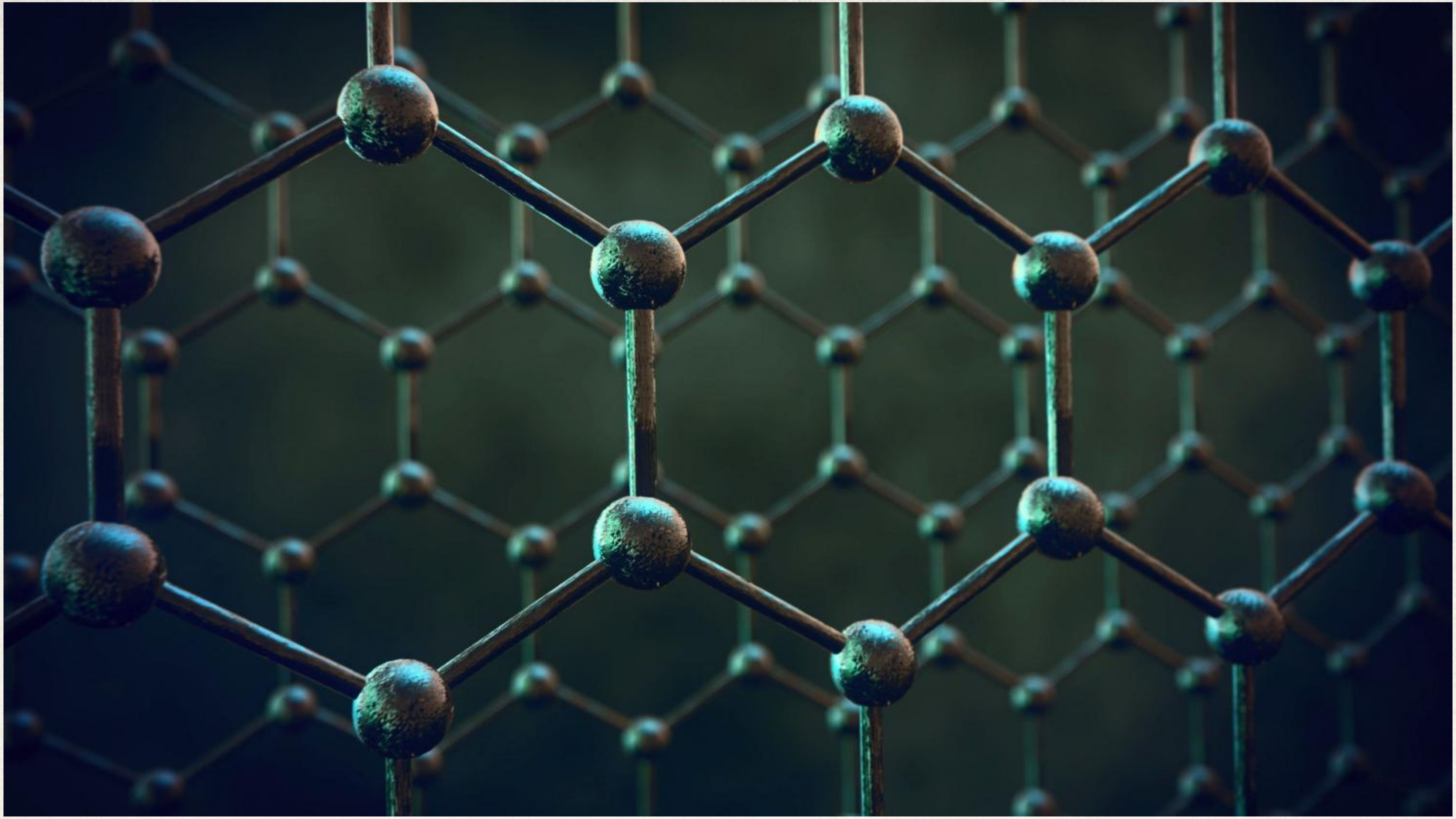

Boundaries and Defects in Quantum Field Theory

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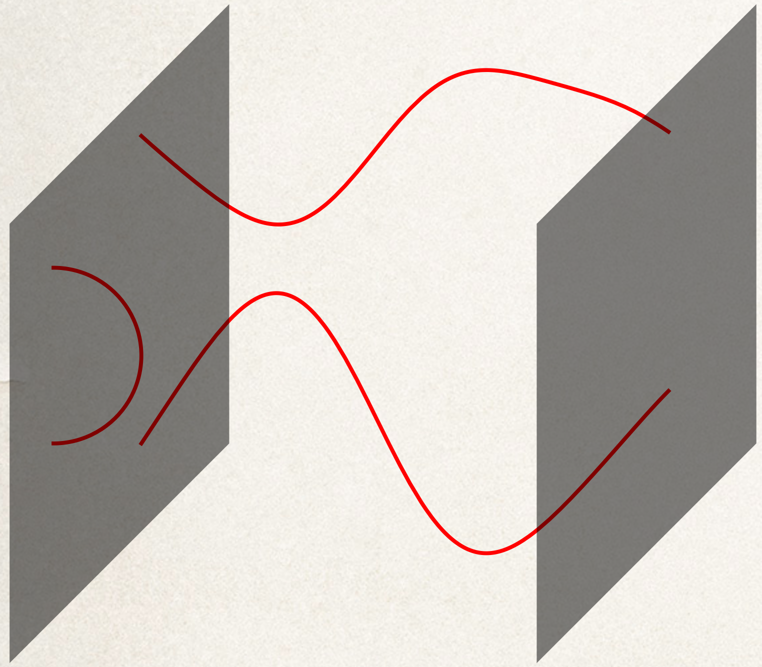
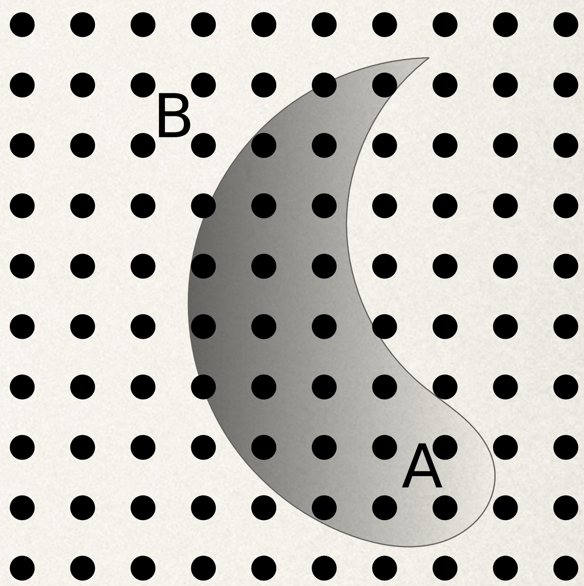
(Huang, Jensen, Shamir, Virrueta)

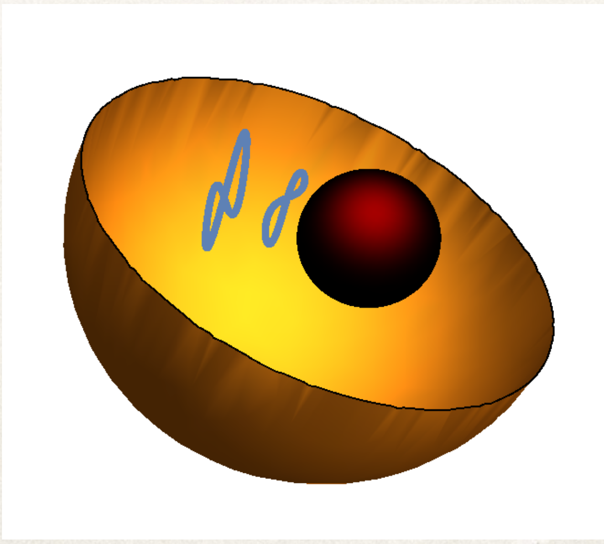
Outline

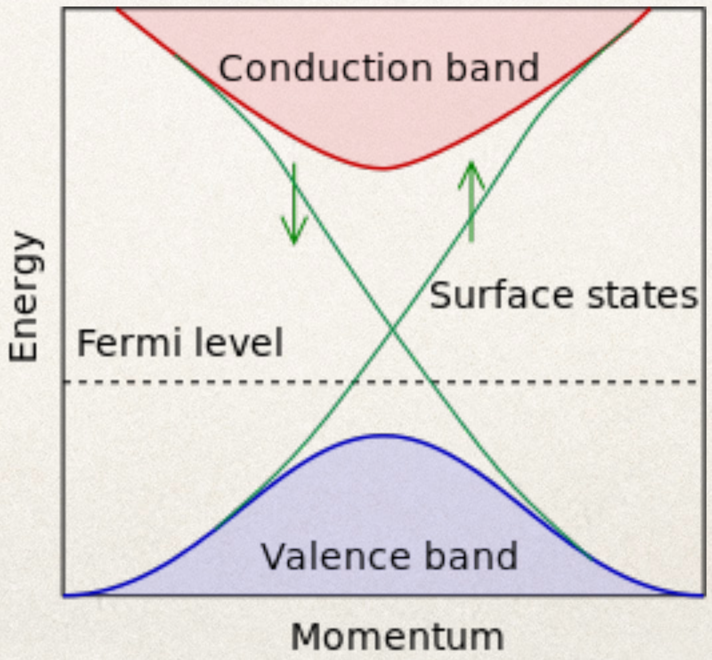
- Motivational remarks about boundaries in quantum field theory and gravity
- Results for boundary contributions to the trace anomaly
- Boundary anomalies in a family of field theories related to graphene

What do D-branes, AdS/CFT, topological insulators, and entanglement entropy for field theories have in common?

