Instability effects in top pair production at threshold

Christoph Reißer

Max-Planck-Institut für Physik, Munich

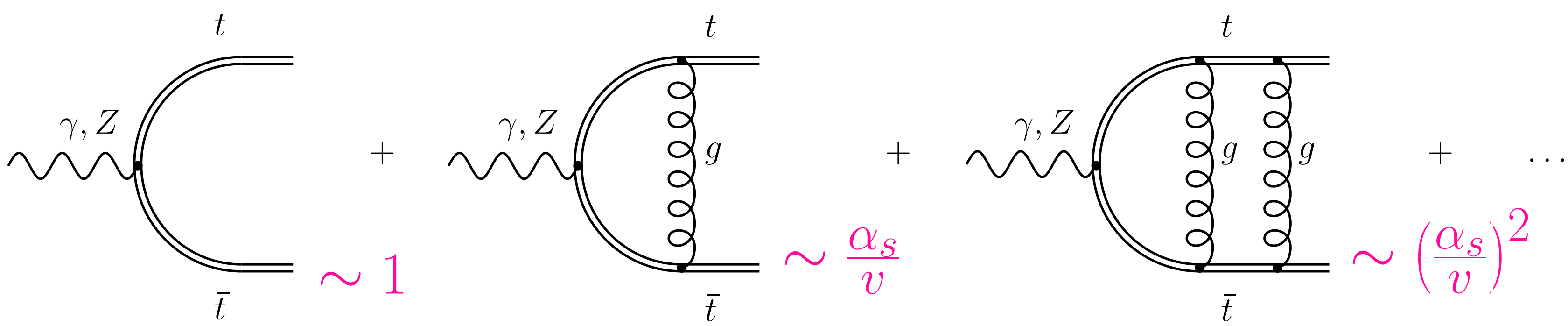
Outline

- Motivation
- Velocity nonrelativistic QCD (vNRQCD)
- Status for $\sigma_{\mathrm{tot}}(\mathrm{e}^+\mathrm{e}^- \to \mathrm{t}\bar{\mathrm{t}})$ at threshold
- Top instability, electroweak effects
- Phase space matching
- Outlook, Summary

Nonrelativistic top pairs

e^+e^- collisions: c.m. energy $\sqrt{s} \approx 340 - 360$ GeV

- Top quarks are nonrelativistic $\boxed{v = \sqrt{1 - \frac{4m_t^2}{s}} \ll 1}$



⇒ Perturbation theory in α_s breaks down $\boxed{v \sim \alpha_s}$

⇒ Nonrelativistic QCD $\simeq$ Schrödinger theory at LO

Nonrelativistic top pairs

e^+e^- collisions: c.m. energy $\sqrt{s} \approx 340 - 360$ GeV

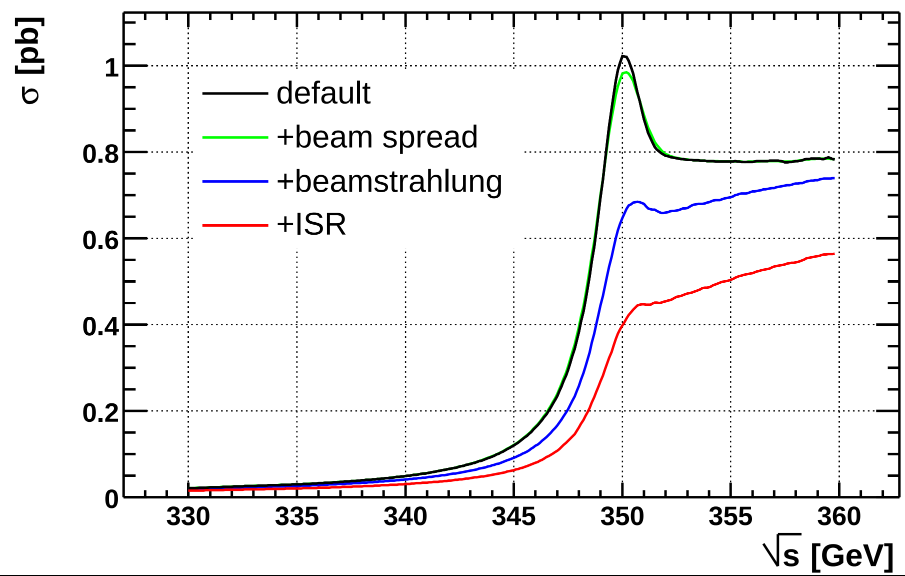
- Top quarks are nonrelativistic $$v = \sqrt{1 - \frac{4m_t^2}{s}} \ll 1$$
- Top quarks decay fast: $t \to Wb$ $$\Gamma_t \approx 1.5\ \text{GeV} \gg \Lambda_{QCD}$$

$\Rightarrow$ No bound states

$\Rightarrow$ Smooth line-shape

$\Rightarrow$ Non-perturbative effects suppressed

Fadin, Khoze (JETP Lett. 46, 1987)



Nonrelativistic top pairs

e^+e^- collisions: c.m. energy $\sqrt{s} \approx 340 - 360$ GeV

- Top quarks are nonrelativistic $\boxed{v = \sqrt{1 - \frac{4m_t^2}{s}} \ll 1}$
- Top quarks decay fast: $t \to Wb$ $\boxed{\Gamma_t \approx 1.5 \text{ GeV} \gg \Lambda_{\mathrm{QCD}}}$

$\Rightarrow$ Instead of process $e^+e^- \to t\bar{t}$ consider $e^+e^- \to bW^+\bar{b}W^-$ or even include W decay products

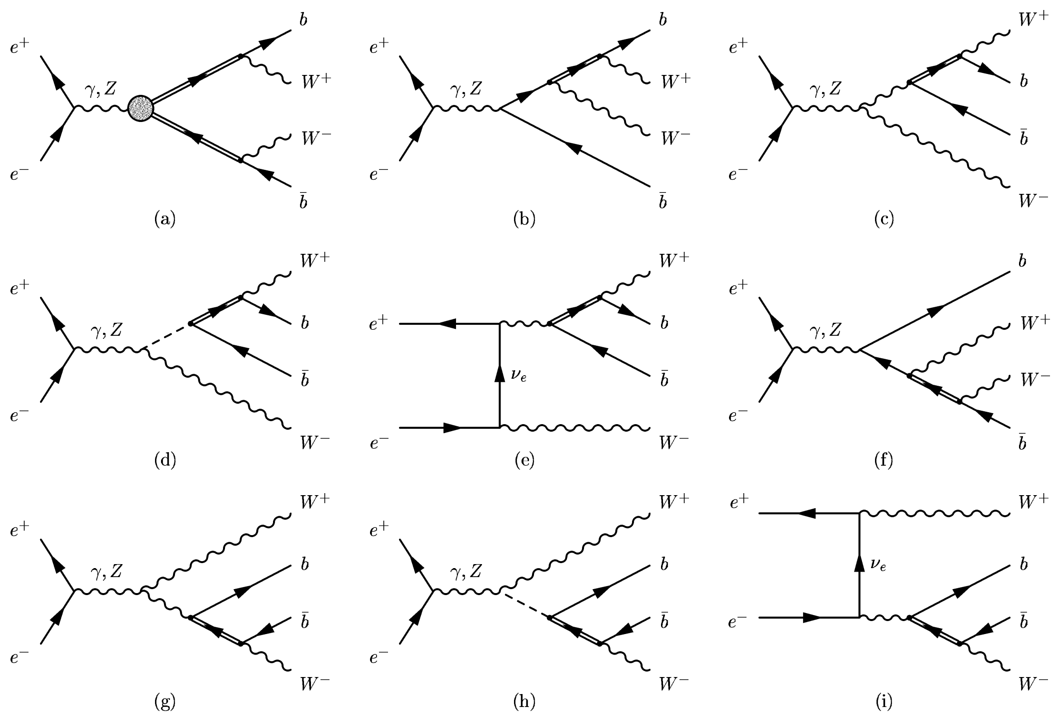
$\Rightarrow$ Interferences of double and single resonant diagrams

$\Rightarrow$ New theoretical concepts for treatment beyond LO

Nonrelativistic top pairs

e^+e^- collisions: c.m. energy $\sqrt{s} \approx 340 - 360$ GeV

- Top quarks are nonrelativistic $v = \sqrt{1 - \frac{4m_t^2}{s}} \ll 1$
- Top quarks decay fast: $t \to Wb$ $\Gamma_t \approx 1.5\,\text{GeV} \gg \Lambda_{\text{QCD}}$



Nonrelativistic top pairs

<u>e^+e^- collisions:</u> c.m. energy $\sqrt{s} \approx 340 - 360$ GeV

- Top quarks are nonrelativistic $\boxed{v = \sqrt{1 - \frac{4m_t^2}{s}} \ll 1}$
- Top quarks decay fast: $t \to Wb$ $\boxed{\Gamma_t \approx 1.5\ \text{GeV} \gg \Lambda_{\mathrm{QCD}}}$

$\Rightarrow$ Instead of process $e^+e^- \to t\bar{t}$ consider $e^+e^- \to bW^+\bar{b}W^-$ or even include W decay products

$\Rightarrow$ Interferences of double and single resonant diagrams

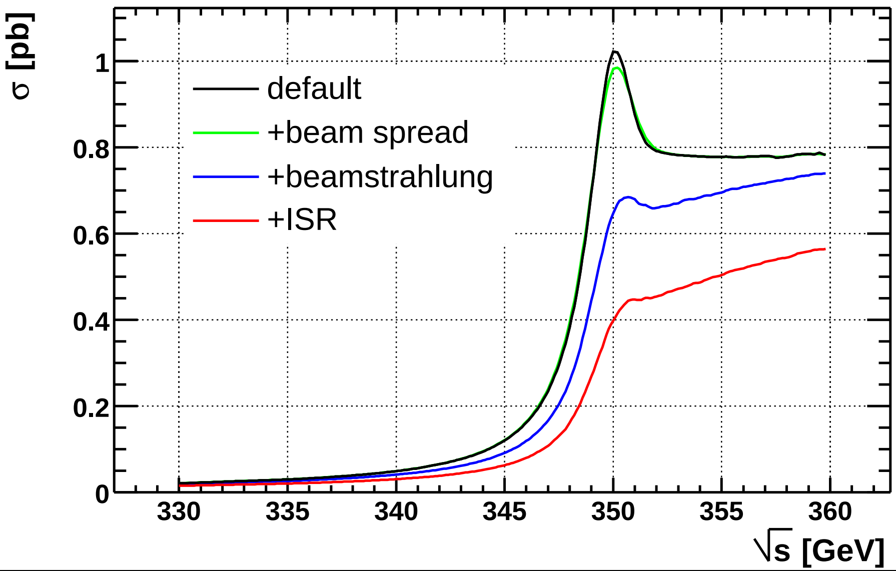
$\Rightarrow$ New theoretical concepts for treatment beyond LO

Nonrelativistic top pairs

<u>e^+e^- collisions:</u> c.m. energy $\sqrt{s} \approx 340 - 360$ GeV

- Top quarks are nonrelativistic $\boxed{v = \sqrt{1 - \frac{4m_t^2}{s}} \ll 1}$
- Top quarks decay fast: $t \to Wb$ $\boxed{\Gamma_t \approx 1.5\ \text{GeV} \gg \Lambda_{QCD}}$
- Measured cross section $\boxed{\sigma^{obs}(s) = \int_0^1 dx\, \mathcal{L}(x) \sigma^{theo}(x^2 s)}$ contains
  - beam spread
  - beamstrahlung
  - ISR

  $\rightarrow$ pure QED not considered in this talk



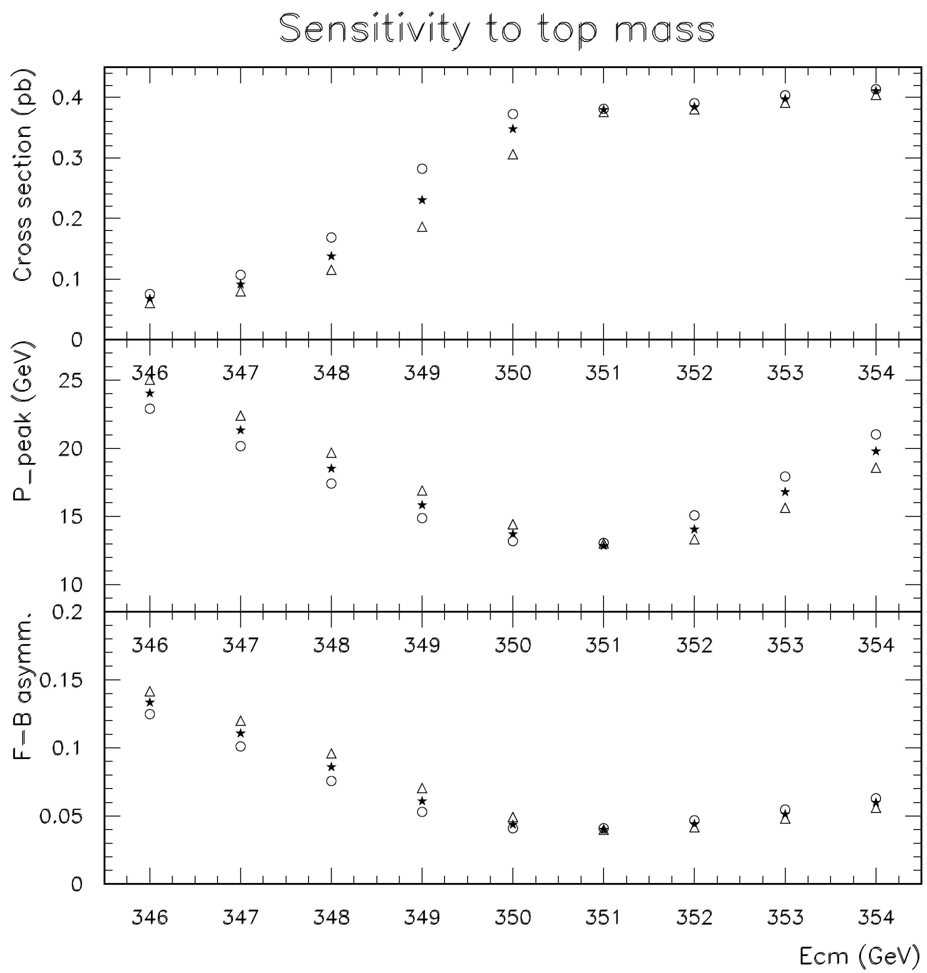
Measurements

- Simulations of Threshold Scan ($\int \mathcal{L}\,\mathrm{dt} \sim 300\,\mathrm{fb}^{-1}$):

  Martinez, Miquel (Eur. Phys. J. C 27, 2003)

  - Top quark mass

    $(\delta m_t)^{\mathrm{exp}} \sim 50$ MeV



Measurements

- Simulations of Threshold Scan ($\int \mathcal{L}\,\mathrm{dt} \sim 300\,\mathrm{fb}^{-1}$):

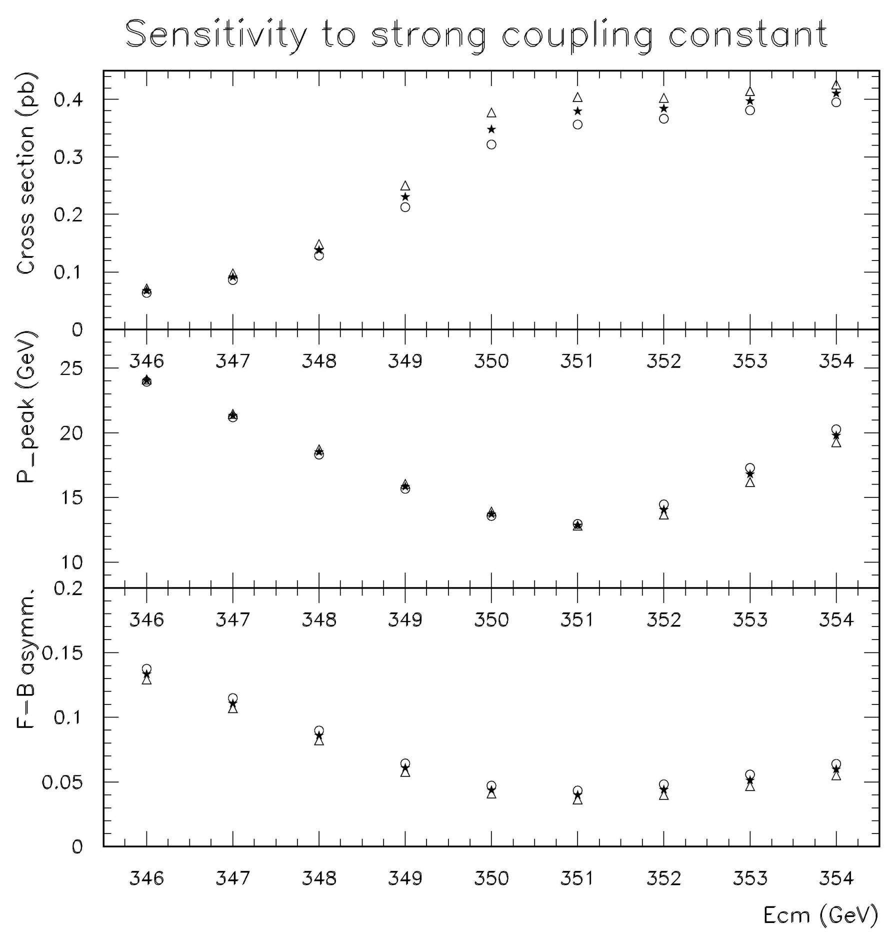
  Martinez, Miquel (Eur. Phys. J. C 27, 2003)

