Parton Distributions, QCD and Electroweak Corrections at the LHC

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Strong force makes it difficult to perform analytic calculations of scattering processes involving hadronic particles.

The weakening of $\alpha_s(\mu^2)$ at higher scales → the **Factorization Theorem**.

Hadron scattering with an electron factorizes.

$Q^2$ – Scale of scattering

$x = \frac{Q^2}{2m\nu}$ – Momentum fraction of Parton ($\nu=$ energy transfer)
The coefficient functions \( C^P_i(x, \alpha_s(Q^2)) \) are process dependent (new physics) but are calculable as a power-series in \( \alpha_s(Q^2) \).

\[
C^P_i(x, \alpha_s(Q^2)) = \sum_k C^P_{i,k}(x) \alpha^k_s(Q^2).
\]

Since the parton distributions \( f_i(x, Q^2, \alpha_s(Q^2)) \) are process-independent, i.e. universal, once they have been measured at one experiment, one can predict many other scattering processes.
Evolve partons upwards using NLO (or NNLO) DGLAP equations. Fit data for scales above $2 - 5\text{GeV}^2$. Need many different types of experiment for full determination.

**H1** $F_2^{e^+p}(x, Q^2)$ 1996-97 moderate $Q^2$ and 1996-97 high $Q^2$, and $F_2^{e^-p}(x, Q^2)$ 1998-99 high $Q^2$ small $x$. **ZEUS** $F_2^{e^+p}(x, Q^2)$ 1996-97 small $x$ wide range of $Q^2$. 1999-2000 high $Q^2$. **H1** and **ZEUS** $F_2^{c,b}(x, Q^2)$.

**NMC** $F_2^{\mu p}(x, Q^2), F_2^{\mu d}(x, Q^2), (F_2^{\mu n}(x, Q^2)/F_2^{\mu p}(x, Q^2))$, **E665** $F_2^{\mu p}(x, Q^2), F_2^{\mu d}(x, Q^2)$ medium $x$.

**BCDMS** $F_2^{\mu p}(x, Q^2), F_2^{\mu d}(x, Q^2)$, **SLAC** $F_2^{\mu p}(x, Q^2), F_2^{\mu d}(x, Q^2)$ large $x$.

**CCFR (NuTeV)** $F_2^{\nu\bar{\nu}p}(x, Q^2), F_3^{\nu\bar{\nu}p}(x, Q^2)$ large $x$, singlet, valence.

**E605 (E866)** $pN \to \mu\bar{\mu} + X$ large $x$ sea.

**E866** Drell-Yan asymmetry $\bar{u}, \bar{d} \bar{d} - \bar{u}$.

**CDF** W-asymmetry $u/d$ ratio at high $x$.

**CDF D0** Inclusive jet data high $x$ gluon.

**CCFR (NuTev)** Dimuon data constrains strange sea.
This procedure is generally successful and is part of a large-scale, ongoing project.

Results in partons of the form shown.

Various choices of partons – MRST, CTEQ Alekhin, ZEUS, H1 .......

All LHC cross-sections rely on our understanding of these partons.
Excellent predictive power – comparison of MRST prediction for $Z$ rapidity distribution with preliminary data.
Interplay of LHC and pdfs/QCD

Make predictions for all processes, both SM and BSM, as accurately as possible given current experimental input and theoretical accuracy.

Check against well-understood processes, e.g. central rapidity $W, Z$ production (luminosity monitor), lowish-$E_T$ jets, ..... 

Compare with predictions with more uncertainty and lower confidence, e.g. high-$E_T$ jets, high rapidity bosons or heavy quarks ..... 

Improve uncertainty on parton distributions by improved constraints, and check understanding of theoretical uncertainties, and determine where NNLO, electroweak corrections, resummations etc. needed.

Make improved predictions for both background and signals with improved partons and surrounding theory.

Spot new physics from deviations in these predictions. As a nice by-product improve our understanding of the strong sector of the Standard Model considerably.

Remainder of talk describes this process in more detail.
LHC Physics

The kinematic range for particle production at the LHC is shown.

Smallish $x \sim 0.001 - 0.01$ partron distributions therefore vital for understanding the standard production processes at the LHC.

