^{n-1}(1/x))$
- large x $(\alpha_s^n \ln^{2n-1}(1-x))$
- low Q^2 (higher twist)

In order to investigate true theoretical error must consider large and small x resummations.

NNLO

Splitting functions now complete. (Moch, Vermaseren and Vogt). Extremely similar to average of best estimates $\rightarrow$ no real change in NNLO partons. Improve quality of fit very slightly (MRST), and reduces $\alpha_S \rightarrow 0.118$.

To do absolutely correct NNLO fit we need not only exact NNLO splitting functions.

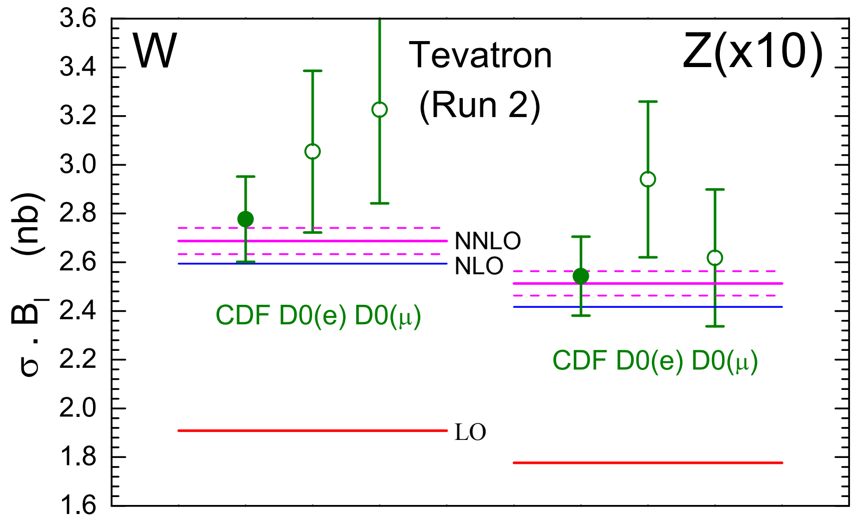
Also require rigorous heavy quark thresholds (partons **discontinuous** at NNLO - see Heavy Flavours talk), NNLO Drell-Yan cross-sections, and a complete treatment of uncertainties. All in hand.

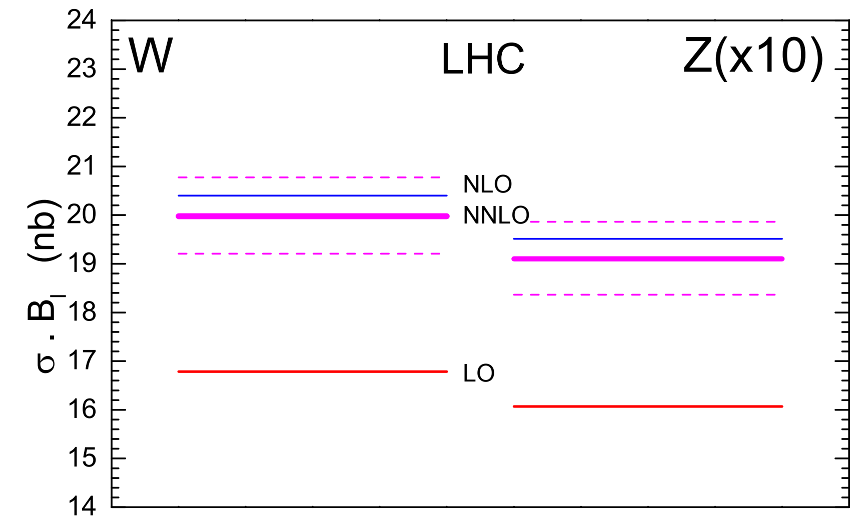
Essentially full NNLO determination of partons very soon.

Only NNLO jet cross-sections missing. Likely to be small correction to NLO.

Reasonable stability order by order for (quark-dominated) W and Z cross-sections.

This fairly good convergence is largely guaranteed because the quarks are fit directly to data. Much worse for gluon dominated quantities e.g. $F_L(x,Q^2)$. Unstable at small x and Q^2.

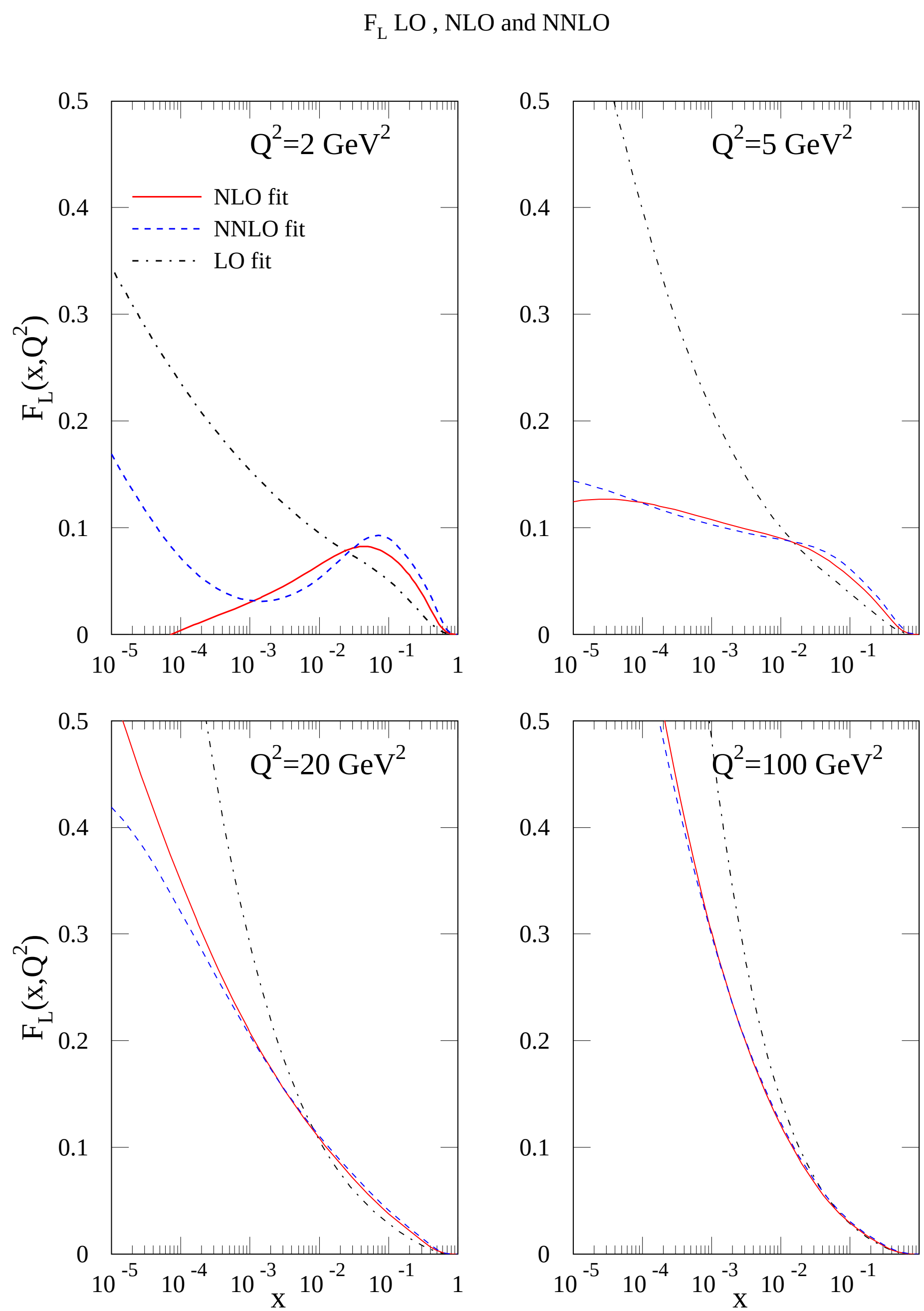




partons: MRST2002
NNLO evolution: van Neerven, Vogt approximation to Vermaseren et al. moments
NNLO W,Z corrections: van Neerven et al. with Harlander, Kilgore corrections

$F_L(x, Q^2)$ predicted from the global fit at LO, NLO and NNLO.

NNLO coefficient functions and splitting functions lead to big changes at small x.



Alternative approach.

In order to investigate real quality of fit and regions with problems vary kinematic cuts on data.

Procedure – change W^2_{cut}, Q^2_{cut} and x_{cut}, re-fit and see if quality of fit to remaining data improves and/or input parameters change dramatically. Continue until quality of fit and partons stabilize.