  - Top quark mass

    $(\delta m_t)^{\mathrm{exp}} \sim 50$ MeV

  - Strong coupling

    $(\delta \alpha_s(M_Z))^{\mathrm{exp}} \sim 0.001$



Measurements

- Simulations of Threshold Scan ($\int \mathcal{L}\,\mathrm{dt} \sim 300\,\mathrm{fb}^{-1}$):

  Martinez, Miquel (Eur. Phys. J. C 27, 2003)

  - Top quark mass

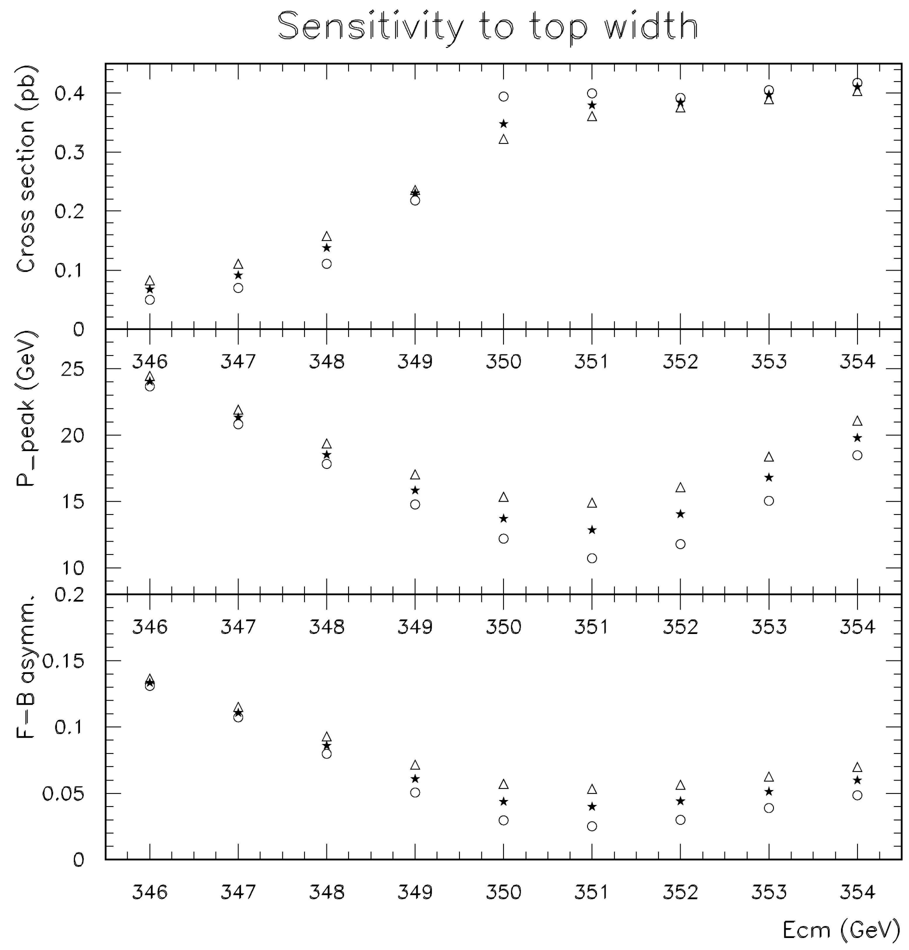
    $(\delta m_t)^{\mathrm{exp}} \sim 50$ MeV

  - Strong coupling

    $(\delta \alpha_s(M_Z))^{\mathrm{exp}} \sim 0.001$

  - Top decay width

    $(\delta \Gamma_t)^{\mathrm{exp}} \sim 50$ MeV



Measurements

- Simulations of Threshold Scan ($\int \mathcal{L}\,\mathrm{dt} \sim 300\,\mathrm{fb}^{-1}$):

  Martinez, Miquel (Eur. Phys. J. C 27, 2003)

  - Top quark mass

    $$(\delta m_t)^{\mathrm{exp}} \sim 50\ \mathrm{MeV}$$

  - Strong coupling

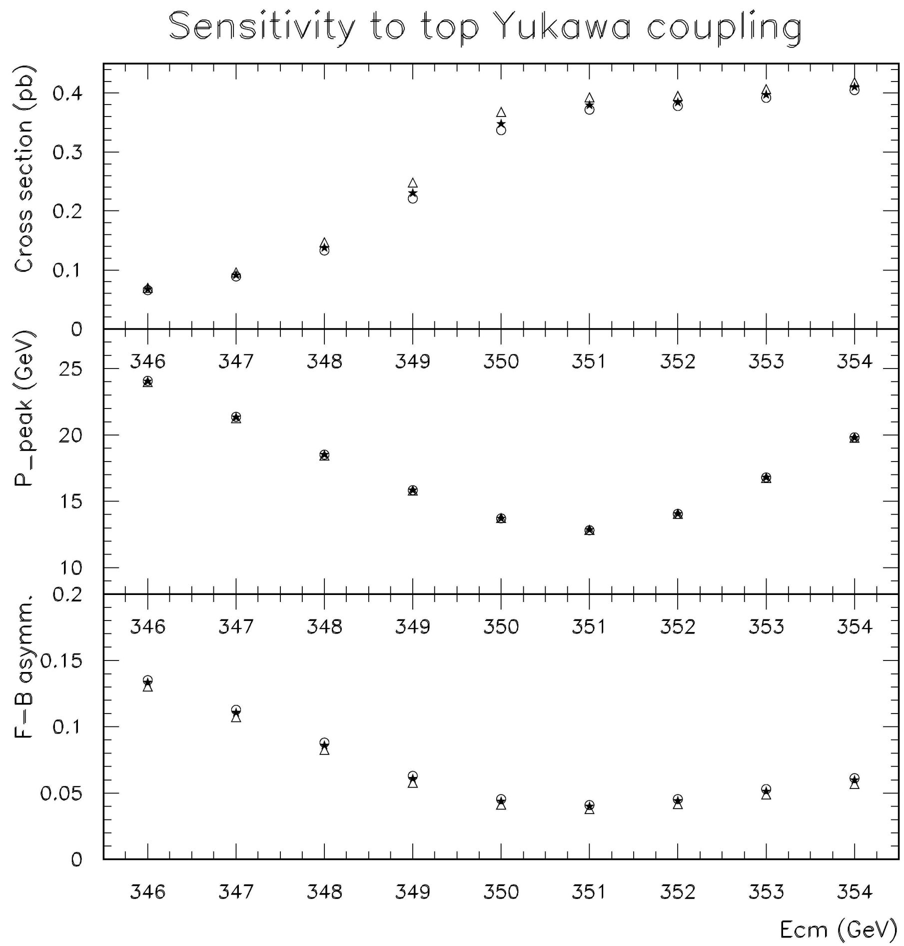
    $$(\delta \alpha_s(M_Z))^{\mathrm{exp}} \sim 0.001$$

  - Top decay width

    $$(\delta \Gamma_t)^{\mathrm{exp}} \sim 50\ \mathrm{MeV}$$

  - Top Yukawa coupling (light Higgs)

    $$(\delta y_t / y_t)^{\mathrm{exp}} \sim 0.35$$



Measurements

- Simulations of Threshold Scan ($\int \mathcal{L}\,\mathrm{dt} \sim 300\,\mathrm{fb}^{-1}$):

  Martinez, Miquel (Eur. Phys. J. C 27, 2003)

  - Top quark mass

    $$(\delta m_t)^{\mathrm{exp}} \sim 50\ \mathrm{MeV}$$

  - Strong coupling

    $$(\delta \alpha_s(M_Z))^{\mathrm{exp}} \sim 0.001$$

  - Top decay width

    $$(\delta \Gamma_t)^{\mathrm{exp}} \sim 50\ \mathrm{MeV}$$

  - Top Yukawa coupling (light Higgs)

    $$(\delta y_t / y_t)^{\mathrm{exp}} \sim 0.35$$

$\Rightarrow$ Theory goal

$$\boxed{(\delta \sigma_{\mathrm{tot}} / \sigma_{\mathrm{tot}}) \leq 3\,\%}$$

Effective theory framework (stable quarks)

- Relevant scales

$$m_t \text{ (hard)} \quad \gg \quad \mathbf{p} \sim m_t v \text{ (soft)} \quad \gg \quad E \sim m_t v^2 \text{ (ultrasoft)}$$

Effective theory framework (stable quarks)

- Relevant scales

$$m_t \text{ (hard)} \quad \gg \quad \mathbf{p} \sim m_t v \text{ (soft)} \quad \gg \quad E \sim m_t v^2 \text{ (ultrasoft)}$$

- Momentum regions   Beneke, Smirnov (Nucl. Phys. B 522, 1998)

hard	$(k^0, \mathbf{k}) \sim (m_t, m_t)$
soft	$(k^0, \mathbf{k}) \sim (m_t v, m_t v)$
potential	$(k^0, \mathbf{k}) \sim (m_t v^2, m_t v)$
ultrasoft	$(k^0, \mathbf{k}) \sim (m_t v^2, m_t v^2)$

Effective theory framework (stable quarks)

- Relevant scales

  m_t (hard) $\gg$ $\mathbf{p} \sim m_t v$ (soft) $\gg$ $E \sim m_t v^2$ (ultrasoft)

- Momentum regions Beneke, Smirnov (Nucl. Phys. B 522, 1998)

hard	$(k^0, \mathbf{k}) \sim (m_t, m_t)$
soft	$(k^0, \mathbf{k}) \sim (m_t v, m_t v)$
potential	$(k^0, \mathbf{k}) \sim (m_t v^2, m_t v)$
ultrasoft	$(k^0, \mathbf{k}) \sim (m_t v^2, m_t v^2)$

- Heavy quark 4-momentum $p^\mu = (m, 0) + (0, \mathbf{p}) + (k^0, \mathbf{k})$

  Heavy quark spinor $\psi \to \sum_{\mathbf{p}} e^{-i\mathbf{p}\cdot\mathbf{x}} \psi_{\mathbf{p}}(x)$

  Luke, Manohar, Rothstein (Phys. Rev. D 61, 2000)

Effective theory framework (stable quarks)

- Relevant scales

  m_t (hard) $\gg$ $\mathbf{p} \sim m_t v$ (soft) $\gg$ $E \sim m_t v^2$ (ultrasoft)

- Momentum regions — Beneke, Smirnov (Nucl. Phys. B 522, 1998)

  hard $(k^0, \mathbf{k}) \sim (m_t, m_t)$

  soft $(k^0, \mathbf{k}) \sim (m_t v, m_t v)$

  potential $(k^0, \mathbf{k}) \sim (m_t v^2, m_t v)$

  ultrasoft $(k^0, \mathbf{k}) \sim (m_t v^2, m_t v^2)$

- Heavy quark 4-momentum $p^\mu = (m, 0) + (0, \mathbf{p}) + (k^0, \mathbf{k})$

  Heavy quark spinor $\psi \to \sum_{\mathbf{p}} e^{-i\mathbf{p}\cdot\mathbf{x}} \psi_{\mathbf{p}}(x)$

- Resonant modes — Luke, Manohar, Rothstein (Phys. Rev. D 61, 2000)

  potential quarks $\psi_{\mathbf{p}}, \chi_{\mathbf{p}}$

  soft gluons A_q^μ

  ultrasoft gluons A^μ

Effective theory framework (stable quarks)

- Relevant scales

$$m_t \text{ (hard)} \gg \mathbf{p} \sim m_t v \text{ (soft)} \gg E \sim m_t v^2 \text{ (ultrasoft)}$$

- Power counting $\boxed{v \sim \alpha_s \ll 1}$

$$\boxed{\left(\frac{\alpha_s}{v}\right) \sim 1} \qquad \boxed{(\alpha_s \ln v) \sim 1}$$

Effective theory framework (stable quarks)

- Relevant scales

$$m_t \text{ (hard)} \gg \mathbf{p} \sim m_t v \text{ (soft)} \gg E \sim m_t v^2 \text{ (ultrasoft)}$$

- Power counting $\boxed{v \sim \alpha_s \ll 1}$

$$\boxed{\left(\frac{\alpha_s}{v}\right) \sim 1} \qquad \boxed{(\alpha_s \ln v) \sim 1}$$

$$\text{LL} \sim \left(\frac{\alpha_s}{v}\right)^n \sum_m (\alpha_s \ln v)^m$$

$$\text{NLL} \sim \{\alpha_s, v\} \left(\frac{\alpha_s}{v}\right)^n \sum_m (\alpha_s \ln v)^m$$

$$\text{NNLL} \sim \{\alpha_s^2, \alpha_s v, v^2\} \left(\frac{\alpha_s}{v}\right)^n \sum_m (\alpha_s \ln v)^m$$

vNRQCD Lagrangian

$$\mathcal{L} = \mathcal{L}_{\rm usoft} + \mathcal{L}_{\rm pot} + \mathcal{L}_{\rm soft}$$

Luke, Manohar, Rothstein (Phys. Rev. D 61, 2000)
Hoang, Stewart (Phys. Rev. D 67, 2003)

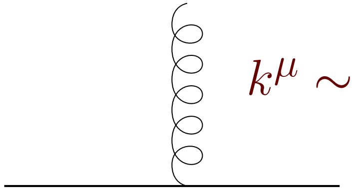
vNRQCD Lagrangian

Luke, Manohar, Rothstein (Phys. Rev. D 61, 2000)
Hoang, Stewart (Phys. Rev. D 67, 2003)

$$\mathcal{L} = \mathcal{L}_{\rm usoft} + \mathcal{L}_{\rm pot} + \mathcal{L}_{\rm soft}$$

- $$\mathcal{L}_{\rm usoft} = \sum_{\mathbf{p}} \psi^\dagger_{\mathbf{p}} \left[\mathsf{i}\mathsf{D}^0 - \frac{(\mathbf{p} - \mathbf{i}\mathbf{D})^2}{2\mathsf{m}} + \frac{\mathbf{p}^4}{8\mathsf{m}^3} + \ldots \right] \psi_{\mathbf{p}}$$

$$\mathsf{D}^\mu = \partial^\mu + \mathsf{i}\mathsf{g}_\mathsf{s}\mathsf{A}^\mu$$



$k^\mu \sim m v^2$

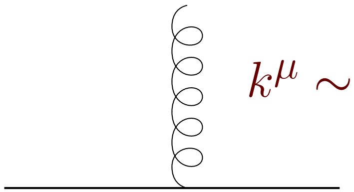
vNRQCD Lagrangian

Luke, Manohar, Rothstein (Phys. Rev. D 61, 2000)
Hoang, Stewart (Phys. Rev. D 67, 2003)

$$\mathcal{L} = \mathcal{L}_{\rm usoft} + \mathcal{L}_{\rm pot} + \mathcal{L}_{\rm soft}$$

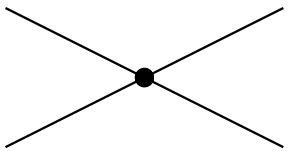
- $$\mathcal{L}_{\rm usoft} = \sum_{\mathbf{p}} \psi^\dagger_{\mathbf{p}} \left[\mathrm{i}D^0 - \frac{(\mathbf{p} - \mathrm{i}\mathbf{D})^2}{2m} + \frac{\mathbf{p}^4}{8m^3} + \ldots \right] \psi_{\mathbf{p}}$$

$$D^\mu = \partial^\mu + \mathrm{i} g_s A^\mu$$



$k^\mu \sim mv^2$

- $$\mathcal{L}_{\rm pot} = - \sum_{\mathbf{p},\mathbf{p}'} \left[\frac{\mathcal{V}_c}{(\mathbf{p}-\mathbf{p}')^2} + \ldots \right] \psi^\dagger_{\mathbf{p}'} \psi_{\mathbf{p}} \chi^\dagger_{-\mathbf{p}'} \chi_{-\mathbf{p}}$$