However, even smaller (and higher) $x$ required when one moves away from zero rapidity, e.g. when calculating total cross-section.
Uncertainty on MRST $\bar{u}$ and $\bar{d}$ distributions, along with CTEQ6. Central rapidity $x = 0.006$ is ideal for MRST uncertainty in $W, Z$ (Higgs?) at the LHC.
Current best (MRST) estimate

\[ \delta \sigma_{W,Z}^{\text{NLO}}(\text{expt pdf}) = \pm 2\% \]

but note that there is a greater theoretical uncertainty in the NLO prediction, mainly due to possible problems at small \( x \) in the global fit to DIS data.

This is because the large rapidity \( W \) and \( Z \) total cross-sections sample very small \( x \)

\[ \frac{\sigma(W^+)/\sigma(W^-)}{\sigma(W^+)/\sigma(W^-)} \text{ is gold-plated} \]

\[ R_{\pm} = \frac{\sigma(W^+)}{\sigma(W^-)} \simeq \frac{u(x_1)d(x_2)}{d(x_1)\bar{u}(x_2)} \simeq \frac{u(x_1)}{d(x_1)} \]

since sea is \( u, d \) symmetric at small \( x \), and using MRST2001E

\[ \delta R_{\pm}(\text{expt. pdf}) = \pm 1.4\% \]

Assuming all other uncertainties cancel, this is probably the most accurate SM cross-section test at LHC.
Could $\sigma(W)$ or $\sigma(Z)$ be used to calibrate other cross-sections, e.g. $\sigma(WH)$, $\sigma(Z')$?

$\sigma(WH)$ more precisely predicted because it samples quark pdfs at higher $x$, and scale, than $\sigma(W)$.

However, ratio shows no improvement in uncertainty, and can be worse.

Partons in different regions of $x$ are often anti-correlated rather than correlated, partially due to sum rules.

This is the end of the story being so simple/straightforward.
Different approaches to fits generally lead to similar uncertainty for measured quantities, but can lead to different central values. Must consider effect of assumptions made during fit and correctness of NLO QCD.

Many can be as important as experimental errors on data used (or more so).
Gluon still very uncertain at low $x$ and $Q^2$.

All partons fit to same small-$x$ HERA data.

Very wide variety in gluon distributions.

Different approaches to fits, much of the uncertainty due to the theoretical errors.
Results from LHC/LP Study Working Group (Bourilkov).

Table 1: Cross-sections for Drell-Yan pairs \((e^+ e^-)\) with PYTHIA 6.206, rapidity < 2.5. The errors shown are the PDF uncertainties.

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<td>1091 ± ...</td>
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<td>Fermi2002</td>
<td>LHAPDF</td>
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Comparison of \(\sigma_W \cdot B_{l\nu}\) for MRST2002 and Alekhin partons.

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<td>± 0.03 (expt)</td>
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<td>Alekhin</td>
<td>LHC</td>
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<td>± 6 (tot)</td>
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<tr>
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<td>LHC</td>
<td>204</td>
<td>± 4 (expt)</td>
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<tr>
<td>CTEQ6</td>
<td>LHC</td>
<td>205</td>
<td>± 8 (expt)</td>
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In both cases differences (mainly) due to detailed constraint (by data) on quark decomposition.
Theoretical Errors

It is vital to consider theoretical corrections. These include . . .

- QED and Weak (comparable to NNLO ?) ($\alpha_s^3 \sim \alpha$). Sometime enhancements – large $E_T$.

- higher orders (NNLO)

- large $x$ ($\alpha_s^n \ln^{2n-1} (1-x)$)

- low $Q^2$ (higher twist)

- small $x$ ($\alpha_s^n \ln^{n-1} (1/x)$)

- possibility of isospin violation, $s(x) \neq \kappa(\bar{u} + \bar{d})$, $s(x) \neq \bar{s}(x)$, etc.

Lead to differences in current partons, and to corrections in predicted cross-sections.
**High-$E_T$ Jets**

The error on predictions for very high-$E_T$ jets at the LHC is dominated by the parton uncertainties.

Sensitive to relatively poorly known high-$x$ gluon.

Improvements to fits in future using new fast cross-section calculation packages, Kluge, Rabbertz, Wobisch, and Carli, Clements *et al*.

Also useful for other processes.
Fit to current Tevatron data excellent.

Comparison to D0 jet data for physical gluon MRST partons.
Comparison of variations in dijet production from large extra dimensions (alters running of $\alpha_S(Q^2)$) with given compactification scale and from uncertainties in $g(x, Q^2)$ (Ferrag).