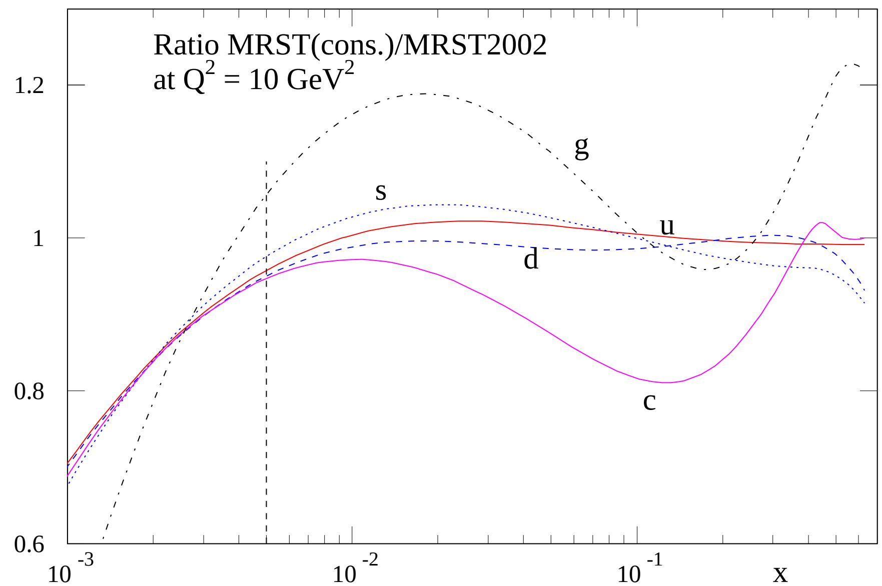
For W^2_{cut} raising from $12.5\mathrm{GeV}^2$ to $15\mathrm{GeV}^2$ sufficient.

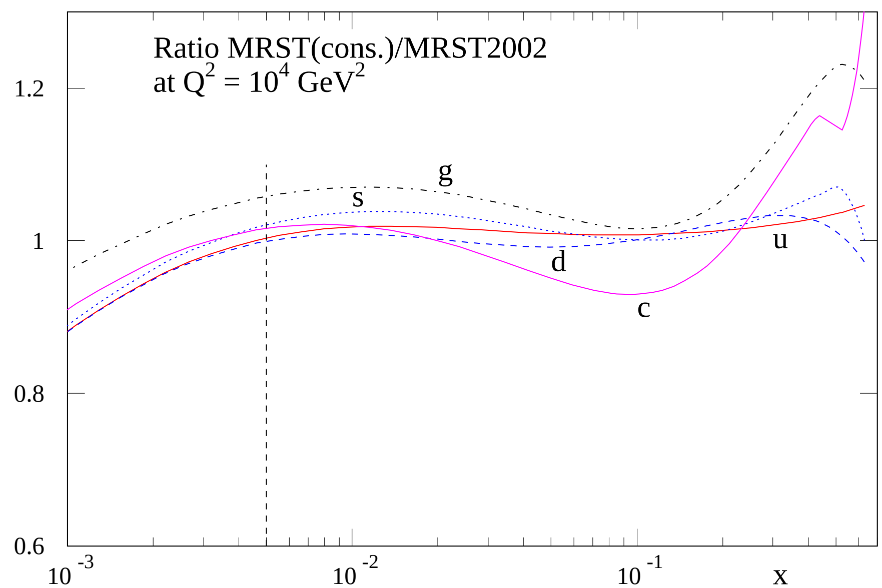
Raising Q^2_{cut} from $2\mathrm{GeV}^2$ in steps there is a slow continuous and significant improvement for higher Q^2 up to $> 10\mathrm{GeV}^2$ (cut 560 data points) – suggests any corrections mainly higher orders not higher twist.

Raising x_{cut} from 0 to 0.005 (cut 271 data points) continuous improvement. At each step moderate x gluon becomes more positive.

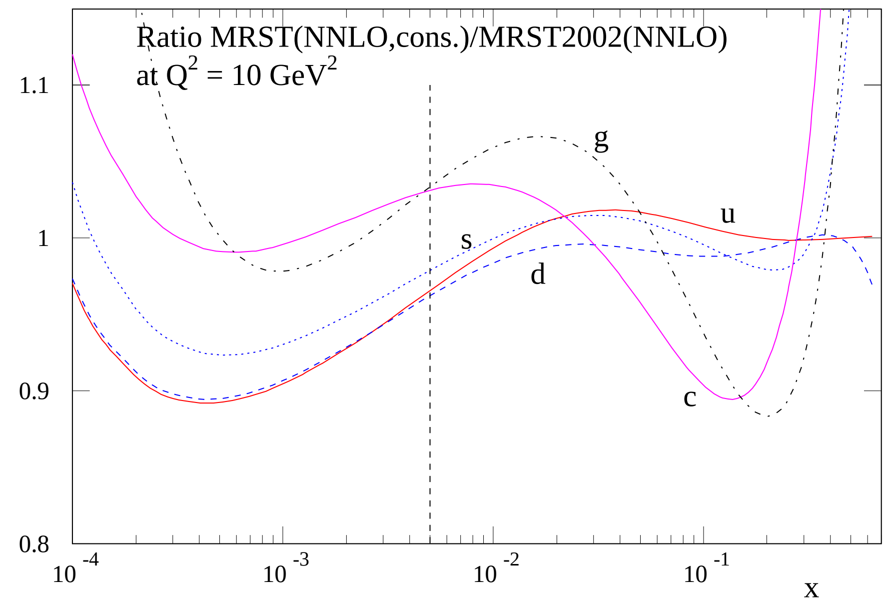
$\rightarrow$ MRST2003 conservative partons. Should be most reliable method of parton determination ($\Delta\chi^2 = -70$ for remaining data), but only applicable for restricted range of x, Q^2. $\rightarrow \alpha_S(M_Z^2) = 0.1165 \pm 0.004$.

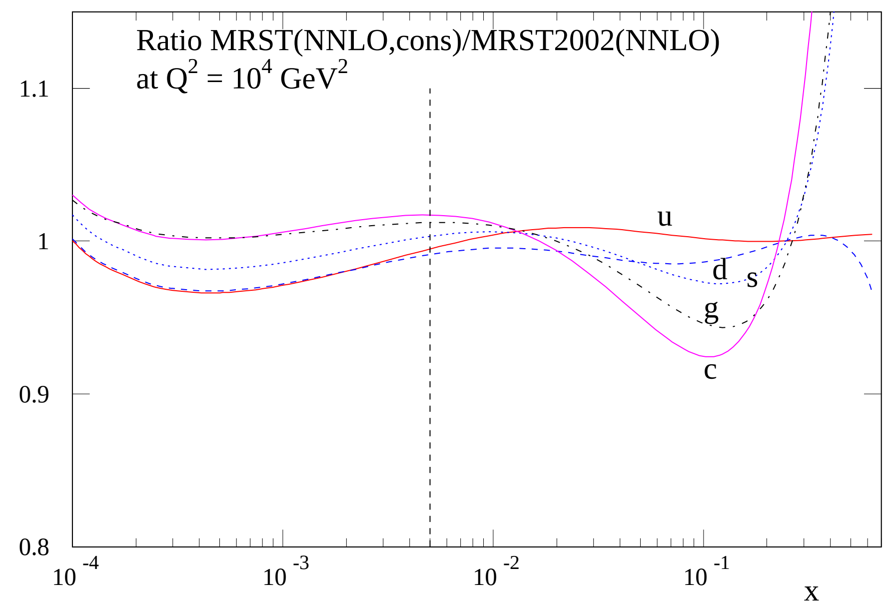
The ratio of the conservative partons to the default partons at NLO. One can see the dip of the conservative partons below $x_{cut} = 0.005$.