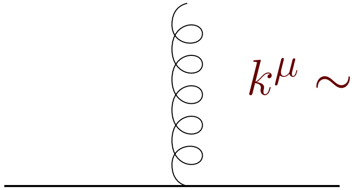
vNRQCD Lagrangian

Luke, Manohar, Rothstein (Phys. Rev. D 61, 2000)
Hoang, Stewart (Phys. Rev. D 67, 2003)

$$\mathcal{L} = \mathcal{L}_{\rm usoft} + \mathcal{L}_{\rm pot} + \mathcal{L}_{\rm soft}$$

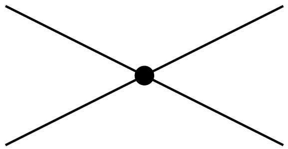
- $$\mathcal{L}_{\rm usoft} = \sum_{\mathbf{p}} \psi^\dagger_{\mathbf{p}} \left[\mathsf{i}\mathsf{D}^0 - \frac{(\mathbf{p} - \mathbf{i}\mathbf{D})^2}{2\mathsf{m}} + \frac{\mathbf{p}^4}{8\mathsf{m}^3} + \ldots \right] \psi_{\mathbf{p}}$$

$$\mathsf{D}^\mu = \partial^\mu + \mathsf{i}\mathsf{g}_\mathsf{s}\mathsf{A}^\mu$$

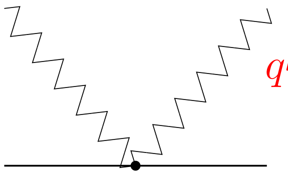


$k^\mu \sim m v^2$

- $$\mathcal{L}_{\rm pot} = -\sum_{\mathbf{p},\mathbf{p}'} \left[\frac{\mathcal{V}_\mathsf{c}}{(\mathbf{p} - \mathbf{p}')^2} + \ldots \right] \psi^\dagger_{\mathbf{p}'} \psi_{\mathbf{p}} \chi^\dagger_{-\mathbf{p}'} \chi_{-\mathbf{p}}$$



- $$\mathcal{L}_{\rm soft} = -\mathsf{g}_\mathsf{s}^2 \sum_{\mathbf{p},\mathbf{p}',\mathsf{q},\mathsf{q}'} \left[\frac{1}{2} \psi^\dagger_{\mathbf{p}'} [\mathsf{A}^\mu_{\mathsf{q}'}, \mathsf{A}^\nu_{\mathsf{q}}] \mathsf{U}_{\mu\nu} \psi_{\mathbf{p}} + \ldots \right]$$



$q^\mu \sim m v$

vNRQCD Lagrangian

Currents for production and annihilation of $t\bar{t}$ pairs:

- $^3S_1$ vector currents $\quad \mathbf{O}^{i}_{\mathbf{p},1} = \psi^{\dagger}_{\mathbf{p}}\, \boldsymbol{\sigma}^{i}\, \tilde{\chi}^{*}_{-\mathbf{p}}, \quad \mathbf{O}^{i}_{\mathbf{p},2} = \psi^{\dagger}_{\mathbf{p}}\, \frac{\mathbf{p}^2}{m_t^2}\, \boldsymbol{\sigma}^{i}\, \tilde{\chi}^{*}_{-\mathbf{p}}$

vNRQCD Lagrangian

Currents for production and annihilation of $\mathrm{t\bar{t}}$ pairs:

- $^3S_1$ vector currents $\quad \mathbf{O}^{\mathrm{i}}_{\mathbf{p},1} = \psi^\dagger_{\mathbf{p}}\, \boldsymbol{\sigma}^{\mathrm{i}}\, \tilde{\chi}^*_{-\mathbf{p}}, \quad \mathbf{O}^{\mathrm{i}}_{\mathbf{p},2} = \psi^\dagger_{\mathbf{p}}\, \frac{\mathbf{p}^2}{\mathrm{m}_\mathrm{t}^2}\, \boldsymbol{\sigma}^{\mathrm{i}}\, \tilde{\chi}^*_{-\mathbf{p}}$
- $^3P_1$ axial vector current $\quad \mathbf{O}^{\mathrm{i}}_{\mathbf{p},3} = \frac{-\mathrm{i}}{2\mathrm{m}_\mathrm{t}}\, \psi^\dagger_{\mathbf{p}}\, [\boldsymbol{\sigma}^{\mathrm{i}}, \boldsymbol{\sigma}\cdot\mathbf{p}]\, \tilde{\chi}^*_{-\mathbf{p}}$

vNRQCD Lagrangian

Currents for production and annihilation of $t\bar{t}$ pairs:

- $^3S_1$ vector currents $\mathbf{O}^{i}_{\mathbf{p},1} = \psi^\dagger_{\mathbf{p}}\, \boldsymbol{\sigma}^i\, \tilde{\chi}^*_{-\mathbf{p}}$, $\mathbf{O}^{i}_{\mathbf{p},2} = \psi^\dagger_{\mathbf{p}}\, \frac{\mathbf{p}^2}{m_t^2}\, \boldsymbol{\sigma}^i\, \tilde{\chi}^*_{-\mathbf{p}}$
- $^3P_1$ axial vector current $\mathbf{O}^{i}_{\mathbf{p},3} = \frac{-i}{2m_t}\, \psi^\dagger_{\mathbf{p}}\, [\boldsymbol{\sigma}^i, \boldsymbol{\sigma}\cdot\mathbf{p}]\, \tilde{\chi}^*_{-\mathbf{p}}$

Attach initial state leptons (gauge invariance if ew. effects beyond LO included):

$$\mathbf{O}_{\mathbf{p},\sigma} \quad = \quad [\, \bar{e}\, \gamma_i (\gamma_5)\, e\,]\, \mathbf{O}^{i}_{\mathbf{p},\sigma}$$

vNRQCD Lagrangian

Currents for production and annihilation of $t\bar{t}$ pairs:

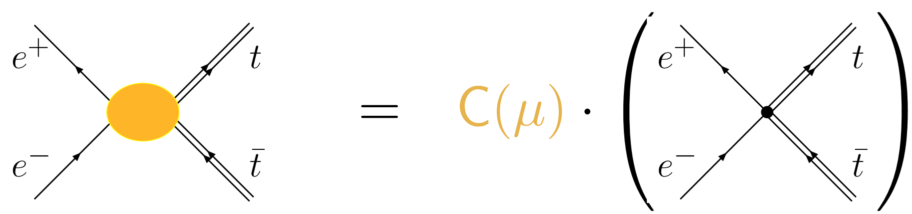
- $^3S_1$ vector currents $\quad \mathbf{O}^{i}_{\mathbf{p},1} = \psi^\dagger_{\mathbf{p}}\, \boldsymbol{\sigma}^i\, \tilde{\chi}^*_{-\mathbf{p}}, \quad \mathbf{O}^{i}_{\mathbf{p},2} = \psi^\dagger_{\mathbf{p}}\, \frac{\mathbf{p}^2}{m_t^2}\, \boldsymbol{\sigma}^i\, \tilde{\chi}^*_{-\mathbf{p}}$
- $^3P_1$ axial vector current $\quad \mathbf{O}^{i}_{\mathbf{p},3} = \frac{-i}{2m_t}\, \psi^\dagger_{\mathbf{p}}\, [\boldsymbol{\sigma}^i, \boldsymbol{\sigma}\cdot\mathbf{p}]\, \tilde{\chi}^*_{-\mathbf{p}}$

Attach initial state leptons (gauge invariance if ew. effects beyond LO included):

$$\mathbf{O}_{\mathbf{p},\sigma} \quad = \quad [\,\bar{e}\,\gamma_i(\gamma_5)\,e\,]\,\mathbf{O}^{i}_{\mathbf{p},\sigma}$$

Contribution to Lagrangian:

$$\Delta\mathcal{L} = \sum_{\mathbf{p},\sigma} C_\sigma(\mu)\mathbf{O}_{\mathbf{p},\sigma}$$



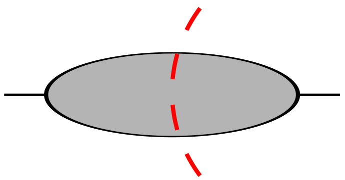
vNRQCD (stable quarks)

Total cross section from $e^+e^- \to e^+e^-$ using the Optical Theorem

Strassler, Peskin (Phys. Rev. D 43, 1991)

$$\sigma_{tot} \quad \propto \quad \mathrm{Im}\left[i \sum_{\mathbf{p},\mathbf{p}'} \int d^4x \, e^{-i\hat{q}\cdot x} \left\langle 0 \,\middle|\, T\left(C(\mu)\mathbf{O}^\dagger_{\mathbf{p}}(0) \right) \left(C(\mu)\mathbf{O}_{\mathbf{p}'}(x) \right) \middle|\, 0 \right\rangle \right]$$

$$\propto \quad \mathrm{Im}\left[C(\mu)^2 \, G(0,0,E) \right]$$



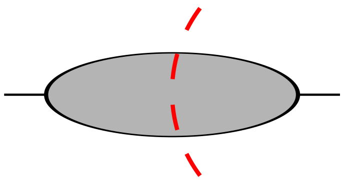
vNRQCD (stable quarks)

Total cross section from $e^+e^- \longrightarrow e^+e^-$ using the Optical Theorem

Strassler, Peskin (Phys. Rev. D 43, 1991)

$$\sigma_{\rm tot} \quad \propto \quad {\rm Im}\left[i \sum_{\mathbf{p},\mathbf{p}'} \int d^4x\, e^{-i\hat{q}\cdot x} \left\langle 0 \left| T\left(C(\mu)\mathbf{O}^\dagger_{\mathbf{p}}(0)\right) \left(C(\mu)\mathbf{O}_{\mathbf{p}'}(x)\right) \right| 0 \right\rangle \right]$$

$$\propto \quad {\rm Im}\left[C(\mu)^2\, G(0,0,E) \right]$$



$$\left(-\frac{\nabla^2}{m_t} - \frac{\nabla^4}{4m_t^3} + V(\mathbf{r}) - E \right) G(\mathbf{r},\mathbf{r}',E) = \delta^{(3)}(\mathbf{r}-\mathbf{r}')$$

$$V(\mathbf{p},\mathbf{p}') = \left[\frac{\mathcal{V}_c}{\mathbf{k}^2} + \frac{\mathcal{V}_k \pi^2}{m_t|\mathbf{k}|} + \frac{\mathcal{V}_r(\mathbf{p}^2+\mathbf{p}'^2)}{2m_t^2\mathbf{k}^2} + \frac{\mathcal{V}_2+2\mathcal{V}_s}{m_t^2} \right], \mathbf{k} = \mathbf{p}-\mathbf{p}'$$

Theory status (QCD)

Fixed order scheme

Hoang, Teubner; Penin et al; Melnikov et al
Beneke, Signer, Smirnov; Sumino et al; Yakovlev et al
Steinhauser, Kniehl, . . .

$$\boxed{\left(\frac{\alpha_s}{v}\right) \sim 1}$$

$$\begin{aligned}
\text{LO} &\sim \left(\frac{\alpha_s}{v}\right)^n \\
\text{NLO} &\sim \{\alpha_s, v\} \times \left(\frac{\alpha_s}{v}\right)^n \\
\text{NNLO} &\sim \{\alpha_s^2, \alpha_s v, v^2\} \times \left(\frac{\alpha_s}{v}\right)^n \\
\text{NNNLO} &\sim \{\alpha_s^3, \alpha_s^2 v, \alpha_s v^2, v^3\} \times \left(\frac{\alpha_s}{v}\right)^n \text{ work in progress}
\end{aligned}$$

– large NNLO correction

– scale dependence $\rightarrow$ large uncertainty in normalization of cross section

Theory status (QCD)

RGE improved computations

$\left(\frac{\alpha_s}{v}\right) \sim 1$ $\qquad$ $\alpha_s \ln v \sim 1$

$$\text{LL} \quad \sim \left(\frac{\alpha_s}{v}\right)^n \sum_m (\alpha_s \ln v)^m$$

$$\text{NLL} \quad \sim \{\alpha_s, v\} \times \left(\frac{\alpha_s}{v}\right)^n \sum_m (\alpha_s \ln v)^m$$

$$\text{NNLL} \quad \sim \{\alpha_s^2, \alpha_s v, v^2\} \times \left(\frac{\alpha_s}{v}\right)^n \sum_m (\alpha_s \ln v)^m$$ work in progress

$$\rightarrow \delta\sigma_{\rm tot}/\sigma_{\rm tot} \sim \pm 6\,\%$$

- log terms summed into coefficients through RGE
- reduced scale dependence

pNRQCD Brambilla, Pineda, Soto, Vairo; Pineda, Signer

vNRQCD Luke, Manohar, Rothstein; Hoang, Stewart

Theory status (electroweak)

Electroweak effects

$$\Gamma_t \sim m_t\alpha \approx E_{\rm kin} \sim m_t\alpha_s^2$$

- LL: $E \to E + i\Gamma_t$ Fadin, Khoze (1987)

Theory status (electroweak)

Electroweak effects

$$\Gamma_t \sim m_t \alpha \approx E_{\rm kin} \sim m_t \alpha_s^2$$

- LL: $E \to E + i\Gamma_t$ Fadin, Khoze (1987)
- NLL:

Theory status (electroweak)

Electroweak effects

$$\Gamma_t \sim m_t \alpha \approx E_{\rm kin} \sim m_t \alpha_s^2$$

- LL: $E \to E + i\Gamma_t$ Fadin, Khoze (1987)
- NLL:

  "only QCD corrections to Γ_t" Melnikov, Yakovlev (1994)

  Fadin, Khoze, Martin, Stirling (1995)

Theory status (electroweak)

Electroweak effects

$$\Gamma_t \sim m_t \alpha \approx E_{\rm kin} \sim m_t \alpha_s^2$$

- LL: $E \to E + i\Gamma_t$ Fadin, Khoze (1987)
- NLL:

  “only QCD corrections to Γ_t” Melnikov, Yakovlev (1994)
  Fadin, Khoze, Martin, Stirling (1995)

  “NNLO phase space divergencies $\to$ NLL RG effects”
  Hoang, CJR (2005)

Theory status (electroweak)

Electroweak effects

$$\Gamma_t \sim m_t \alpha \approx E_{\rm kin} \sim m_t \alpha_s^2$$

- LL: $E \to E + i\Gamma_t$ Fadin, Khoze (1987)
- NLL:

  "only QCD corrections to Γ_t" Melnikov, Yakovlev (1994)
  Fadin, Khoze, Martin, Stirling (1995)

  "NNLO phase space divergencies $\to$ NLL RG effects"
  Hoang, CJR (2005)

  "phase space matching" $\to$ w.i.p.

Theory status (electroweak)

Electroweak effects

$$\Gamma_t \sim m_t \alpha \approx E_{\rm kin} \sim m_t \alpha_s^2$$

- LL: $E \to E + i\Gamma_t$ Fadin, Khoze (1987)
- NLL:

  “only QCD corrections to Γ_t” Melnikov, Yakovlev (1994)
  Fadin, Khoze, Martin, Stirling (1995)

  “NNLO phase space divergencies $\to$ NLL RG effects”
  Hoang, CJR (2005)

  “phase space matching” $\to$ w.i.p.
- NNLL:

Theory status (electroweak)

Electroweak effects

$$\Gamma_t \sim m_t \alpha \approx E_{\rm kin} \sim m_t \alpha_s^2$$

- LL: $E \to E + i\Gamma_t$ Fadin, Khoze (1987)
- NLL:

  "only QCD corrections to Γ_t" Melnikov, Yakovlev (1994)
  Fadin, Khoze, Martin, Stirling (1995)

  "NNLO phase space divergencies $\to$ NLL RG effects"
  Hoang, CJR (2005)

  "phase space matching" $\to$ w.i.p.
- NNLL:

  Matrixelement corrections (real & imaginary)
  Hoang, CJR (2005, 2006)

Theory status (electroweak)

Electroweak effects

$$\Gamma_t \sim m_t\alpha \approx E_{\rm kin} \sim m_t\alpha_s^2$$

- LL: $E \to E + i\Gamma_t$ Fadin, Khoze (1987)
- NLL:

  "only QCD corrections to Γ_t" Melnikov, Yakovlev (1994)
  Fadin, Khoze, Martin, Stirling (1995)

  "NNLO phase space divergencies $\to$ NLL RG effects"
  Hoang, CJR (2005)

  "phase space matching" $\to$ w.i.p.

