Parton Showers

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- motivating parton showers
- current parton showers (LO)
- parton shower accuracy
- improving parton showers
- persistent problems
- harsh realities and wild dreams

motivating parton showers

motivation: why care?

- QCD radiation omnipresent at the LHC
- enters as signal (and background) in high-$p_\perp$ analyses
  - multi-jet signatures

    $\longrightarrow$ multijet merging & higher-order matching (not the topic today)
  - inner-jet structures e.g. from "fat jets"

    $\longrightarrow$ parton shower algorithms
- begs the question:

  can we improve on parton showers and increase their precision?

  (keep in mind: accuracy vs. precision)

another systematic uncertainty

- parton showers are approximations, based on

  leading colour, leading logarithmic accuracy, spin-average
- parametric accuracy by comparing Sudakov form factors:

$$\Delta = \exp\left\{-\int \frac{\mathrm{d}k_\perp^2}{k_\perp^2}\left[A\log\frac{k_\perp^2}{Q^2}+B\right]\right\},$$

  where A and B can be expanded in $\alpha_S(k_\perp^2)$
- Q_T resummation includes $A_{1,2,3}$ and $B_{1,2}$

  (transverse momentum of Higgs boson etc.)
- showers usually include terms $A_{1,2}$ and B_1

  A = cusp terms (“soft emissions”), $B \sim$ anomalous dimensions γ

current parton showers (LO)

implementation in DIRE

- evolution and splitting parameter $((ij)+k \to i+j+k)$:

$$\kappa^2_{j,ik} = \frac{4(p_i p_j)(p_j p_k)}{Q^4} \quad \text{and} \quad z_j = \frac{2(p_j p_k)}{Q^2} \,.$$

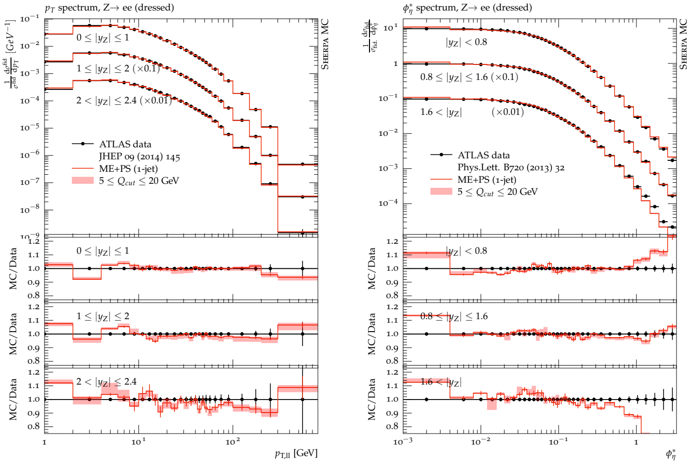
- splitting functions including IR regularisation
(a la Curci, Furmanski & Petronzio, Nucl.Phys. B175 (1980) 27-92)

$$\begin{aligned}
P^{(0)}_{qq}(z,\kappa^2) &= 2C_F\left[\frac{1-z}{(1-z)^2+\kappa^2}-\frac{1+z}{2}\right]\,,\\
P^{(0)}_{qg}(z,\kappa^2) &= 2C_F\left[\frac{z}{z^2+\kappa^2}-\frac{2-z}{2}\right]\,,\\
P^{s(0)}_{gg}(z,\kappa^2) &= 2C_A\left[\frac{1-z}{(1-z)^2+\kappa^2}-1+\frac{z(1-z)}{2}\right]\,,\\
P^{(0)}_{gq}(z,\kappa^2) &= T_R\left[z^2+(1-z)^2\right]
\end{aligned}$$

- renormalisation/factorisation scale given by $\mu = \kappa^2 Q^2$
- combine gluon splitting from two splitting functions with different spectators $k \to$ accounts for different colour flows

LO results for Drell-Yan

(example of accuracy in description of standard precision observable)



parton shower accuracy

how to assess formal precision?

