



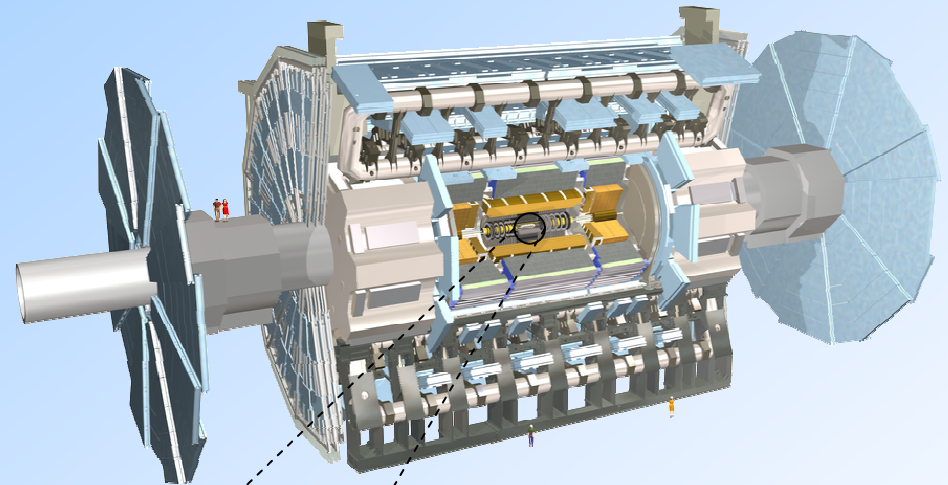
Determination Of Parton Density Functions At The LHC

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(With the help of G. Salam and Z. Nagy)*

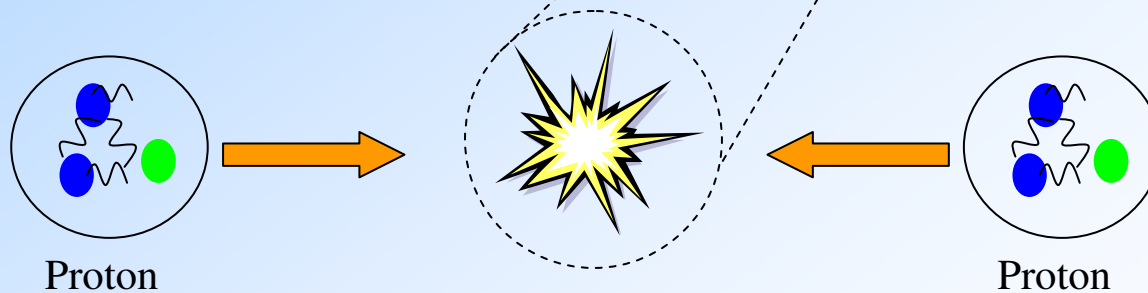
The LHC Project

- The LHC is designed to collide protons together at a centre of mass energy of 14TeV, and a luminosity of $10^{34}\text{cm}^{-2}\text{s}^{-1}$ (after 3 years of running).

- ATLAS (A Toroidal Lhc Apparatus) is a general purpose detector designed to explore the new energy reach available to the LHC.



The ATLAS detector



- Knowledge of proton content (i.e. PDFs) is important to understand cross-sections at the LHC, and can limit sensitivity to new physics (e.g. compositeness)

Measurement Of PDFs At A Hadron Collider

- Most PDF data comes from Deep Inelastic Scattering experiments (e.g. HERA) but hadron colliders can provide additional information particularly for the gluon distribution.
- Hadron collider cross-sections are sensitive to PDFs through the initial state of partons which interact in a given event for a $2 \rightarrow 2$ scattering i.e:

Hard $2 \rightarrow 2$ scattering

$$\sigma(P_1, P_2) = \sum_{i,j} \int dx_1 dx_2 f_i(x_1, \mu^2) f_j(x_2, \mu^2) \hat{\sigma}_{ij}(x_1 P_1, x_2 P_2, \alpha_s(\mu^2), Q^2 / \mu^2)$$

Diagram illustrating the components of the cross-section formula:

- PDFs**: Points to the parton distribution functions $f_i(x_1, \mu^2)$ and $f_j(x_2, \mu^2)$.
- Parton momenta**: Points to the parton momenta $x_1 P_1$ and $x_2 P_2$.
- Parton Level Cross-Section**: Points to the hard scattering cross-section $\hat{\sigma}_{ij}$.
- Sum over parton types**: Points to the summation over parton types i, j .
- Hadron momenta**: Points to the hadron momenta P_1 and P_2 .

