Axion dark matter minicluster

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with Jonas Enander, Andreas Pargner, 1708.04466

Unravelling the dark matter mystery — IPPP Durham, UK, March 2018







The strong CP problem

$$\mathcal{L}_{\theta \text{QCD}} = \frac{\theta_{\text{QCD}}}{32\pi^2} \text{Tr } G_{\mu\nu} \tilde{G}^{\mu\nu} \qquad \qquad \theta_{\text{QCD}} = \tilde{\theta}_{\text{QCD}} + \text{arg } \det M_u M_d$$

• limit on neutron electric dipole moment:

$$\theta_{\rm QCD} \lesssim 10^{-10}$$

 requires cancellation between bare angle and contribution from quark masses at the 10⁻¹⁰ level.

symmetry (PQ)

 gets broken at high scale f_{PQ}

receives a mass by

introduce a global U(1)

Axion solution

 axion is the p-Goldstone of the U(1)

non-perturbative QCD instanton effects

axion potential drives the theta-angle dynamically to zero

plots from G. Raffelt



The QCD Axion

• mass determined by PQ breaking scale:

$$m_0 \simeq \frac{m_\pi f_\pi}{f_{\rm PQ}} \frac{\sqrt{m_u m_d}}{m_u + m_d} \simeq 10^{-4} \text{ eV} \frac{6 \times 10^{10} \text{ GeV}}{f_{\rm PQ}}$$

• all interactions with SM suppressed by $f_{\rm PQ}$

• single parameter model!



experimentally excluded, astro/cosmo excluded, sensitivity of planned experiments, "preferred" region

Axion cosmology

the action (ignoring gravity for the moment)

$$S = f_a^2 \int d^4 x \sqrt{-g} \left[-\frac{1}{2} g_{\mu\nu} \partial^{\mu} \theta \partial^{\nu} \theta - V(\theta, T) \right]$$

equation of motion

$$\ddot{\theta} + 3H\dot{\theta} - \frac{\nabla^2}{a^2}\theta + V'(\theta, T) = 0$$

$$\phi = f_a \theta$$

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$$\ddot{\theta} + 3H\dot{\theta} - \frac{\nabla^2}{a^2}\theta + V'(\theta, T) = 0$$

- periodic potential $V(\theta, T) = m^2(T)(1 \cos \theta)$
- harmonic approximation

$$V(\theta,T) \approx \frac{1}{2}m^2(T)\theta^2$$

→ Fourier-modes evolve independently

 $\phi = f_a \theta$

$$\begin{vmatrix} \ddot{\theta}_k + 3H(T)\dot{\theta}_k + \omega^2\theta_k = 0 \qquad \qquad \omega^2 = \frac{k^2}{a^2} + m(T)^2$$

- modes outside the horizon 3H(T) > ω: over-damped oscillator, field frozen: θ_k = const
- modes inside the horizon 3H(T) < ω: oscillator with frequency ω, amplitude decays with expansion relativistic modes are red shifted relative to non-relativistic modes



$$\rho(\vec{x}) = \frac{f_a^2}{2} \left[\dot{\theta}^2 - \frac{(\vec{\nabla}\theta)^2}{a^2} + m^2(T)\theta^2 \right]$$

Example:

homogeneous field (no gradient terms), adiabatic evolution

$$\theta \sim \frac{\overline{\theta}}{a^{3/2}} \cos[m(T)t]$$

late time energy density:

$$\rho \sim f_a^2 m(T_{\rm osc}) m_0 \left(\frac{a_{\rm osc}}{a}\right)^3 \overline{\theta}^2 \qquad m(T_{\rm osc}) = 3H(T_{\rm osc})$$

behaves as cold dark matter (also true in perturbation theory)

Initial conditions



pre-inflation case: PQ phase transition before end of inflation \Rightarrow constant initial field value $\overline{\theta} \in [-\pi, \pi]$ in whole observable Universe

$$\Omega_a h^2 pprox 0.24 \,\overline{\theta}^2 g(\overline{\theta}) \left(rac{f_a}{10^{12}\,{
m GeV}}
ight)^{7/6} \ pprox 0.13 \,\overline{\theta}^2 g(\overline{\theta}) \left(rac{10^{-5}\,{
m eV}}{m_0}
ight)^{7/6}$$