- PS proven to be NLL accurate for simple observables, provided

  Catani,Marchesini,Webber, NPB349(1991)635

  - soft double-counting removed ($\nearrow$ before) and
  - 2-loop cusp anomalous dimension included
- not entirely clear what this means numerically, because
  - parton shower is momentum conserving, NLL is not
  - parton shower is unitary, NLL approximations break this
- differences can be quantified by
  - designing an MC that reproduces NLL exactly
  - removing NLL approximations one-by-one
- employ well-established NLL result as an example
  - observable: thrust in $e^+e^- \to$hadrons
  - method: CAESAR

  Banfi,Salam,Zanderighi, hep-ph/0407286

this discussion is technical, but necessary to show that equivalence at NLL does not imply identical numerical results

differences between pure NLL and parton shower

Hoeche, Reichelt, Siegert, arXiv:1711.03497

- consider observable V depending additively on emission i:

$$V_i(k_i) \;=\; \left(\frac{k_{i\perp}}{Q}\right)^a \, e^{-b_i\eta_i}$$

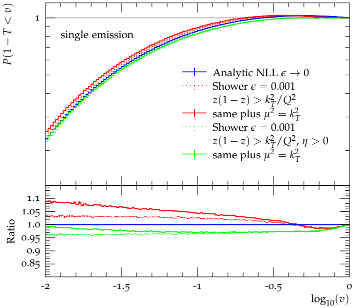
- isolated differences in resolved/unresolved splitting probability:

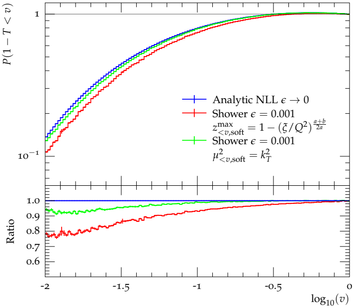
$$R_{\rm PS}(v) \;=\; 2\int\limits_{Q^2 v^{\frac{2}{a+b}}}^{Q^2} \frac{{\rm d}\xi}{\xi}\int\limits_{z_{\rm min}}^{z_{\rm max}} {\rm d}z\,\frac{\alpha_S\left(\xi(1-z)^{\frac{2b}{a+b}}\right)}{2\pi}\,C_F\left[\frac{2}{1-z}-(1+z)\right]\,\Theta\left(\log\frac{(1-z)^{\frac{2a}{a+b}}Q^2}{\xi}\right)$$

$$R_{\rm NLL}(v) \;=\; 2\int\limits_{Q^2 v^{\frac{2}{a+b}}}^{Q^2} \frac{{\rm d}\xi}{\xi}\left[\int\limits_0^1 {\rm d}z\,\frac{\alpha_S\left(\xi(1-z)^{\frac{2b}{a+b}}\right)}{2\pi}\,\frac{2C_F}{1-z}\,\Theta\left(\log\frac{(1-z)^{\frac{2a}{a+b}}Q^2}{\xi}\right)-\frac{C_F\,\alpha_S(\xi)}{\pi}\,B_q\right]$$

  - z-integration in soft terms (violation of local 4-momentum)
  - potential double counting in anti-collinear direction (sectorize)
  - vanilla parton shower uses different scale definition

local momentum conservation and unitarity

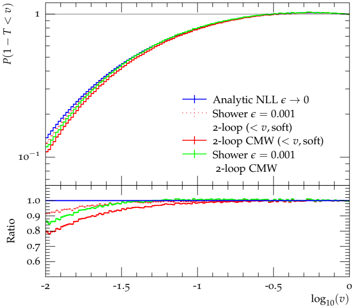


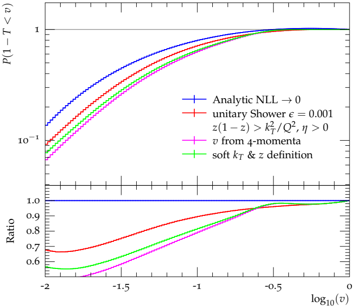


- NLL→PS in $z_{\min/\max}$ (4-momentum conservation)
- NLL→PS in $z^{\mathrm{coll}}_{>v,\max}$ (phase-space sectorization)
- NLL→PS in $\mu^2_{>v,\mathrm{coll}}$ (conventional)

- NLL→PS in $z^{\mathrm{soft}}_{<v,\max}$ (from PS unitarity)
- NLL→PS in $\mu^2_{<v,\mathrm{soft}}$ (from PS unitarity)

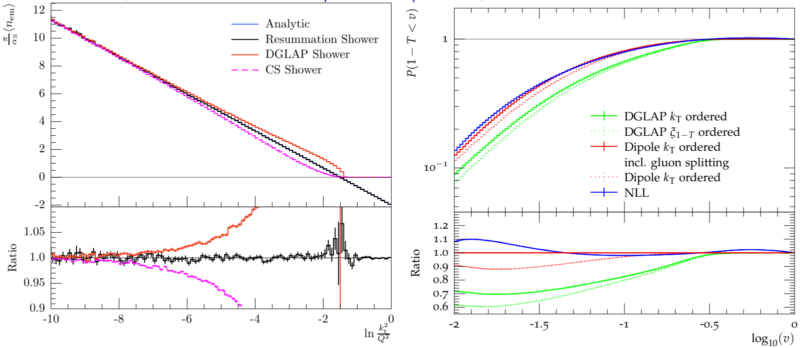
running coupling and global momentum conservation