Limit on $M_C$ changes from $5\text{TeV} \rightarrow 2\text{TeV}$. Depends on particular parton set and uncertainties.

Horizontal line — one year projected LHC running.
Weak corrections

Jet cross-section a major example – calculation by Moretti, Nolten, Ross, goes like \( (1 - \frac{1}{3} C_F \frac{\alpha_W}{\pi} \log^2 \left( \frac{E_T^2}{M_W^2} \right) ) \).

Dominated by quark-(anti)quark processes \( \rightarrow \approx 6\% \) correction at \( E_T = 450\text{GeV} \).
Phenomenological impact not huge - movement of both CDF and D0 data small.

MRST 2004 NNLO DIS-type and D0 jet data, $\alpha_s(M_Z)=0.1167$, $\chi^2=64/82$ pts

MRST 2004 NNLO DIS type and D0 jet data, $\alpha_s(M_Z)=0.1167$, $\chi^2=75/82$ pts
Much bigger at LHC energies. Up to 30%. Bigger than NLO QCD.

\[ \log^2 \left( \frac{E_T^2}{M_W^2} \right) \] a very large number.

Similar results for corrections to other processes with a hard scale, e.g. Di-boson production (Accomando et al), large-\(p_T\) vector bosons (Kühn et al, Maina et al)...
Only virtual corrections. Must have contributions of the form

\[ \ln(s/m_W^2) \]

terms can also affect \( \Gamma_W \) extraction from the transverse mass distribution.
Prompt Photons

Also possible to determine the gluon at high $x$ via prompt photon production.

In principle this is a direct test of the large $x$ gluon - $x_T = 2p_T/\sqrt{s}$.

However, at low $p_T \sim 5 - 10\text{GeV}$ $d^2\sigma/dEdp_T$ has been sensitive to nonperturbative information about the intrinsic $k_T$ of the gluons in the proton, to resummation of threshold logarithms, i.e. $\ln(1 - x_T)$, and to the interplay between the two.
Far cleaner probe of the perturbative gluon at the LHC at much higher $p_T \geq 330\text{GeV}$. Also sensitive to electroweak corrections (Kühn et al), → consistency check.

Study by Hollins notices differences between MRST and CTEQ gluons. At $p_T = 350\text{GeV}$, $\eta = 0$ corresponds to $x = 0.05$. 
NNLO

Default has long been NLO. Essentially well understood. Now starting to go further.

NNLO coefficient functions for structure functions know for many years.

Splitting functions now complete. (Moch, Vermaseren and Vogt). Extremely similar to average of best estimates → no significant change in NNLO partons. Improve quality of fit very slightly (MRST), and reduces $\alpha_S$. Can be big change from NLO → NNLO
To do absolutely correct NNLO fit we need not only exact NNLO splitting functions. NNLO differential Drell-Yan cross-sections recently calculated in terms of $y$ by Anastasiou, Dixon, Melnikov and Petriello.

Decreases sea quarks. Implemented by Alekhin and MRST.
Do not know NNLO corrections to jet production in $pp(\bar{p})$ collisions. Stumbling block?

NLO corrections themselves not large, except at high rapidities. At central rapidities $\leq 10\%$. Similar to correlated errors.

Also good NNLO estimates Kidonakis, Owens. Calculated threshold correction logarithms. Expected to be significant component of total NNLO correction. (Issue concerning application within given jet definition – non-global logarithms.)

→ Flat 3 – 4% correction. Consistent with what is known from NLO. Smaller than systematics on data.
Also require rigorous treatment of heavy quark thresholds – partons discontinuous at this order. Rather significant effect (RT).

Essentially full NNLO determination of partons possible. Surely this is best, i.e. most accurate.

Yes, but … only know some hard cross-sections at NNLO.

Processes with two strongly interacting particles largely completed

DIS coefficient functions and sum rules

\[ pp(\bar{p}) \rightarrow \gamma^*, W, Z \] (including rapidity dist.), \( H, A^0, WH, ZH \).

But for many other final states NNLO not known. NLO still more appropriate.

(If NLO known. NLO calculations largely complete for \( 2 \rightarrow 2 \) and \( 2 \rightarrow 3 \) processes. Beyond this only LO. Absolute cross-section not under control. e.g. \( pp \rightarrow t\bar{t} + b\bar{b} \) or \( \rightarrow t\bar{t} +\) jets, background to \( t\bar{t} + H \).)