- By comparing the experimental cross-section with NLO predictions, knowledge of the PDFs can be extracted. NLO Cross-Sections however can take 1-2 CPU days to calculate.

Separating PDFs From The Integral

- A NLO Cross-Section for DIS is normally calculated using MC by:

$$W = \sum_{m=1}^N w_m \left(\frac{\alpha_s(Q_m^2)}{2\pi} \right)^{p_m} q(x_m, Q_m^2)$$

For events $m=1 \dots N$, (w_m is an MC weight, $q(x, Q^2)$ a PDF).

- Can instead define a **weight grid** in (x, Q^2) , which is updated for each event m :

$$W_{i,j}^{(p)} = W_{i,j}^{(p)} + w_m$$

Where i, j define a discrete point in x, Q^2 space relating to the event.

- A **PDF grid** is also defined in x, Q^2 as $q_{i,j}$.

- Cross-Section can be reproduced by combining the PDF and weight grids **after** the Monte-Carlo run:

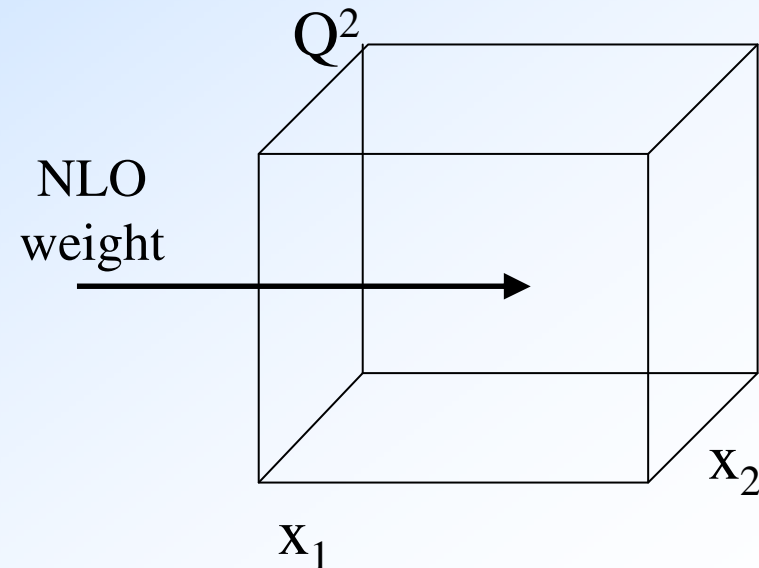
$$W = \sum_i \sum_j W_{i,j}^{(p)} \left(\frac{\alpha_s(Q^2)}{2\pi} \right)^p q_{i,j}$$

Separating PDFs From The Integral

- This method can recreate the Monte-Carlo cross-section exactly assuming grids could be made with an infinitely small spacing in (x, Q^2) .
- Instead grids with a finite spacing in x, Q^2 are used and interpolation methods used between points.

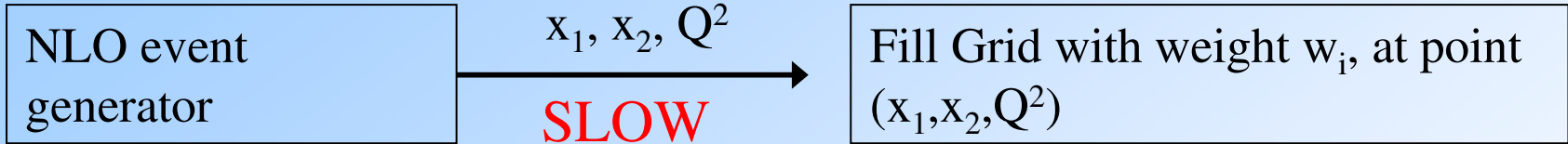
D.Graudenz, M.Hampel, A. Vogt, C Berger, D.A. Kosower, C. Adloff, S.Chekanov, M Wobisch.....

- The situation is a little more complicated in the case of hadron-hadron collisions as PDFs have to be considered for both incoming particles, hence the grid is three dimensional (x_1, x_2, Q^2) .