- NNLL:

  Matrixelement corrections (real & imaginary)
  Hoang, CJR (2005, 2006)

  NNLL running from phase space divergencies $\to$ not yet started

Electroweak corrections in vNRQCD

Electroweak effects

i) Usual (non-imaginary) electroweak effects

Electroweak corrections in vNRQCD

Electroweak effects

i) Usual (non-imaginary) electroweak effects

ii) Wb cuts, interference effects

Electroweak corrections in vNRQCD

Electroweak effects

i) Usual (non-imaginary) electroweak effects

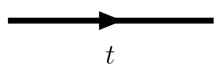
ii) Wb cuts, interference effects

iii) Phase space matching

Unstable quarks in vNRQCD

Power counting: $\boxed{D^0 \sim m_t v^2, \quad \mathbf{p}^2 \sim m_t^2 v^2, \quad \Gamma_t \sim m_t \alpha \sim m_t \alpha_s^2}$

<u>Full theory:</u>



$\bar{\psi}(\not{p} - m_t)\psi$

<u>Effective theory:</u>

$$\mathcal{L}_{\rm bil} = \psi_{\mathbf{p}}^\dagger(x)\left[iD^0 - \frac{\mathbf{p}^2}{2m_t} + \frac{\mathbf{p}^4}{8m_t^3}\right]\psi_{\mathbf{p}}(x)$$

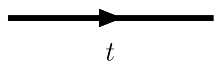
$\sim m_t v^2$

stable propagator: $\dfrac{i}{p^0 - \frac{\mathbf{p}^2}{2m_t}}$

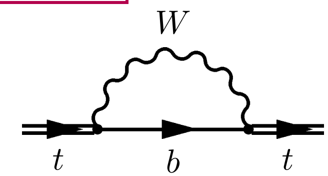
Unstable quarks in vNRQCD

Power counting: $D^0 \sim m_t v^2, \quad \mathbf{p}^2 \sim m_t^2 v^2, \quad \Gamma_t \sim m_t \alpha \sim m_t \alpha_s^2$

Full theory:



$\bar{\psi}(\not{p} - m_t)\psi$

$i\,\Sigma_t =$ 

$\mathrm{Im}\,\Sigma_t = \frac{\Gamma_t}{2}$

Effective theory:

$$\mathcal{L}_{\rm bil} = \psi_{\mathbf{p}}^\dagger(x)\left[iD^0 - \frac{\mathbf{p}^2}{2m_t} + \frac{\mathbf{p}^4}{8m_t^3}\right]\psi_{\mathbf{p}}(x), \quad \delta\mathcal{L}_{\rm bil} = \psi_{\mathbf{p}}^\dagger\left[\delta m_t + i\frac{\Gamma_t}{2} - i\frac{\Gamma_t}{2}\frac{\mathbf{p}^2}{2m_t^2}\right]\psi_{\mathbf{p}}$$

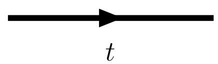
$\sim m_t v^2$

stable propagator: $\dfrac{i}{p^0 - \frac{\mathbf{p}^2}{2m_t}}$

Unstable quarks in vNRQCD

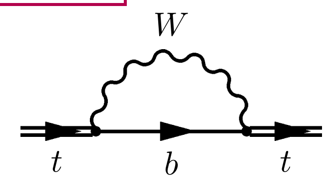
Power counting: $D^0 \sim m_t v^2, \quad \mathbf{p}^2 \sim m_t^2 v^2, \quad \Gamma_t \sim m_t \alpha \sim m_t \alpha_s^2$

Full theory:



$\bar{\psi}(\not{p} - m_t)\psi$

$i\,\Sigma_t =$



$\mathrm{Im}\,\Sigma_t = \frac{\Gamma_t}{2}$

Effective theory:

$$\mathcal{L}_{\rm bil} = \psi_{\mathbf{p}}^\dagger(x)\left[iD^0 - \frac{\mathbf{p}^2}{2m_t} + \frac{\mathbf{p}^4}{8m_t^3}\right]\psi_{\mathbf{p}}(x), \quad \delta\mathcal{L}_{\rm bil} = \psi_{\mathbf{p}}^\dagger\left[\delta m_t + i\frac{\Gamma_t}{2} - i\frac{\Gamma_t}{2}\frac{\mathbf{p}^2}{2m_t^2}\right]\psi_{\mathbf{p}}$$

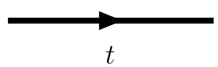
$\sim m_t v^2$ $\qquad$ $v \sim \alpha_s$ $\qquad$ $\sim m_t \alpha_s^2$ $\qquad$ $\sim m_t \alpha_s^4$

stable propagator: $\dfrac{i}{p^0 - \frac{\mathbf{p}^2}{2m_t}}$

Unstable quarks in vNRQCD

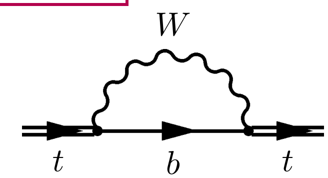
Power counting: $D^0 \sim m_t v^2, \quad \mathbf{p}^2 \sim m_t^2 v^2, \quad \Gamma_t \sim m_t\alpha \sim m_t\alpha_s^2$

Full theory:



$\bar\psi(\not p - m_t)\psi$

$i\,\Sigma_t =$



$\mathrm{Im}\,\Sigma_t = \frac{\Gamma_t}{2}$

Effective theory:

$$\mathcal{L}_{\rm bil} = \psi^\dagger_{\mathbf{p}}(x)\left[iD^0 - \frac{\mathbf{p}^2}{2m_t} + \frac{\mathbf{p}^4}{8m_t^3}\right]\psi_{\mathbf{p}}(x), \quad \delta\mathcal{L}_{\rm bil} = \psi^\dagger_{\mathbf{p}}\left[\delta m_t + i\frac{\Gamma_t}{2} - i\frac{\Gamma_t}{2}\frac{\mathbf{p}^2}{2m_t^2}\right]\psi_{\mathbf{p}}$$

$\sim m_t v^2$ $\qquad$ $v \sim \alpha_s$ $\qquad$ $\sim m_t\alpha_s^2$ $\qquad$ $\sim m_t\alpha_s^4$

stable propagator: $\dfrac{i}{p^0 - \frac{\mathbf{p}^2}{2m_t}}$

unstable propagator:

$$\frac{i}{p^0 - \frac{\mathbf{p}^2}{2m_t} + i\frac{\Gamma_t}{2}}$$

NNLL time dilatation correction

Unstable quarks in vNRQCD

Effective theory for unstable particles

- Replacement rule $E \rightarrow E + i\Gamma_t$ at LL   Fadin, Khoze (JETP Lett. 46, 1987)
- Complex matching conditions
  $\longrightarrow$ at NNLL contain interferences (in a few slides)
  $\longrightarrow$ UV phase space divergencies arise (in a few slides)
  $\longrightarrow$ Phase space matching necessary (end of talk)
- Effective Lagrangian non-hermitian
- Total cross section through the optical theorem using unitarity of the underlying theory

Unstable quarks in vNRQCD

Effective theory for unstable particles

- Replacement rule $E \longrightarrow E + i\Gamma_t$ at LL   Fadin, Khoze (JETP Lett. 46, 1987)
- Complex matching conditions
  $\longrightarrow$ at NNLL contain interferences (in a few slides)
  $\longrightarrow$ UV phase space divergencies arise (in a few slides)
  $\longrightarrow$ Phase space matching necessary (end of talk)
- Effective Lagrangian non-hermitian
- Total cross section through the optical theorem using unitarity of the underlying theory

$\Rightarrow$ Contributions from real Wb final states included in EFT matching conditions

$\Rightarrow$ EFT does not describe details of decay mechanism
$\longrightarrow$ inclusive treatment

$\gg$ In analogy to absorptive processes in the optical theory

Instability beyond LL (inclusive)

Quark bilinear operators:

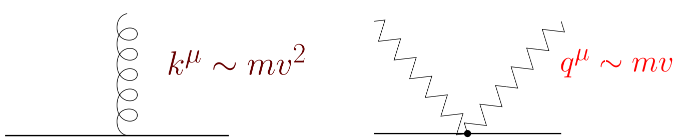
- Dilatation of lifetime at NNLL
- $O(\alpha_s)$ QCD corrections to Γ_t at NLL   Ježabek, Kühn (Nucl. Phys. B 314, 1989)
  $O(\alpha_s^2)$ QCD and $O(\alpha)$ electroweak corrections to Γ_t at NNLL
  Blokland, Czarnecki, Ślusarczyk, Tkachov (Phys. Rev. Lett. 93, 2004)

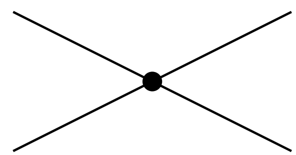
Instability beyond LL (inclusive)

Quark bilinear operators:

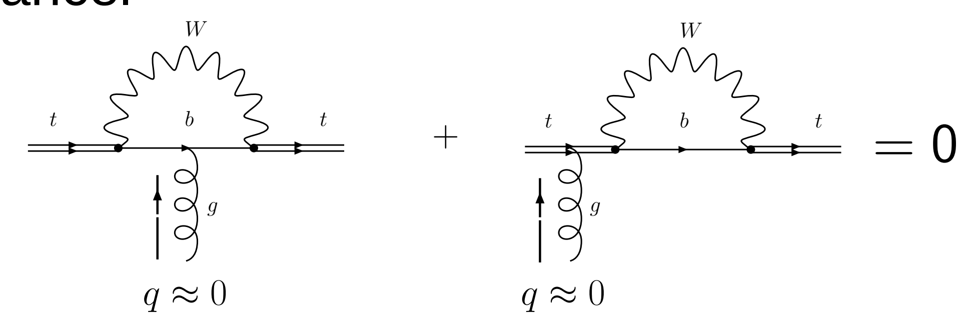
- Dilatation of lifetime at NNLL
- $O(\alpha_s)$ QCD corrections to Γ_t at NLL   Jeżabek, Kühn (Nucl. Phys. B 314, 1989)
  $O(\alpha_s^2)$ QCD and $O(\alpha)$ electroweak corrections to Γ_t at NNLL
  Blokland, Czarnecki, Ślusarczyk, Tkachov (Phys. Rev. Lett. 93, 2004)

Gluon interactions and potentials:





- electroweak corrections either beyond NNLL or contributions to $\sigma_{\rm tot}$ cancel due to gauge invariance

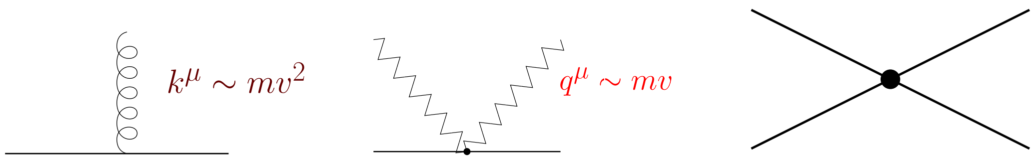


Instability beyond LL (inclusive)

Quark bilinear operators:

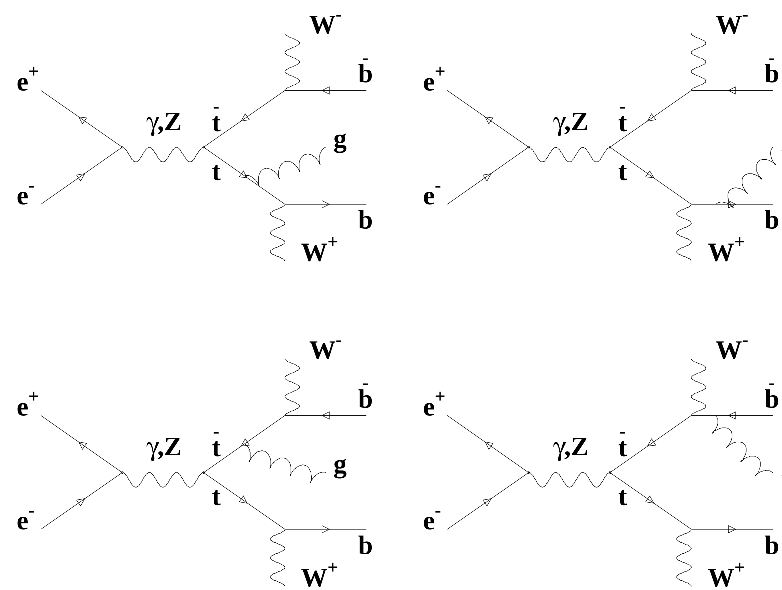
- Dilatation of lifetime at NNLL
- $O(\alpha_s)$ QCD corrections to Γ_t at NLL   Jeżabek, Kühn (Nucl. Phys. B 314, 1989)
  $O(\alpha_s^2)$ QCD and $O(\alpha)$ electroweak corrections to Γ_t at NNLL
  Blokland, Czarnecki, Ślusarczyk, Tkachov (Phys. Rev. Lett. 93, 2004)

Gluon interactions and potentials:



- electroweak corrections either beyond NNLL or contributions to σ_{tot} cancel due to gauge invariance

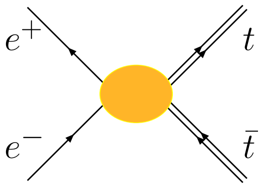
$\longrightarrow$ no non-factorizable effects from ultrasoft gluons
Melnikov, Yakovlev (Phys. Lett. B324, 1994)
Fadin, Khoze, Martin, Stirling (1995)



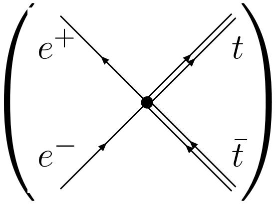
Real electrow. matching beyond LL

Currents:

$m_t\,\alpha \sim m_t\,\alpha_s^2$



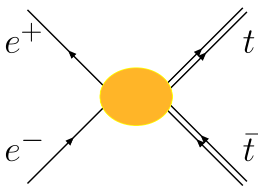
$$= \left[C_{LL}^{\mathrm{born}} + C_{NLL}^{\mathrm{QCD}} + C_{NNLL}^{\mathrm{QCD}} + C_{NNLL}^{\mathrm{ew}} + i\, C_{NNLL}^{\mathrm{bW,abs}} + \ldots \right] \cdot \left(\quad \right)$$



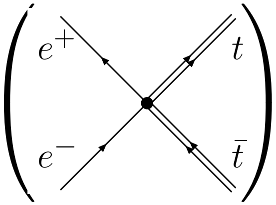
Real electrow. matching beyond LL

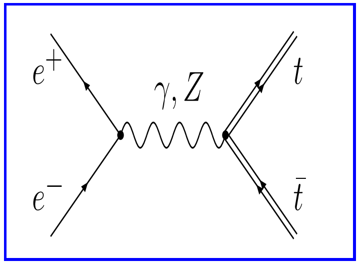
Currents:

$m_t\,\alpha \sim m_t\,\alpha_s^2$



$$= \left[C_{LL}^{born} + C_{NLL}^{QCD} + C_{NNLL}^{QCD} + C_{NNLL}^{ew} + i\, C_{NNLL}^{bW,abs} + \dots \right] \cdot \left(\quad \right)$$

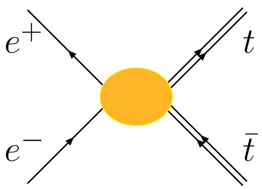




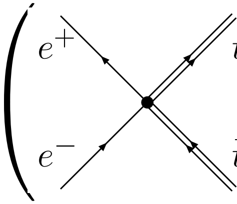
Real electrow. matching beyond LL

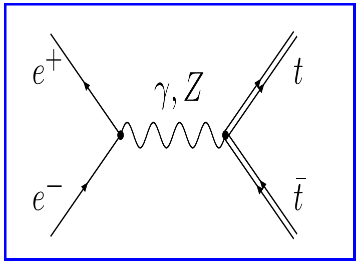
$m_t\,\alpha \sim m_t\,\alpha_s^2$

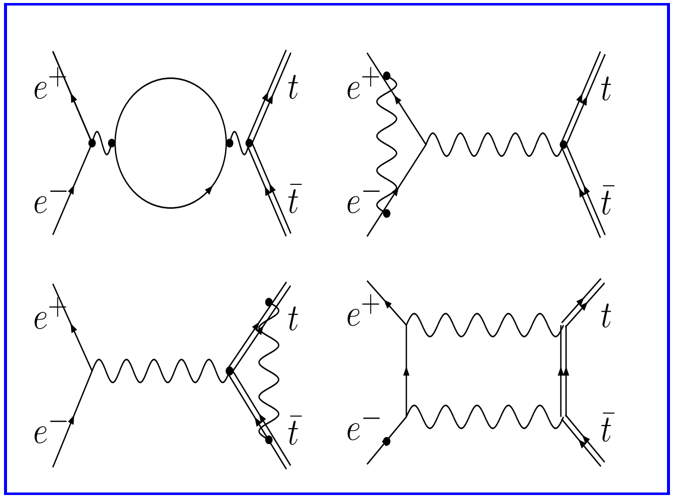
Currents:



$$= \left[C^{\text{born}}_{\text{LL}} + C^{\text{QCD}}_{\text{NLL}} + C^{\text{QCD}}_{\text{NNLL}} + C^{\text{ew}}_{\text{NNLL}} + i\,C^{\text{bW,abs}}_{\text{NNLL}} + \dots \right] \cdot \left(\right)$$