Resummations may be important even beyond NNLO in some regions, as may higher twist.
Reasonable stability order by order for (quark-dominated) $W$ and $Z$ cross-sections.

This fairly good convergence is largely guaranteed because the quarks are fit directly to data.
Stability much worse for gluon dominated quantities. Unstable at small $x$ and $Q^2$.

$xg(x, Q^2)$ going from LO $\rightarrow$ NLO $\rightarrow$ NNLO.
Small-$x$ Theory

Reason for this problem.

It is known that at each order in $\alpha_S$ each splitting function and coefficient function obtains an extra power of $\ln(1/x)$ (some accidental zeros in $P_{gg}$), i.e.

$$P_{ij}(x, \alpha_s(Q^2)), \quad C^P_i(x, \alpha_s(Q^2)) \sim \alpha_s^m(Q^2) \ln^{m-1}(1/x).$$

→ no guarantee of convergence at small $x$!

$$x < 0.01, \quad \ln(1/x) > 5, \quad \rightarrow \alpha_S \ln(1/x) > 1.$$  

The global fits usually assume that this turns out to be unimportant in practice, and proceed regardless.

Fits work fairly well at small $x$, but could be better.

Good recent progress in incorporating $\ln(1/x)$ resummation into global fits – now at next-to-leading $\ln(1/x)$ level (White, RT). More work needed for reliable predictions.
Higgs production known at NNLO (with assumptions – large $m_t$) – Catani et al, Harlander and Kilgore, Anastasiou et al, Ravindran et al.

Uncertainty $8 - 10\%$ mainly due to cross-section, not partons or really low $x$ region due to high mass.

Slightly less convergence at highest rapidity?
Instability in physical, gluon dominated, quantity $F_L(x, Q^2)$ going from LO $\rightarrow$ NLO $\rightarrow$ NNLO.

Gluon at NLO $\rightarrow F_L(x, Q^2)$ dangerously small at smallest $x, Q^2$.

Note very large effect of exact NNLO coefficient function.

Possible sign of required $\ln(1/x)$ corrections.

Similar problems possible for charm and/or bottom production, and low-mass Drell-Yan ($\gamma$) production at the LHC.
Study by Rizvi. Good reach at ATLAS if low $p_T$-trigger works well.

**Measurement Feasibility**

08/05/06

**x of parton producing Drell-Yan pair**

After $P_T > 6$ GeV for one fermion

Can reach $x \sim 10^{-5}$

After $P_T > 20$ GeV for one fermion

Need low $P_T$ trigger
Possible to get to very low values of $x$ at the LHC.

ALICE in $pp$ mode at $10^{31} cm^{-2} s^{-1}$ with forward muon detection.

Can probe below $x = 10^{-5}$ - beyond range tested at HERA.
At LHCb even probe very small $x$ with high rapidity $Z$ (Lastovicka, Ferro-Luzzi).

**Kinematic coverage**

- Reconstructed events overlayed
  - $Q^2 = M_{Z^0}^2$
  - Leading order Bjorken $x$
- LHCb at high $x$ overlaps with D0/CDF and HERA
- A very nice opportunity to pinpoint/cross-check PDFs at low $x$!
- Overlap between LHC experiments?
- Expected reconstructed rate? $10^6$ / year?

At lowest rapidity $y = 1.8$ could be luminosity monitor if cross-checked with ATLAS, CMS and QCD calculations.
Some doubt in predictions at high rapidity.

Comparison of prediction for $(d\sigma_W/dy_W)$ for the standard MRST partons and a set which represents the possible type of theoretical uncertainty in this region when working at NLO.

Good stability at central rapidity—$x = 0.005$.

Increased uncertainty if worrying about theory for very small $x$. 
**Heavy Quarks** – Essential to treat these correctly. Two distinct regimes:

Near threshold $Q^2 \sim m_H^2$ massive quarks not partons. Created in final state. Described using **Fixed Flavour Number Scheme (FFNS)**.

$$F(x, Q^2) = C_k^{FF}(Q^2/m_H^2) \otimes f_k^{nf}(Q^2)$$

High scales $Q^2 \gg m_H^2$ massless partons. Behave like up, down, strange. Sum $\ln(Q^2/m_H^2)$ terms via evolution. **Zero Mass Variable Flavour Number Scheme (ZM-VFNS)**. Ignores $O(m_H^2/Q^2)$ corrections.

$$F(x, Q^2) = C_j^{ZMVF} \otimes f_j^{nf+1}(Q^2).$$

Partons in different number regions related to each other perturbatively.

$$f_k^{nf+1}(Q^2) = A_{jk}(Q^2/m_H^2) \otimes f_k^{nf}(Q^2),$$

$A_{jk}(Q^2/m_H^2)$ contain $\ln(Q^2/m_H^2)$ terms $\to$ correct evolution.