 $g(\overline{\theta})$ anharmonic correction

▶ $m_0 \ll 10^{-5} \, \text{eV}$: tune $\overline{\theta}$ to small values

• $m_0 \gg 10^{-5} \,\mathrm{eV}$: not enough dark matter

possibly constrained by iso-curvature perturbations

Gondolo, Visinelli, 09

Initial conditions

pre-inflation case: PQ phase tr constant initial field value $\overline{\theta} \in$

post-inflation case: PQ phase 1 random field value $\overline{\theta} \in [-\pi, \pi]$ in causally disconnected regions

After inflation, PQ phase transition, misaligned patches

. Redondc

rse

ed patches

▶ before axion mass turns on: random field values θ
∈ [-π, π] in regions of order of horizon size at QCD epoch → order one density fluctuations ⇒ axion minicluster

Axion minicluster

Hogan, Rees, 1988; Kolb, Tkachev, 93, 94, 95 Zurek, Hogan, Quinn, 06

- PQ symmetry breaking after inflation: Axion field takes random values in different Hubble volumes
- O(1) density fluctuations when Axion mass switches on
- expect gravitationally bound objects with size ~ Hubble volume @ QCD PT

$$M \sim \frac{4\pi}{3} d_H^3(T_{\rm osc}) \overline{\rho}(T_{\rm osc}) \qquad d_H \sim 1/H$$

 $M \sim 10^{-12} M_{\odot} (f_{\rm PQ}/10^{11} \,{\rm GeV})^2$

 $\left[10^{-12} M_{\odot} \sim 10^{18} \,\mathrm{kg}\,, M_{\mathrm{Earth}} \simeq 6 \times 10^{24} \,\mathrm{kg}\,, M_{\mathrm{Moon}} \simeq 10^{23} \,\mathrm{kg}\right]$

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 $[10^{-12} M_{\odot} \sim 10^{18} \,\mathrm{kg}\,, M_{\mathrm{Earth}} \simeq 6 \times 10^{24} \,\mathrm{kg}\,, M_{\mathrm{Moon}} \simeq 10^{23} \,\mathrm{kg}]$









Initial condition



From the PQ PT down to the QCD PT

- random values of periodic field → cosmic strings
- Kibble mechanism: `scaling' of string network: roughly one string per Hubble volume during expansion of Universe



J. Redondo

 Before onset of QCD PT remains one string per Hubble volume and

$$\langle \theta(\vec{x})^2 \rangle = \pi^2/3$$

String-domain wall network at QCD PT



white: strings blue: domain walls $\tau_f/\tau_i = 24$ T = 0 mass at $\tau = 3.3$

Hiramatsu, Kawasaki, Saikawa, Sekiguchi, 12

string/domain wall system decays by radiating axions

significant contribution to the axion energy density e.g. up to 75% KSS14 large uncertainty

String-domain wall network at QCD PT



Initial condition

- axion field smooth on scales < horizon uncorrelated on scales > horizon
- assume power spectrum for axion field w Gaussian cut-off



$$\langle \theta_k \theta_{k'}^* \rangle = (2\pi)^3 \,\delta^3(\vec{k} - \vec{k'}) P_\theta(k)$$

$$P_{\theta}^{\rm G}(k) = \frac{8\pi^4}{3\sqrt{\pi}K^3} \exp\left(-\frac{k^2}{K^2}\right)$$

- normalization: fixed by flat distribution
- cut-off: comoving horizon wave-number K

$$\langle \theta(\vec{x})^2 \rangle = \pi^2/3$$

 $K = a_i H_i$

Axion field evolution



- harmonic approximation of axion potential
- equation of motion including gradient terms:

$$\ddot{\theta}_k + 3H(T)\dot{\theta}_k + \omega_k^2\theta_k = 0, \qquad \omega_k^2 \equiv \frac{k^2}{a^2} + m(T)^2$$

