Challenges to parameter reconstruction for Direct Detection

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Based on arXiv:1802.03174 with D. Cerdeño, E. Reid and H. Schulz

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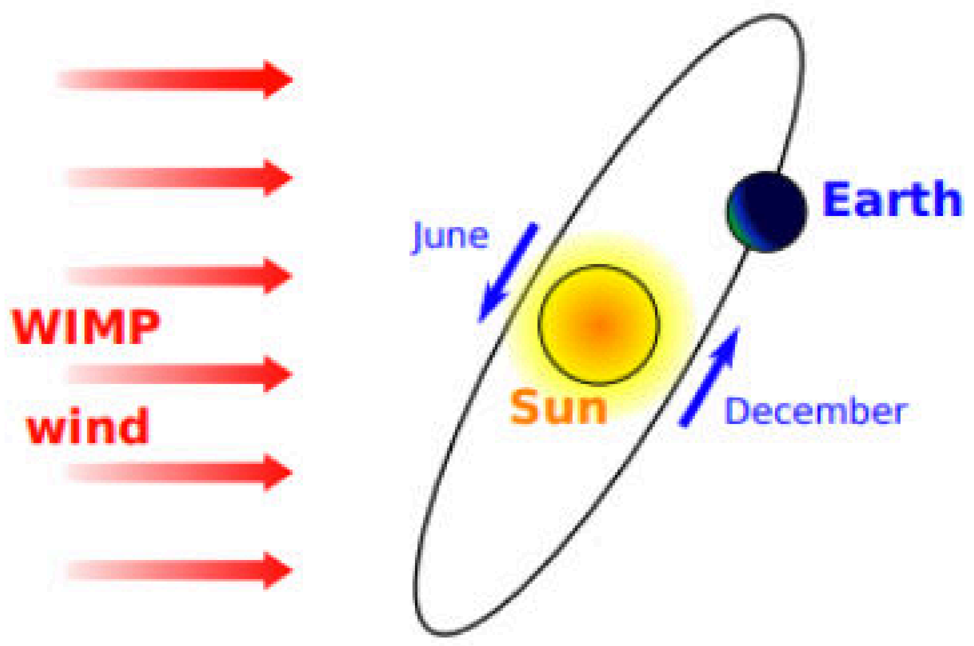
Why should we care about parameter reconstruction

- Understanding the UV models that could show up in DD experiments.
- By taking a general approach to analysis in DD we can perhaps learn the limits of the technology.
- We can learn precisely what is excluded by experimental results including relevant uncertainties.
- With a positve signal, we can determine what the likely nature of DM is. Connecting with indirect and collider searches.
- In this talk I will focus primarily on RAPIDD, which tries to eleviate some of the issues that can arise in more general parameter reconstruction.
- RAPIDD stands for Reconstruction Analysis using Polynomials In Direct Detection.

The Dark Matter Direct Detection Calculation

- DD exploits the relative velocity of Earth and the Dark Matter halo to tell us something about the interactions DM has with ordinary matter.
- In order to calculate the number of recoils in a given energy bin, one typically needs to evaluate these nested integrals.

$$N_k = \frac{\rho_0 \epsilon}{m_T\, m_\chi} \int_{E_k}^{E_{k+1}} dE_R\, \varepsilon(E_R) \int_{E'_R} dE'_R\, Res(E'_R, E_R) \int_{v_{min}} d\vec{v}\, v f(\vec{v})\, \frac{d\sigma_{\chi T}}{dE'_R}$$



Dealing with the halo velocity distribution may not be simple

- The velocity distribution of incident DM is often assumed to be maxwellian $f(v) = \left(\frac{1}{N}\right) \exp(-v^2/v_0^2)\Theta(v_{esc} - v)$, which can be integrated analytically.
- However, to account for uncertainties in halo parameters, and unknowns about the shape of this distribution, one could take results from simulations or astrophysical data, which could be more complicated.
- To account for moderate variations in distributions we have used

$$f(v) = N_k^{-1} \left[e^{-v^2/kv_0^2} - e^{-v_{esc}^2/kv_0^2}\right]^k \Theta(v_{esc} - v)$$

The particle interactions may be more complicated

- General particle interactions are not fully encapsulated by the canonical spin-(in-)dependent parametrisation and could misrepresent the physics of DM.
- A Non-Relavistic Effective Field Theory has been developed for the 4 field DM-Nucleon interaction,

$$\mathcal{L}_{\text{int}} = \chi \mathcal{O}_\chi \chi N \mathcal{O}_N N = \sum_{N=n,p} \sum_i c_i^{(N)} \mathcal{O}_i \, \chi^+ \chi^- N^+ N^-$$

- Like all EFTs they describe the physics by only using the relevant degrees of freedom.
- In Direct Detection the quantities that are relavent are velocity v, the tranfer momentum q and the spins of DM and the nucleon S_χ and S_N.

Generality comes with its usual drawbacks

- The more complex NREFT basis will allow analysis to be more general and model independent.
- By widening the parameter space, we can test what DD experiments could tell us thing about the particle nature of Dark Matter in general.
- A computational drawback to the NREFT is that we're going from σ_0^{SI} and σ_0^{SD} to

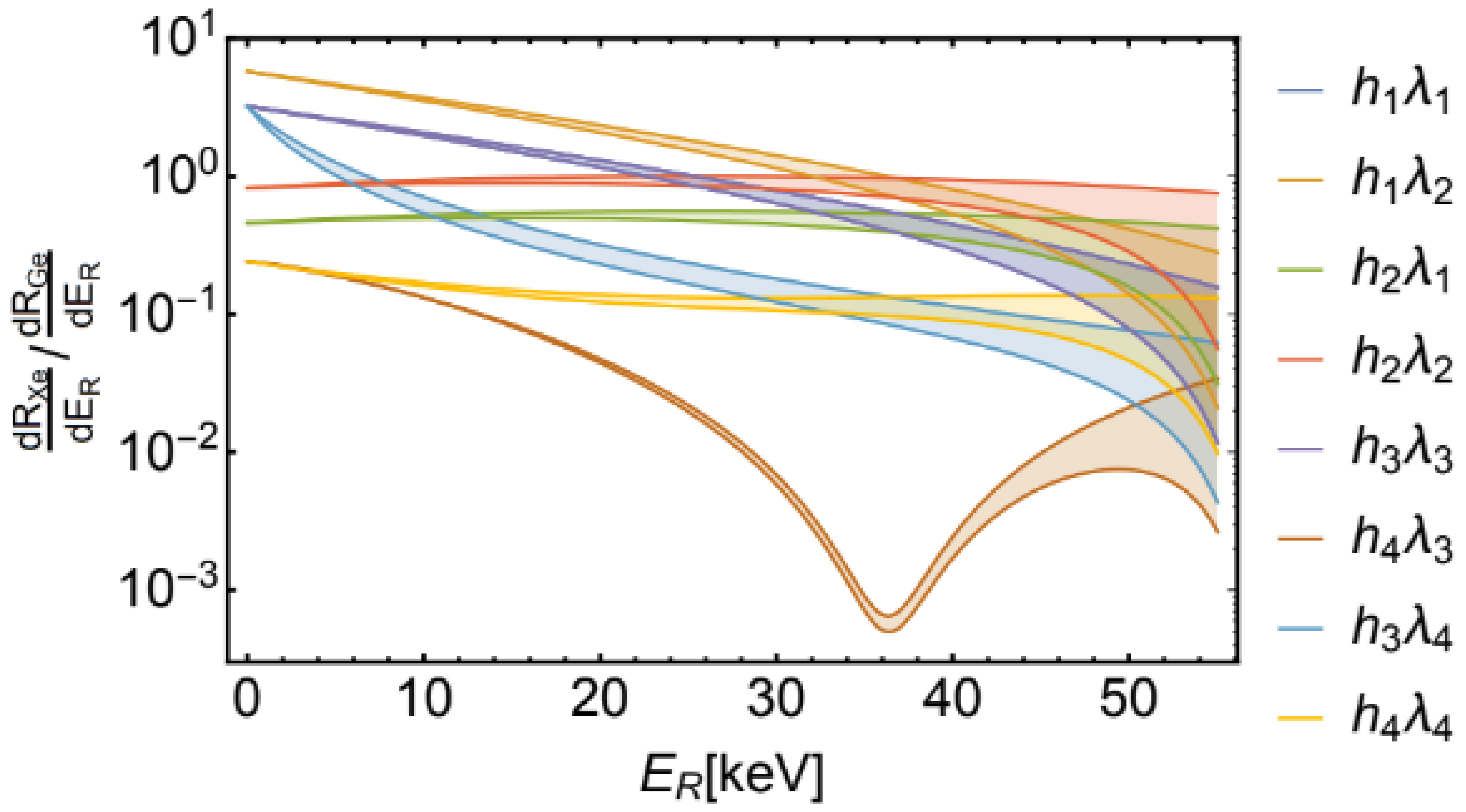
$$\hat{\mathcal{O}}_1 = \mathbb{1}_\chi \mathbb{1}_N \qquad \hat{\mathcal{O}}_{10} = i\hat{\mathbf{S}}_N \cdot \frac{\hat{\mathbf{q}}}{m_N}\mathbb{1}_\chi$$
$$\hat{\mathcal{O}}_3 = i\hat{\mathbf{S}}_N \cdot \left(\frac{\hat{\mathbf{q}}}{m_N} \times \hat{\mathbf{v}}^\perp\right)\mathbb{1}_\chi \qquad \hat{\mathcal{O}}_{11} = i\hat{\mathbf{S}}_\chi \cdot \frac{\hat{\mathbf{q}}}{m_N}\mathbb{1}_N$$
$$\hat{\mathcal{O}}_4 = \hat{\mathbf{S}}_\chi \cdot \hat{\mathbf{S}}_N \qquad \hat{\mathcal{O}}_{12} = \hat{\mathbf{S}}_\chi \cdot \left(\hat{\mathbf{S}}_N \times \hat{\mathbf{v}}^\perp\right)$$
$$\hat{\mathcal{O}}_5 = i\hat{\mathbf{S}}_\chi \cdot \left(\frac{\hat{\mathbf{q}}}{m_N} \times \hat{\mathbf{v}}^\perp\right)\mathbb{1}_N \qquad \hat{\mathcal{O}}_{13} = i\left(\hat{\mathbf{S}}_\chi \cdot \hat{\mathbf{v}}^\perp\right)\left(\hat{\mathbf{S}}_N \cdot \frac{\hat{\mathbf{q}}}{m_N}\right)$$
$$\hat{\mathcal{O}}_6 = \left(\hat{\mathbf{S}}_\chi \cdot \frac{\hat{\mathbf{q}}}{m_N}\right)\left(\hat{\mathbf{S}}_N \cdot \frac{\hat{\mathbf{q}}}{m_N}\right) \qquad \hat{\mathcal{O}}_{14} = i\left(\hat{\mathbf{S}}_\chi \cdot \frac{\hat{\mathbf{q}}}{m_N}\right)\left(\hat{\mathbf{S}}_N \cdot \hat{\mathbf{v}}^\perp\right)$$
$$\hat{\mathcal{O}}_7 = \hat{\mathbf{S}}_N \cdot \hat{\mathbf{v}}^\perp \mathbb{1}_\chi \qquad \hat{\mathcal{O}}_{15} = -\left(\hat{\mathbf{S}}_\chi \cdot \frac{\hat{\mathbf{q}}}{m_N}\right)\left[\left(\hat{\mathbf{S}}_N \times \hat{\mathbf{v}}^\perp\right) \cdot \frac{\hat{\mathbf{q}}}{m_N}\right]$$
$$\hat{\mathcal{O}}_8 = \hat{\mathbf{S}}_\chi \cdot \hat{\mathbf{v}}^\perp \mathbb{1}_N \qquad \hat{\mathcal{O}}_{17} = i\frac{\hat{\mathbf{q}}}{m_N} \cdot \mathcal{S} \cdot \hat{\mathbf{v}}^\perp \mathbb{1}_N$$
$$\hat{\mathcal{O}}_9 = i\hat{\mathbf{S}}_\chi \cdot \left(\hat{\mathbf{S}}_N \times \frac{\hat{\mathbf{q}}}{m_N}\right) \qquad \hat{\mathcal{O}}_{18} = i\frac{\hat{\mathbf{q}}}{m_N} \cdot \mathcal{S} \cdot \hat{\mathbf{S}}_N$$