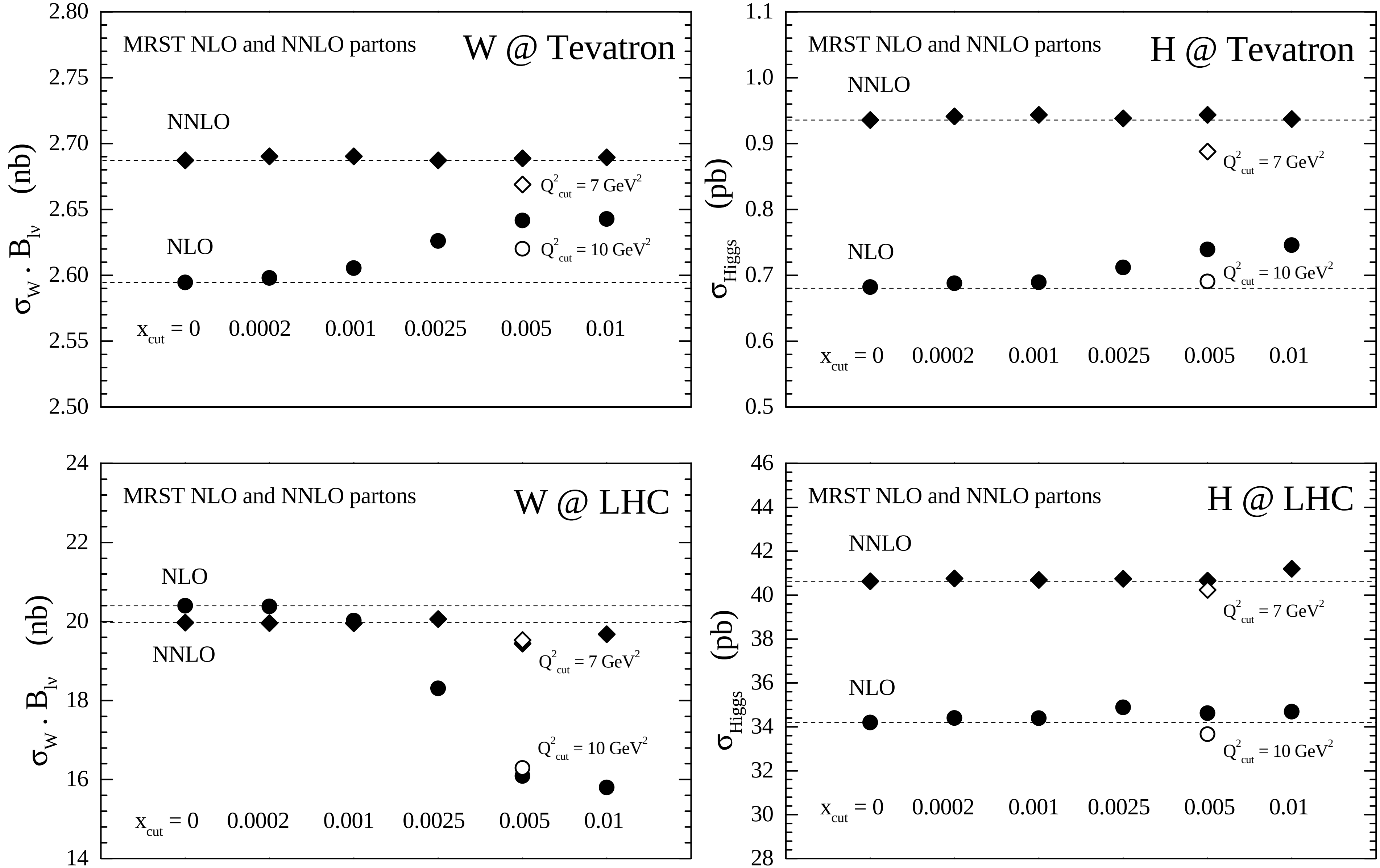


The ratio of the conservative partons to the default partons at NNLO. This time the partons are similar below $x_{cut} = 0.005$, but the degree of similarity may be partially accidental.

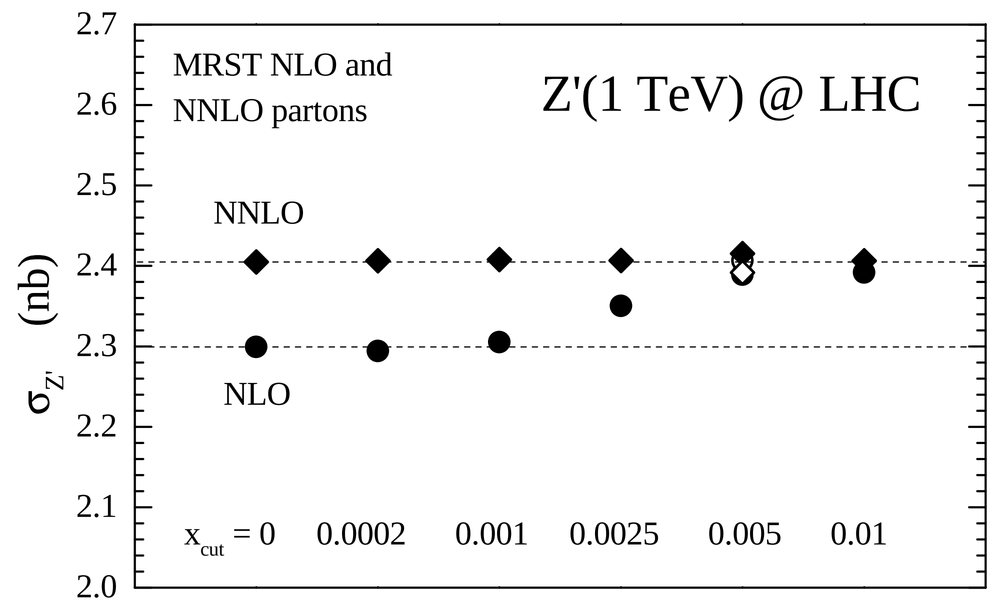


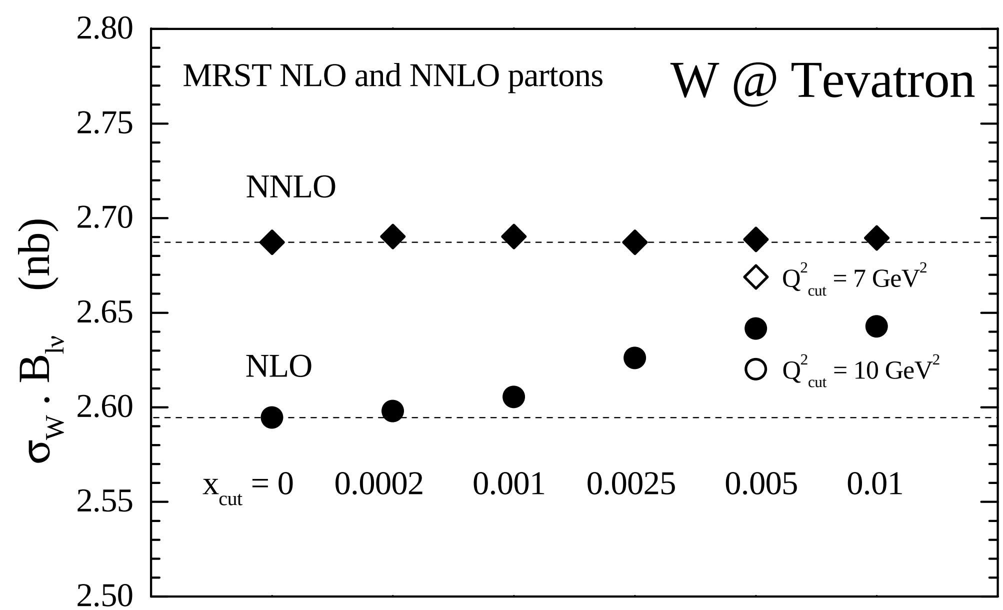


Variation in predictions with cuts. Follows patterns expected. Range of possible theoretical error.



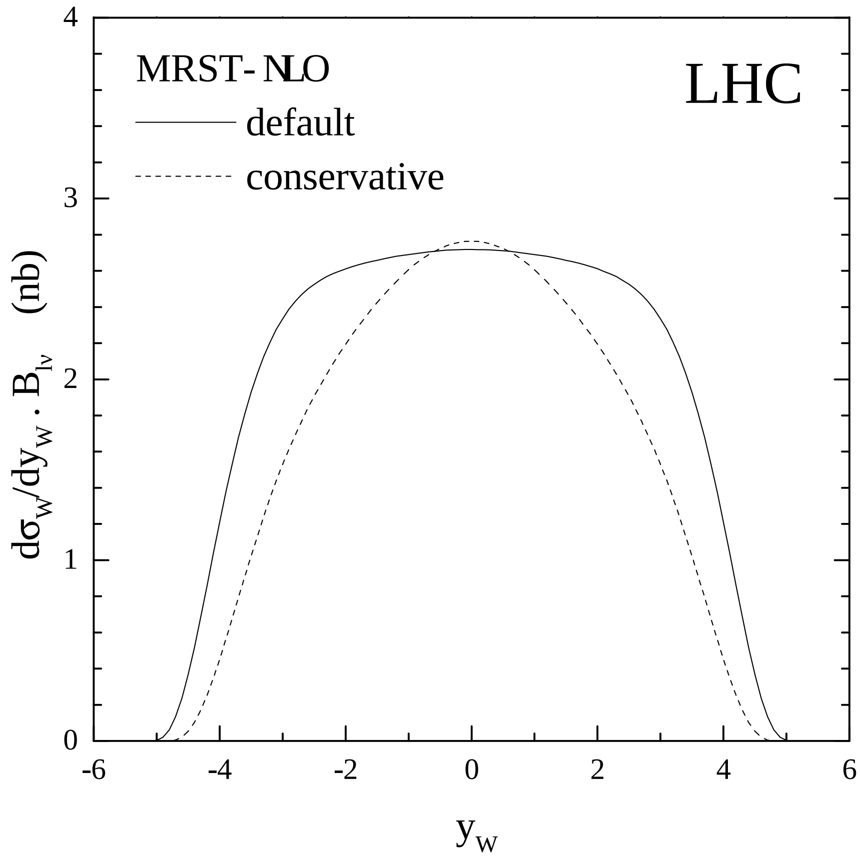
A change in the mass of the vector boson is very similar to a change in centre of mass energy for a fixed mass. Hence the variation with cuts for Z' with mass $1000\ \mathrm{GeV}$ is similar to the that for W production at the Tevatron rather than the LHC.