Axion field evolution



- harmonic approximation of axion potential
- equation of motion including gradient terms:

$$\ddot{\theta}_k + 3H(T)\dot{\theta}_k + \omega_k^2\theta_k \neq 0, \qquad \omega_k^2 \equiv \frac{k^2}{a^2} + m(T)^2$$

 solve EoM to calculate axion energy density and density power spectrum

$$\rho(\vec{x}) = \frac{f_{\rm PQ}^2}{2} \left[\dot{\theta}^2 - \frac{1}{a^2} (\vec{\nabla}\theta)^2 + m^2 \left(T\right) \theta^2 \right]$$

$$\theta_k(a) = \theta_k f_k(a)$$

go to Fourier space for field

$$\rho(\vec{x}) = \frac{1}{(2\pi)^6} \frac{f_{PQ}^2}{2} \int d^3k d^3k' \,\theta_k \theta_{k'}^* F(k,k') e^{-i\vec{x}(\vec{k}-\vec{k'})}$$
$$F(k,k') = \dot{f}_k \dot{f}_{k'} + \left(\frac{\vec{k} \cdot \vec{k'}}{a^2} + m^2(T)\right) f_k f_{k'}$$

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• average energy density (without string contribution):

$$\overline{\rho} \equiv \langle \rho(\vec{x}) \rangle = \frac{1}{2\pi^2} \frac{f_{\rm PQ}^2}{2} \int_0^\infty dk \, k^2 \, P_\theta(k) F(k,k)$$

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Density power spectrum

Fourier transform of the density:

$$\rho_q = \frac{1}{(2\pi)^3} \frac{f_{\mathrm{PQ}}^2}{2} \int d^3k \,\theta_k \theta_{k-q}^* F(k,k-q)$$

variance (use Wick's theorem):

$$\begin{split} \langle |\rho_q|^2 \rangle &= \left[\frac{1}{(2\pi)^3} \frac{f_{\rm PQ}^2}{2} \right]^2 \int d^3k d^3k' \, \langle \theta_k \theta_{k-q}^* \theta_{k'}^* \theta_{k'-q} \rangle F(k,k-q) F^*(k',k'-q) \\ &= 2 \frac{V}{(2\pi)^3} \left(\frac{f_{\rm PQ}^2}{2} \right)^2 \int d^3k \, P_\theta(|\vec{k}|) P_\theta(|\vec{k}-\vec{q}|) \, F(k,k-q)^2 \end{split}$$

this gives for the power spectrum

$$P(q) = \frac{1}{V} \frac{\langle |\rho_q|^2 \rangle}{\overline{\rho}^2} = 2(2\pi)^3 \frac{\int d^3k \, P_\theta(|\vec{k}|) P_\theta(|\vec{k} - \vec{q}|) \, F(k, k - q)^2}{\left[\int d^3k \, P_\theta(k) F(k, k)\right]^2}$$

Density power spectrum

 $\Delta^2(q) = \frac{q^3}{2\pi^2} P(q)$ 10 $- f_{PQ} = 10^{10} \text{ GeV}$ $T_i = 2T_{\rm osc}$ 5 1.5 TH $- f_{PQ} = 10^{11} \text{ GeV}$ $f_{\rm PQ}$ =10¹² GeV P(q) $[K_1^{-3}]$ 1.0 0.50 $\Delta^2(q)$ 0.10 0.5 0.05 0.0 0.01 0.5 0.5 5 10 5 10 1 1 q [*K*₁] q [*K*₁]

- density fluctuations of order one
- charact. size a few times smaller than horizon @ Tosc

Enander, Pargner, TS, 1708.04466

Collapse of over-densities



- spherical collapse model for non-linear density fluctuations during radiation domination [Kolb, Tkachev, 94]
- modified Press-Schechter ansatz to calculate doubledifferential minicluster distribution in mass and size