- NLL→PS in 2-loop CMW $< v, \mathrm{soft}$ (from PS unitarity)
- NLL→PS in 2-loop CMW overall (conventional)
- NLL→PS in observable (use experimental definition)
- NLL→PS in evolution variable

overall comparison NLL / PS / Dipole Shower



- tuned comparison of differences between formally equivalent calculations
- simplest process and simplest observable, but still large differences
- origin of differences traced to treatment of kinematics & unitarity
- at NLL accuracy, none of the methods is formally superior
  $\rightarrow$ Difference is a systematic uncertainty & needs to be kept in mind

improving parton showers

including NLO splitting kernels

(Hoeche, FK & Prestel, 1705.00982, and Hoeche & Prestel, 1705.00742)

- expand splitting kernels as

$$P(z,\,\kappa^2)\;=\;P^{(0)}(z,\,\kappa^2)\,+\,\frac{\alpha_S}{2\pi}\,P^{(1)}(z,\,\kappa^2)$$

- aim: reproduce DGLAP evolution at NLO
  include all NLO splitting kernels
- three categories of terms in $P^{(1)}$:
  - cusp (universal soft-enhanced correction) (already included in original showers)
  - corrections to $1\to 2$
  - new flavour structures (e.g. $q\to q'$), identified as $1\to 3$
- new paradigm: two independent implementations

subtle symmetry factors

- observations for LO PS in final state:
  - only $P^{(0)}_{qq}$ used but not $P^{(0)}_{qg}$
  - $P^{(0)}_{gg}$ comes with "symmetry factor" 1/2
- challenge this way of implementing symmetry through:

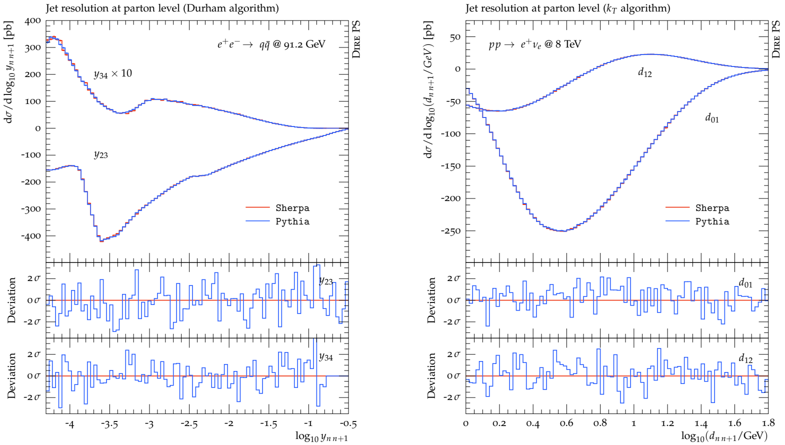
(Jadach & Skrzypek, hep-ph/0312355)

$$\sum_{i=q,g}\int_0^{1-\epsilon} \mathrm{d}z\, z\, P^{(0)}_{qi}(z) = \int_\epsilon^{1-\epsilon} \mathrm{d}z\, P^{(0)}_{qq}(z) + \mathcal{O}(\epsilon)$$

$$\sum_{i=q,g}\int_0^{1-\epsilon} \mathrm{d}z\, z\, P^{(0)}_{gi}(z) = \int_\epsilon^{1-\epsilon} \mathrm{d}z\, \left[\frac{1}{2}P^{(0)}_{gg}(z) + n_f P^{(0)}_{gq}(z)\right] + \mathcal{O}(\epsilon)$$