Caveat to Complexity

- Just like in the canonical case, $\mathcal{O}_1$, the spin independent response is usually dominant (enhanced by A^2).
- This enhancement can lead to loop generated $\mathcal{O}_1$ responses being the dominant contribution in DD.
- Its been shown for certain simplified models, running from LHC scales to DD scales, operators that aren't present at tree level will be at DD. See D'Eramo et al arXiv:1605.04917.
- In the similar vain, a full UV complete pseudo-scalar dark matter model has been studied at 1-loop in Bell et al arXiv:1803.01574. A tree level $\mathcal{O}_6$ is dominated by a $\mathcal{O}_1$ response which is generated at 1-loop.
- Are these considerations enough to give us to power to descriminate between all models?

Not quite

- Dent et al arXiv:1505.03117 suggested that for a DM mass, complimentarity of different experimental targets will be able to descriminate between all signals in DD.

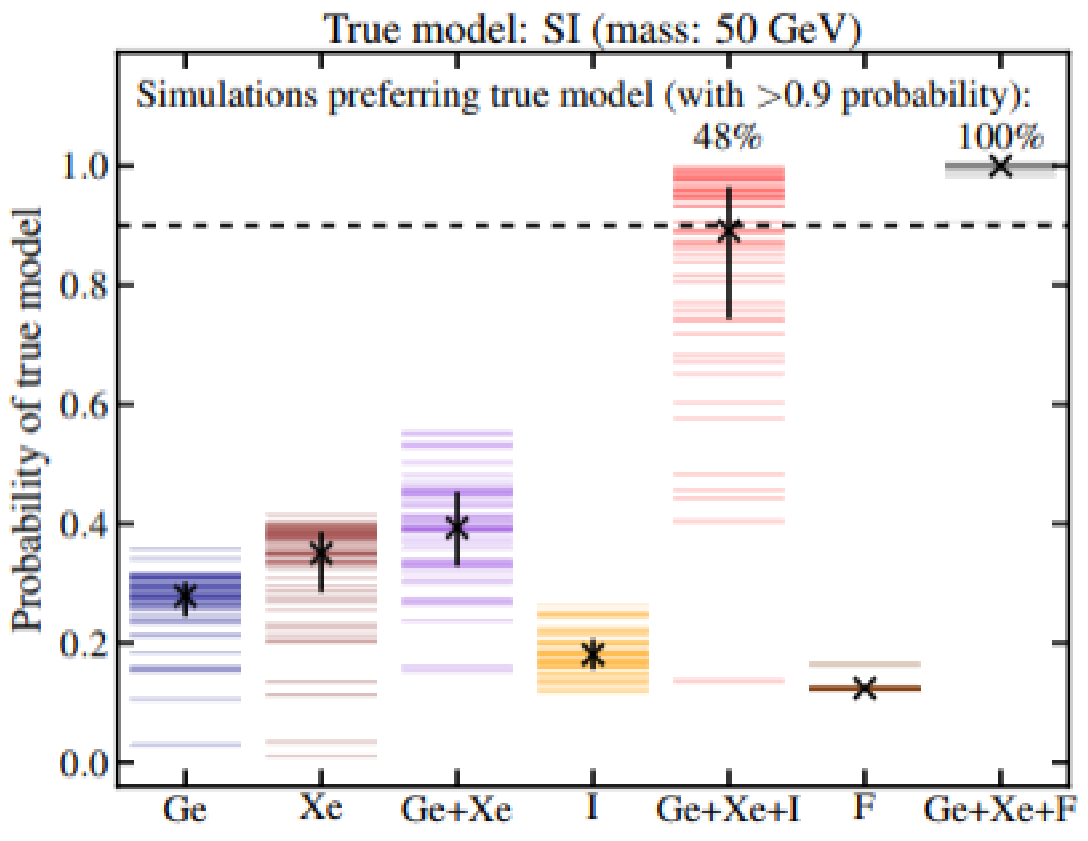


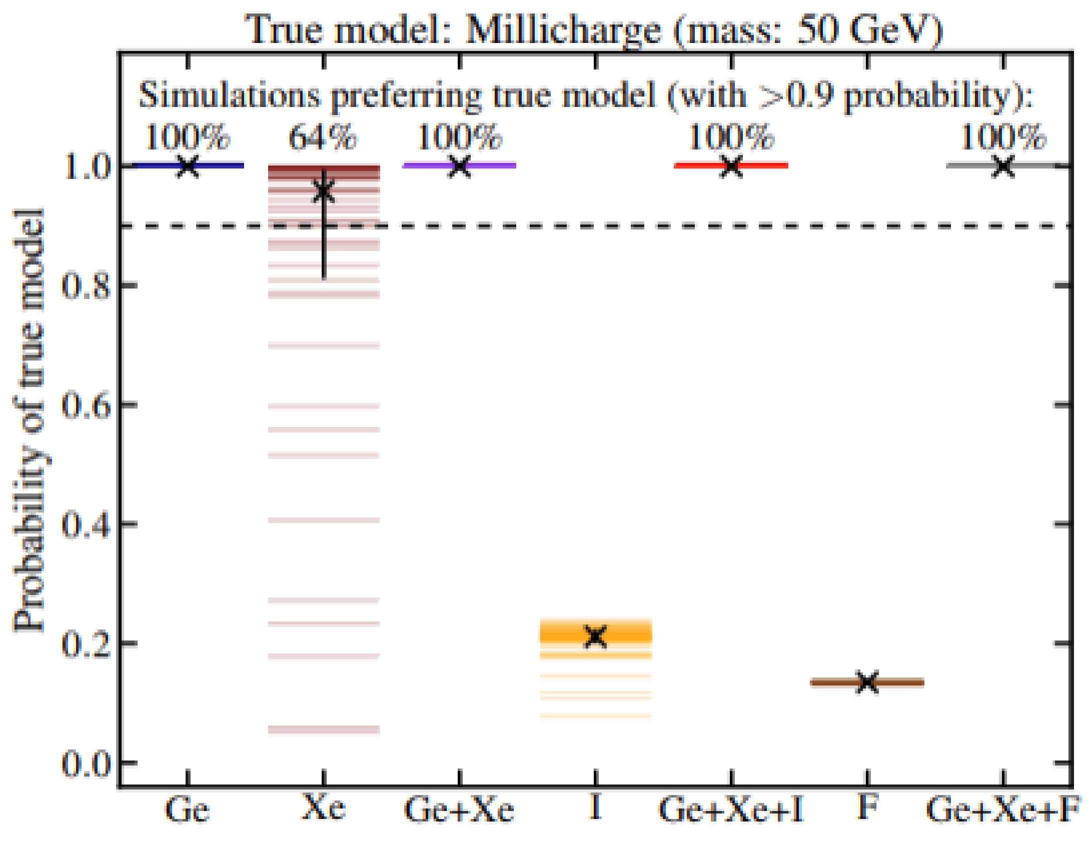
- However, DD would be fairly poor at determining the mass of DM. When the mass of the DM is unknown, different operators can mimick the 'true signal'.
- Furthermore, multiple operators acting at the same time were not considered.
- So how should we move forward in discriminating models or pinning down parameter values?