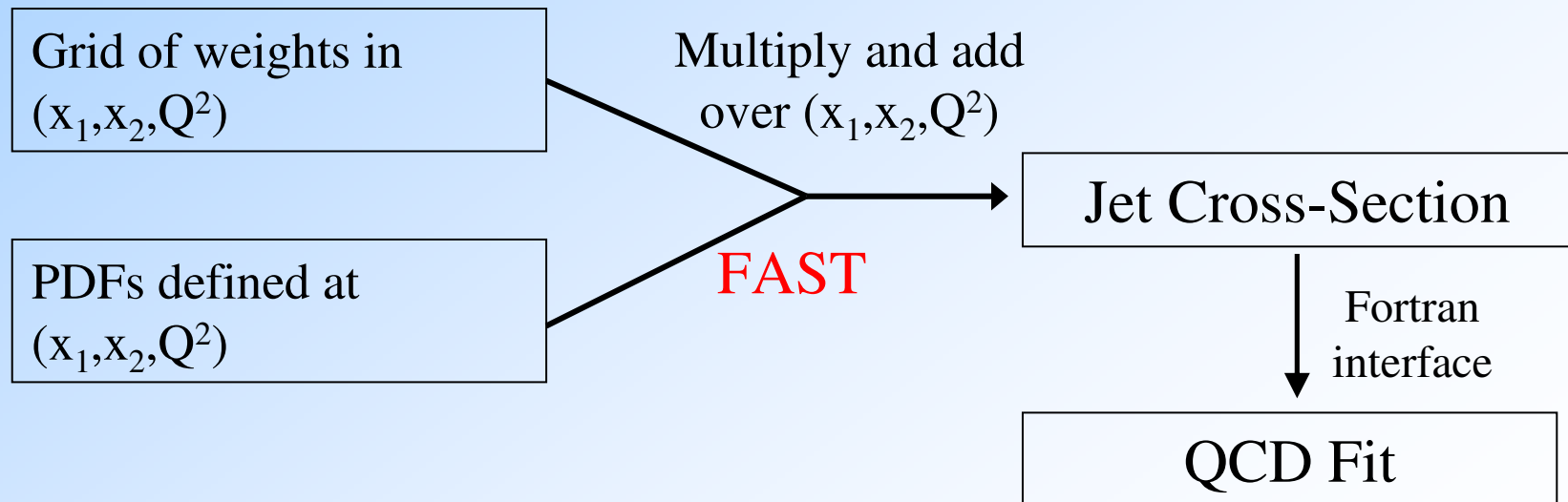


Using Integration Grids

Step 1: Fill the Grid



Step 2: Multiply grid by PDFs to generate Cross-Section



Grid Structure

- The grid implementation used was developed by Carli, Salem, Siegert and is described in (hep-ph/0510324) [1].

Main Features Of A Grid

- A co-ordinate transform is applied to the the x and Q^2 bins, to increase the density of bins at low x and high Q^2 .