Double-differential pdf for density fluctuations

variance of the smoothed density contrast

$$\sigma_R^2 \equiv \left\langle \delta_R(\vec{x})^2 \right\rangle = \frac{1}{2\pi^2} \int_0^\infty dk \, k^2 P(k) \left| \tilde{W}_R(k) \right|^2$$

Gaussian distribution for the smoothed contrast

$$f_{\rm sm}(\delta; R) = \frac{1}{\sqrt{2\pi\sigma_R}} \exp\left(-\frac{\delta^2}{2\sigma_R^2}\right)$$

derive distribution in δ and R:

$$f(\delta, R) = -\frac{1}{\sigma_0} \frac{d\sigma_R}{dR} \frac{\delta^2}{\sigma_R^2} f_{\rm sm}(\delta; R)$$

Variance of the smoothed density field



Spherical collapse model

 EoM for spherical shell of matter on homogeneous radiation background

$$\ddot{r} = -\frac{8\pi G}{3}\rho_{\rm rad}r - \frac{GM}{r^2} \qquad \qquad M = \frac{4\pi}{3}\overline{\rho}\left(1+\delta\right)r^3$$

- over-density `turn around' for $\dot{r} = 0$
- an initial over-density $\delta(x_i)$ is collapsed at $x > x_i$ if

$$\delta > \delta_c$$
 with $\delta_c(x) \approx \frac{0.7}{x}$ $x \equiv a/a_{\rm eq}$

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$$\delta > \delta_c \text{ with } \delta_c(x) \approx \frac{0.7}{x} \qquad x \equiv a/a_{\text{eq}}$$

• holds for radiation as well as matter domination!

Double-differential mass function

given distribution in δ and R (before self-gravity becomes imporant):

$$f(\delta, R) = -\frac{1}{\sigma_0} \frac{d\sigma_R}{dR} \frac{\delta^2}{\sigma_R^2} f_{\rm sm}(\delta; R)$$

 derive double-differential distribution of collapsed objects with mass M and size R using:

$$M = \frac{4\pi}{3}\overline{\rho}\left(1+\delta\right)r^3$$

$$\frac{dn}{dMdR} = \frac{3}{2\pi MR^3} f(\delta, R)\Theta[\delta - \delta_c(x)]$$

Dimensionless double-differential mass function

physical sizes at turn around

$f_{\rm PQ} \ [{\rm GeV}]$	$M_{ m peak}\left[M_{\odot} ight]$	M range $[M_{\odot}]$	$r_{\mathrm{ta}}^{\mathrm{peak}} \; \mathrm{[km]}$	$r_{\rm ta}$ range [km]
10^{10}	4×10^{-16}	$[2 \times 10^{-17}, 1 \times 10^{-14}]$	4×10^4	$[2\times 10^4, 2\times 10^5]$
10^{11}	2×10^{-14}	$[5 \times 10^{-16}, 3 \times 10^{-13}]$	2×10^5	$[4 imes 10^4, 7 imes 10^5]$
10^{12}	8×10^{-13}	$[6 \times 10^{-14}, 2 \times 10^{-11}]$	2×10^6	$[7\times 10^5, 7\times 10^6]$

- MC masses span 3 orders, sizes span 1 order of magn.
- peak-masses 2 orders of mag. smaller than naive estimates

 $M \sim 10^{-12} M_{\odot} (f_{\rm PQ} / 10^{11} \, {\rm GeV})^2$

(typical fluctuations smaller than horizon at T_{osc})



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Minicluster mass function



Figure 6. Dimensionless minicluster mass function $X_M \equiv M^2/\overline{\rho}(dn/dM)$ for three choices of $f_{\rm PQ}$. The different line-styles indicate the mass function at different times: dotted x = 0.2, dashed x = 0.5, solid x = 1, dot-dashed x = 5, where $x = a/a_{\rm eq}$.