We can pick a set of our favourite models and fix couplings through the lagrangian

- By comparing fits of specific models to some simulated data, we can get an idea for whether these are distinguishable.
- For example the Lagrangian, $\mathcal{L}_{int} = a\bar{\chi}\gamma^{\mu}\gamma^{5}\chi\partial_{\nu}F_{\mu\nu}$, gives rise to specific c_i^N values for a given DM mass.
- With some statistical techniques, you can try to compare models arXiv:1506.04454

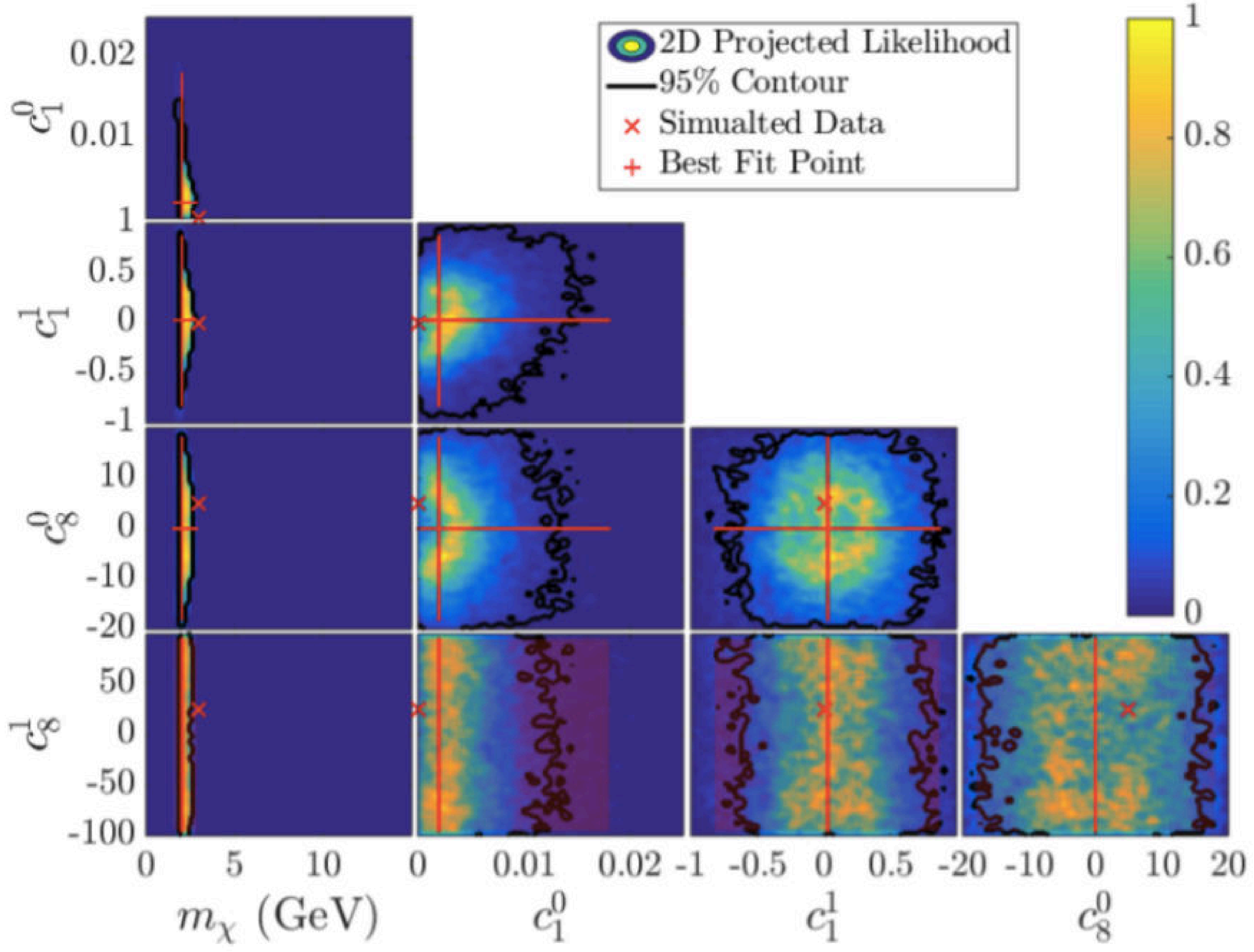
$$Pr(\mathcal{M}_i) = \frac{\varepsilon_i(\mathbf{X}|\mathcal{M}_i)}{\sum_j \varepsilon_j(\mathbf{X}|\mathcal{M}_j)}$$





We can focus on operator responses of a certain type

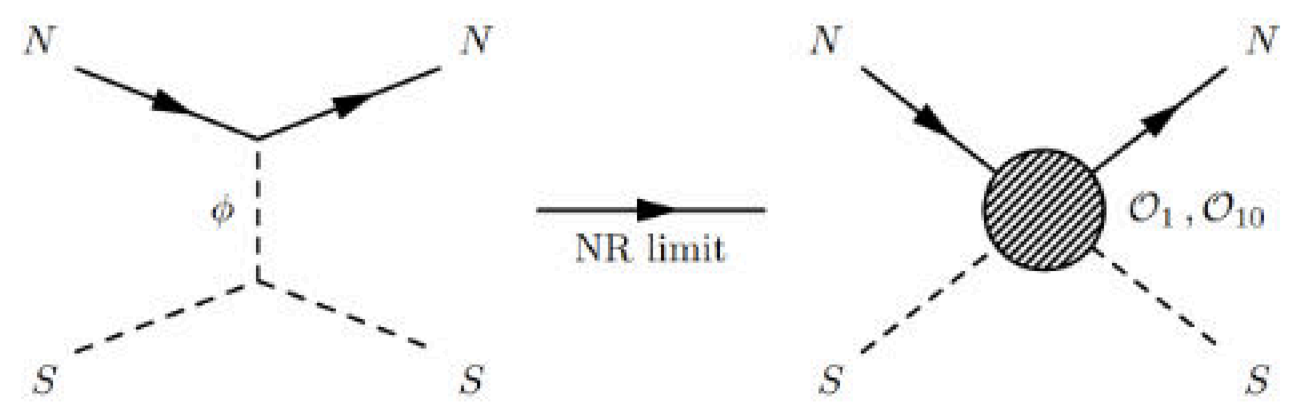
- We can pick a couple operators and determine whether we can differentiate.
- If you wanted to remain general in your analysis though, you'd have to deal with a large parameter space. arXiv:1612.09038



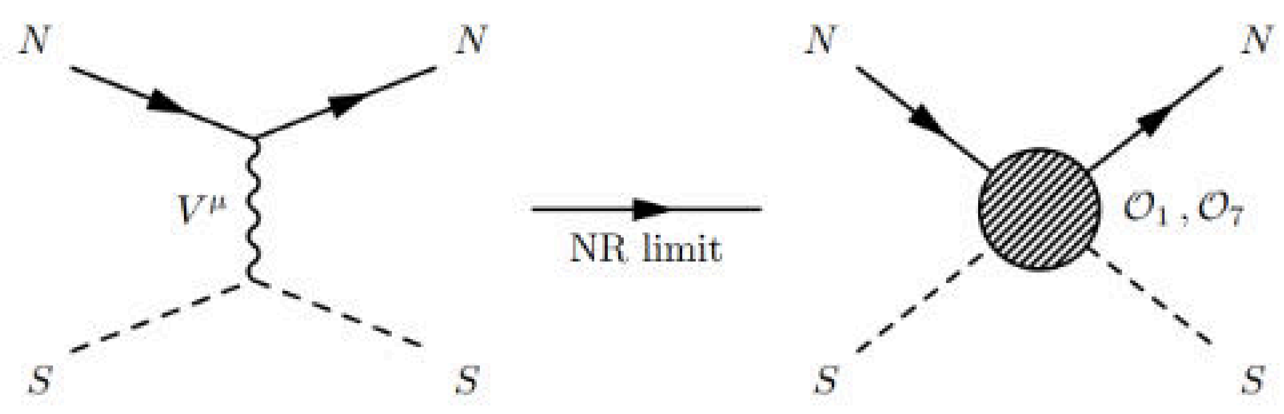
By using simplified models we can have the best of both worlds

- In principle each operator coefficient could be treated as a free parameter (except in FV).
- Simplified models allow us to incorporate multiple models at the same time, i.e. the pseudo-scalar mediated DM model is just the fermion-scalar DM with certain operators turned off.
- Including the four extra dimensions from the halo function, we have up to 9 dimensions.