Real parts of ew. 1-loop diagrams $O(\alpha)$

(pure QED diagrams not included)

Grzadkowski, Kühn, Krawczyk, Stuart (Nucl. Phys. B 281,1987)

Hoang, CJR (Phys. Rev. D 74, 2006)

$\Rightarrow$ NNLL hard usual electroweak effects

Real electrow. matching beyond LL

NNLL usual hard corrections (real parts) (pure QED effects not included)

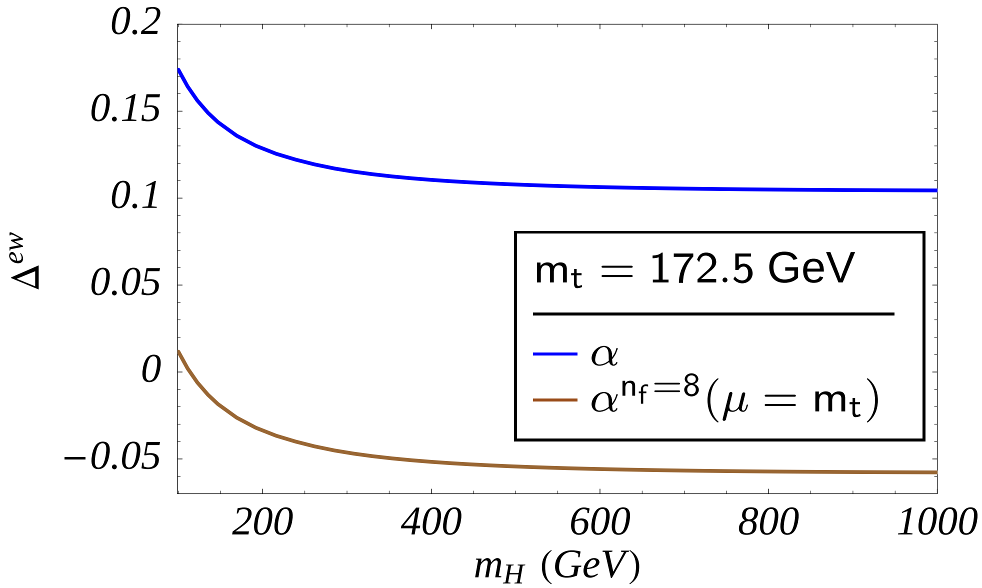
- $\delta\sigma^{\rm ew}_{\rm tot} = 2\,N_c\,{\rm Im}\left[2\,C^{\rm born}_{\rm LL}\,C^{\rm ew}_{\rm NNLL}\,G_{\rm LL}(0,0,E+i\Gamma_t)\right]$
  $= \sigma_{\rm tot,LL}\cdot\Delta^{\rm ew}$
- $\overline{\rm MS}$ definition for α:

$$\alpha^{n_f=8}(\mu) = \frac{\alpha}{1-\frac{\alpha}{3\pi}\sum_{i=e,\mu,\tau}Q_i^2\ln\left(\frac{\mu^2}{m_i^2}\right)-\frac{\alpha}{3\pi}\sum_{i=u,d,c,s,b}N_cQ_i^2\ln\left(\frac{\mu^2}{m_i^2}\right)}$$

$\Rightarrow$ $\alpha^{n_f=8}(\mu=m_t)$ absorbs LL vacuum polarization through leptons and quarks

$\Rightarrow$ Remaining correction $\Delta^{{\rm ew},\overline{\rm MS}}$ characterized by Higgs exchange

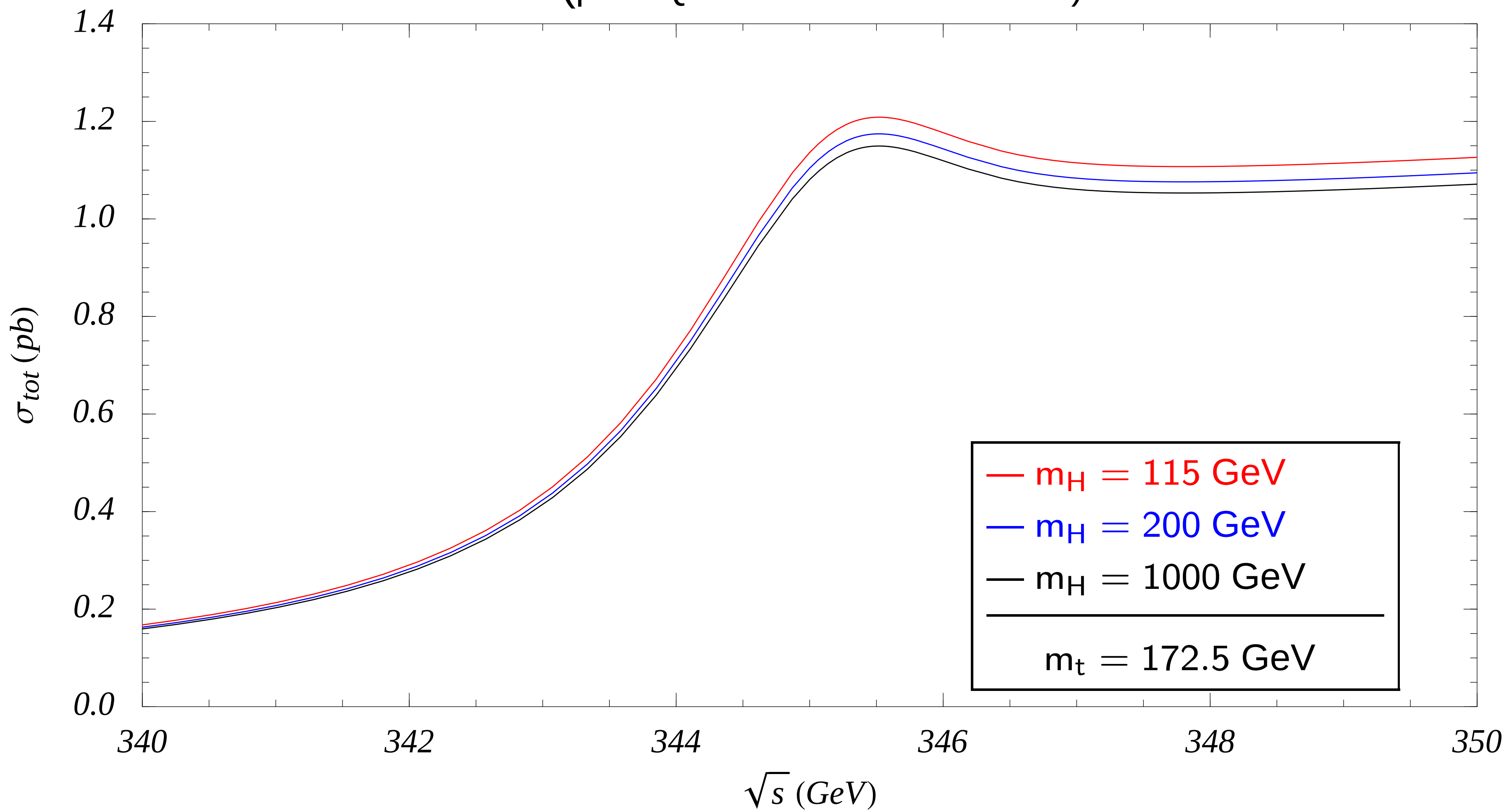
Jeżabek, Kühn (Phys. Lett. B 316, 1993)



Real electrow. matching beyond LL

Total cross section: LL + NNLL hard electroweak effects (real parts)

(pure QED effects not included)

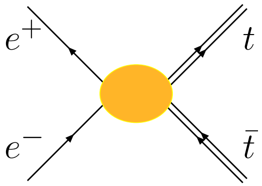


$\Rightarrow$ Shift of normalization by 5.7 % ($m_H = 115$ GeV) or 2.4 % ($m_H = 200$ GeV) with respect to $m_H = 1000$ GeV

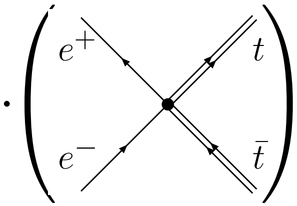
Imaginary electrow. matching beyond LL

Currents:

$m_t\,\alpha \sim m_t\,\alpha_s^2$

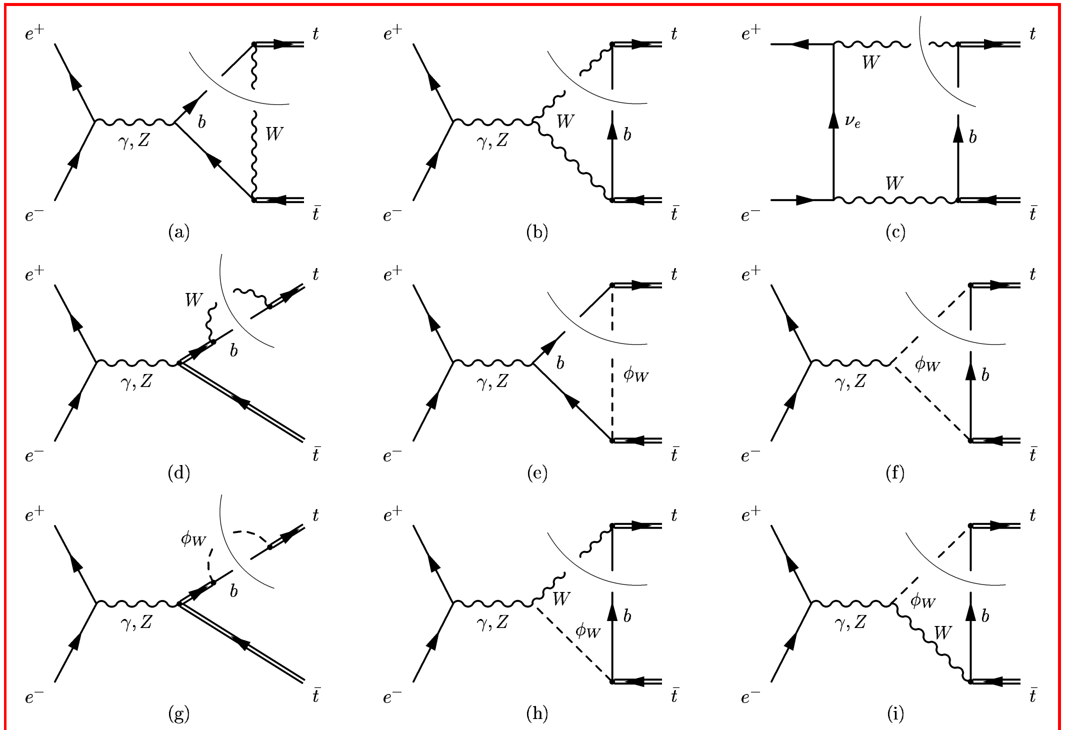


$$= \left[C^{\rm born}_{\rm LL} + C^{\rm QCD}_{\rm NLL} + C^{\rm QCD}_{\rm NNLL} + C^{\rm ew}_{\rm NNLL} + i\, C^{\rm bW,abs}_{\rm NNLL} + \ldots \right] \cdot \left(\; \right)$$



- ≫ bW-cuts of electroweak 1-loop diagrams $O(\alpha)$
- ≫ bW-cuts are gauge invariant
- ≫ bW treated as stable particles

⇒ NNLL instability effects



Hoang, CJR (Phys. Rev. D 71, 2005)

Total cross section

Optical Theorem $\Rightarrow$ $\boxed{\sigma_{\mathrm{tot}} = 2\,N_c\ \mathrm{Im}\left[\,C(\mu)^2\,G(0,0,E+i\Gamma_t)\,\right]}$

$$\sigma_{\mathrm{tot}} = 2\,N_c\,\mathrm{Im}\left[C_{\mathrm{LL}}^{\mathrm{born}}\,(C_{\mathrm{LL}}^{\mathrm{born}} + 2\,C_{\mathrm{NNLL}}^{\mathrm{ew}} + 2\,i\,C_{\mathrm{NNLL}}^{\mathrm{abs,bW}})\,G_{\mathrm{LL}} + \ldots\right]$$

Total cross section

Optical Theorem $\Rightarrow$ $\boxed{\sigma_{\rm tot} = 2\,\mathsf{N_c}\;{\rm Im}\left[\,\mathsf{C}(\mu)^2\,\mathsf{G}(0,0,\mathsf{E}+\mathsf{i}\Gamma_\mathsf{t})\,\right]}$

$$\sigma_{\rm tot} = 2\,\mathsf{N_c}\,{\rm Im}\left[\mathsf{C}_{\rm LL}^{\rm born}\left(\mathsf{C}_{\rm LL}^{\rm born} + 2\,\mathsf{C}_{\rm NNLL}^{\rm ew} + 2\,\mathsf{i}\,\mathsf{C}_{\rm NNLL}^{\rm abs,bW}\right)\mathsf{G}_{\rm LL} + \ldots\right]$$

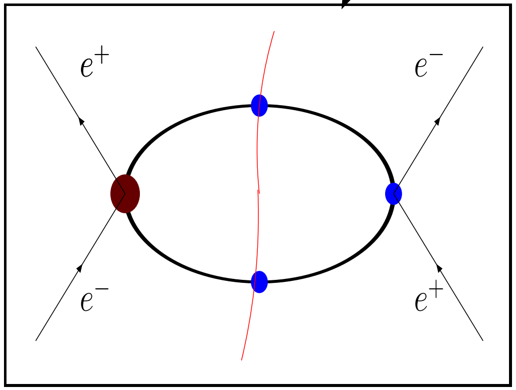
$$= 2\,\mathsf{N_c}\left\{\left[\left(\mathsf{C}_{\rm LL}^{\rm born}\right)^2 + 2\,\mathsf{C}_{\rm LL}^{\rm born}\,\mathsf{C}_{\rm NNLL}^{\rm ew}\right]{\rm Im}[\mathsf{G}_{\rm LL}] + 2\,\mathsf{C}_{\rm LL}^{\rm born}\,\mathsf{C}_{\rm NNLL}^{\mathsf{abs,bW}}\,{\rm Re}[\mathsf{G}_{\rm LL}] + \ldots\right\}$$

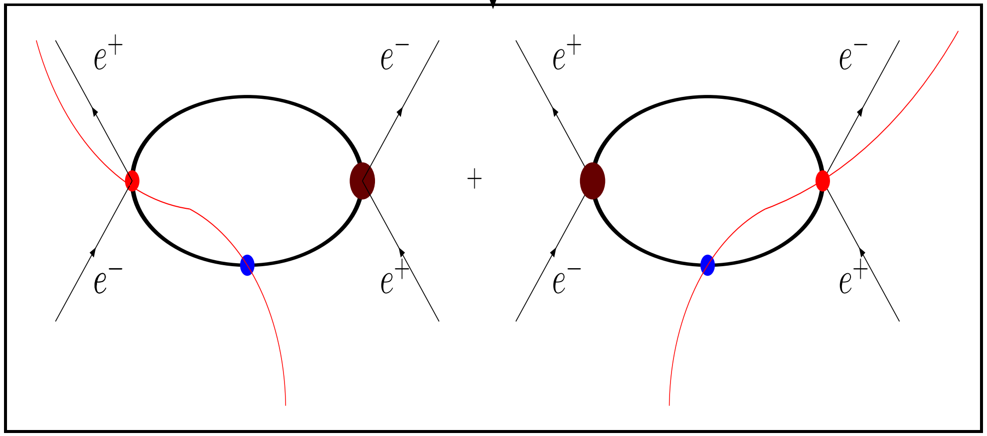
Total cross section

Optical Theorem $\Rightarrow$ $\sigma_{\rm tot} = 2\,N_c\ {\rm Im}\left[\,C(\mu)^2\,G(0,0,E+i\Gamma_t)\,\right]$

$$\sigma_{\rm tot} = 2\,N_c\,{\rm Im}\left[C_{\rm LL}^{\rm born}\left(C_{\rm LL}^{\rm born} + 2\,C_{\rm NNLL}^{\rm ew} + 2\,i\,C_{\rm NNLL}^{\rm abs,bW}\right)G_{\rm LL} + \ldots\right]$$

$$= 2\,N_c\left\{\underbrace{\left[\left(C_{\rm LL}^{\rm born}\right)^2 + 2\,C_{\rm LL}^{\rm born}\,C_{\rm NNLL}^{\rm ew}\right]{\rm Im}[G_{\rm LL}]}\ + \underbrace{2\,C_{\rm LL}^{\rm born}\,C_{\rm NNLL}^{\rm abs,bW}\,{\rm Re}[G_{\rm LL}]} + \ldots\right\}$$