Comparison of prediction for $(d\sigma_W/dy_W)$ for the standard MRST partons and the conservative set. The reduction in the total cross-section in the latter case is clearly due to the huge reduction at high y_W and represents the possible type of theoretical uncertainty in this region when working at NLO.

Note a slight increase in cross-section for $y_W = 0$ $(x = 0.006)$. Due to increased evolution of quarks here.



CTEQ **results**

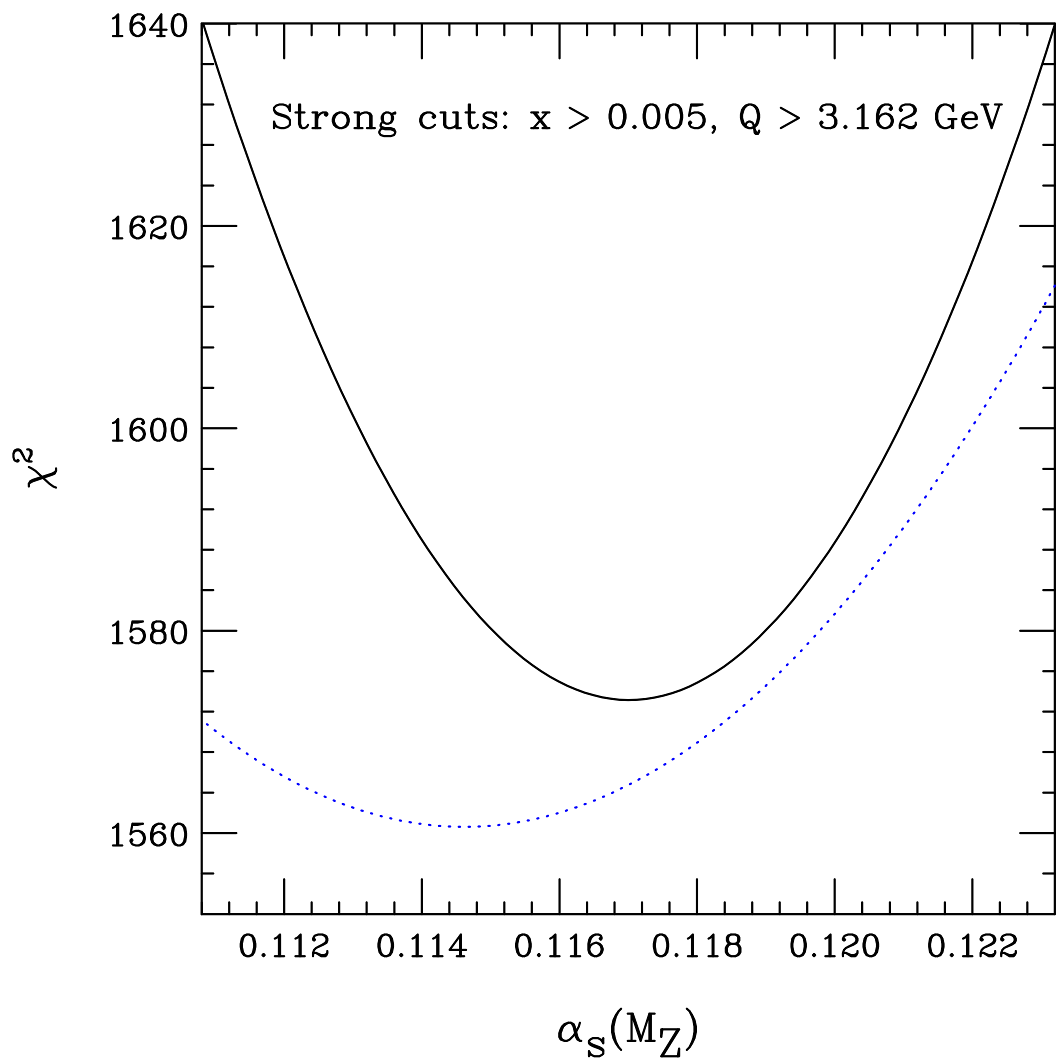
CTEQ see similar type of behaviour with cuts, though not as dramatic.

With conservative cuts on data their input gluon is as keen to have negative component (remember $Q_0^2 = 1.69\mathrm{GeV}^2$), and best value of $\alpha_S(M_Z^2)$ moves lower.

Blue line – negative gluon allowed.

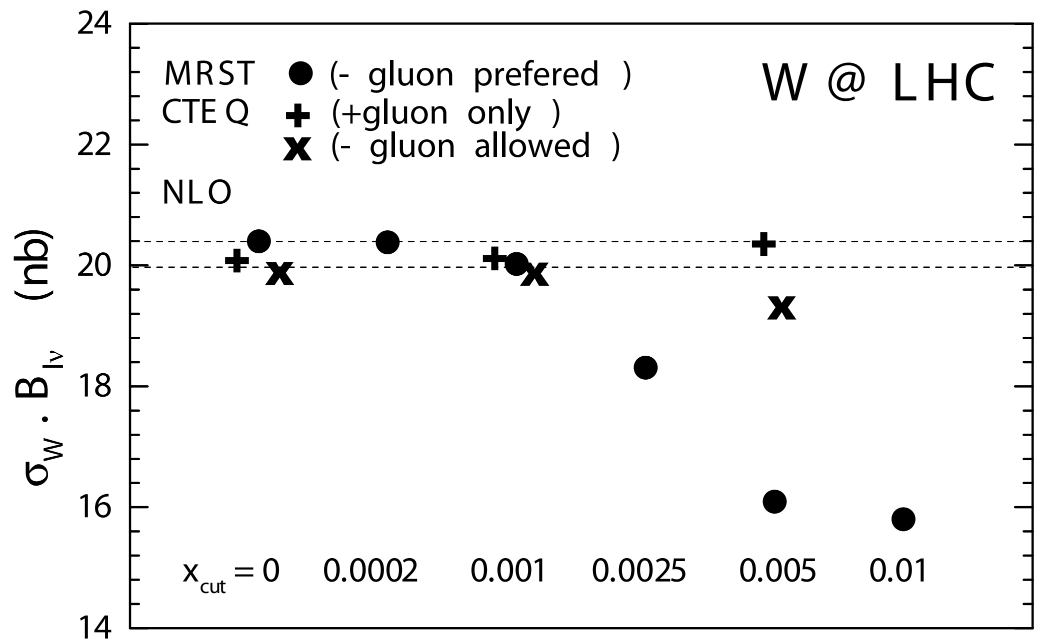
Black line – positive definite gluon.

Verifies negative/small gluon at low x and Q^2 **not** due to data at low x and Q^2.



Prediction stability.

Also find prediction for σ_W at the LHC moves down a little as more cuts imposed. Not as significant as MRST by a long way, it appears.



However, loss of data leads to larger errors, and χ^2 profile is very flat indeed in the downwards direction.

Not really any inconsistency with MRST.

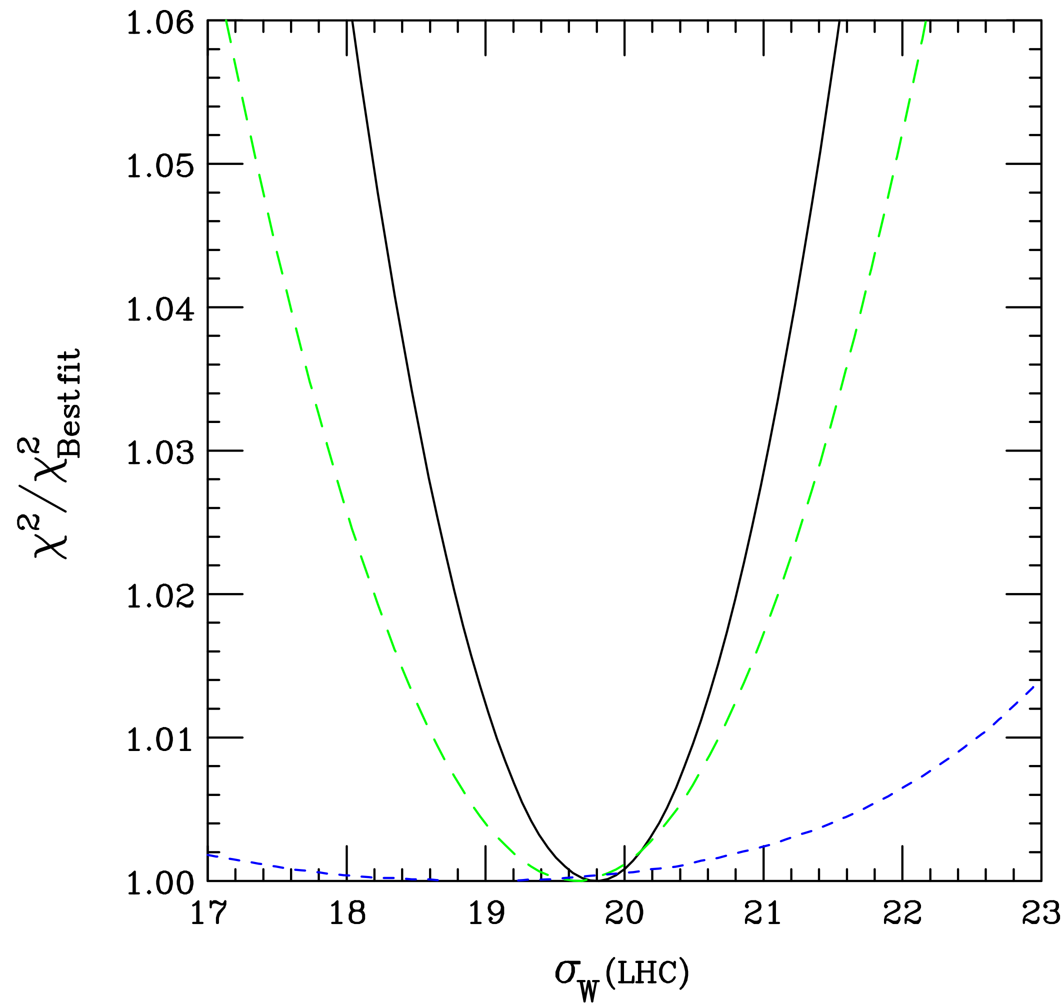
If one is cautious about accuracy of theory at low x and Q^2, conclusion that uncertainty large on small x-sensitive quantities holds. CTEQ claim no reason to be cautious.