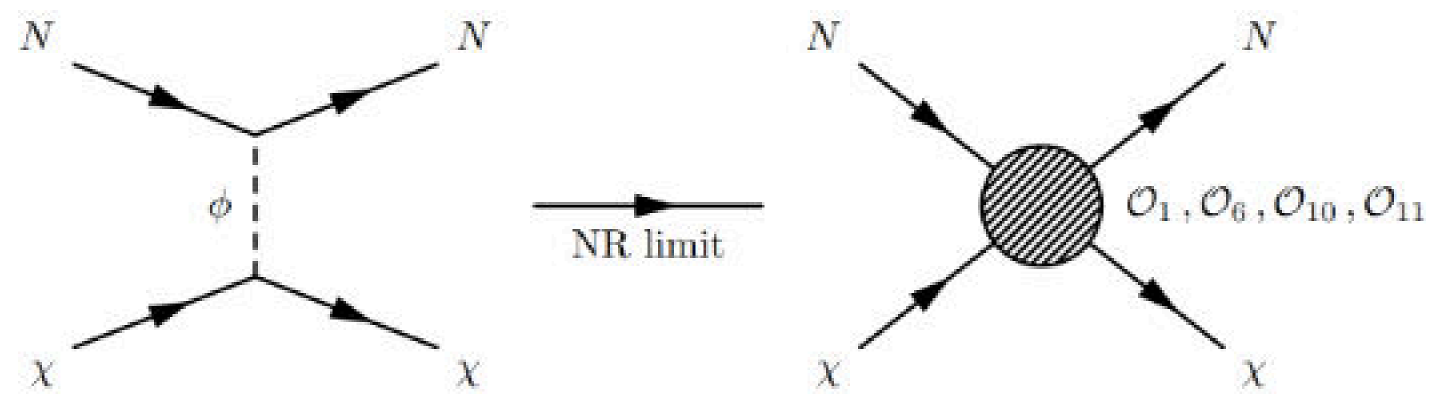
Scalar - Scalar



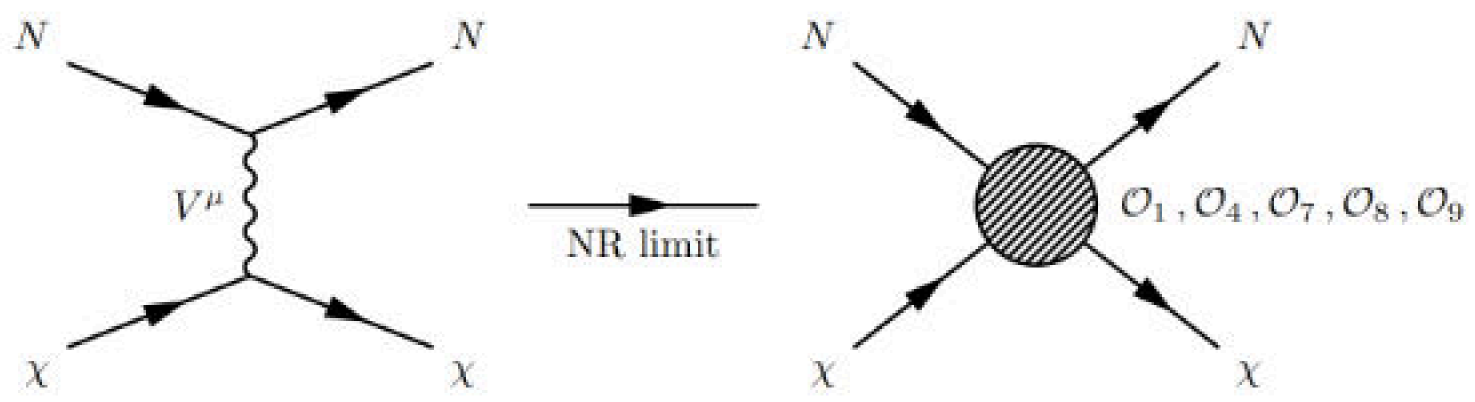
Scalar- Vector



Fermion- Scalar

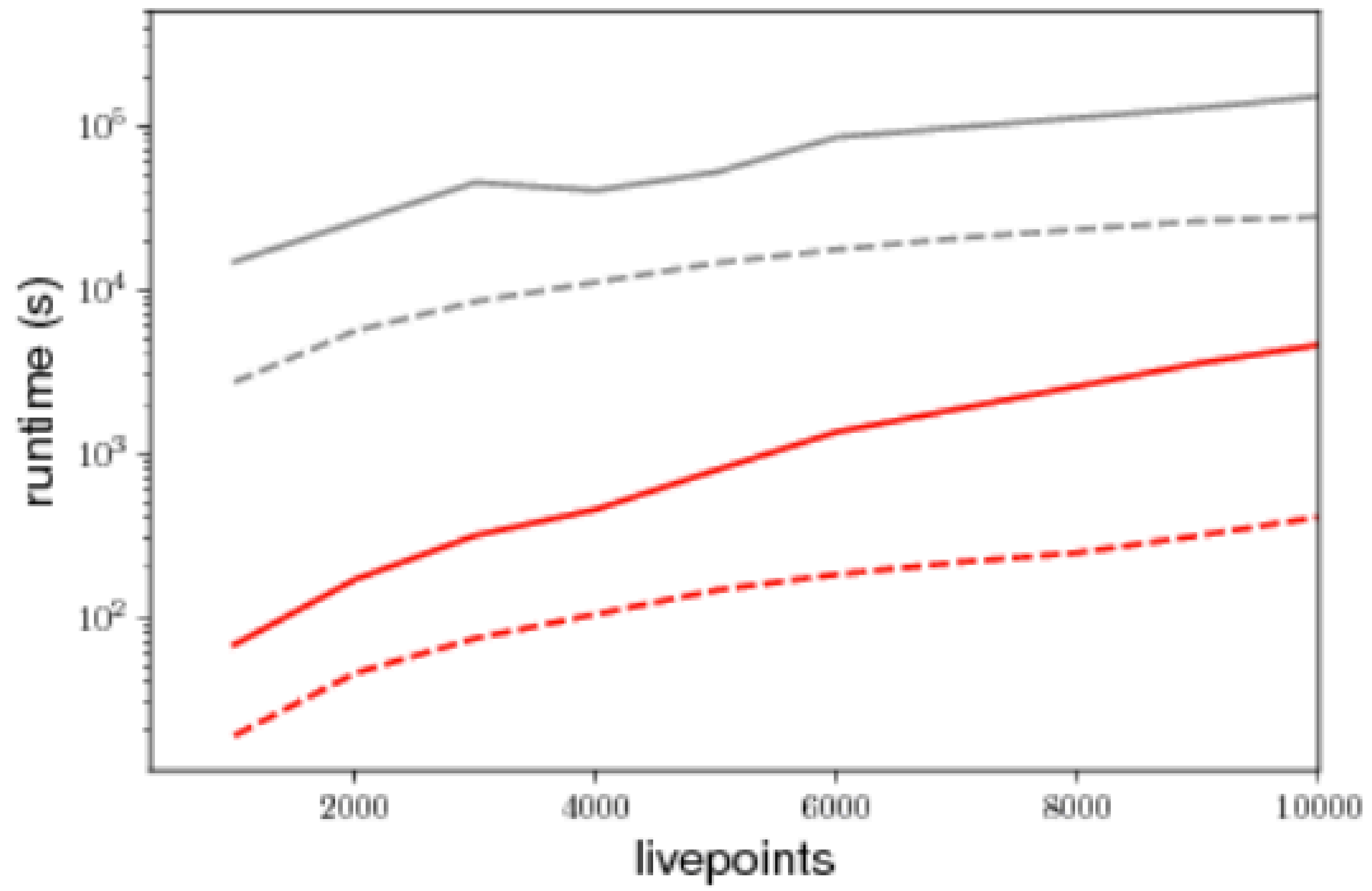


Fermion- Vector



We have developed RAPIDD for fast and general analysis

- We trained the surrogate model on a C code and used MultiNest to compare performance over a number of livepoints.
- RAPIDD at worst sees a speed up factor of ~ 20. At best above 200.