$$y(x) = \ln \frac{1}{x}$$

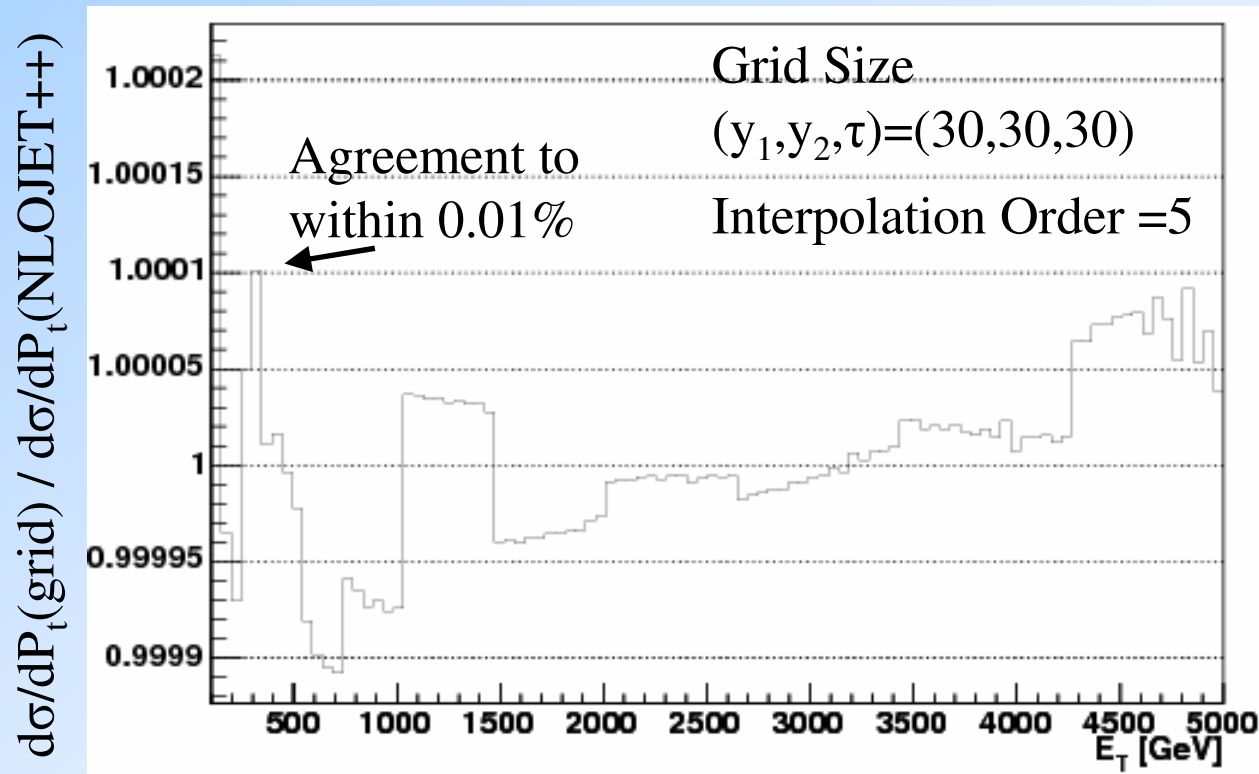
$$\tau(Q^2) = \ln \ln \frac{Q^2}{\Lambda^2}$$

$$\Lambda \approx \Lambda_{QCD}$$

- The grid is equidistant in the new variables.
- A high order interpolation method is used when filling the grid to provide increased accuracy without increasing CPU memory consumption.
- The grid software is written in C++ and uses ROOT [2] libraries.
- The NLOJET++ event generator [3] (Z.Nagy) is used to generate MC weights and a FORTRAN interface to the grid software is used for the fitting algorithm.

