Lattice QCD beyond the valence approximation

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How fast are lattice QCD simulations?

Simulations of lattice QCD with light sea quarks turn out to be much more feasible than previously thought

No of operations [in Tflops×year] required for an ensemble of 100 gauge fields*

$$5\left[\frac{20\ \mathrm{MeV}}{m}\right]^3\left[\frac{L}{3\ \mathrm{fm}}\right]^5\left[\frac{0.1\ \mathrm{fm}}{a}\right]^7$$

Ukawa, Berlin 2001

*Two-flavour QCD, O(a) improved Wilson quarks, quark mass m, $2L\times L^3$ lattice, spacing a

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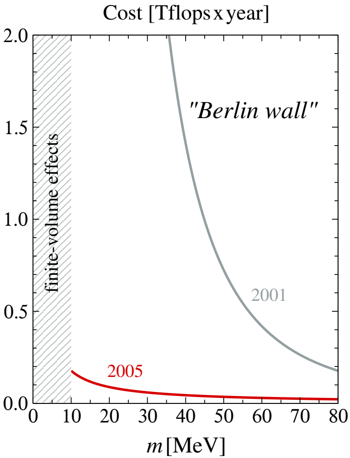
$$0.05\left[\frac{20\ \mathrm{MeV}}{m}\right]^{1}\left[\frac{L}{3\ \mathrm{fm}}\right]^{5}\left[\frac{0.1\ \mathrm{fm}}{a}\right]^{6}$$	Giusti, Tucson 2006

*Two-flavour QCD, O(a) improved Wilson quarks, quark mass m, $2L\times L^3$ lattice, spacing a

Example:

64×32^3 lattice

$L \simeq 2.5$ fm, $a \simeq 0.08$ fm



- ★ *The acceleration is due to progress in algorithms*

  Sexton & Weingarten '92, Hasenbusch '01,
  ML '03f, Urbach et al. '05, Clark & Kennedy '06

- ★ *Better program efficiency & faster computers speed up the simulations by another big factor*

- ★ *Many teams around the world, strong competition*

  ALPHA, BGR, CERN–Tor Vergata, ETMC, HPQCD, JLQCD, MILC, PACS-CS, QCDSF, RBC, UKQCD, . . .

Outline

1. Sea-quark effects we expect to see
2. Which lattice QCD?
3. Domain decomposition, a new technique in lattice QCD
4. First experiences and physics results

Sea-quark effects we expect to see

In the presence of light sea quarks, the ρ-meson becomes a $\pi\pi$ resonance

Ex: $M_\rho/M_\pi = 3$, $L \sim 2 \to 6$ fm

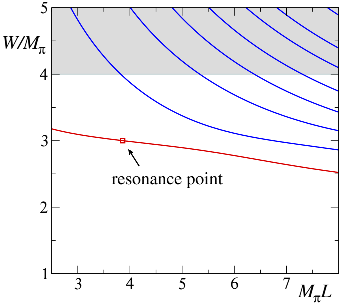
Energy spectrum in the ρ-channel

$$W = 2\sqrt{M_\pi^2 + k^2}, \quad k > 0,$$

Quantization condition

$$\pi - \delta_{11}(k) = \phi(q), \qquad q \equiv \frac{kL}{2\pi}$$

$\Rightarrow$ resonance mass & width



ML '90 [Nucl. Phys. B354 (1990) 222 and B364 (1991) 237]

Ishizuka et al. (CP-PACS collab.) '06

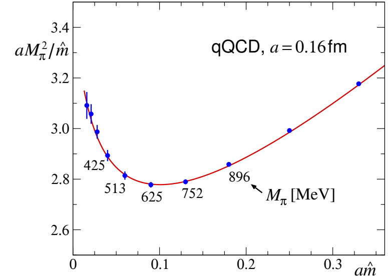
Chiral symmetry breaking

Expect

$$M_\pi^2 = 2B\hat{m} + \ldots, \quad \hat{m} = \tfrac{1}{2}(m_u + m_d)$$

NB: this is not so in quenched QCD!

Hasenfratz et al. '02, Hauswirth '03



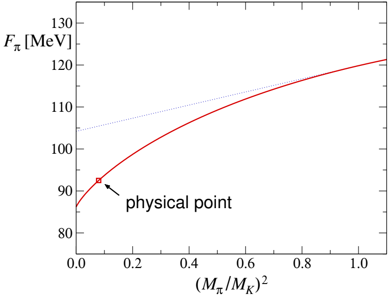
Chiral logarithms such as the one in

$$F_\pi = F - \frac{M_\pi^2}{16\pi^2 F} \ln(M_\pi^2/\Lambda_4^2) + \ldots$$

depend on the sea-quark content

Gasser & Leutwyler '84

Colangelo, Gasser & Leutwyler '01



Effects of the axial anomaly at fixed m_s and $m_u = m_d \to 0$

In this limit, the topological susceptibility

$$\chi_t = \tfrac{1}{4} M_\pi^2 F_\pi^2 + \ldots$$

is suppressed by the quark determinant

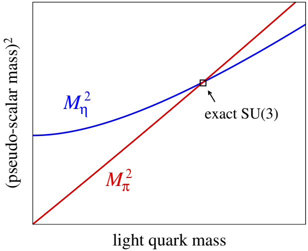
Leutwyler & Smilga '92, Giusti, Rossi, Testa & Veneziano '02