How RAPIDD works

- Instead of using the physics code to produce a result for a given energy bin N_k^a we call a polynomial $\mathcal{P}_k^a$.
- To do so we first choose a polynomial order $\mathcal{O}$ appropriate for the physics problem at hand. With $\mathcal{O}$ and the parameter point $\mathbf{\Theta}$ given, the structure of the polynomial is fixed. What remains to be done is to determine the coefficients, $d_{k,l}^a$, that allow to approximate the true behaviour of $N_k^a(\mathbf{\Theta})$ such that

$$N_k^a(\mathbf{\Theta}) \approx \mathcal{P}_k^a(\mathbf{\Theta}) = \sum_{l=1}^{N_{\text{coeffs}}} d_{k,l}^a \, \tilde{\Theta}_l \equiv \mathbf{d_k^a} \cdot \tilde{\mathbf{\Theta}}$$

- For example, for a quadratic polynomial in a two dimensional parameter space $\mathbf{\Theta} = (m_\chi,\ c_1) = (x, y)$, the coefficients take on the form $\mathbf{d_k^a} = (\alpha, \beta_x, \beta_y, \gamma_{xx}, \gamma_{xy}, \gamma_{yy})$

How RAPIDD works

- This is done by collecting each $N_k^a(\mathbf{\Theta})$ for the set of sample points and solving this matrix equation

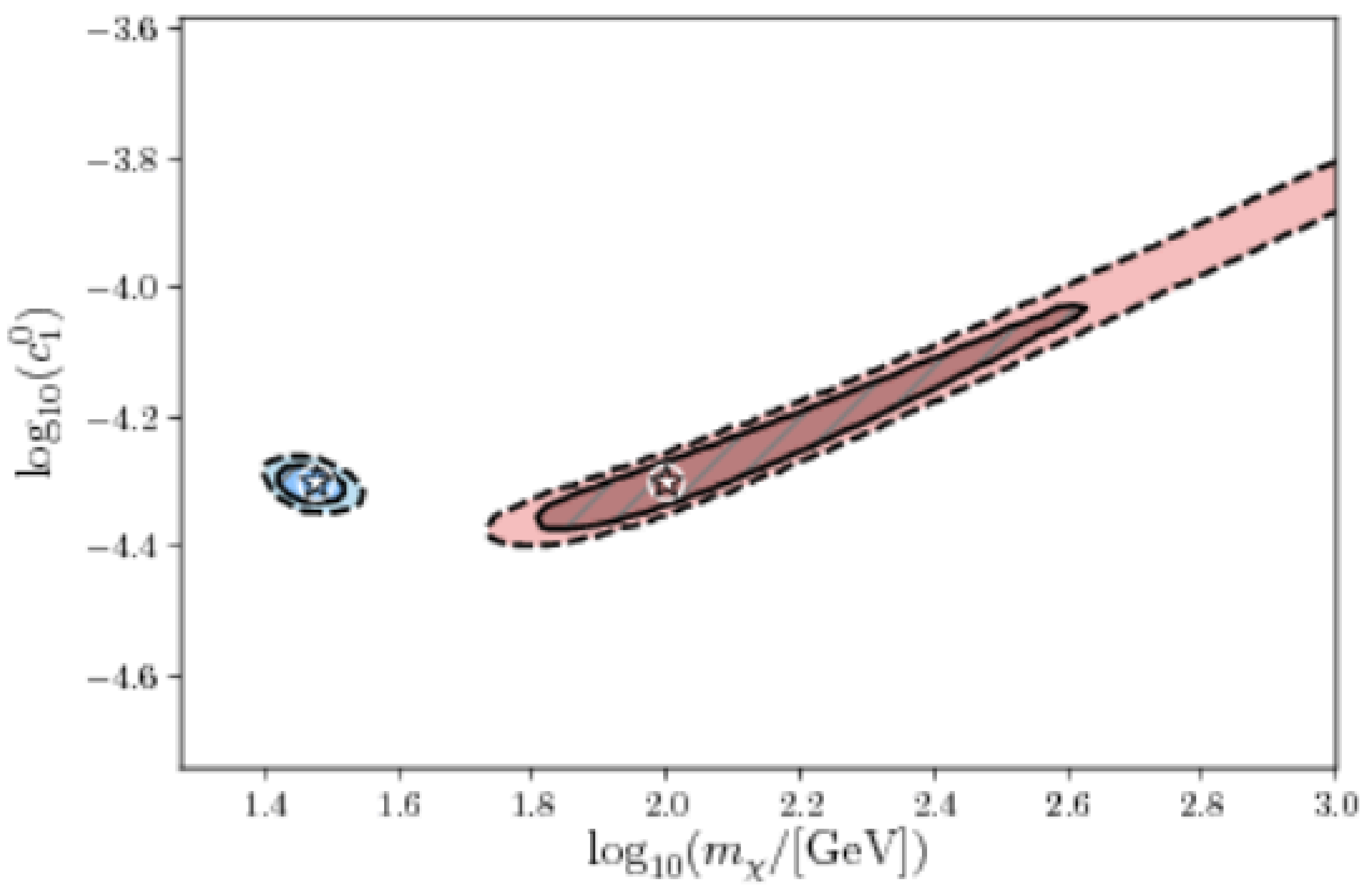
$$\overrightarrow{N_k^a} = M_{\tilde{\mathbf{\Theta}}} \cdot \mathbf{d_k^a}$$

- Where $M_{\tilde{\mathbf{\Theta}}}$ is a quantity similar to a Vandermorde matrix where each row contains the values of $\tilde{\mathbf{\Theta}}$ for each sampled point, and $\overrightarrow{N_k^a}$ is a vector of the resulting number of events. This allows us to solve for $\mathbf{d_k^a}$ using the (pseudo-) inverse of $M_{\tilde{\mathbf{\Theta}}}$, which in the PROFESSOR program is evaluated by means of a singular value decomposition.

Tests

- In order to test our code we used RAPIDD and the physics code for some canonical examples.
- The first of which was to test in 2-D, scanning in the (m_χ, c_1^0) plane, which is just the NREFT equivalent to the spin independent case,

$$\sigma_{\chi N} = \frac{\mu_{\chi N}^2}{\pi\, m_v^4} \left(c_1^0\right)^2$$

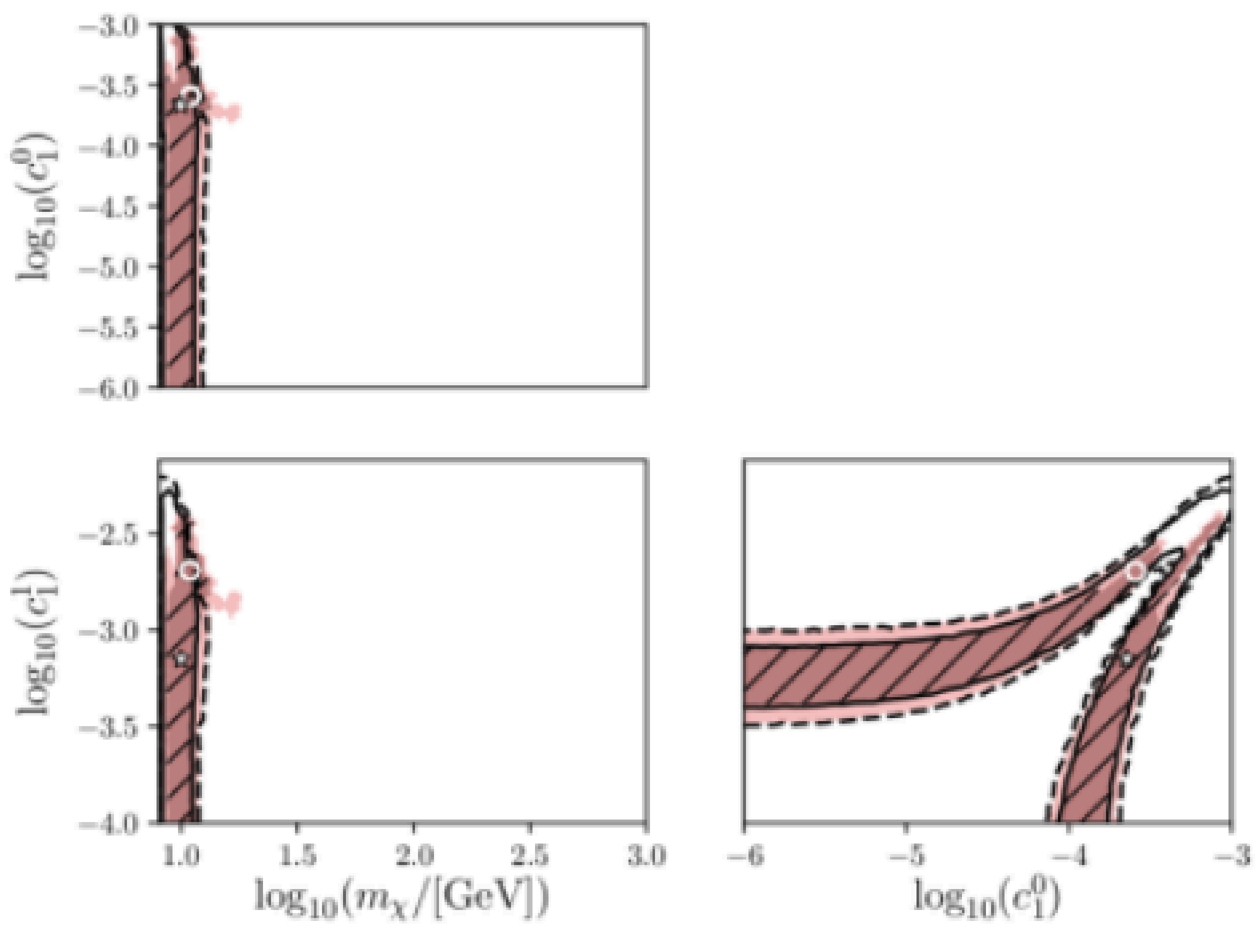


3-D Test 2 (Cancellation)