blue line - conservative cuts

green line - semi-conservative cuts

black line - normal cuts.

Not so much of an issue at NNLO though.



Gluon outside conservative range very negative, and $dF_2(x,Q^2)/d\ln Q^2$ incorrect, (NNLO much more stable than NLO). Theory corrections could cure this (quite plausible). Empirical resummation corrections improve global fit, e.g.

$$P_{gg} \rightarrow .... + \frac{3.86\bar{\alpha}_S^4}{x}\left(\frac{\ln^3(1/x)}{6} - \frac{\ln^2(1/x)}{2}\right),$$

$$P_{qg} \rightarrow .... + 5.12\alpha_S\frac{N_f\bar{\alpha}_S^4}{3\pi x}\left(\frac{\ln^3(1/x)}{6} - \frac{\ln^2(1/x)}{2}\right).$$

Cuts suggestive of possible/probable theoretical errors for small x and/or small Q^2.

Much explicit work on $\ln(1/x)$-resummation in structure functions and parton distributions - RT, Ciafaloni, Colferai, Salam and and Stasto, Altarelli, Ball and Forte, ...

Can suggest improvements to fit and changes in predictions.

Resummations

Try alternative perturbative organization – resummation of leading $\ln(1/x)$ terms. Obtained by solving BFKL equation for unintegrated gluon distribution $f(k^2)$.

$$f(k^2,x) = f_I(Q_0^2) + \int_x^1 \frac{dx'}{x'}\bar{\alpha}_S \int_0^\infty \frac{dq^2}{q^2} K_0(q^2,k^2) f(q^2,x)$$

Iterative equation sums terms of form $\frac{\alpha_S^n \ln^{n-1}(1/x)}{x}$ in gluon distribution

For fixed coupling and at LO $\ln(1/x)$-resummation $xg(x,Q^2) \sim x^{-12\ln 2/\pi\alpha_S}$.

$12\ln 2/\pi\alpha_s \geq 0.5$ – far too steep in practice.

NLO correction to kernel $\rightarrow$ change in asymptotic power

$$xg(x,Q^2) \sim x^{-\lambda}, \qquad \lambda = \frac{12\ln 2}{\pi}\alpha_S(1-6.4\alpha_S)$$

Seemingly unstable expansion.

Beyond LO must remember $\alpha_s(Q^2)$ runs with scale.

$\rightarrow \infty$ as $Q^2 \rightarrow \Lambda^2_{QCD}$, $\rightarrow 0$ as $Q^2 \rightarrow \infty$.

In BFKL low scales affect normalization of gluon. divergent coupling render input gluon incalculable (nonperturbative).

Evolution upwards involves sensitivity to higher scales, i.e. weaker coupling.

Don't know input gluon but in splitting functions (very roughly)

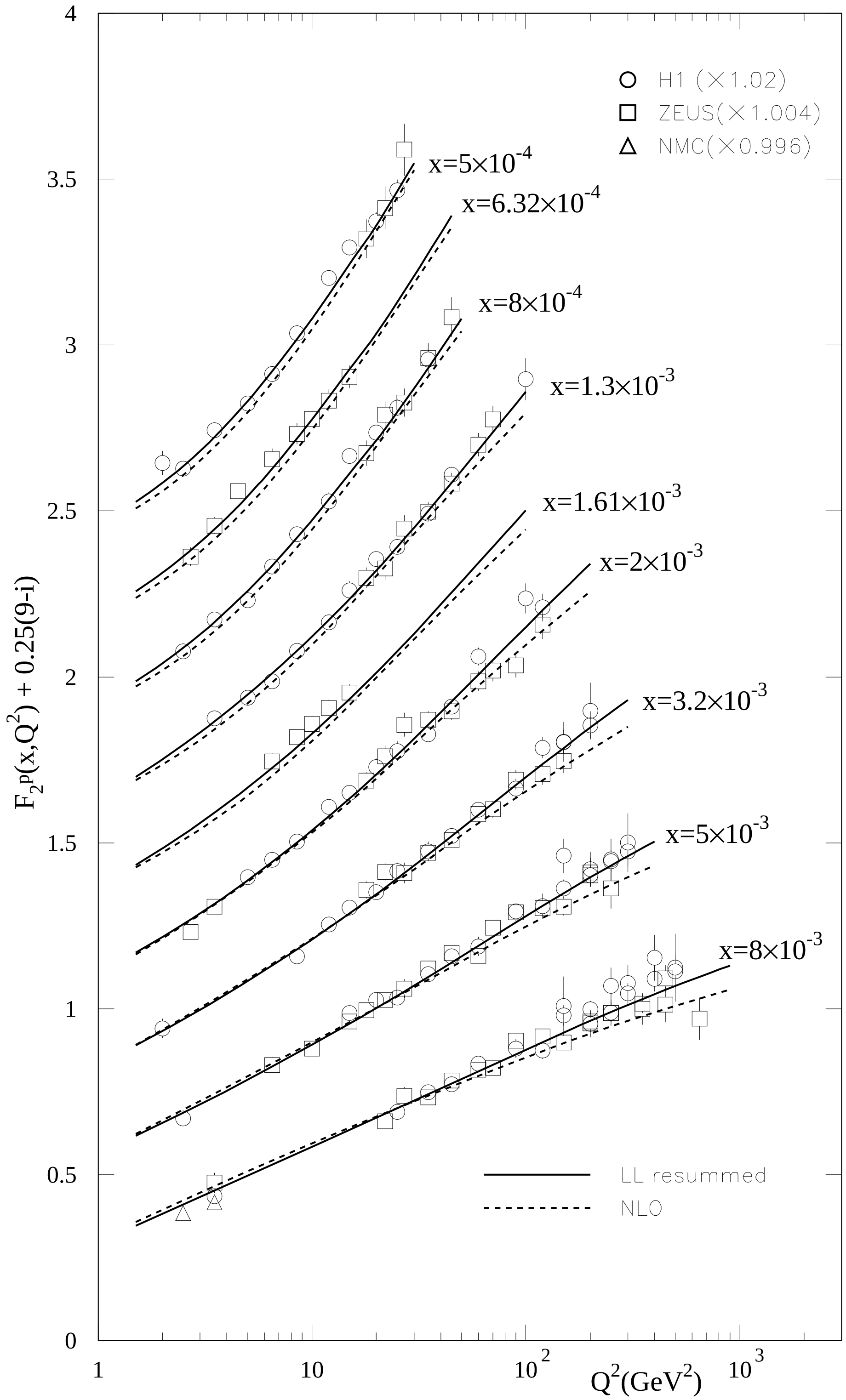
$$xP_{gg}(x, Q^2) \sim x^{-\lambda}, \qquad \lambda \sim \alpha_S(Q^2)(1 - 1.5\alpha_S(Q^2))$$

Qualitatively running coupling leads to change in coupling scale

Both conventional α_S expansion and fixed coupling $\ln(1/x)$ expansion show instability. The doubly resummed calculation (plus other possible higher order improvements to resummation) stabilizes the perturbative expansion. (RT, White, Altarelli, Ball and Forte, Ciafaloni, Colferai, Salam and Stasto).

Also leads to better fit than NLO-in-α_S, particularly in terms of $dF_2(x,Q^2)/d\ln Q^2$. Empirically, global fits prefer small x resummations which speed evolution, dislike higher twist (saturation) corrections which slow evolution.