double-resonant

$e^+e^- \to t\bar{t} \to bW^+\bar{b}W^-$

interference: double- & single-resonant

$e^+e^- \to t\bar{t} \to bW^+\bar{b}W^-$

$e^+e^- \to t\bar{b}W^- \to bW^+\bar{b}W^-$

$e^+e^- \to bW^+\bar{t} \to bW^+\bar{b}W^-$

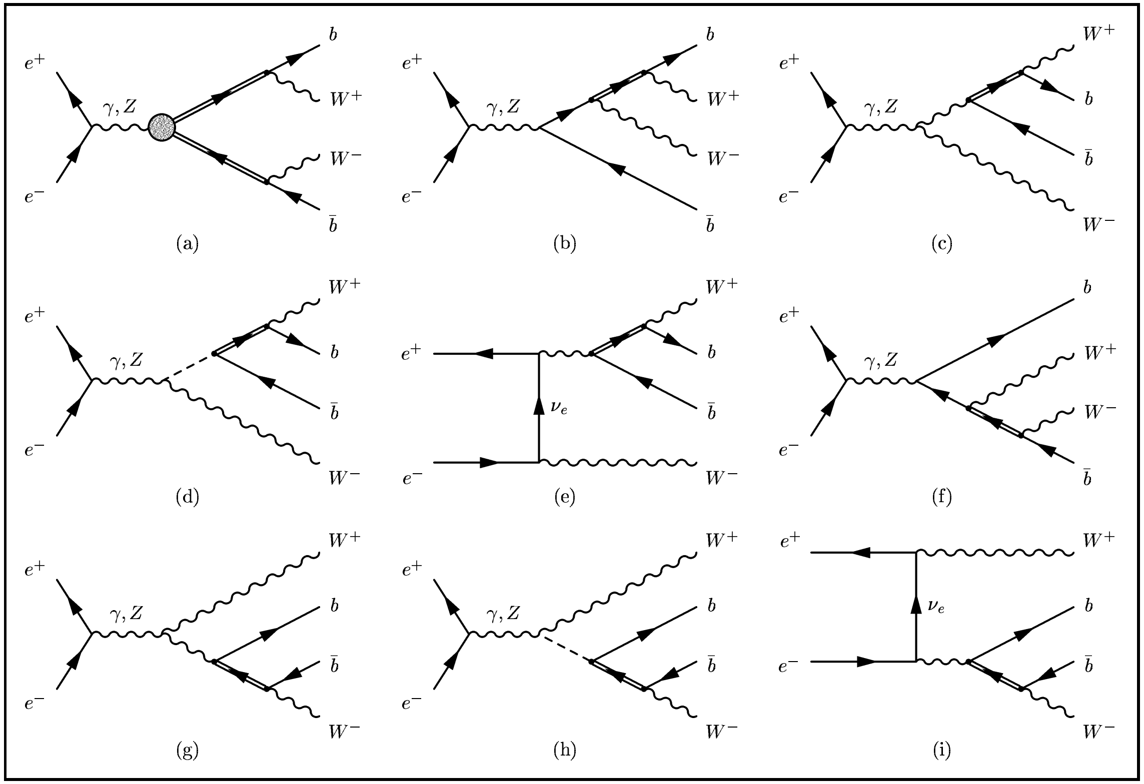
Total cross section

Optical Theorem $\Rightarrow$ $\boxed{\sigma_{\rm tot} = 2\,N_c\ \mathrm{Im}\left[\,C(\mu)^2\,G(0,0,E+i\Gamma_t)\,\right]}$

$$\sigma_{\rm tot} = 2\,N_c\,\mathrm{Im}\left[C_{\rm LL}^{\rm born}\,(C_{\rm LL}^{\rm born} + 2\,C_{\rm NNLL}^{\rm ew} + 2\,i\,C_{\rm NNLL}^{\rm abs,bW})\,G_{\rm LL} + \ldots\right]$$

$$= 2\,N_c\left\{\left[\left(C_{\rm LL}^{\rm born}\right)^2 + 2\,C_{\rm LL}^{\rm born}\,C_{\rm NNLL}^{\rm ew}\right]\mathrm{Im}[G_{\rm LL}] + \underbrace{2\,C_{\rm LL}^{\rm born}\,C_{\rm NNLL}^{\rm abs,bW}\,\mathrm{Re}[G_{\rm LL}]} + \ldots\right\}$$

- ≫ Interference of double-resonant and single-resonant $bW^+\bar{b}W^-$ final state diagrams



Phase space divergence

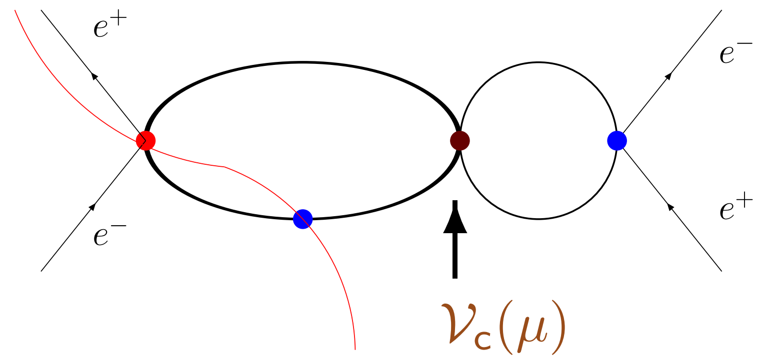
Optical Theorem $\Rightarrow$ $\boxed{\sigma_{\rm tot} = 2\,N_c\ \mathrm{Im}\left[\,C(\mu)^2\,G(0,0,E+i\Gamma_t)\,\right]}$

- NNLL decay correction

$$\Delta^{\Gamma,1}\sigma_{\rm tot} = 2\,N_c\Big\{2\,C_{\rm LL}^{\rm born}\,C_{\rm NNLL}^{\rm abs,bW}\,\mathrm{Re}[G_{\rm LL}] + \ldots\Big\}$$

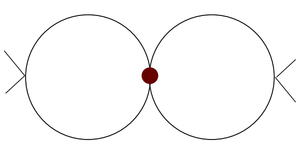
contains logarithmic UV phase space divergence

$$\boxed{C_{\rm NNLL}^{\rm abs,bW}\,\mathcal{V}_c(\mu)\,\frac{1}{\epsilon}} \quad \sim$$



$$\mathcal{V}_c(\mu) = -4\pi C_F\,\alpha_s(\mu)$$

from $\mathcal{O}(\alpha_s)$ term in Green function



$$= G_{\rm LL}^{\mathcal{O}(\alpha_s)} = \alpha_s(\mu)\,C_F\,\frac{m_t^2}{4\pi}\left[\frac{1}{4\epsilon} - \ln\left(\frac{-im_t v}{\mu}\right) + \frac{1}{2} - \ln 2\right]$$

Phase space divergence

Optical Theorem $\Rightarrow$ $\boxed{\sigma_{\rm tot} = 2\,N_c\,\mathrm{Im}\left[\,C(\mu)^2\,G(0,0,E+i\Gamma_t)\,\right]}$

- NNLL decay correction

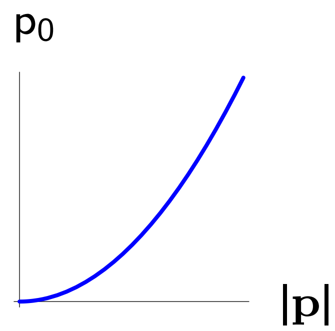
$$\Delta^{\Gamma,1}\sigma_{\rm tot} = 2\,N_c\left\{2\,C_{\rm LL}^{\rm born}\,C_{\rm NNLL}^{\rm abs,bW}\,\mathrm{Re}[G_{\rm LL}] + \ldots\right\}$$

contains logarithmic UV phase space divergence

Phase space:

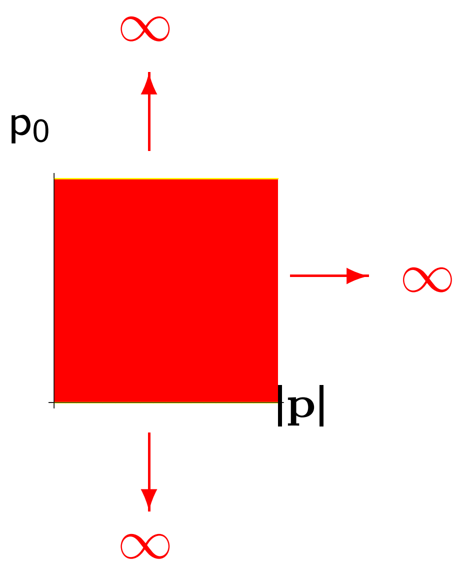
stable tops

$$\frac{i}{p^0 - \frac{\mathbf{p}^2}{2m_t} + i\epsilon} \longrightarrow$$



unstable tops

$$\frac{i}{p^0 - \frac{\mathbf{p}^2}{2m_t} + i\frac{\Gamma_t}{2}} \longrightarrow$$



Phase space divergence

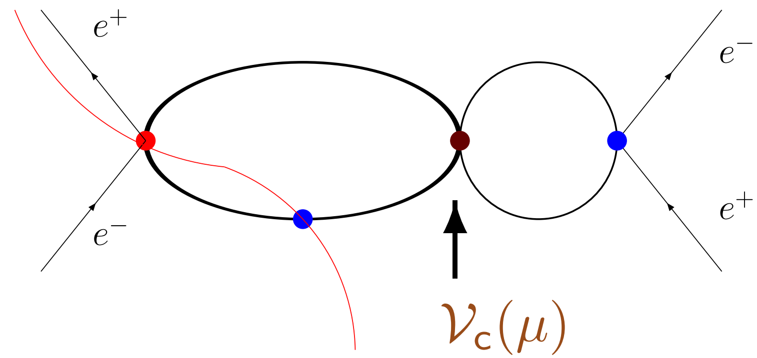
Optical Theorem $\Rightarrow$ $\boxed{\sigma_{\rm tot} = 2\,N_c\ \mathrm{Im}\left[\,C(\mu)^2\,G(0,0,E+i\Gamma_t)\,\right]}$

- NNLL decay correction

  $$\Delta^{\Gamma,1}\sigma_{\rm tot} = 2\,N_c\left\{2\,C^{\rm born}_{\rm LL}\,C^{\rm abs,bW}_{\rm NNLL}\,\mathrm{Re}[G_{\rm LL}] + \ldots\right\}$$

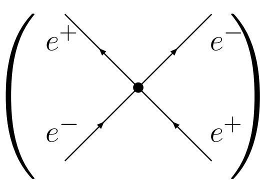
  contains logarithmic UV phase space divergence

$$\boxed{C^{\rm abs,bW}_{\rm NNLL}\,\mathcal{V}_c(\mu)\,\frac{1}{\epsilon}} \quad \sim$$



- NLL mixing effect:

  $\Rightarrow$ Anomalous dimension for operator:

  $i\,C(\mu)\cdot$ 

  $\Rightarrow$ Running $\rightarrow$ correction $\Delta^{\Gamma,2}\sigma_{\rm tot}$

  - $\sqrt{s}$-independent
  - scale-dependent

$\gg$ Matching coefficient

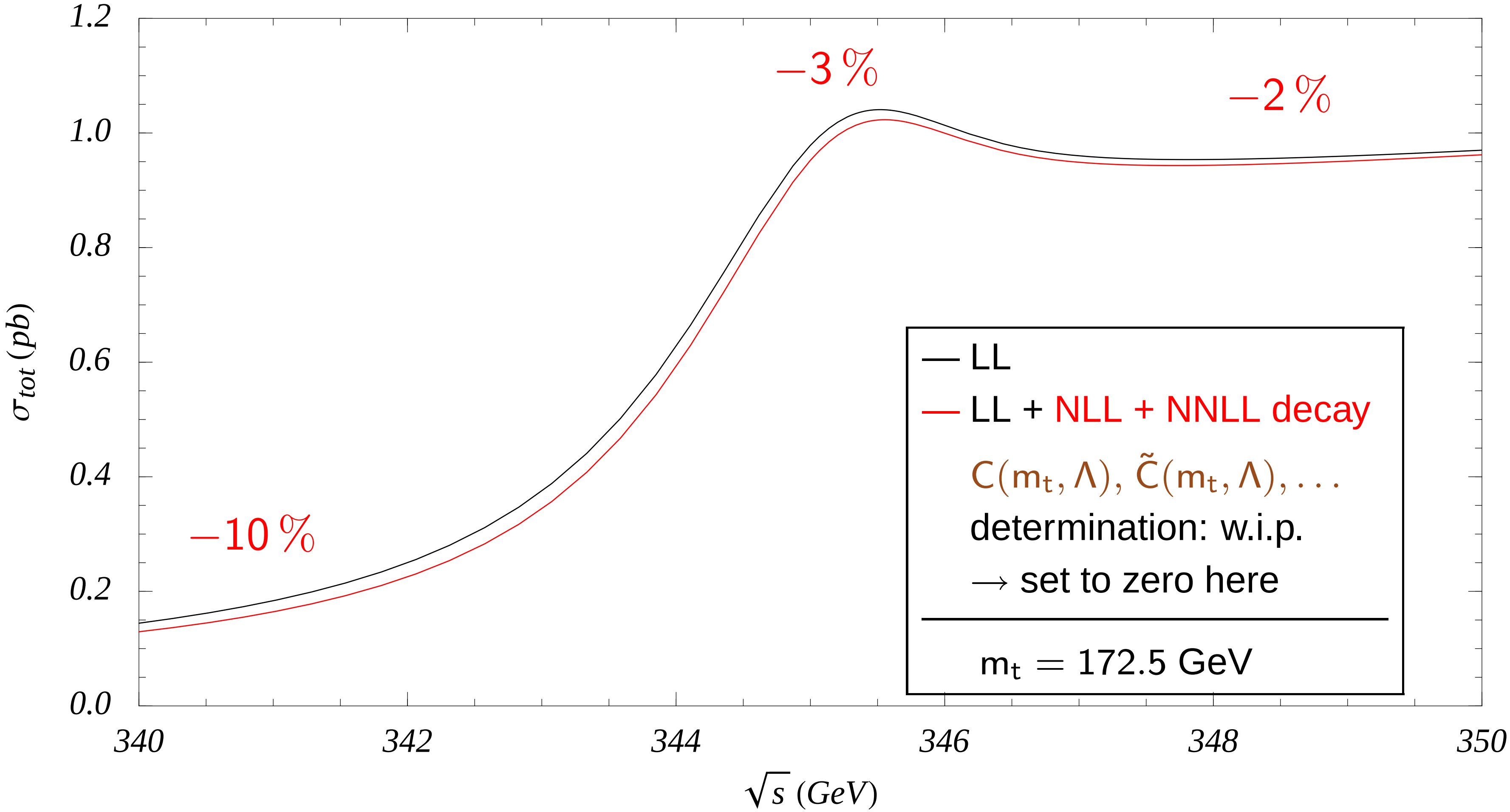
$C(\mu = m_t, \Lambda)$

determination by

phase space matching

Imaginary matching: Numerical analysis

Total cross section: LL + NLL + NNLL decay effects (absorptive parts)



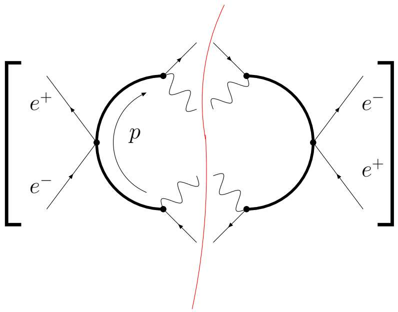
⇒ Comparable to NNLL QCD corrections

⇒ LL peak position shifted by 30 − 50 MeV

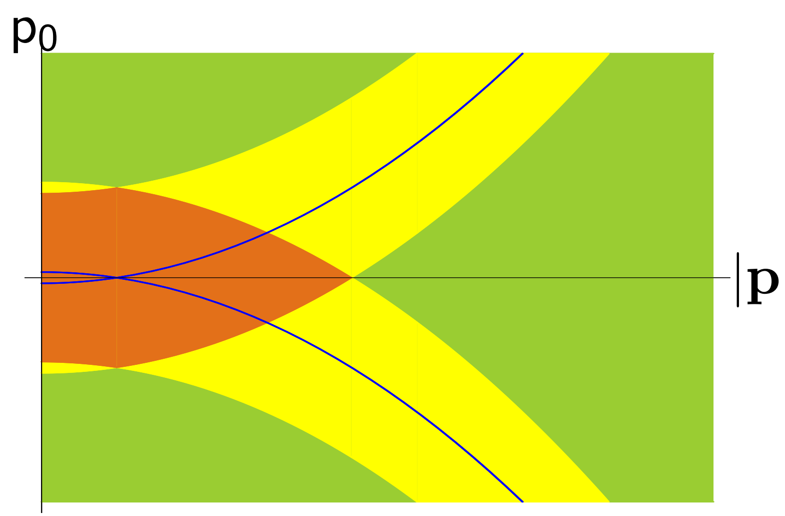
Phase space matching

Hoang, Ruiz-Femenía, CJR (w.i.p.)

$$\sigma_{\rm tot} \sim$$

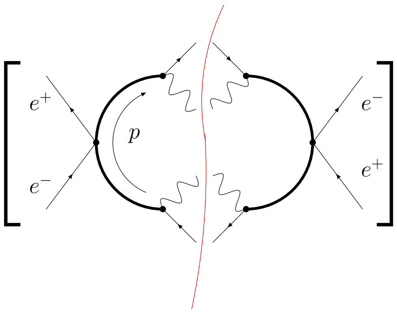


$$\sim \int_{-\infty}^{+\infty} \mathsf{d}p_0 \int_0^{+\infty} \mathsf{d}|\mathbf{p}|\,|\mathbf{p}|^2 \frac{\Gamma_t^2}{|\underbrace{\frac{E}{2} + p_0 - \frac{\mathbf{p}^2}{2m_t}}_{(q^2-m_t^2)/2m_t} + i\frac{\Gamma_t}{2}|^2\,|\frac{E}{2} - p_0 - \frac{\mathbf{p}^2}{2m_t} + i\frac{\Gamma_t}{2}|^2}$$

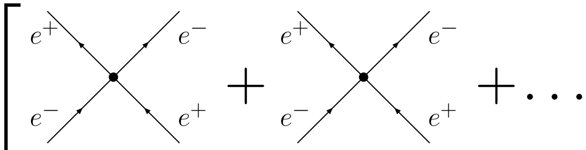


Phase space matching

Hoang, Ruiz-Femenía, CJR (w.i.p.)