- We also wanted to test RAPIDD in specific cases where finely tuned cancellations were possible.
- This inspired us to build the different polynomials contributions seperately

$$N_k^a(\mathbf{\Theta}) \approx \sum_{ij} \sum_{\tau,\tau'=0,1} \mathcal{P}_k^{a,i,j,\tau,\tau'}(\mathbf{\Theta})$$

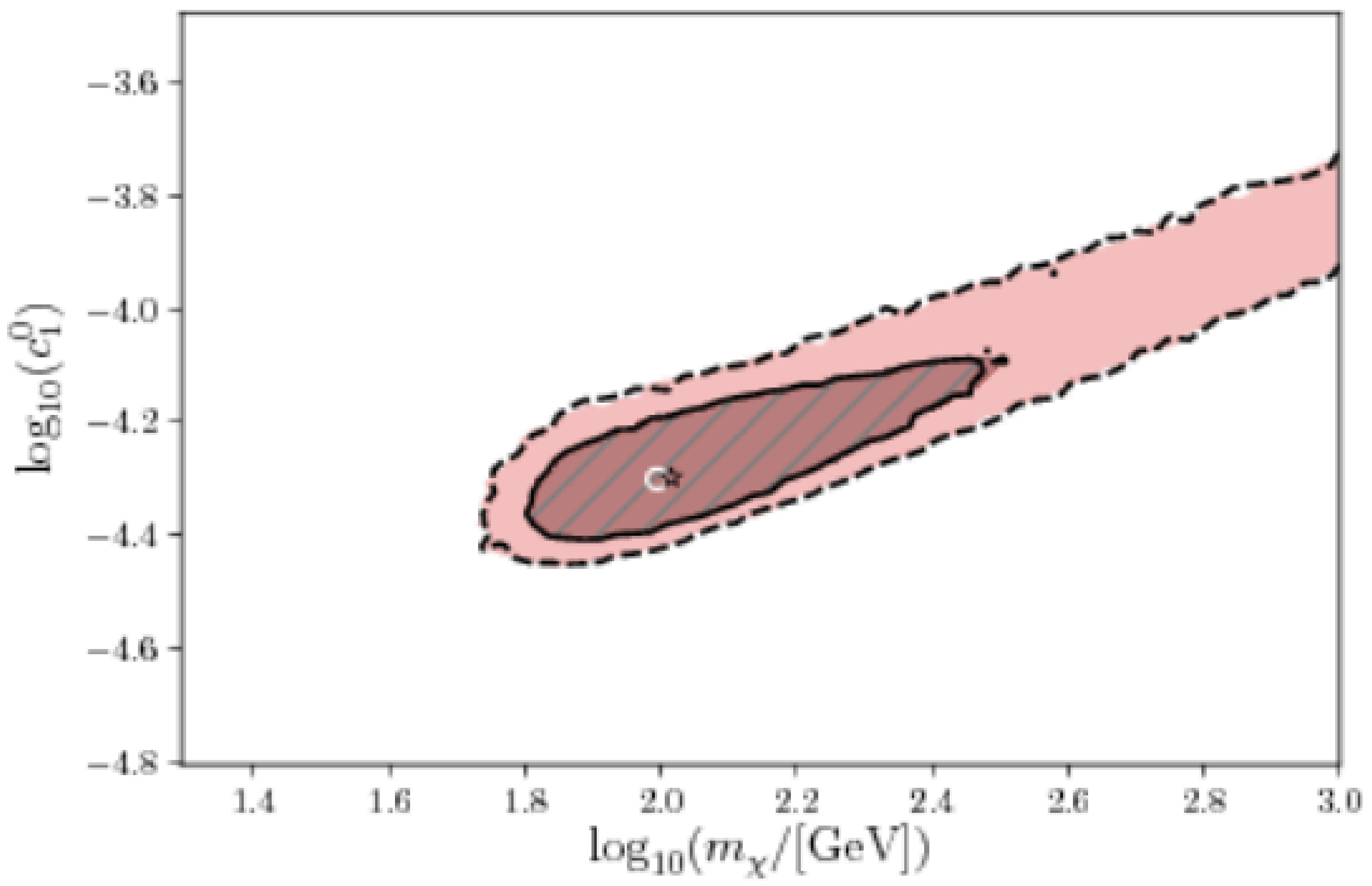
- For example when isoscalar and isovector couplings are free (would cause problems with quark universality).



6-D Test (with Halo)

- Finally we tested how RAPIDD works with the general halo function

$$f(v) = N_k^{-1}\left[e^{-v^2/kv_0^2} - e^{-v_{esc}^2/kv_0^2}\right]^k \Theta(v_{esc} - v)$$



Using RAPIDD to Constrain Models

- We wanted to provide a case study of how our code could be used in future analysis.
- We took the following detector variables

Target	Exposure	Energy window	Bin No
Xe	5.6×10^6 kg days	3-30 keV	27
Ge	91250 kg days	0.35-50 keV	49
Ar	7.3×10^6 kg days	5.0-30 keV	24

- Then we took three benchmark points, which are accessible by future detectors.

Name	Model	DM Parameters	N_{Xe}	N_{Ge}	N_{Ar}
BP1	SS	$m_\chi = 10$ GeV $c_1 = 1\times10^{-4}$ $c_{10} = 5$	93	10	50
BP2	SS	$m_\chi = 100$ GeV $c_1 = 3\times10^{-5}$ $c_{10} = 5\times10^{-1}$	206	2	30
BP3	FS	$m_\chi = 30$ GeV $c_1 = 0.0$ $c_6 = 60$ $c_{10} = 0.0$ $c_{11} = 0.0$	256	1	0

Using RAPIDD to Constrain Models

- Essentially, our results concluded with this

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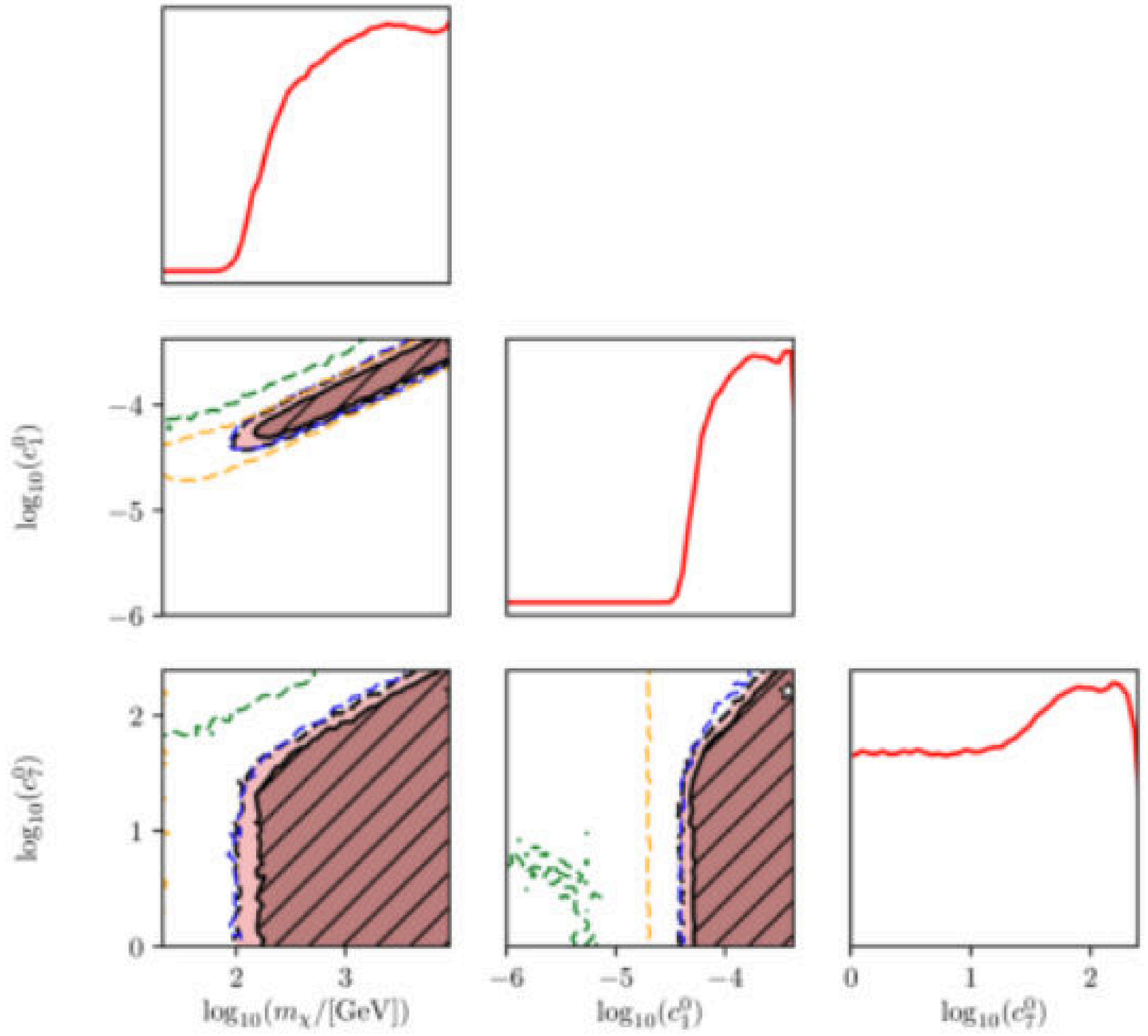
BP1, BP2: Fully Degenerate