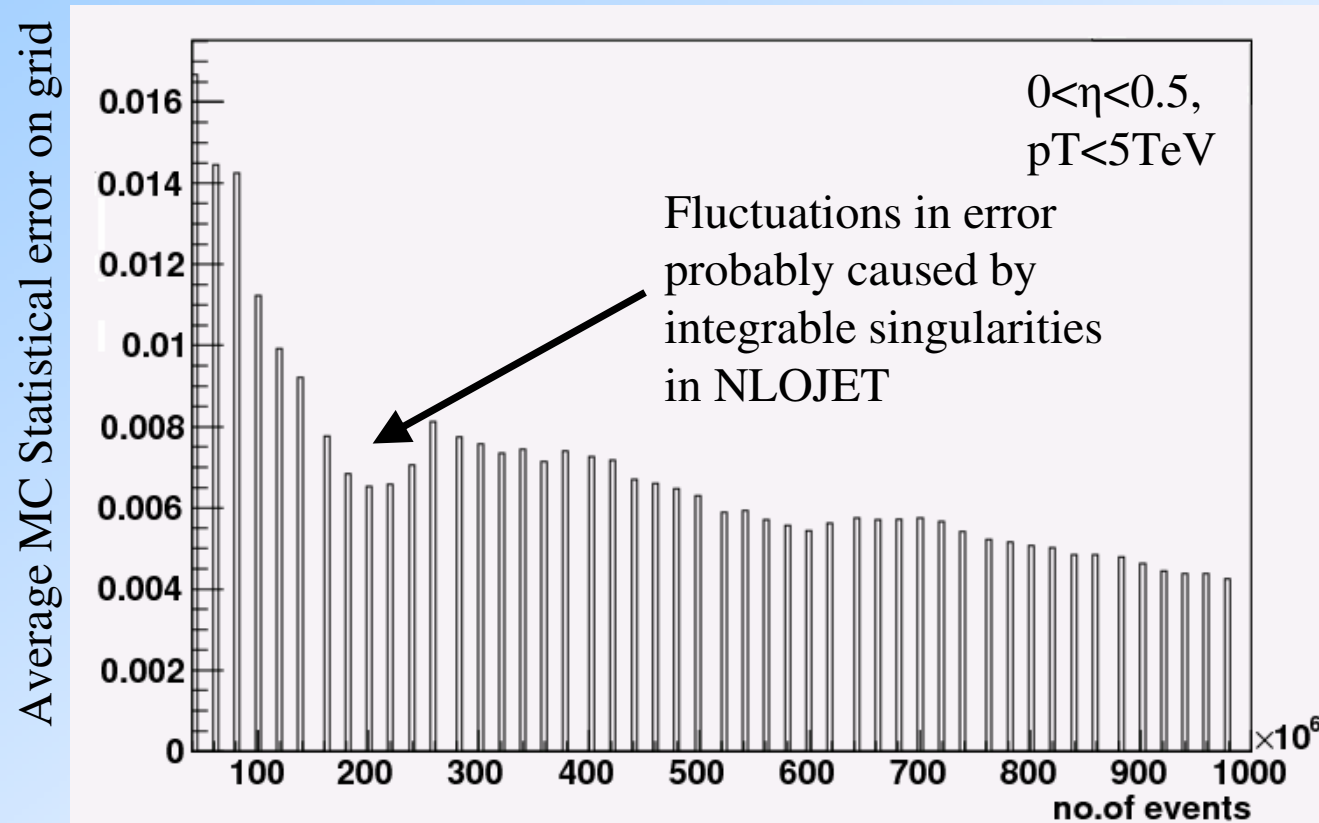
A Grid Generated Cross-Section

- Tests were carried out to test the Grid's ability to recreate the inclusive jet cross-section.
- Here the ratio of the grid generated and standard NLOJET cross-section is shown for the inclusive jet cross-section $0 < \eta < 0.5$:



NLO-Monte Carlo Errors

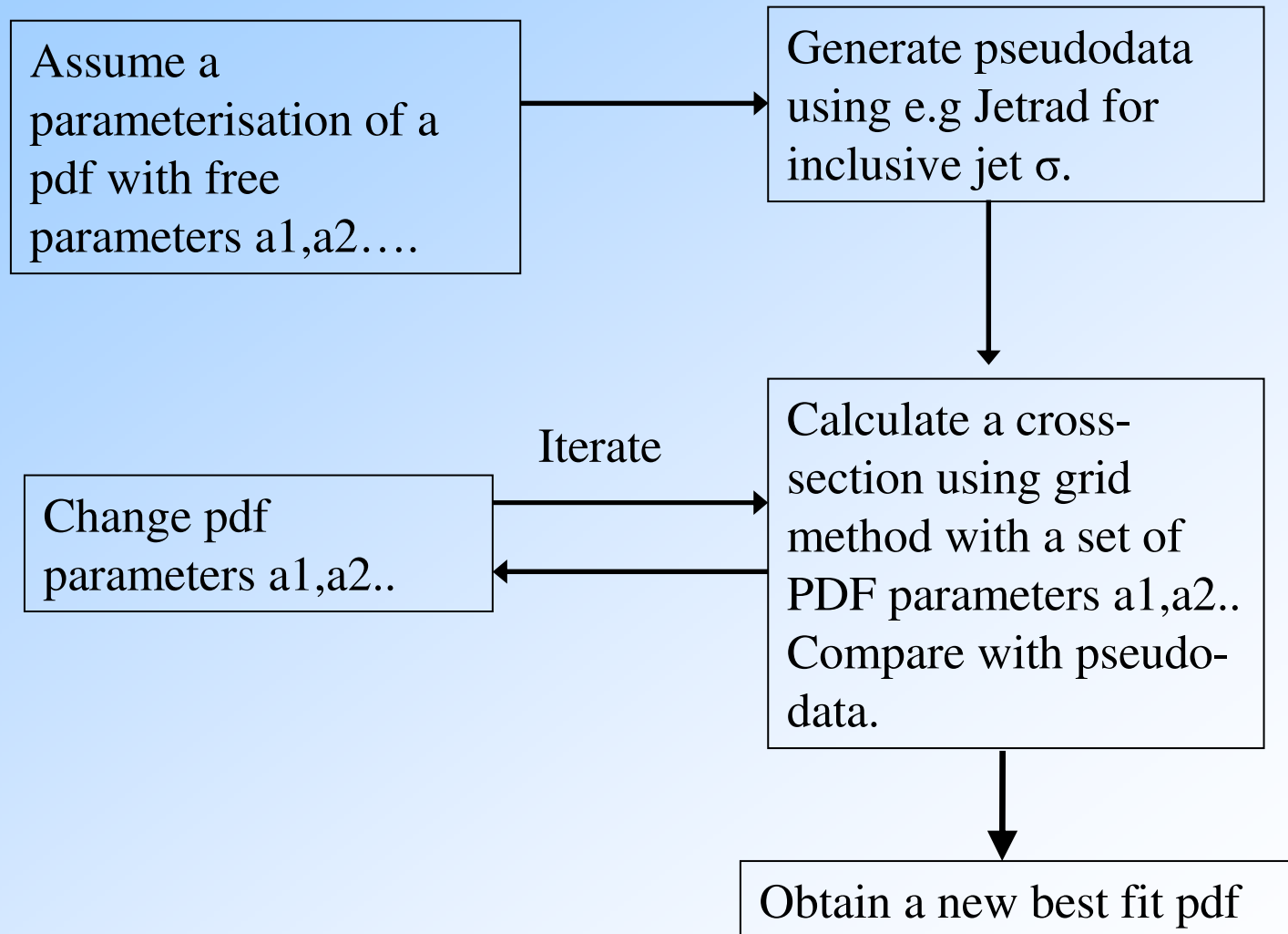
- Tests were carried out comparing the grid generated cross-section and an independently generated standard NLO cross-section for the inclusive jet cross-section.