Improvement of fit to small x HERA data (within global fit), due to $\ln(1/x)$ and β_0 double resummation compared to standard NLO in α_S. (White, RT)

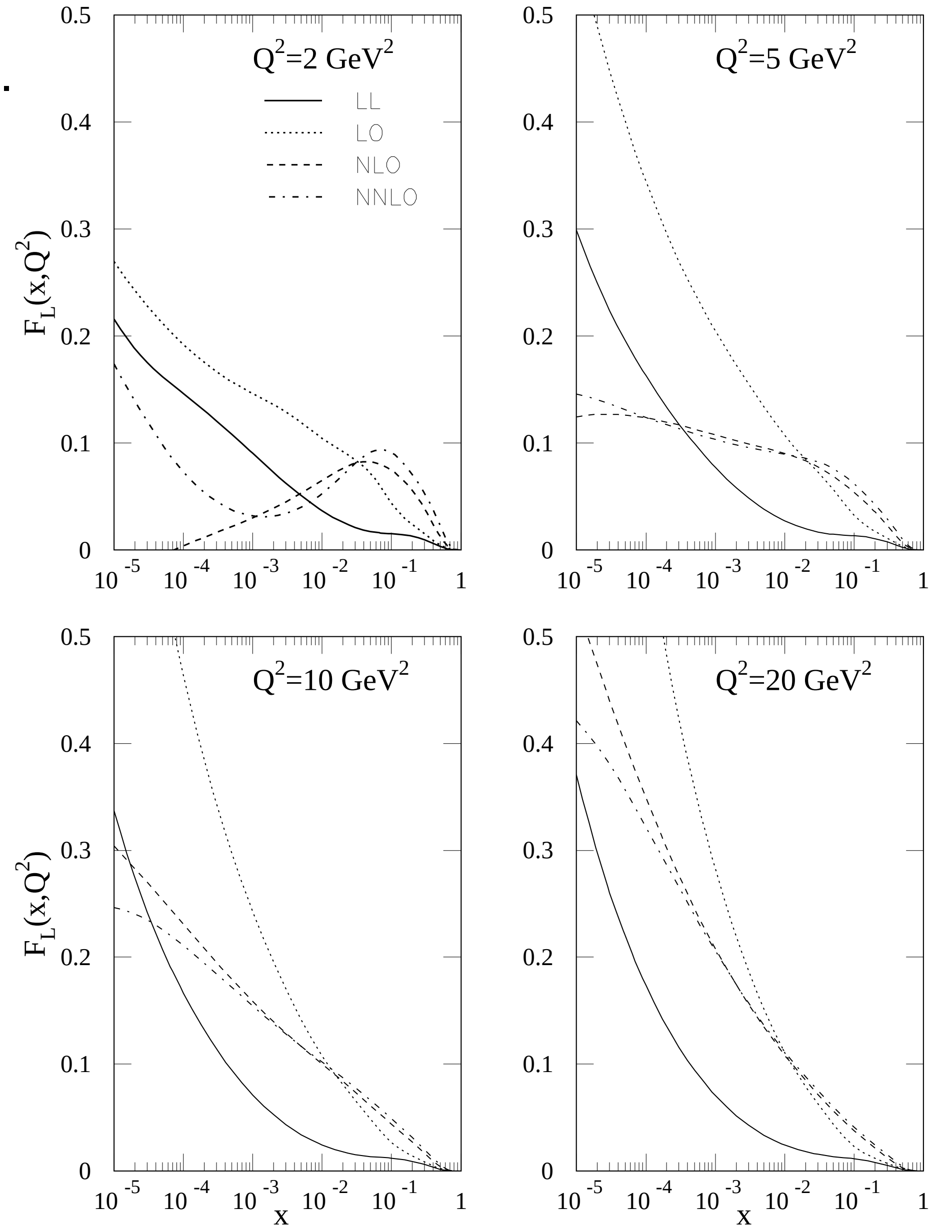


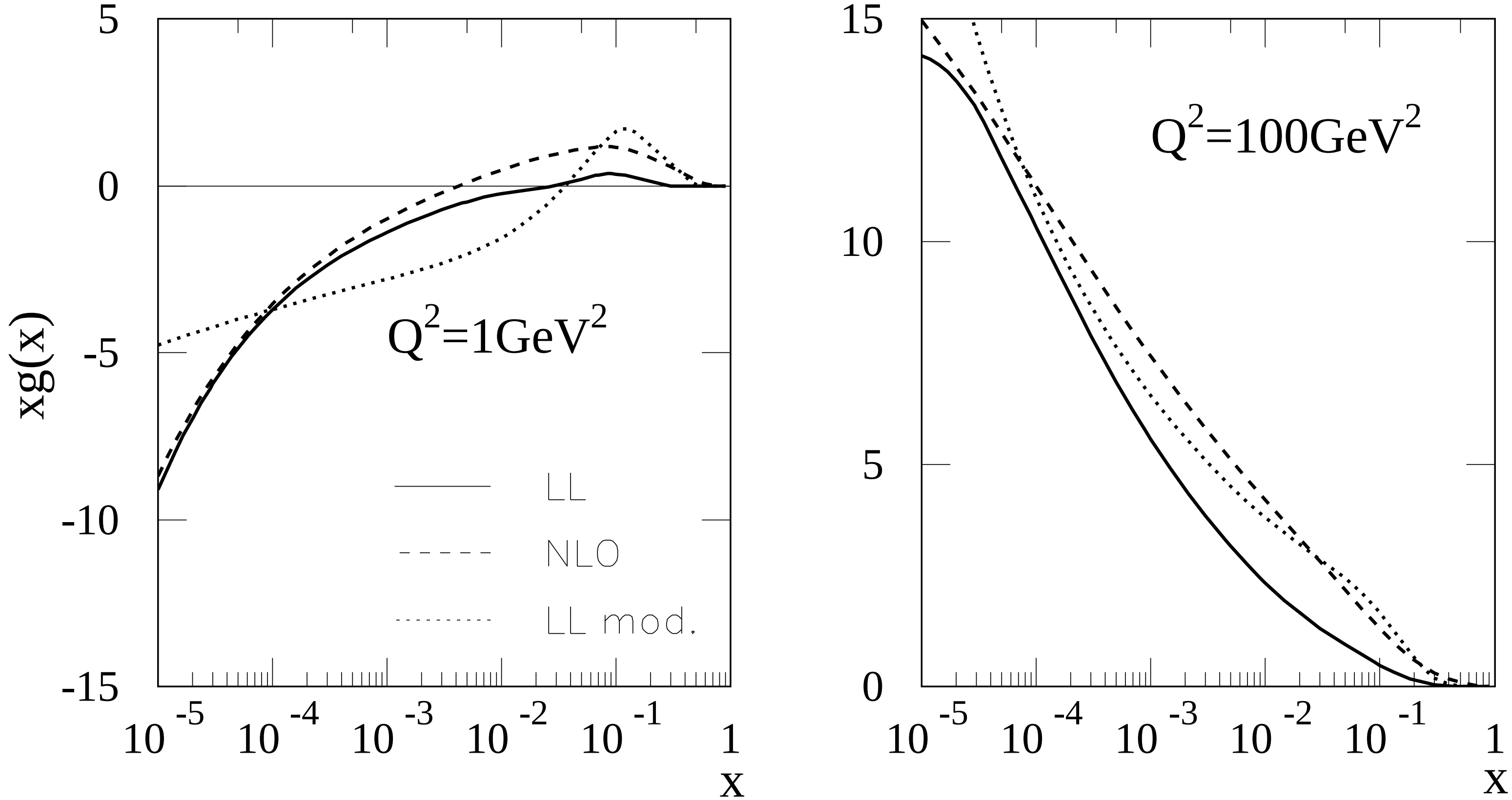
main
LHC
region

Predictions Change!

Stabilization of prediction for the gluon-dominated quantity, $F_L(x, Q^2)$. (White, RT)

Complete resummation only strictly at LO. Enhancement of evolution too great at small x. Gluon and $F_L(x, Q^2)$ too small at moderate x. Need the full NLO generalization. Very detailed calculations still required.





Also requires extension beyond current concentration on Deep Inelastic Scattering – particularly towards hadron-hadron colliders, and combination with heavy flavour expansions. LO resummation gluon too small for Tevatron jets.

Absorptive corrections.

Possible effect at small x due to absorptive corrections, i.e. bilinear parton distributions mixing with normal partons due to higher twist corrections.

$$\frac{\partial(xg(x,Q^2))}{\partial \ln Q^2} = \ldots - 3\frac{\alpha_S^2(Q^2)}{R^2Q^2}\int_x^1 \frac{dx'}{x'}\,[x'g(x',Q^2)]^2$$

$$\frac{\partial(xq(x,Q^2))}{\partial \ln Q^2} = \ldots - \frac{1}{10}\frac{\alpha_S^2(Q^2)}{R^2Q^2}[xg(x,Q^2)]^2$$

Slows evolution of small x partons at low Q^2. However, dies away quickly at higher Q^2.