BP3: Partially Degenerate

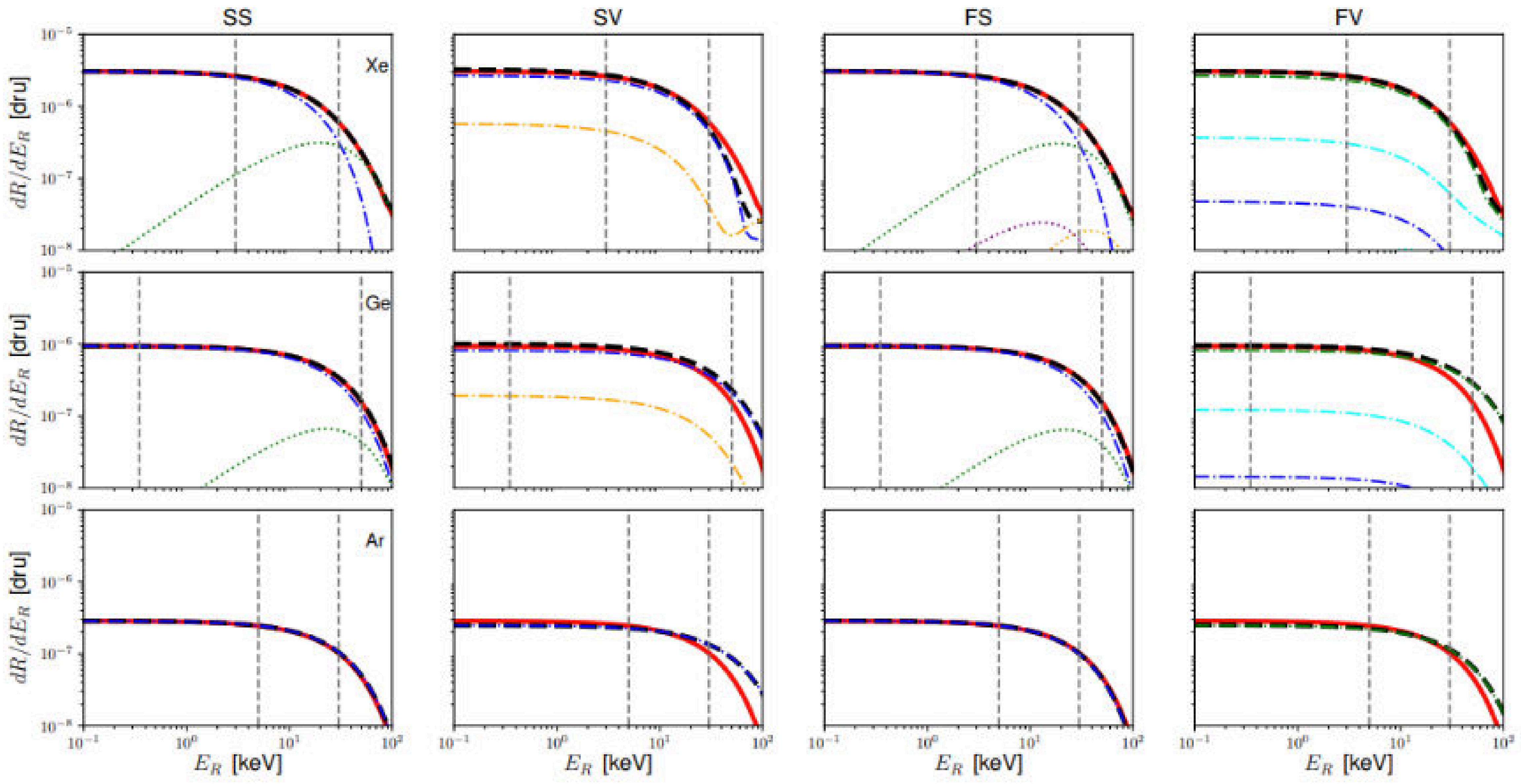
For the degenerate cases

- You get profile likelihoods with no tensions between experiments.



For the degenerate cases

- You get good data reconstuction for each model.

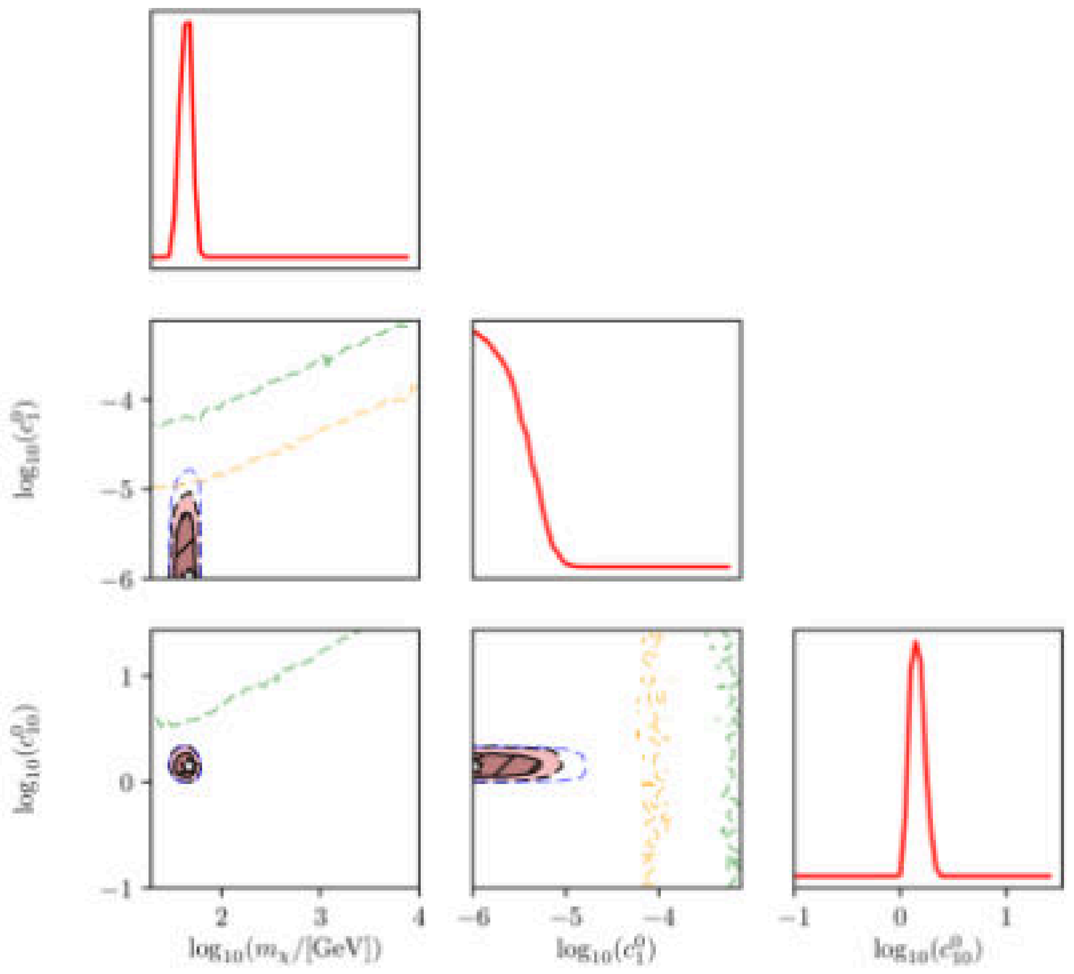


SS	SV	FS	FV
$m_\chi = 105$ GeV $c_1 = 4.11 \times 10^{-5}$ $c_{10} = 0.595$	$m_\chi = 989$ GeV $c_1 = 3.79 \times 10^{-4}$ $c_7 = 161$	$m_\chi = 95.7$ GeV $c_1 = 3.90 \times 10^{-5}$ $c_6 = 3.22$ $c_{10} = 0.570$ $c_{11} = 1.85 \times 10^{-4}$	$m_\chi = 1403$ GeV $c_1 = 1.81 \times 10^{-5}$ $c_4 = 5.14 \times 10^{-2}$ $c_7 = 1.69$ $c_8 = 3.04 \times 10^{-1}$ $c_9 = 0.303$
$\log \mathcal{L}(SS) = -85.2$	$\log \mathcal{L}(SV) = -86.0$	$\log \mathcal{L}(FS) = -85.2$	$\log \mathcal{L}(FV) = -85.4$