However, the η-meson mass

$$M_\eta^2 = \begin{cases} \dfrac{4}{F_\pi^2} \chi_t|_{\text{no quarks}} + \ldots & (N_\text{f} = 2) \\ \tfrac{4}{3} M_K^2 + \ldots & (N_\text{f} = 3) \end{cases}$$

is expected to remain large

Witten '79, Veneziano '79f, Giusti et al. '02



- ⋆ *All these sea-quark effects have not or only barely been seen on the lattice*
- ⋆ *However, such studies are now feasible, using the available techniques and resources*
- ⋆ *Will be an important step forward for numerical lattice QCD*

Which lattice QCD?

Currently many different forms of lattice QCD are in use

- Wilson action
- Symanzik, LW, Iwasaki actions
- DBW2 action
- O(a) improvement
- Tadpole improvement
- Fat links
- Improved staggered quarks
- Domain-wall fermions
- Neuberger fermions
- Perfect action
- 4th root trick
- p4fat7 action
- Twisted-mass QCD
- Topology-conserving actions

The more complicated forms were introduced with the intention to

- ⋆ *reduce the lattice effects*

  e.g. by adding higher-dimensional counterterms

  $$\Delta S = aS_1 + a^2 S_2 + \ldots$$

  Symanzik '81
  ML & Weisz '85
  Sheikholeslami & Wohlert '85

- ⋆ *have exact chiral symmetry*

  by constructing a lattice Dirac operator D that satisfies the GW relation

  $$\gamma_5 D + D\gamma_5 = aD\gamma_5 D$$

  Ginsparg & Wilson '82
  Kaplan '92
  Hasenfratz et al. '98
  Neuberger '98
  ML '98

- ⋆ *accelerate the simulations*

  Fat-link actions & "highly improved" staggered fermions (conceptually problematic, however)

  Lepage '99
  Orginos et al. '99


Museum Tinguely, Basel
The ultimate formulation of lattice QCD

An alternative philosophy is to

- ⋆ *keep things simple at the fundamental level*
  $\Rightarrow$ *essentially stick to Wilson's formulation*

- ⋆ *develop powerful algorithms and adapted computational strategies*

- ⋆ *simulate at small lattice spacings using these*
  $\Rightarrow$ *large lattices, distributed over many processors*

ALPHA, CERN–Tor Vergata, ETMC, PACS-CS, QCDSF–UKQCD, . . .

Domain decomposition, a new technique in lattice QCD

Lattice QCD is relatively easy to simulate on small blocks of lattice points

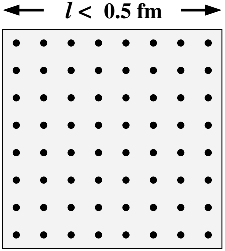
On such blocks, an infrared cutoff

$$q \geq \pi/l > 1\,\mathrm{GeV}$$

is implied (assuming Dirichlet b.c.)

⇒ asymptotic freedom implies that the theory is weakly coupled

⇒ easy to simulate at all quark masses



Exploiting locality, lattice QCD naturally decomposes into block systems

The gauge action decomposes into a sum

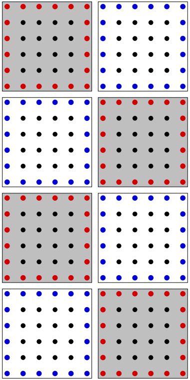
$$S_{\text{gauge}} = \sum_{\text{blocks } \Lambda} S_\Lambda$$

and the quark determinant factorizes

$$\det D = \prod_{\text{blocks } \Lambda} \det D_\Lambda \times \det R$$

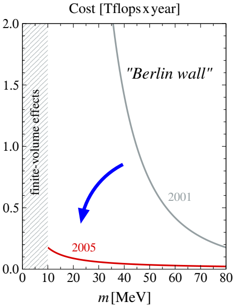
↑ Dirichlet b.c.

$$R \equiv 1 - \sum_{\text{pairs } \Lambda, \Lambda^*} D_\Lambda^{-1} D_{\partial\Lambda} D_{\Lambda^*}^{-1} D_{\partial\Lambda^*}$$



Why is this useful?