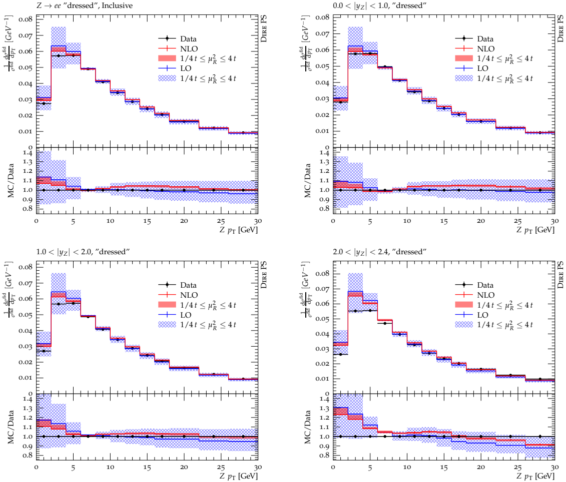
- net effect: replace symmetry factors by parton marker z

validation of $1 \to 3$ splittings



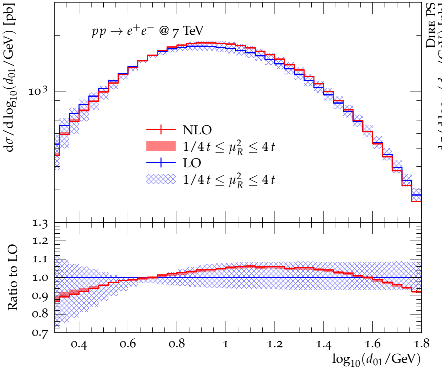
physical results: DY at LHC

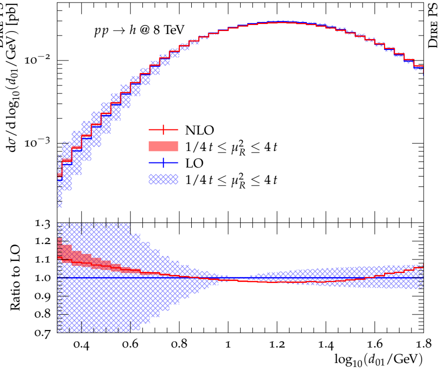
(untuned showers vs. 7 TeV ATLAS data, optimistic scale variations)



physical results: differential jet rates at LHC

(untuned showers, optimistic scale variations)





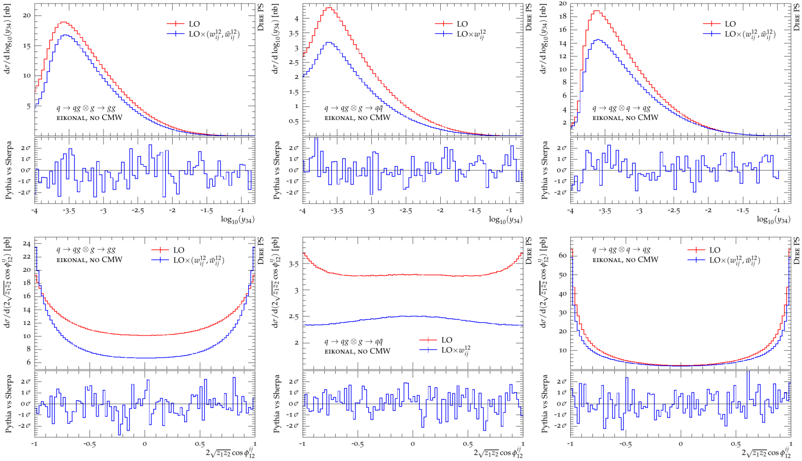
leading colour differential two-loop soft corrections

(Dulat, Hoeche & Prestel, 1805.03757)

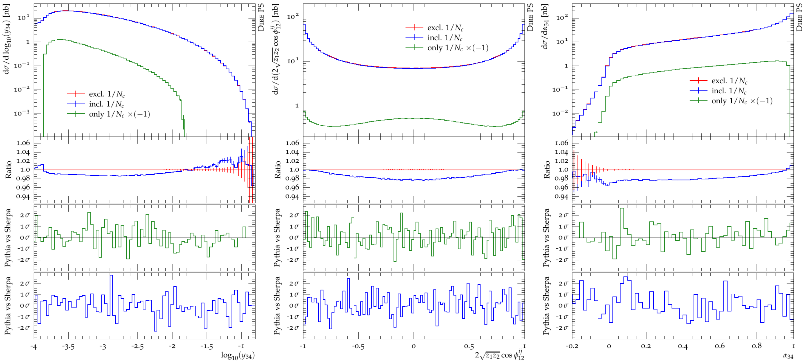
- analyse two-emission soft contribution and compare with iterated single emissions
- capture effect by reweighting original parton shower, with
  - accounting for finite recoil
  - including first $1/N_c$ corrections