Enander, Pargner, TS, 1708.04466

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Assumptions

- initial power spectrum: should follow from evolution of string network (Kibble mechanism)
- harmonic approximation: anharmonic effects may lead to spikes in axion density [Kolb, Tkachev, 93]
- contribution from string/domain wall decays: likely to introduce additional energy density & fluctuations

Axion DM today - in our galaxy?



- Do minicluster survive non-linear structure formation?
- Do they collapse to dense Axion-stars? Are Axion-stars stable?

Bad news for direct detection

- suppose a fraction f_{MC} of DM is bound in objects of mass 10⁻¹³ M_{solar}
- the density in the Galaxy would be f_{MC}/solar system
- such a clump would pass through the Earth with a frequency

$$\frac{f_{\rm MC}}{t_{\rm Univ}} \left(\frac{\rm clump \ size}{10^6 \,\rm km}\right)^2$$

How large are Minicluster today?

- do axions (particles) virialize?
 Transition from classical field to collection of particles?
- virialized system would have a size $\sim 10^4 10^6$ km
- do they survive hierarchical structure formation?
- may be disrupted and form tidal streams stream crossing 1/(20 yr) lasting for 2 to 3 days Tinyakov, Tkachev, Zioutas, 15 Green, O'Hare, 17

Do minicluster condense to bose stars?

- static solution of a scalar field coupled to gravity (Klein-Gordon — Einstein / Schrödinger — Poisson)
- large literature (very incomplete)

Ruffini, Bonazzola, 1969 Rindler-Daller, Shapiro, 10, 12, 14 Chavanis, 11,17 **applied to QCD axion:** J. Barranco and A. Bernal, 2011 J. Barranco, A. C. Monteverde and D. Delepine, 2013 J. Eby, P. Suranyi, L. Wijewardhana et al., 14, 15, 16, 17 Davidson, Schwetz, 2016 Braaten, Mohapatra, Zhang, 15, 16, 16 Hertzberg, Schiappacasse 17 Baum, Freese, Redondo, Visinelli, Wilczek, 17

Bose star — dimensional analysis

consider static solution of Euler equation, replace $\nabla \to 1/R$:

$$\frac{1}{2m^2R^2} - \frac{M}{8f_a^2m^2R^3} - G_N\frac{M}{R} \simeq 0$$

balance quantum pressure, axion self-interactions, gravity

has a solution only if $M < M_{max}$: Chavanis 2011; Davidson, TS, 16;...

$$M \lesssim M_{max} \sim rac{m_{
m Pl} f_a}{m} \,, \qquad R \sim rac{m_{
m Pl}^2}{4m^2 M}$$

putting numbers:

$$egin{split} M_{max} &\simeq 10 rac{m_{
m Pl} f_a}{m} \sim 10^{-13} M_{\odot} \, (!) \ R &\sim 100 \, {
m km} \, , \qquad
ho &\sim 3 \, {
m g} \, {
m cm}^{-3} \left(rac{f_a}{10^{11} \, {
m GeV}}
ight)^2 \end{split}$$

Baum, Freese, Redondo, Visinelli, Wilczek, 17





distribution at turn-around

How can minicluster condense into bose stars?

- gravitational cooling Seidel, Suen, 1994: radiate away scalar field
- significant fraction of minicluster mass maybe radiated away in terms of (semi)relativistic axions
- possible collapse to a black hole (at much larger mass than MC)



see also:

Helfer, Marsh, Clough, Fairbairn, Lim, Becerril, 1609.04724; Levkov, Panin, Tkachev, 1609.03611

Sensitivity in lensing?

 depending on structure formation history, potentially interesting lensing signatures: femto-lensing Kolb, Tkachev, 95 micro-lensing Fairbairn, Marsh et al, 17



Fairbairn, Marsh, Quevillon, Rozier, 1707.03310



Fairbairn, Marsh, Quevillon, Rozier, 1707.03310

Summary – outlook

- Axion miniclusters are expected in the post-inflation scenario
- evolution till today not well understood
- generic feature also for ALP dark matter scenarios beyond the QCD axion
- important for direct detection
- potential lensing signature

P-spectrum dependence on assumptions