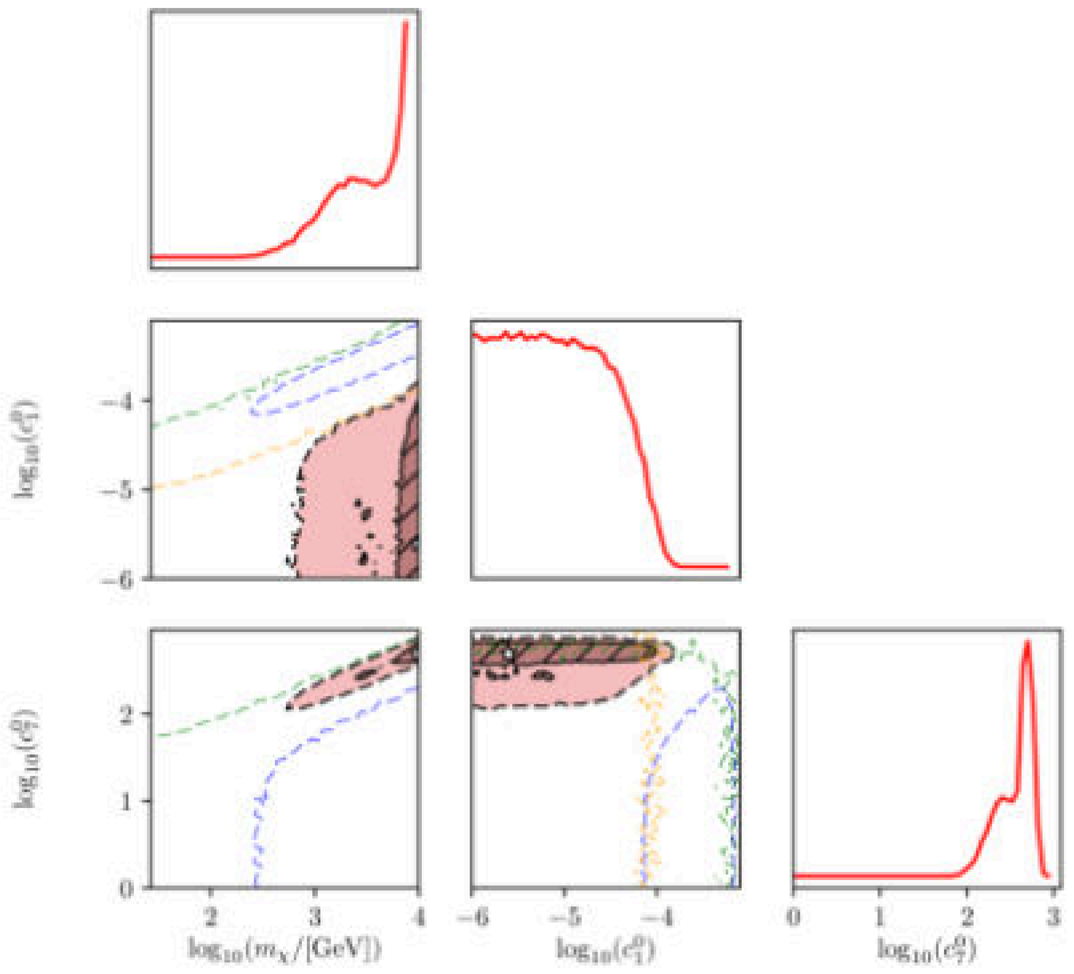
For the non-degenerate case

- You get profile likelihoods with tensions between experiments for some models, but not for others.

Scalar - Scalar

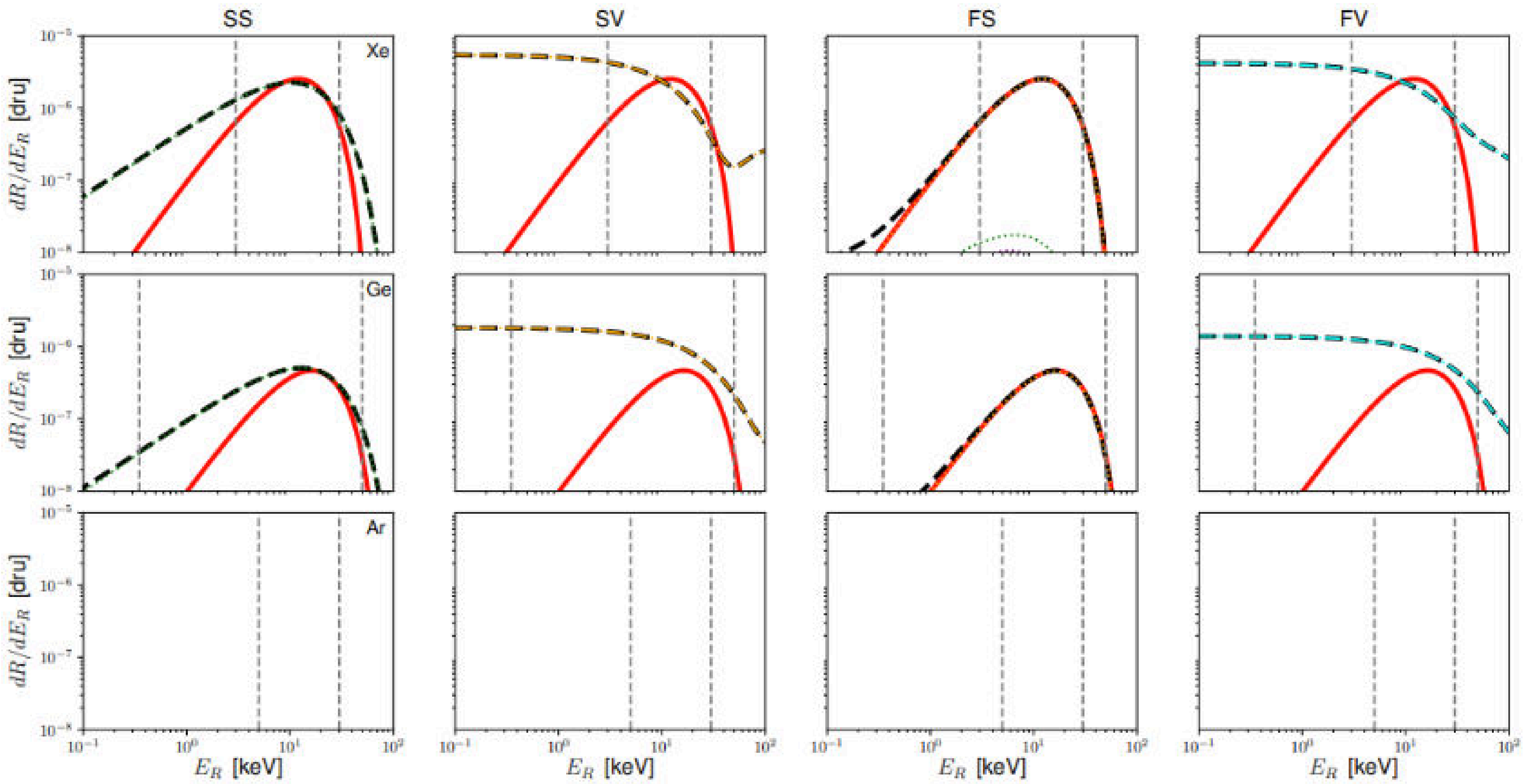


Scalar - Vector



For the non-degenerate case

- You get poor reconstructions of the data.



$m_\chi = 47.0$ GeV	$m_\chi = 9920$ GeV	$m_\chi = 30.9$ GeV	$m_\chi = 9966$ GeV
$c_1 = 1.06 \times 10^{-6}$	$c_1 = 2.48 \times 10^{-6}$	$c_1 = 1.00 \times 10^{-6}$	$c_1 = 2.35 \times 10^{-6}$
$c_{10} = 1.39$	$c_7 = 4.91 \times 10^2$	$c_6 = 59, 8$	$c_4 = 0.469$
		$c_{10} = 0.126$	$c_7 = 1.40$
		$c_{11} = 1.02 \times 10^{-4}$	$c_8 = 1.21 \times 10^{-2}$
			$c_9 = 1.20 \times 10^{-2}$
$\log \mathcal{L}(SS) = -62.6$	$\log \mathcal{L}(SV) = -103$	$\log \mathcal{L}(FS) = -58.9$	$\log \mathcal{L}(FV) = -84.4$

Conclusions and future work

- There are many challenges to parameter reconstruction in DD, RAPIDD will hopefully help us adress some of them.
- Rapidd, is a new tool that enables quick general analysis at high-dimensionality. Something required to compare different models of DM in the face of data.
- Tools like this can examine the best way to break signal degeneracies in experiments, informing experimental parameters and perhaps the next generation of experiments.
- RAPIDD is not public yet, we've been delayed by advances in the methods we use to build the polynomials. But it'll be out before the end of the summer.
- Thank you