- The statistical errors introduced by the Monte-Carlo calculation can be much greater than that caused by using the grid even after generating a large no. of events

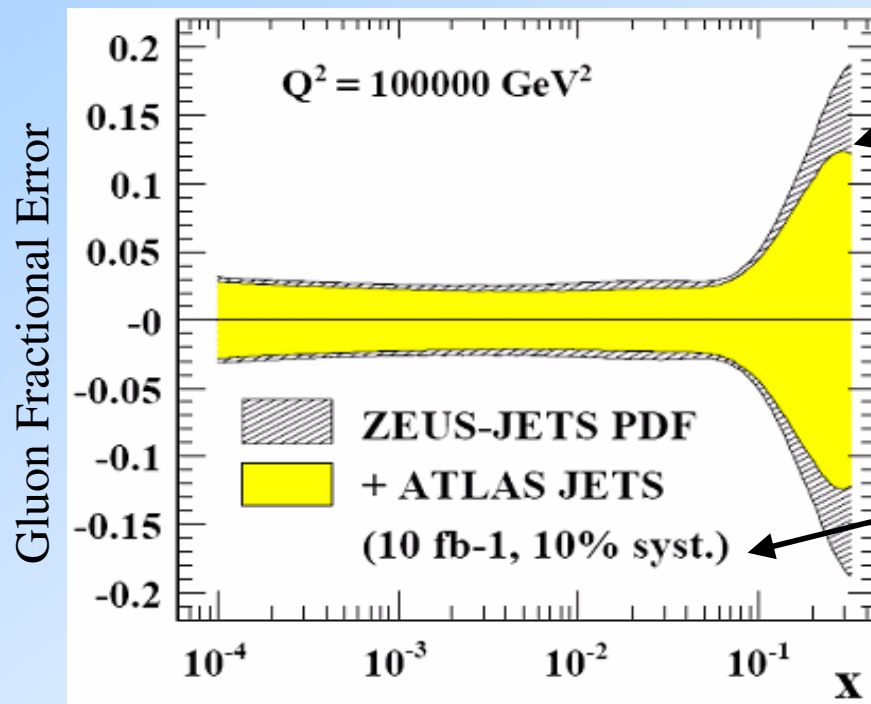
PDF Fitting Using Grids - Basic Idea

- Grid methods allow hadron collider data to be used directly in QCD fits.



PDF Fitting Using Pseudodata

- Grids were generated for the inclusive jet cross-section at ATLAS in the pseudorapidity ranges $0 < \eta < 1$, $1 < \eta < 2$, and $2 < \eta < 3$ up to $p_T = 3 \text{ TeV}$ (NLOJET).
- In addition pseudodata for the same process was generated using JETRAD [4].
- The pseudo-data was then used in a global fit to assess the impact of ATLAS data on constraining PDFs:

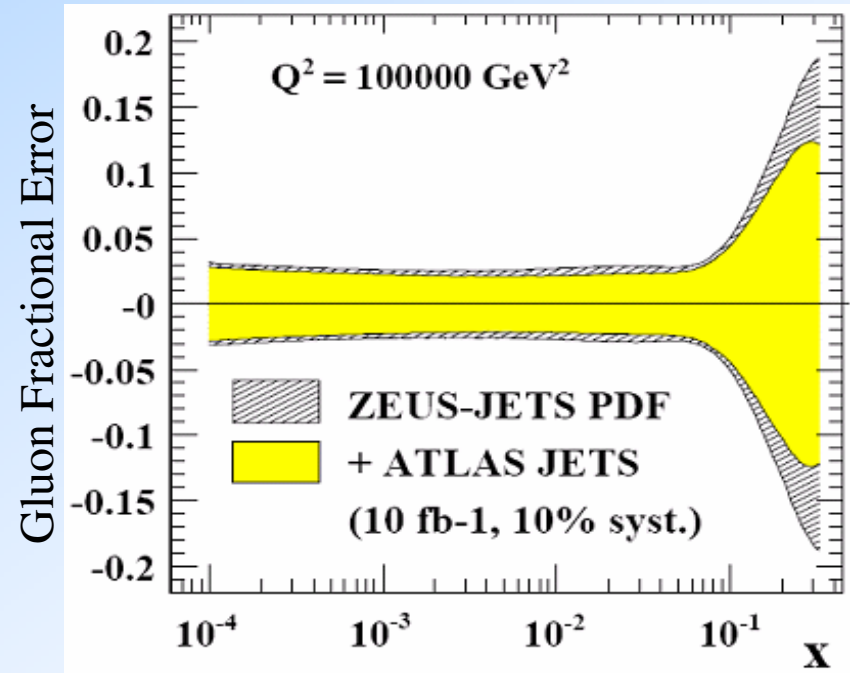
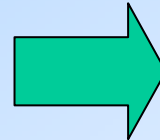
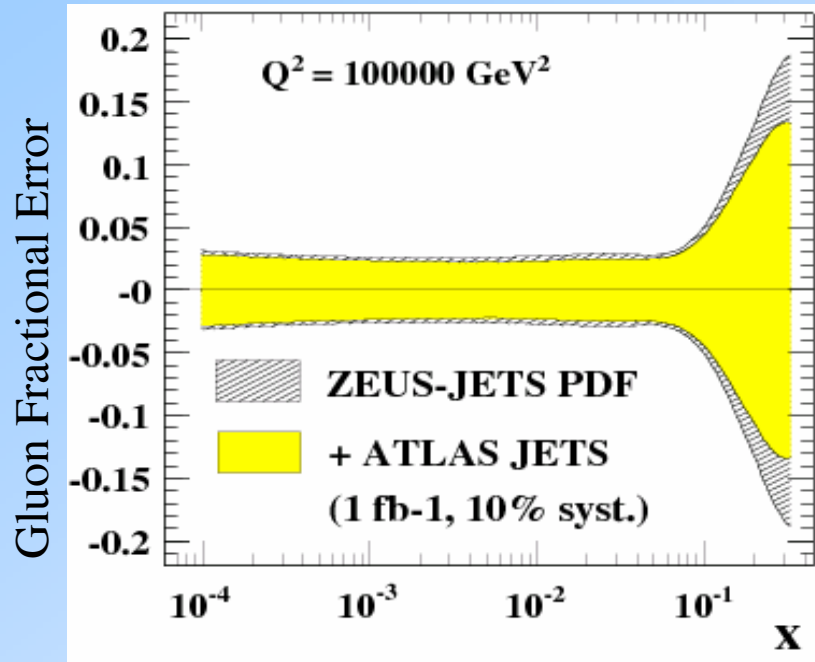


Preliminary indications suggest that ATLAS data can constrain the high x -gluon.

Systematic errors are uncorrelated, $10 \text{ fb}^{-1} = 1$ year of nominal data-taking at $10^{33} \text{ cm}^{-2} \text{ s}^{-1}$