Need a general **Variable Flavour Number Scheme (VFNS)** interpolating between the two well-defined limits of $Q^2 \leq m_H^2$ and $Q^2 \gg m_H^2$.

Inclusive processes (mainly $ep \rightarrow X$) ACOT, TR (now to NNLO) .... prescriptions. In LO Monte Carlo AcerMC (Kerseven and Hinchliffe).
Example of need to understand both heavy flavours and small-$x$ physics for LHC.

Production of supersymmetric Higgs depends on parton uncertainties (Belyaev, Pumplin, Tung and Yuan), heavy flavour procedure and high-energy (small-$x$) treatment.
Consider bottom production along with a Higgs boson.

\[ g_{b\bar{b}h} = \frac{m_b}{v}, \quad (v \text{ Higgs vacuum expectation value.}) \quad m_b = 4.5\text{GeV}, \quad v = 246\text{GeV}. \]

In Standard Model tiny since Higgs-bottom coupling \( g_{b\bar{b}h} \) is small. Expectation values \( v_d \) and \( v_u \).

Ratio \( \tan \beta = \frac{v_u}{v_d} \).

Enhancement of Higgs-bottom coupling

\[ g_{b\bar{b}h} \propto \frac{g_{b\bar{b}h}^{SM}}{\cos \beta}. \]

Bounds from LEP, \( \tan \beta \) large \( \rightarrow \) \( \cos \beta \) small. Enhancement of Higgs-bottom coupling.
Search at Tevatron for enhancement in jets with $b$ quarks.

Produces upper limit on parameter $\tan \beta$. 

$MSSM$ $Higgs$ $bosons$
$bb\phi(\rightarrow bb)$, $\phi = h, H, A$
Conclusions

One can determine the parton distributions and predict cross-sections at the LHC, and the fit quality using NLO or NNLO QCD is fairly good.

Various ways of looking at uncertainties due to errors on data. Uncertainties rather small – \( \sim 1 - 5\% \)for most LHC quantities. Ratios often don’t reduce uncertainties unless theoretical uncertainties cancel. Ratio \( W^+/W^- \) tight constraint on partons.

Uncertainty from input assumptions e.g. cuts on data, data used, etc., comparable and potentially larger. Can shift central values of predictions significantly. Electroweak corrections potentially large at very high energies – \( \ln^2(E^2/M_W^2) \). Requires careful definitions of theory and measurement.

Errors from higher orders/resummation potentially large. Direct measurement of \( F_L(x, Q^2) \) at HERA an important means of testing this. At LHC measurement at high rapidities, e.g. \( W, Z \) would be useful in testing understanding of QCD, and particularly quantities sensitive to low \( x \) at low scales, e.g. low mass Drell-Yan.

Theory often the dominant source of uncertainty. Much progress – more processes at NLO, some NNLO, heavy flavours treatments, resummations .... Pretty much full NNLO parton determinations now possible. Should become new standard. Important to have data to check if even further corrections needed for real precision.
In full **global** fit art in choosing “correct” $\Delta \chi^2$ given complication of errors. Ideally $\Delta \chi^2 = 1$, but unrealistic.

Many approaches use $\Delta \chi^2 \sim 1$. CTEQ choose $\Delta \chi^2 \sim 100$ for 90% confidence limit, i.e. $\sim 40$ for $1 - \sigma$ error. MRST choose $\Delta \chi^2 \sim 20$ for $1 - \sigma$ error.
LO partons in some regions qualitatively different to all NLO and NNLO partons. Due to important missing NLO corrections in splitting functions.

Can lead to wrong conclusions on size of small-\(x\) gluon, and conclusions on shadowing etc.

Nevertheless, LO partons are the appropriate ones to use with many LO Monte Carlo programs.

All such results should be treated with care.
Treatment of errors.

Exercise for *HERA – LHC* meeting. Fit proton and deuteron structure function data from H1, ZEUS, NMC and BCDMS, for \( Q^2 > 9 \text{GeV}^2 \) using *ZM – VFNS* and same form of parton inputs at same \( Q_0^2 = 1 \text{GeV}^2 \).