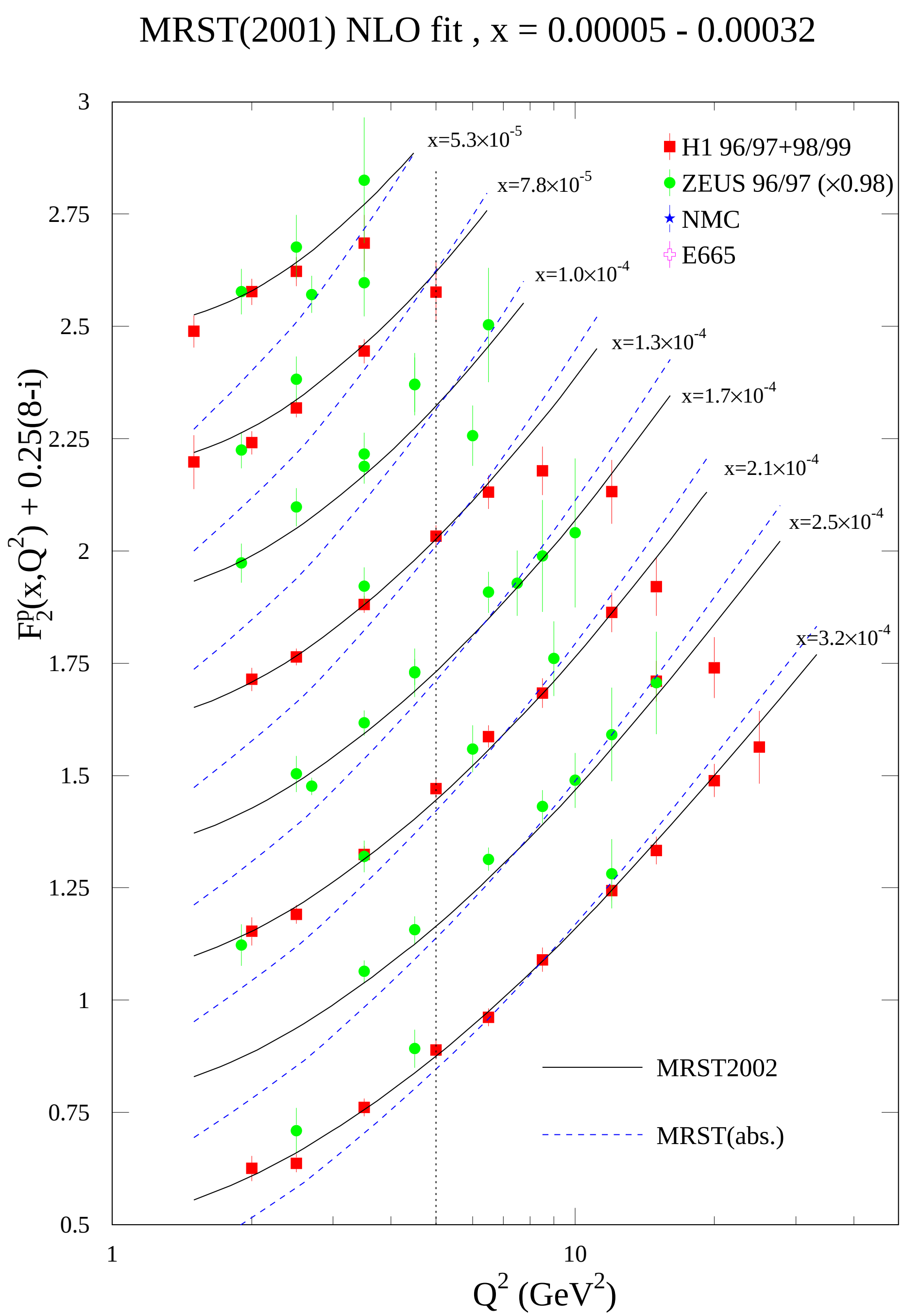
Help solve problem of negative input gluon distributions?

MRST fit with shadowing corrections and flattish input gluon extrapolated to $Q^2 \leq 5\mathrm{GeV}^2$.

Absorptive corrections do not solve problem at NLO or NNLO (misleading at LO).

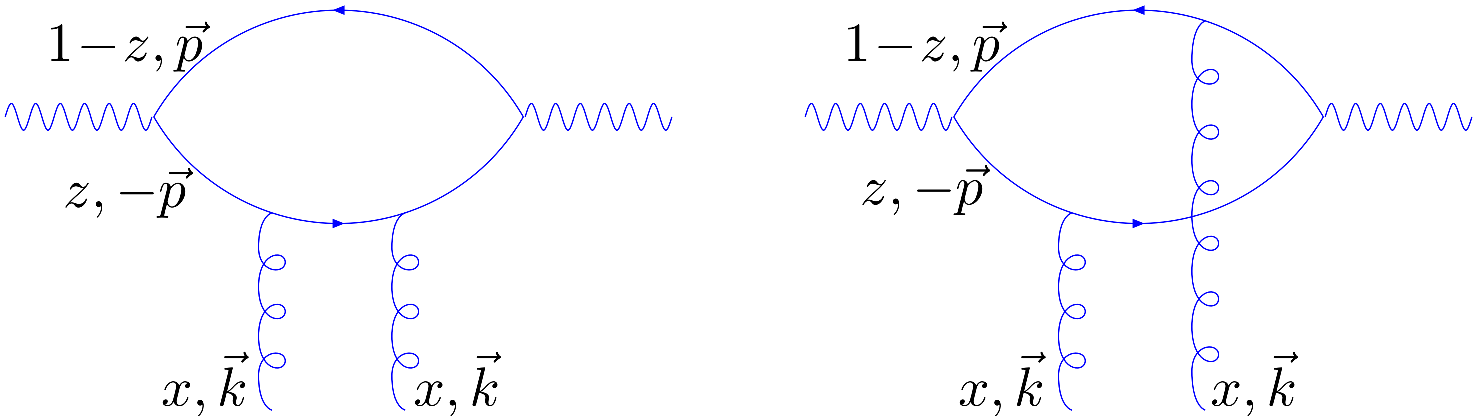
With standard gluons absorptive corrections small effect.

Can help a little (Martin, Ryskin, Watt).



Dipole cross-section and small-x saturation.

can think of the incoming parton as fluctuating into quark-antiquark pair before interacting with proton.



The cross-section can be written as

$$\sigma = \int_0^1 dz \int d^2r |\Psi(r,z,Q)|^2 \hat{\sigma}(x,r^2).$$

r^2 is conjugate to k_T^2, and is the transverse dipole size.

$\Psi(r,z,Q)$ is the wavefunction for dipole formation.

$\sigma(x,r^2)$ is the dipole scattering cross-section – related to gluon.

$\sigma(x, r^2)$ often calculated, or modelled, using small-x, high parton density limit, incorporating saturation of gluon density at low x and Q^2. $\Psi(r, z, Q)$ well-behaved as $Q^2 \to 0$.

Incorporates leading $\ln(1/x)$ information and higher twist information and absorptive corrections. Ignores higher-x information.

Procedure phenomenologically successful when fitting to HERA data - like resummation. Evidence for saturation? Some, but not overwhelming.

Also like resummation, high-x gluon badly reproduced. No real way of extrapolating properly to high-x.

Needs to be combined with fixed-order approach for quantitative physics. Not as easy as for resummation. Also not easy to use for proton-proton collisions.

Conclusions

One can determine the parton distributions and predict boson cross-sections, by performing global fits to all up-to-date data over wide range of parameter space. The fit quality using NLO or NNLO QCD is fairly good.

Uncertainty from input assumptions e.g. cuts on data, data used, ..., comparable and potentially larger. Can shift central values of predictions significantly.

Errors from higher orders/resummation potentially large in some regions of parameter space - most important at high rapidity. Cutting out low x and/or Q^2 allows much improved fit to remaining data, and altered partons. NNLO appears to be much more stable than NLO.

Theory often the dominant source of uncertainty at present. Resummation, higher twist, saturation ? Systematic study needed. Much progress – NNLO more-or-less complete, resummations combined with fixed order expansions (personal preference) ..., but much still to do. Both for theory and in obtaining useful new data.