    (another way to solve "problem" in Dasgupta et al., 1805.09327)
  - incorporating spin correlations

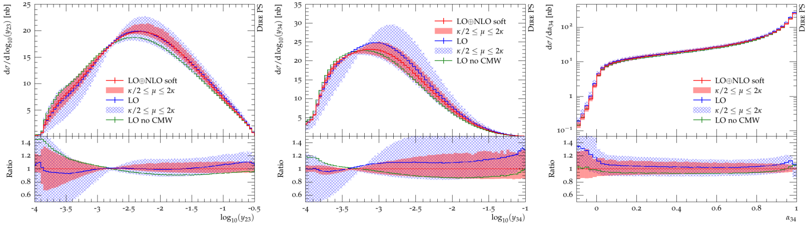
reweighting



including $1/N_c$ effects



scale uncertainties



persistent problems

dealing with heavy quarks

- $g \to q\bar{q}$ beyond "shower-approximation" $\longrightarrow$ no soft gluon
- recent analyses showed problems in ll showers (see below)
- heavy quarks also problematic in initial state:
  no PDF support for $Q^2 \leq m_Q^2 \longrightarrow$ quarks stop showering
- possible solutions:
  - naive: ignore and leave for beam remnants (SHERPA)
  - better: enforce splitting in region around m_Q^2 (PYTHIA)
    longrightarrow effectively produces collinear Q and gluon in IS

$g \to Q\bar{Q}$ — a systematic nightmare

- parton showers geared towards collinear & soft emissions of gluons

  (double log structure)

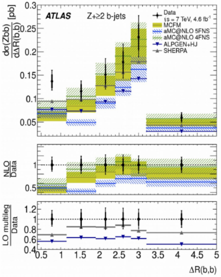
- $g \to q\bar{q}$ only collinear

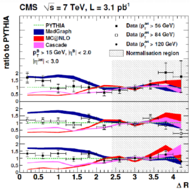
- old measurements at LEP of $g \to b\bar{b}$ and $g \to c\bar{c}$ rate
- fix this at LHC for modern showers

  (important for $t\bar{t}b\bar{b}$)

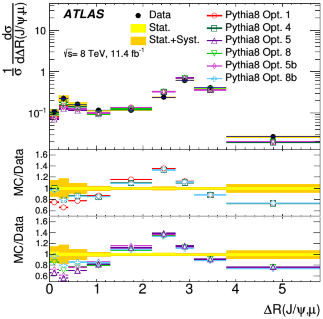
- questions: kernel, scale in α_S

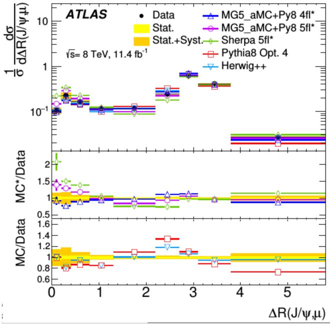
  (example: $k_\perp$ vs. m_{bb})





- ATLAS measurement in $b\bar{b}$ production
- use decay products in $B \to J/\Psi(\mu\mu) + X$ and $B \to \mu + X$
- use muons as proxies, most obvious observable $\Delta R(J\Psi, \mu)$





harsh realities and wild dreams

Summary

- implemented NLO DGLAP kernels into two independent showers
  will allow cross checks/validation of NP effects
- cross-validated implementations PYTHIA $\longleftrightarrow$ SHERPA
- matching to NNLO/multijet merging at NLO ongoing work
- extension to include loop-corrections to $1 \to 2$ straightforward
  will allow to use triple-collinear splitting functions throughout
- future plans: soft-gluon emissions and non-trivial colour correlations
- in SHERPA: implement forced splittings for heavy quarks at threshold

Points for further investigation

- compare shower with analytic reummation

  maybe in the spirit of Hoeche, Reichelt & Siegert, 1711.03497 (e^+e^- there, shower vs. CAESAR)

- compare two shower implementations in SHERPA, HERWIG, PYTHIA
- treatment of heavy flavours in IS:

  $\longrightarrow$ forced transitions to gluons at/around mass threshold

  (different in Z w.r.t. W production)

  probably need to check y-dependence of flavour composition

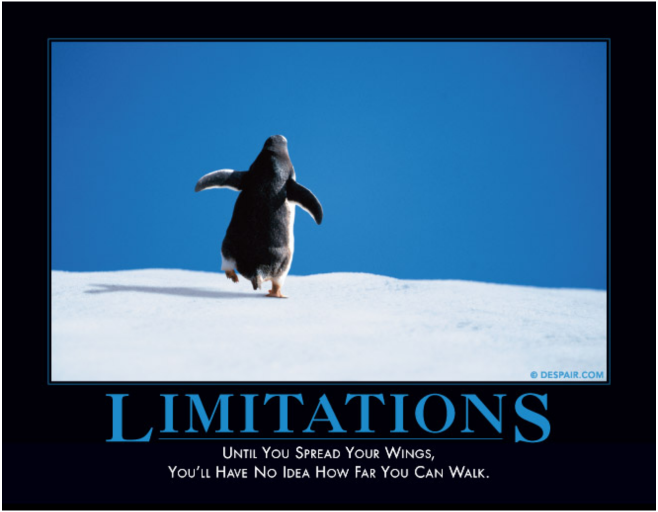
- non-perturbative effects: intrinsic $k_\perp$:
  - initial state partons "kicked": $\langle k_\perp\rangle \approx 1-2$ GeV

    (usually parametrised by Gaussian and tuned to Z-$p_\perp$)

  - usually flavour-blind and x–independent

    (non-default option of x-dependent in PYTHIA)

- mind the gap: accuracy vs. precision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LIMITATIONS
Until You Spread Your Wings,
You'll Have No Idea How Far You Can Walk.

connection to fragmentation functions

- DGLAP for FFs:

$$\frac{\mathrm{d}\, xD_a(x,\, t)}{\mathrm{d}\log t} = \sum_{b=q,g} \int_0^1 \mathrm{d}\tau \int_0^1 \mathrm{d}z\, \delta(x-\tau z)\, \frac{\alpha_S}{2\pi}\, [zP_{ab}(z)]_+\, \tau D_b(\tau, t)\,.$$

- rewrite for definition of "+"-function, $[zP_{ab}(z)]_+ = \lim\limits_{\epsilon\to 0} zP_{ab}(z,\epsilon)$:

$$P_{ab}(z,\,\epsilon) = P_{ab}(z)\,\Theta(1-z-\epsilon) - \delta_{ab}\sum_{c=q,g}\frac{\Theta(1-z-\epsilon)}{\epsilon}\int_0^1 \mathrm{d}\xi\, \xi P_{ac}(\xi)$$

$$\frac{\mathrm{d}\,\log D_a(x,t)}{\mathrm{d}\,\log t} = \underbrace{-\sum_{c=q,g}\int_0^{1-\epsilon}\mathrm{d}\xi\,\frac{\alpha_S}{2\pi}\,\xi P_{ac}(\xi)}_{\text{derivative of Sudakov}} + \sum_{b=q,g}\int_x^{1-\epsilon}\frac{\mathrm{d}z}{z}\,\frac{\alpha_S}{2\pi}\,P_{ac}(z)\,\frac{D_b(\frac{x}{z},t)}{D_a(x,t)}$$

- re-introduce Sudakov form factor

$$\Delta_a(t,t_0) = \exp\left\{-\int\limits_{t_0}^{t}\frac{\mathrm{d}t'}{t'}\sum_{c=q,g}\int\limits_0^{1-\epsilon}\mathrm{d}\xi\,\frac{\alpha_S}{2\pi}\,\xi P_{ac}(\xi)\right\}$$

to express equation above through generating functional $\mathcal{D}_a(x,t,\mu^2) = D_a(x,t)\Delta_a(\mu^2,t)$:

$$\frac{\mathrm{d}\,\log\mathcal{D}_a(x,t,\mu^2)}{\mathrm{d}\,\log t} = \sum_{b=q,g}\int\limits_x^{1-\epsilon}\frac{\mathrm{d}z}{z}\frac{\alpha_S}{2\pi}\,P_{ac}(z)\,\frac{D_b(\frac{x}{z},t)}{D_a(x,t)}$$

- add initial states (PDFs) & arrive at argument(s) for Sudakov form factors when jets not measured

$$\sum_{i\in IS}\sum_{b=q,g}\int\limits_{x_i}^{1-\epsilon}\frac{\mathrm{d}z}{z}\frac{\alpha_S}{2\pi}\,P_{ba_i}(z)\,\frac{f_b(\frac{x_i}{z},t)}{f_{a_i}(x,t)}+\sum_{j\in FS}\sum_{b=q,g}\int\limits_{x_i}^{1-\epsilon}\mathrm{d}z\,z\frac{\alpha_S}{2\pi}\,P_{a_jb}(z)\,.$$