Very conservative fit.

Compare rigorous treatment of all systematic errors (*Alekhin*) with simple quadratures approach (*MRST*), both with \( \Delta \chi^2 = 1 \).

→ some difference in central values (other possible reasons) and similar errors.

Fairly consistent.
Back to HERA-LHC benchmark partons.

How do partons from very conservative, structure function only data compare to global partons?

Compare to MRST01 partons with uncertainty from $\Delta \chi^2 = 50$.

Enormous difference in central values.

Errors similar.
No obvious advantage in using $\sigma(tt)$ as a calibration SM cross-section, except maybe for very particular, and rather large, $M_H$.

However, a light (SM or MSSM) Higgs dominantly produced via $gg \rightarrow H$ and the cross-section has small pdf uncertainty because $g(x)$ at small $x$ is well constrained by HERA DIS data.

Current best (MRST) estimate, for $M_H = 120$ GeV: $\delta\sigma_{H}^{NLO}(\text{expt pdf}) = \pm 2 - 3\%$ with less sensitivity to small $x$ than $\sigma(W)$.

Much smaller than the uncertainty from higher-order corrections, for example, Catani et al,

$\delta\sigma_{H}^{NNLL}(\text{scale variation}) = \pm 8\%$
Table 2: Cross sections for Drell-Yan pairs \((e^+e^-)\) with PYTHIA 6.206. The errors shown are the statistical errors of the Monte-Carlo generation.

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Note anti-correlation between deviations at high and low mass, i.e. high and low \(x\). Typical result from sum rules and evolution.
Other groups find similar problems with gluon at low $x$.

**CTEQ** have valence-like input gluon at $Q_0^2 = 1.69\text{GeV}^2$ which would like (at least a little) to be negative. *(Blue line – negative gluon allowed, black line – positive definite gluon.)*
Approach to Look for Safe Theoretical Regions.

In order to investigate real quality of fit and regions with problems vary kinematic cuts on data.

Procedure – change $W_{cut}^2$, $Q_{cut}^2$ and $x_{cut}$, re-fit and see if quality of fit to remaining data improves and/or input parameters change dramatically. Continue until quality of fit and partons both stabilize.

Raising $Q_{cut}^2$ from $2\text{GeV}^2$ in steps there is a slow continuous and significant improvement for higher $Q^2$ up to $>10\text{GeV}^2$.

Raising $x_{cut}$ from 0 to 0.005 continuous improvement. At each step moderate $x$ gluon becomes more positive.

$\rightarrow$ MRST2003 conservative partons. Should be most reliable method of parton determination ($\Delta\chi^2 = -70$ for remaining data), but only applicable for restricted range of $x$, $Q^2$. $\rightarrow \alpha_S(M_Z^2) = 0.1165 \pm 0.004$.

Also NNLO conservative partons. Similar cuts and improvement in fit quality (bit smaller), but change in partons considerably less. Already includes important theoretical corrections.
Theories with extensions at small $x$, both resummations and higher twist, produce rather different shape and size prediction for $F_L(x, Q^2)$ from that at NLO and NNLO.

Similar variation expected for other gluon-sensitive quantities.

Currently working with HERA to determine if this can be measured if beam energy is lowered to measure $F_L(x, Q^2)$. Now very likely.

Clearly some reasonable power to differentiate.

Important for understanding LHC physics.
Need a general **Variable Flavour Number Scheme (VFNS)** interpolating between the two well-defined limits of $Q^2 \leq m_H^2$ and $Q^2 \gg m_H^2$.

Conclusion easily reached by looking at the extrapolation between the two simple kinematic regimes for $xF_3$, measured using neutrino scattering at NuTeV.
At NNLO additional complications – partons become discontinuous.

ZM-VFNS leads to peculiar, unphysical results. FFNS not known at this order.

Makes need for **Variable Flavour Number Scheme** more vital but also more difficult to implement.
Difference in charm procedure affects gluon compared to approx MRST2004 NNLO fit.

Change greater than uncertainty in some places. Correct heavy flavour treatment vital.
Instability in physical, gluon dominated, quantity $F_L(x, Q^2)$ going from LO $\to$ NLO $\to$ NNLO.

Improved by next-to-leading $\ln(1/x)$ resummation in the global fit and prediction (White, RT).