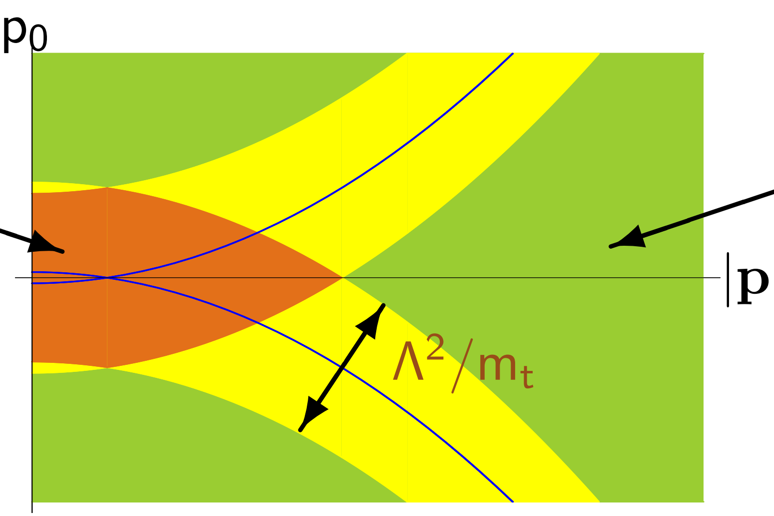
$\sigma_{\mathrm{tot},\Lambda} \sim \Big[$  $\Big]$

$+\ \mathrm{Im}\Big[$ $iC(\mu,\Lambda)$ $+$ $i\tilde{C}(\mu,\Lambda)\frac{\hat{E}}{m_t}$ $+ \ldots \Big]$



$$\sim \int dp_0 \int d|\mathbf{p}|\,|\mathbf{p}|^2 \frac{\Gamma_t^2}{|\underbrace{\frac{E}{2} + p_0 - \frac{\mathbf{p}^2}{2m_t}}_{(q^2-m_t^2)/2m_t} + i\frac{\Gamma_t}{2}|^2\,|\frac{E}{2} - p_0 - \frac{\mathbf{p}^2}{2m_t} + i\frac{\Gamma_t}{2}|^2}$$

$(q^2 - m_t^2) \ll \Lambda^2 \lesssim m_t^2$
double resonant
nonrel. expansion valid
Λ: cut on top and antitop invariant masses



unphysical region of EFT
single/non-resonant
subtracted in local exp.
by matching conditions
$C(m_t,\Lambda), \tilde{C}(m_t,\Lambda), \ldots$

alternative approach see Beneke, Falgari, Schwinn, Signer, Zanderighi (2007)

Phase space cutoff

Cutoff scaling: $\boxed{\Lambda^2 \lesssim m_t^2}$

- Captures resonance region, excludes unphysical parts of the phase space

Phase space cutoff

Cutoff scaling: $\boxed{\Lambda^2 \lesssim m_t^2}$

- Captures resonance region, excludes unphysical parts of the phase space
- Good convergence of the $\left(\frac{E}{\Lambda}\right)^n \left(\frac{\Gamma_t}{\Lambda}\right)^m$ expansion

Phase space cutoff

Cutoff scaling: $\boxed{\Lambda^2 \lesssim m_t^2}$

- Captures resonance region, excludes unphysical parts of the phase space
- Good convergence of the $\left(\frac{E}{\Lambda}\right)^n \left(\frac{\Gamma_t}{\Lambda}\right)^m$ expansion
- Power counting breaking: natural scaling $\Lambda^2 \sim m_t^2 v^2$

Phase space cutoff

Cutoff scaling: $\boxed{\Lambda^2 \lesssim m_t^2}$

- Captures resonance region, excludes unphysical parts of the phase space
- Good convergence of the $\left(\frac{E}{\Lambda}\right)^n \left(\frac{\Gamma_t}{\Lambda}\right)^m$ expansion
- Power counting breaking: natural scaling $\Lambda^2 \sim m_t^2 v^2$
  - Higher dimensional operators will not be suppressed

Phase space cutoff

Cutoff scaling: $\boxed{\Lambda^2 \lesssim m_t^2}$

- Captures resonance region, excludes unphysical parts of the phase space
- Good convergence of the $\left(\frac{E}{\Lambda}\right)^n \left(\frac{\Gamma_t}{\Lambda}\right)^m$ expansion
- Power counting breaking: natural scaling $\Lambda^2 \sim m_t^2 v^2$
  - – Higher dimensional operators will not be suppressed
  - + But: $\frac{\Lambda}{m} < 1$ yields sufficient suppression (choose e.g. $\Lambda \approx 0.6\, m_t$)

    $\rightarrow$ $\boxed{\text{mild power counting breaking}}$

Phase space cutoff

Cutoff scaling: $\boxed{\Lambda^2 \lesssim m_t^2}$

- Captures resonance region, excludes unphysical parts of the phase space
- Good convergence of the $\left(\frac{E}{\Lambda}\right)^n \left(\frac{\Gamma_t}{\Lambda}\right)^m$ expansion
- Power counting breaking: natural scaling $\Lambda^2 \sim m_t^2 v^2$
  - – Higher dimensional operators will not be suppressed
  - + But: $\frac{\Lambda}{m} < 1$ yields sufficient suppression (choose e.g. $\Lambda \approx 0.6\, m_t$)

    $\rightarrow$ mild power counting breaking

Physical cutoff Λ

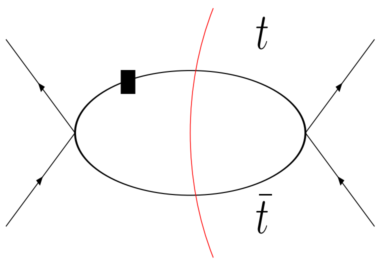
- Cutoff corresponds to maximal invariant mass of an experimentally measured Wb pair that is assigned to a top decay event

Cross section is differential in experimental parameter Λ: $\sigma(\Lambda)$

Finite renormalization

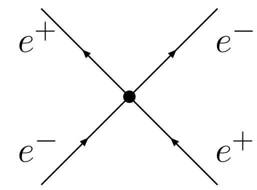
How to incorporate into effective theory framework?

- Phase space effects arise at the level of $\mathrm{e}^+\mathrm{e}^-$ forward scattering (optical theorem) $\rightarrow$ Matching conditions for $(\mathrm{e}^+\mathrm{e}^-)(\mathrm{e}^+\mathrm{e}^-)$ operators
  e.g. kinetic energy insertion



$$\rightarrow \quad i\,\Gamma_t \left[\frac{\#\Lambda}{m_t^2} + \frac{\#\hat{E}+\#\Gamma_t}{m_t\Lambda} + \dots \right]$$

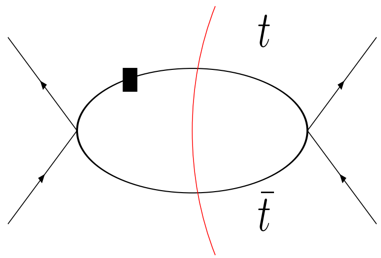
NLL N^{3}LL



Finite renormalization

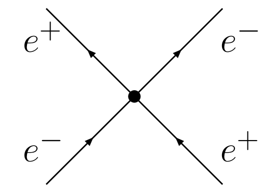
How to incorporate into effective theory framework?

- Phase space effects arise at the level of e^+e^- forward scattering (optical theorem) $\rightarrow$ Matching conditions for $(e^+e^-)(e^+e^-)$ operators
  e.g. kinetic energy insertion

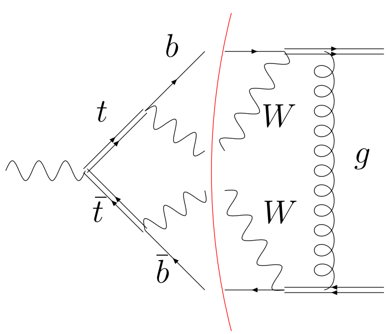


$$\rightarrow \quad i\,\Gamma_t \left[\frac{\#\Lambda}{m_t^2} + \frac{\#\hat{E} + \#\Gamma_t}{m_t \Lambda} + \ldots \right]$$

NLL $\qquad$ N^3LL

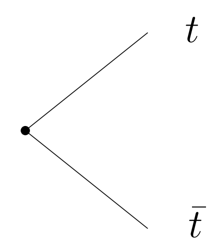


- Finite imaginary renormalization of every effective theory operator that corresponds to a full theory diagram with a cut through bW lines, e.g.



$$\rightarrow \quad \alpha_s\, i\,\Gamma_t \left[\frac{\#m_t^2}{\Lambda^3} + \frac{\#m_t^2 \hat{E} \Gamma_t + \#m_t^2 \Gamma_t^2}{\Lambda^5} + \ldots \right]$$

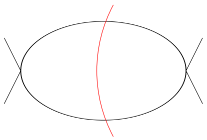
NNLL $\qquad$ N^4LL



Phase space corrections

Suppose formal counting $\boxed{\Lambda^2 \lesssim m_t^2}$

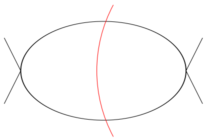
NLL Born level (leading $^3S_1$ current correlator)



$\sim \frac{\Gamma_t}{\Lambda} + \ldots$

Phase space corrections

Suppose formal counting $\Lambda^2 \lesssim m_t^2$

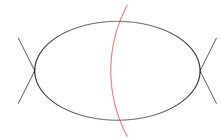
NLL Born level (leading $^3S_1$ current correlator)  $\sim \frac{\Gamma_t}{\Lambda} + \ldots$

NLL Insertions of bilinear operators and higher order current correlators

Phase space corrections

Suppose formal counting $\Lambda^2 \lesssim m_t^2$

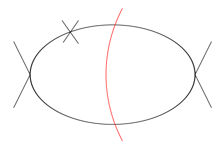
NLL Born level (leading $^3S_1$ current correlator)



$\sim \frac{\Gamma_t}{\Lambda} + \ldots$

NLL Insertions of bilinear operators and higher order current correlators

- kinetic energy insertions $\frac{\mathbf{p}^4}{8m_t^3}$

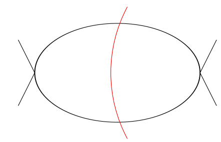


$\sim \frac{\Gamma_t \Lambda}{m_t^2} + \ldots$

Phase space corrections

Suppose formal counting $\Lambda^2 \lesssim m_t^2$

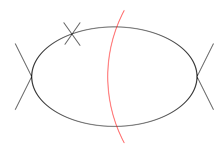
NLL Born level (leading $^3S_1$ current correlator)



$\sim \frac{\Gamma_t}{\Lambda} + \dots$

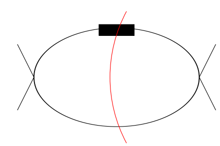
NLL Insertions of bilinear operators and higher order current correlators

- kinetic energy insertions $\frac{\mathbf{p}^4}{8m_t^3}$



$\sim \frac{\Gamma_t \Lambda}{m_t^2} + \dots$

- lifetime dilatation insertions $(-i\Gamma_t)\frac{\mathbf{p}^2}{4m_t^2}$

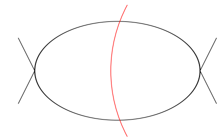


$\sim \frac{\Gamma_t \Lambda}{m_t^2} + \dots$

Phase space corrections

Suppose formal counting $\Lambda^2 \lesssim m_t^2$

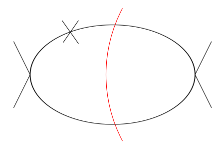
NLL Born level (leading $^3S_1$ current correlator)



$\sim \frac{\Gamma_t}{\Lambda} + \ldots$

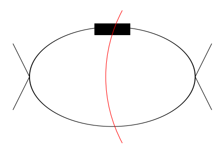
NLL Insertions of bilinear operators and higher order current correlators

- kinetic energy insertions $\frac{\mathbf{p}^4}{8m_t^3}$



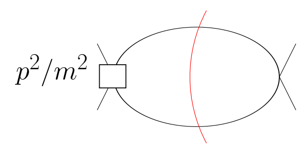
$\sim \frac{\Gamma_t \Lambda}{m_t^2} + \ldots$

- lifetime dilatation insertions $(-i\Gamma_t)\frac{\mathbf{p}^2}{4m_t^2}$



$\sim \frac{\Gamma_t \Lambda}{m_t^2} + \ldots$

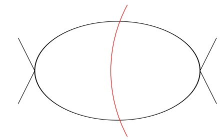
- correlator of leading and subleading $^3S_1$



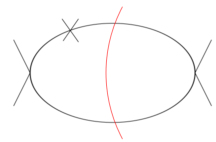
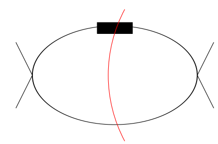
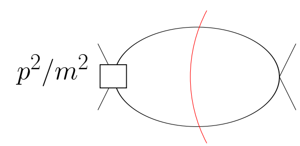
$\sim \frac{\Gamma_t \Lambda}{m_t^2} + \ldots$

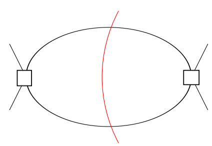
Phase space corrections

Suppose formal counting $\boxed{\Lambda^2 \lesssim m_t^2}$

NLL Born level (leading ${}^3S_1$ current correlator)  $\sim \frac{\Gamma_t}{\Lambda} + \ldots$

NLL Insertions of bilinear operators and higher order current correlators

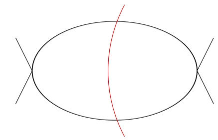
- kinetic energy insertions $\frac{\mathbf{p}^4}{8m_t^3}$  $\sim \frac{\Gamma_t \Lambda}{m_t^2} + \ldots$
- lifetime dilatation insertions $(-i\Gamma_t)\frac{\mathbf{p}^2}{4m_t^2}$  $\sim \frac{\Gamma_t \Lambda}{m_t^2} + \ldots$
- correlator of leading and subleading ${}^3S_1$ 

$\sim \frac{\Gamma_t \Lambda}{m_t^2} + \ldots$
- ${}^3P_1$ correlator  $\sim \frac{\Gamma_t \Lambda}{m_t^2} + \ldots$

Phase space corrections

Suppose formal counting $\boxed{\Lambda^2 \lesssim m_t^2}$

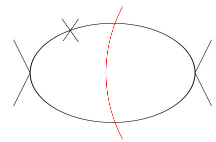
NLL Born level (leading $^3S_1$ current correlator)



$\sim \frac{\Gamma_t}{\Lambda} + \ldots$

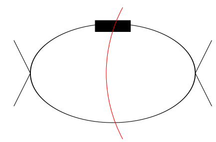
NLL Insertions of bilinear operators and higher order current correlators

- kinetic energy insertions $\frac{\mathbf{p}^4}{8m_t^3}$



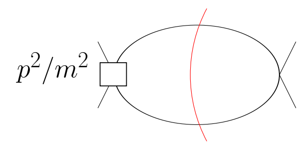
$\sim \frac{\Gamma_t \Lambda}{m_t^2} + \ldots$