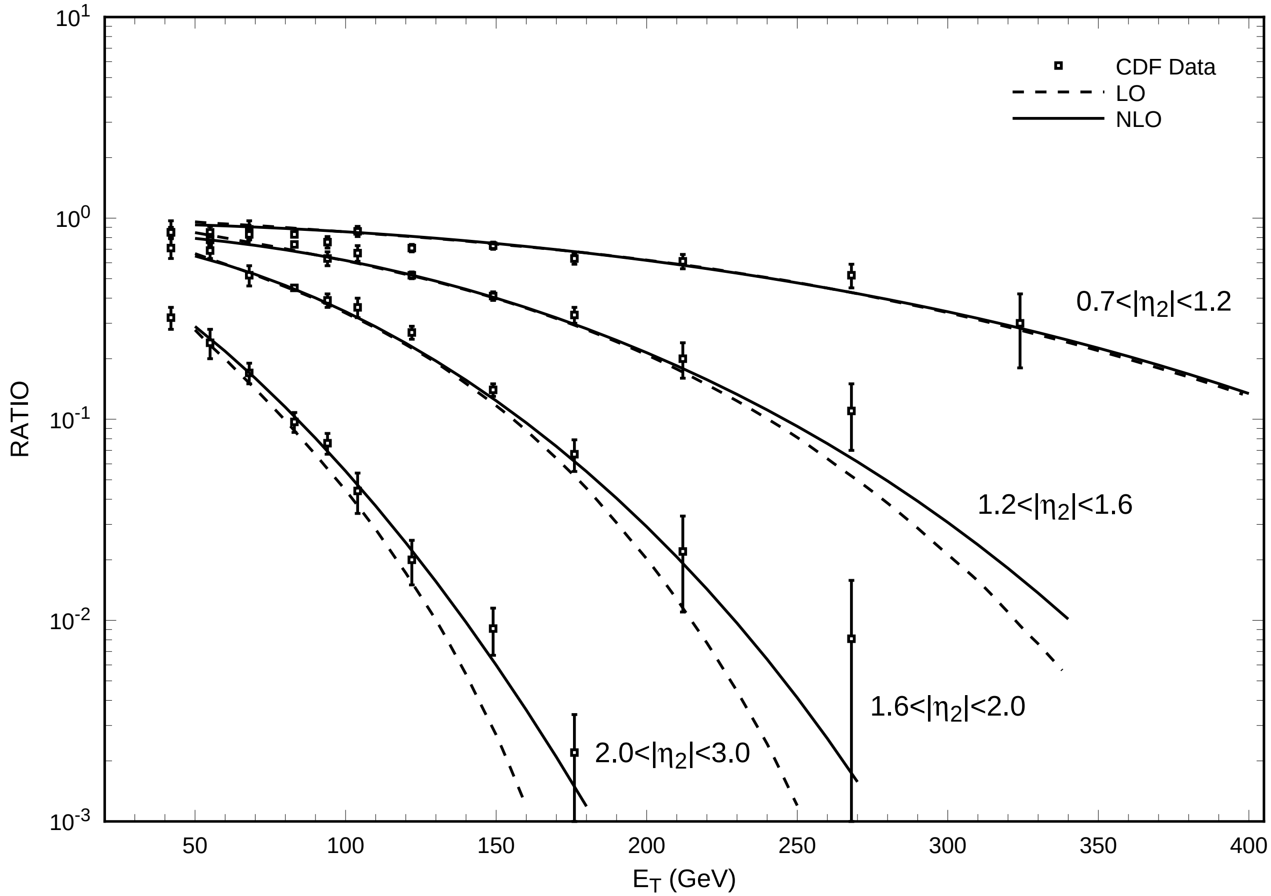
Table 2: Cross sections for Drell-Yan pairs (e^+e^-) with `PYTHIA` 6.206. The errors shown are the statistical errors of the Monte-Carlo generation.

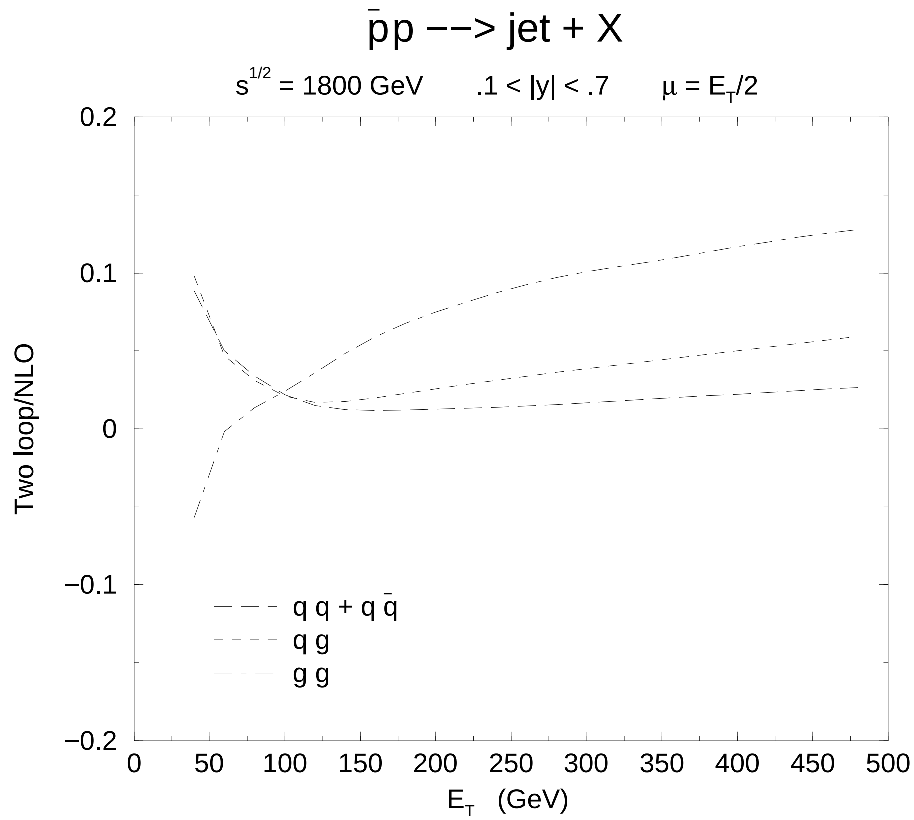
PDF set	Comment	xsec
$81 < M < 101$ GeV		
CTEQ5L	PYTHIA internal	1516 ± 5 pb
CTEQ5L	PDFLIB	1536 ± 5 pb
CTEQ6	LHAPDF	1564 ± 5 pb
MRST2001	LHAPDF	1591 ± 5 pb
Fermi2002	LHAPDF	1299 ± 4 pb
$M > 1000$ GeV		
CTEQ5L	PYTHIA internal	6.58 ± 0.02 fb
CTEQ5L	PDFLIB	6.68 ± 0.02 fb
CTEQ6	LHAPDF	6.76 ± 0.02 fb
MRST2001	LHAPDF	7.09 ± 0.02 fb
Fermi2002	LHAPDF	7.94 ± 0.03 fb

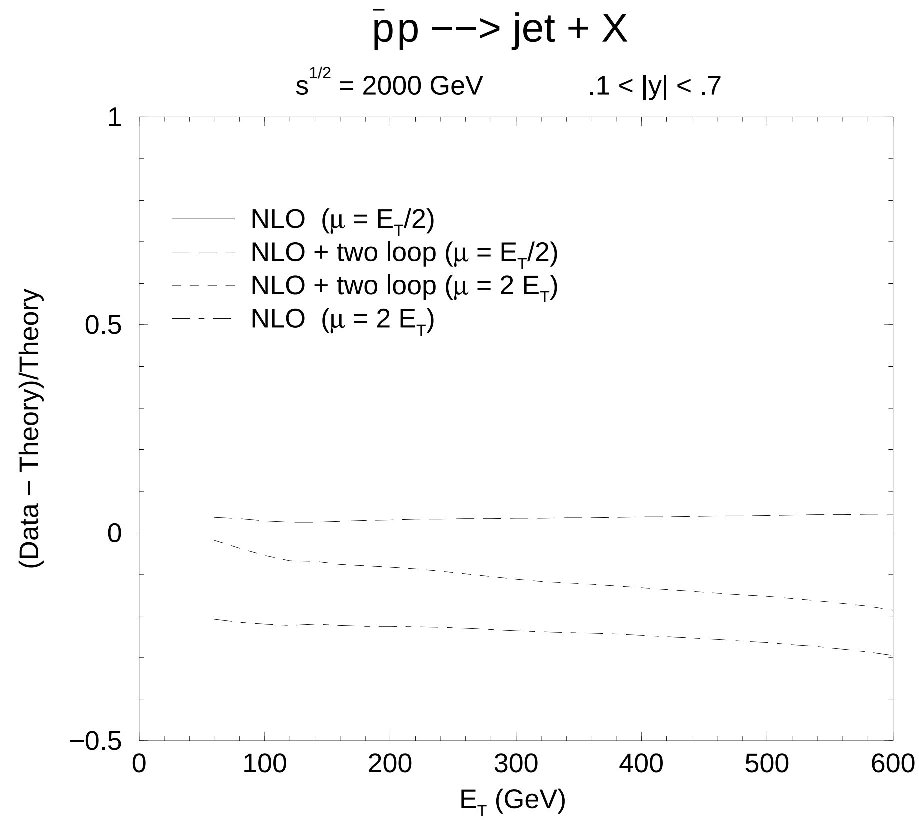
Note anti-correlation between deviations at high and low mass, i.e. high and low x. Typical result from sum rules and evolution.

NLO corrections themselves not large, except at high rapidities.

At central rapidities $\leq 10\%$. Similar to correlated errors.







Also good NNLO estimates Kidonakis, Owens. Calculated threshold correction logarithms. Expected to be major component of total NNLO correction.

$\rightarrow$ Flat $3-4\%$ correction. Consistent with what is known from NLO. Smaller than systematics on data.

Mistakes from ignoring jets in fits bigger than mistakes made at NNLO by not knowing exact hard cross-section.