- ★ *R couples the gauge-field variables in distant blocks, but only weakly so*
- ★ *Large speed-up factors can be achieved by treating these interactions separately*
- ★ *The block decomposition is also very suitable for parallel processing*



ML '04f [Comput. Phys. Commun. 156 (2004) 209 and 165 (2005) 199]

First experiences and physics results

Del Debbio, Giusti, ML, Petronzio, Tantalo [CERN – Tor Vergata] '05f

Simulations of two-flavour QCD, including a valence s-quark

- $32 \times 24^3, \ldots, 64 \times 32^3$ lattices
- $a = 0.052, \ldots, 0.078\,\mathrm{fm}$
- $m_{\rm sea} \equiv m = \frac{1}{4}m_s, \ldots, m_s$
- 100–170 field configurations at each mass

Performed on PC clusters at Bern, CERN, Rome and on a CRAY XT3 at CSCS Manno

Physical sea-quark effects?

0^- meson propagator at $\vec{p} = 0$

$$\langle PP \rangle \sim \mathrm{e}^{-Mt} + c\,\mathrm{e}^{-M't} + \ldots$$

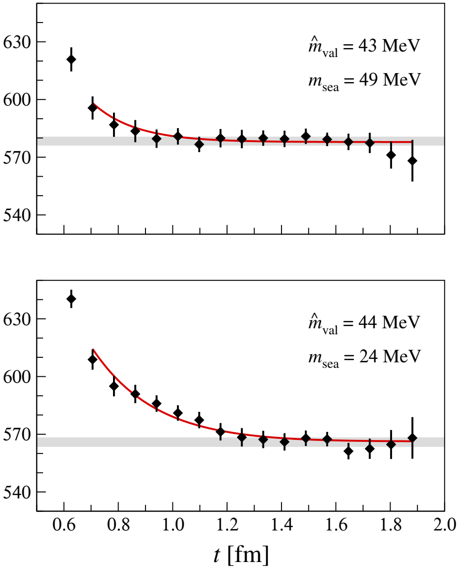
Expect

$$M' = M + 2M_\pi$$

Plots of the effective mass

$$M_{\text{eff}}(t) = -\frac{\mathrm{d}}{\mathrm{d}t} \ln\langle PP \rangle$$

confirm this



Chiral behaviour of the pion mass & decay constant

SU(2) ChPT predicts

$$M_\pi^2 = M^2 R_\pi, \quad M^2 = 2Bm$$

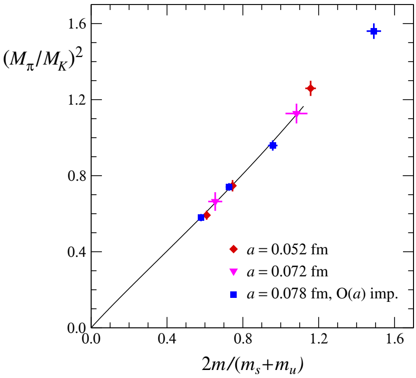
$$R_\pi = 1 + \frac{M^2}{32\pi^2 F^2} \ln(M^2/\Lambda_3^2) + \ldots$$

where, in real-world QCD,

$$F = 86.2 \pm 0.5\,\mathrm{MeV}$$

$$\ln(\Lambda_3^2/M^2)\big|_{M=140\,\mathrm{MeV}} \equiv \bar{l}_3 = 2.9 \pm 2.4$$

Gasser & Leutwyler '84



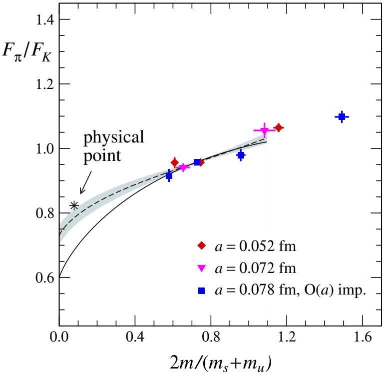
Fits the data very well, with $\bar{l}_3 = 3.0(5)(1)$, up to $M_\pi \simeq 550$ MeV

For the decay constant, ChPT predicts

$$F_\pi = F - \frac{M^2}{16\pi^2 F}\ln(M^2/\Lambda_4^2) + \ldots$$

$$\ln(\Lambda_4^2/M^2)\big|_{M=140\,\mathrm{MeV}} \equiv \bar{l}_4 = 4.4 \pm 0.2$$

Colangelo, Gasser & Leutwyler '01



The extrapolation to the chiral limit is ambiguous in this case

- Will need more data at smaller quark masses
- Finite-volume and lattice effects must be understood

Conclusions

- *Thanks to recent technical advances, the valence approximation is now practically overcome*
- *However, the most important effects of the light sea quarks still have not been studied*
- *Accurate computations of*

$$\Lambda_{\overline{\mathrm{MS}}}, m_u, m_d, m_s, m_c, m_b, M_\pi, M_K, M_\rho, M_N, f_\pi, f_K, f_D, f_B, B_K, g_A, \sigma, \ldots$$

will require the systematics to be understood

Progress is likely to be accelerated through the upcoming "International Lattice Data Grid"

ILDG