- lifetime dilatation insertions $(-i\Gamma_t)\frac{\mathbf{p}^2}{4m_t^2}$



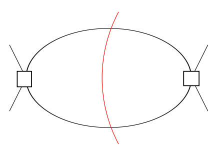
$\sim \frac{\Gamma_t \Lambda}{m_t^2} + \ldots$

- correlator of leading and subleading $^3S_1$



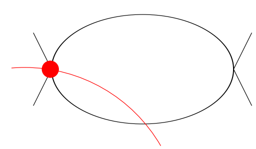
$\sim \frac{\Gamma_t \Lambda}{m_t^2} + \ldots$

- $^3P_1$ correlator



$\sim \frac{\Gamma_t \Lambda}{m_t^2} + \ldots$

- interference diagrams

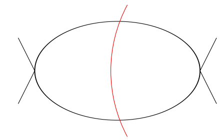


$\sim \frac{\Gamma_t \Lambda}{m_t^2} + \ldots$

Phase space corrections

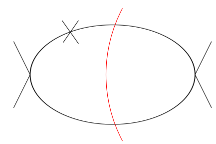
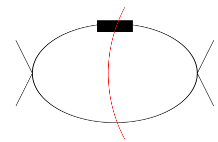
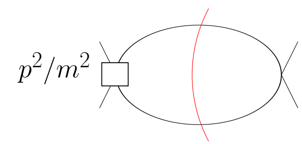
Suppose formal counting $\boxed{\Lambda^2 \lesssim m_t^2}$

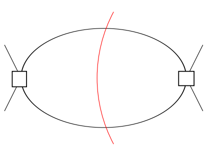
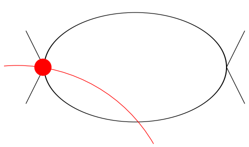
NLL Born level (leading $^3S_1$ current correlator)



$\sim \frac{\Gamma_t}{\Lambda} + \ldots$

NLL Insertions of bilinear operators and higher order current correlators

- kinetic energy insertions $\frac{\mathbf{p}^4}{8m_t^3}$  $\sim \frac{\Gamma_t \Lambda}{m_t^2} + \ldots$
- lifetime dilatation insertions $(-i\Gamma_t)\frac{\mathbf{p}^2}{4m_t^2}$  $\sim \frac{\Gamma_t \Lambda}{m_t^2} + \ldots$
- correlator of leading and subleading $^3S_1$ 

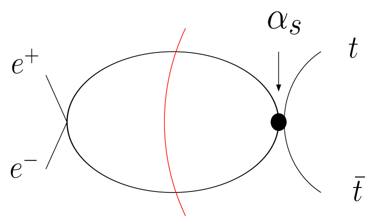
$\sim \frac{\Gamma_t \Lambda}{m_t^2} + \ldots$
- $^3P_1$ correlator  $\sim \frac{\Gamma_t \Lambda}{m_t^2} + \ldots$
- interference diagrams  $\sim \frac{\Gamma_t \Lambda}{m_t^2} + \ldots$

$\Rightarrow$ Matching conditions for $(e^+e^-)(e^+e^-)$ operators

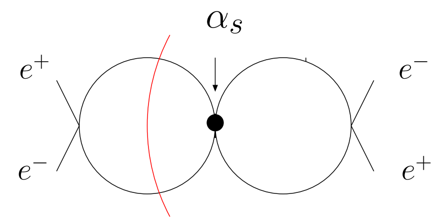
Phase space corrections

Suppose formal counting $\Lambda^2 \lesssim m_t^2$

NNLL $\mathcal{O}(\alpha_s)$



$\sim \alpha_s \frac{m_t^2 \Gamma_t}{\Lambda^3}$

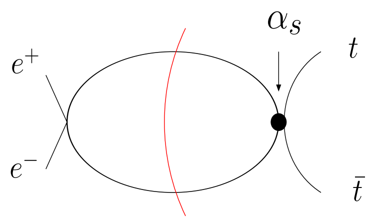


$\sim \alpha_s \frac{m_t \Gamma_t}{\Lambda^2}$

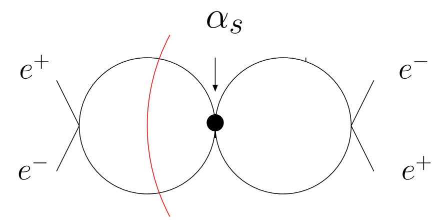
Phase space corrections

Suppose formal counting $\boxed{\Lambda^2 \lesssim m_t^2}$

NNLL $\mathcal{O}(\alpha_s)$

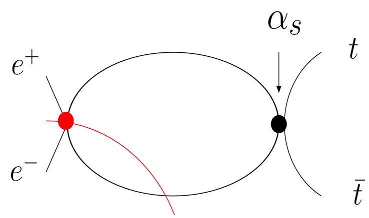


$\sim \alpha_s \frac{m_t^2 \Gamma_t}{\Lambda^3}$

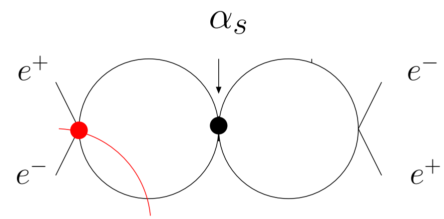


$\sim \alpha_s \frac{m_t \Gamma_t}{\Lambda^2}$

$\mathcal{O}(\alpha_s)$ interference



$\sim \alpha_s \frac{C^{\mathrm{bW,abs}} m_t}{C^{\mathrm{born}} \Lambda}$

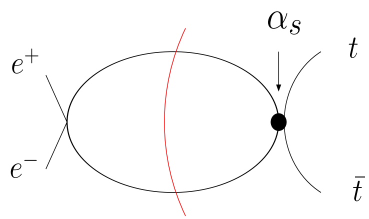


$\sim \alpha_s \frac{C^{\mathrm{bW,abs}}}{C^{\mathrm{born}}} \left[\ln \frac{m}{\Lambda} + \mathrm{const.}\right]$

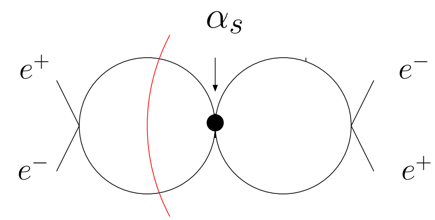
Phase space corrections

Suppose formal counting $\Lambda^2 \lesssim m_t^2$

NNLL $\mathcal{O}(\alpha_s)$

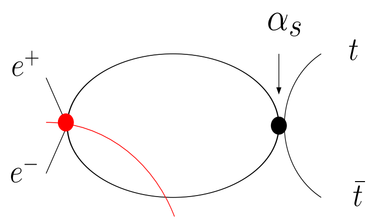


$\sim \alpha_s \frac{m_t^2 \Gamma_t}{\Lambda^3}$

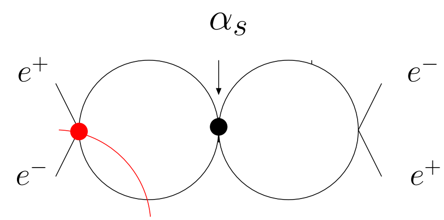


$\sim \alpha_s \frac{m_t \Gamma_t}{\Lambda^2}$

$\mathcal{O}(\alpha_s)$ interference



$\sim \alpha_s \frac{C^{\mathrm{bW,abs}} m_t}{C^{\mathrm{born}} \Lambda}$



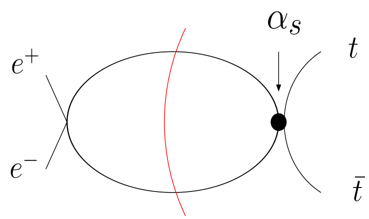
$\sim \alpha_s \frac{C^{\mathrm{bW,abs}}}{C^{\mathrm{born}}} \left[\ln \frac{m}{\Lambda} + \mathrm{const.}\right]$

- Potential insertions: $\frac{\mathcal{V}_c}{\mathbf{k}^2}, \quad \frac{\pi^2 \mathcal{V}_k}{m_t |\mathbf{k}|}, \quad \frac{\mathcal{V}_2 + 2\mathcal{V}_s}{m_t^2}, \quad \frac{(\mathbf{p}^2 + \mathbf{q}^2)\mathcal{V}_r}{2 m_t^2 \mathbf{k}^2}$

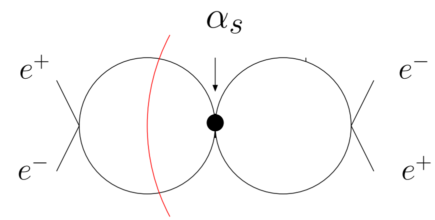
Phase space corrections

Suppose formal counting $\boxed{\Lambda^2 \lesssim m_t^2}$

NNLL $\mathcal{O}(\alpha_s)$

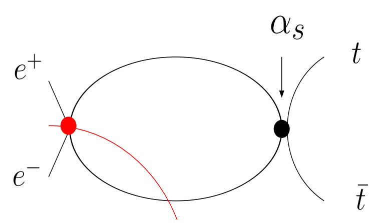


$\sim \alpha_s \frac{m_t^2 \Gamma_t}{\Lambda^3}$

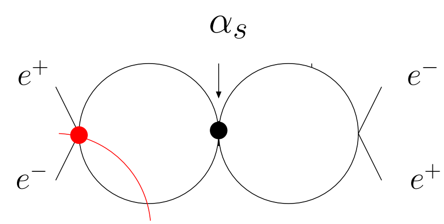


$\sim \alpha_s \frac{m_t \Gamma_t}{\Lambda^2}$

$\mathcal{O}(\alpha_s)$ interference



$\sim \alpha_s \frac{C^{\mathrm{bW,abs}} m_t}{C^{\mathrm{born}} \Lambda}$



$\sim \alpha_s \frac{C^{\mathrm{bW,abs}}}{C^{\mathrm{born}}} \left[\ln \frac{m}{\Lambda} + \mathrm{const.}\right]$

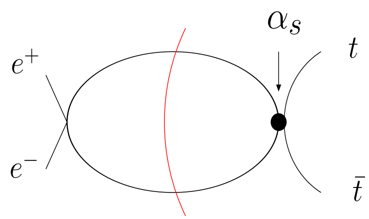
- Potential insertions: $\frac{\mathcal{V}_c}{\mathbf{k}^2}, \quad \frac{\pi^2 \mathcal{V}_k}{m_t |\mathbf{k}|}, \quad \frac{\mathcal{V}_2 + 2\mathcal{V}_s}{m_t^2}, \quad \frac{(\mathbf{p}^2 + \mathbf{q}^2)\mathcal{V}_r}{2 m_t^2 \mathbf{k}^2}$

NNLL Combinations of $\mathcal{O}(\alpha_s)$ corrections and bilinear operators or subleading currents

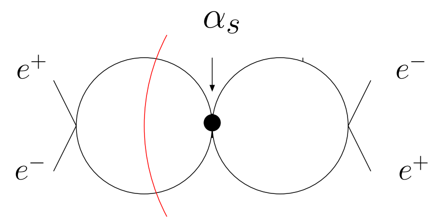
Phase space corrections

Suppose formal counting $\Lambda^2 \lesssim m_t^2$

NNLL $\mathcal{O}(\alpha_s)$

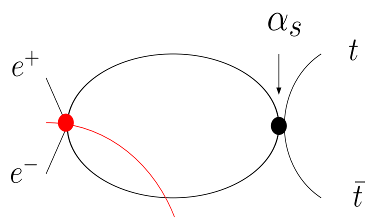


$\sim \alpha_s \frac{m_t^2 \Gamma_t}{\Lambda^3}$

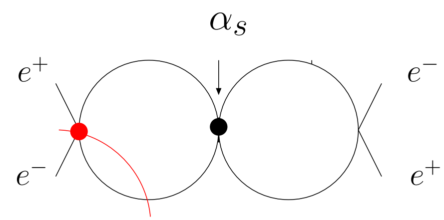


$\sim \alpha_s \frac{m_t \Gamma_t}{\Lambda^2}$

$\mathcal{O}(\alpha_s)$ interference



$\sim \alpha_s \frac{C^{\mathrm{bW,abs}} m_t}{C^{\mathrm{born}} \Lambda}$



$\sim \alpha_s \frac{C^{\mathrm{bW,abs}}}{C^{\mathrm{born}}} \left[\ln \frac{m}{\Lambda} + \mathrm{const.}\right]$

- Potential insertions: $\frac{\mathcal{V}_c}{\mathbf{k}^2}$, $\frac{\pi^2 \mathcal{V}_k}{m_t |\mathbf{k}|}$, $\frac{\mathcal{V}_2 + 2\mathcal{V}_s}{m_t^2}$, $\frac{(\mathbf{p}^2 + \mathbf{q}^2)\mathcal{V}_r}{2 m_t^2 \mathbf{k}^2}$

NNLL Combinations of $\mathcal{O}(\alpha_s)$ corrections and bilinear operators or subleading currents

$\Rightarrow$ Imaginary matching conditions for currents and $(e^+e^-)(e^+e^-)$ operators

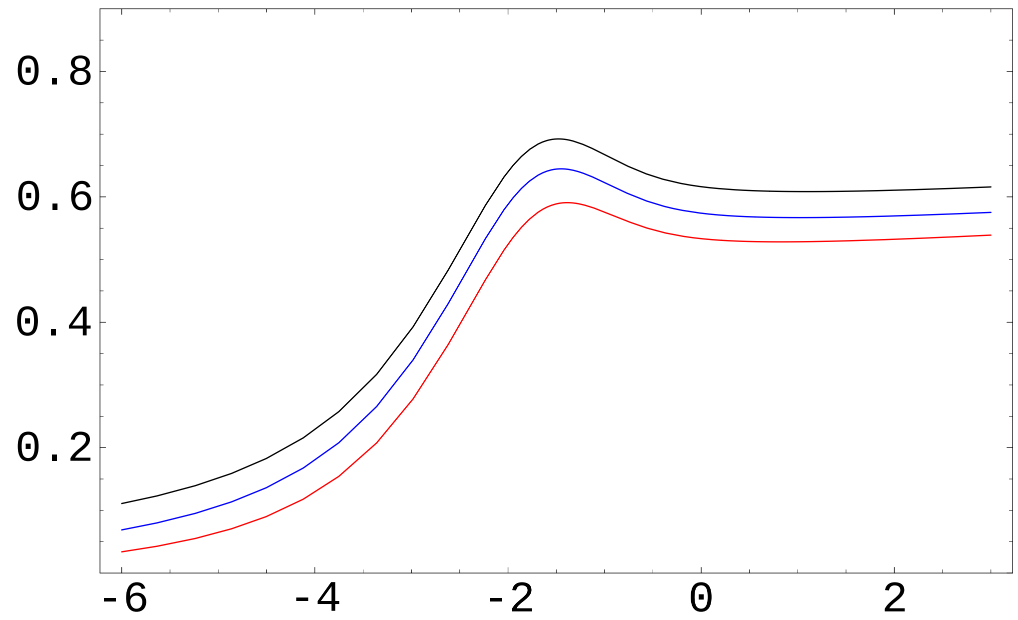
Phase space corrections

Numerical effects

Example: Full LL Green function

- — No phase space cut
- — $\Lambda^2 = 2\,m_t \times 20\,\text{GeV}$
- — $\Lambda^2 = 2\,m_t \times 10\,\text{GeV}$

work in progress ...



Outlook

- Completion of phase space matching
  $\longrightarrow$ publication

Outlook

- Completion of phase space matching
  $\longrightarrow$ publication
- Investigate effects of ultrasoft gluons in phase space matching

Outlook

- Completion of phase space matching
  $\longrightarrow$ publication
- Investigate effects of ultrasoft gluons in phase space matching
- $\mathcal{O}(\alpha_s)$ corrections to imaginary current matching conditions $\longrightarrow$ NNLL running of $(e^+e^-)(e^+e^-)$ operators

Outlook

- Completion of phase space matching
  $\rightarrow$ publication
- Investigate effects of ultrasoft gluons in phase space matching
- $\mathcal{O}(\alpha_s)$ corrections to imaginary current matching conditions $\rightarrow$ NNLL running of $(e^+e^-)(e^+e^-)$ operators
- QED contributions: ISR, Coulomb singularities

Summary

- Threshold scan allows for precise $m_t, y_t, \Gamma_t, \alpha_s$ determination
- Effective theory approach crucial to sum up threshold contributions

Summary

- Threshold scan allows for precise $m_t, y_t, \Gamma_t, \alpha_s$ determination
- Effective theory approach crucial to sum up threshold contributions

Unstable top leads to

- Complex matching conditions
- UV divergencies
- Matching conditions for the $t\bar{t}$ phase space that depend on defi nition of “threshold top pair event”
- Cutoff involves mild power counting breaking
- Corrections at NLL and NNLL order