Effect Of Increased Statistics on PDF Fits

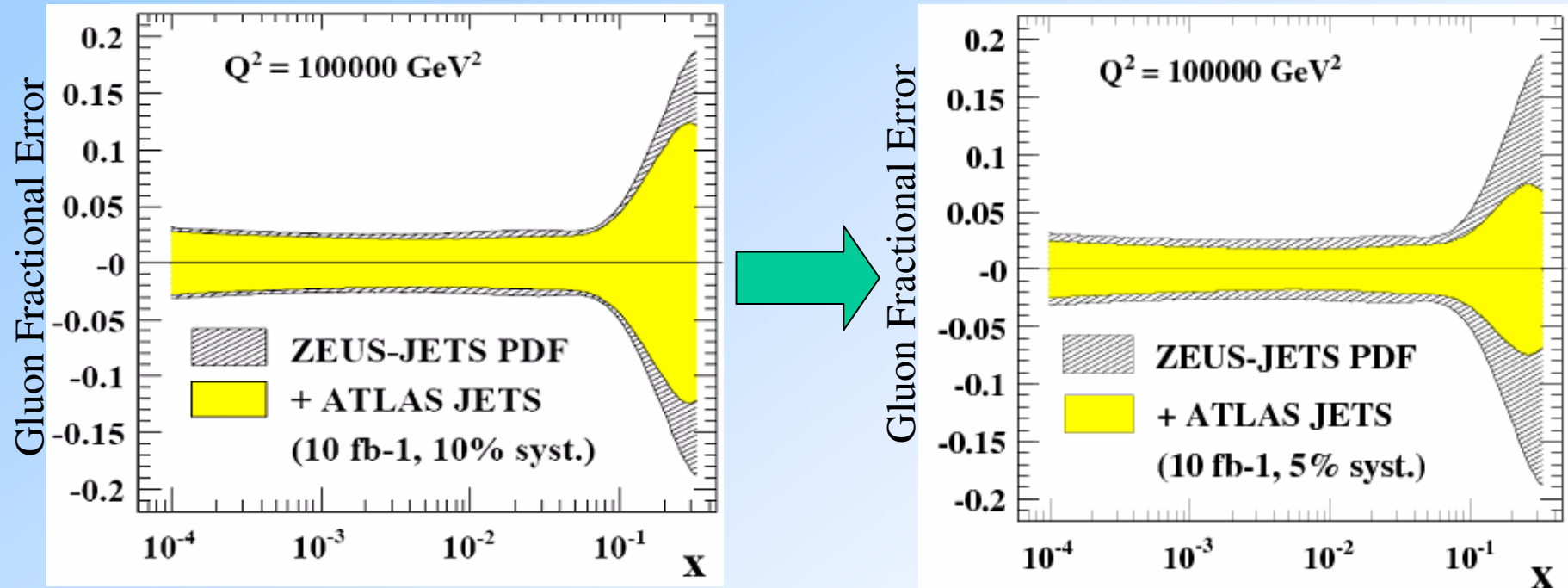
Increase 10×statistics



- Increasing the statistics from 1 fb^{-1} to 10 fb^{-1} has little effect on improving the constraining of PDFs at ATLAS.

Effect Of Decreased Systematic Errors On PDF Fits

Decrease (uncorrelated) Systematic errors
10% → 5%



- Decreasing the systematic errors (on the ATLAS experiment) creates a significant improvement in constraining the PDFs.

Summary

- Integration grids using high order interpolation methods can be used to recreate NLO cross-sections to accuracies of better than 0.01%.
- Preliminary results using pseudodata indicate that ATLAS jet data will be useful to constrain the gluon PDF.
- Error on gluon PDF that can be extracted from the jet cross-section is dominated by systematics, the statistical error being negligible even for 1fb^{-1}

Ongoing Developments With Grids And Analysis

- Variable grid spacing (in x and Q^2) is being developed to better model PDFs at large x .
- Post grid generation changing of the renormalisation and factorisation scales being developed.
- Extension of grid interface to other NLO QCD programs..
- Software freely available from the authors.

References

- [1] “A Posteriori Inclusion Of PDFs in NLO QCD Final-State Calculations”, Tancredi Carli, Gavin P Salam, Frank Siegert, hep-ph/0510324
- [2] “*ROOT - An Object Oriented Data Analysis Framework*”, Rene Brun and Fons Rademakers, Proceedings AIHENP'96 Workshop, Lausanne, Sep. 1996, Nucl. Inst. & Meth. in Phys. Res. A 389 (1997) 81-86. See also <http://root.cern.ch/>.
- [3] “Next-to-leading order calculation of three-jet observables in hadron-hadron collisions”, Z Nagy, Phys Rev D68, 094002 (2003)
- [4] Higher Order Corrections To Jet Cross-Section In Hadron Colliders, W.T. Giele, E.W.N. Glover, David A. Kosower, Nucl.Phys.B403:633-670,1993, [hep-ph/ 9302225].