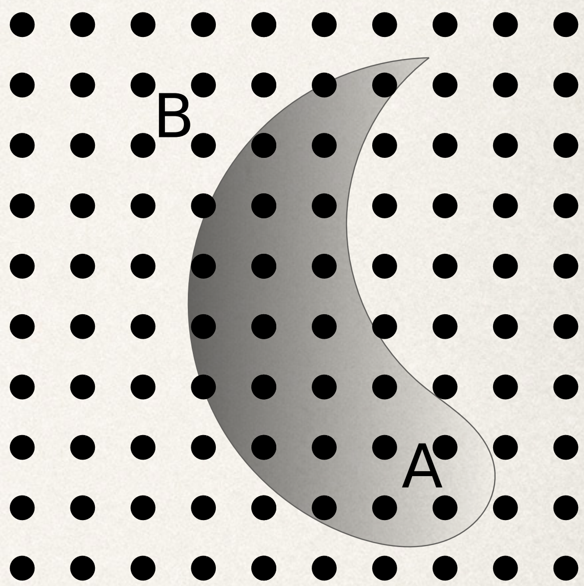




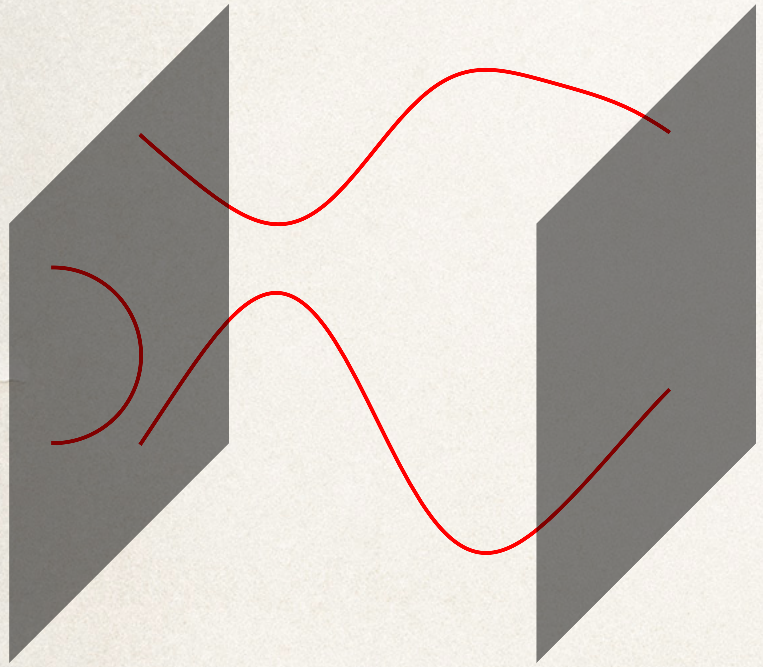


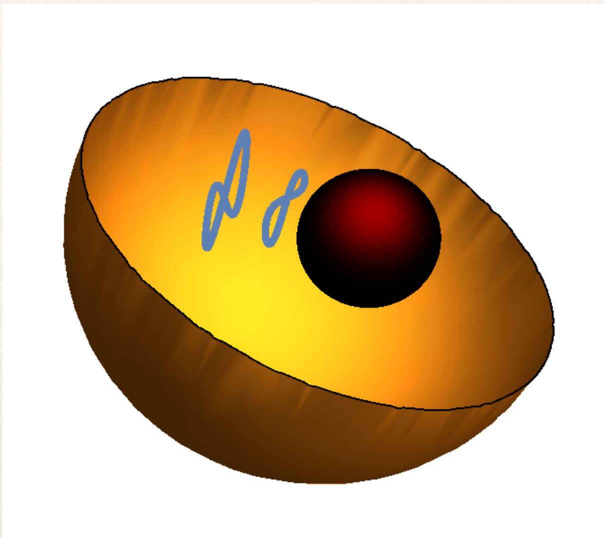


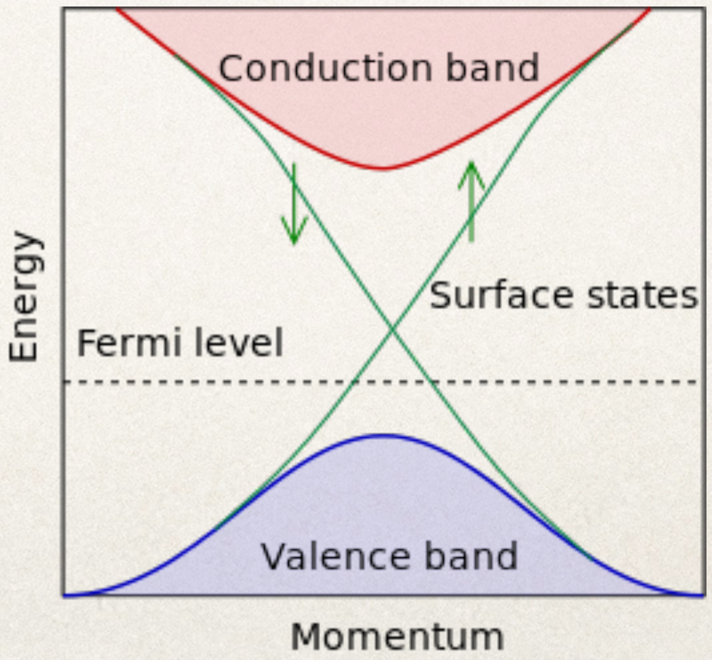
What do D-branes, AdS/CFT, topological insulators, and entanglement entropy for field theories have in common?



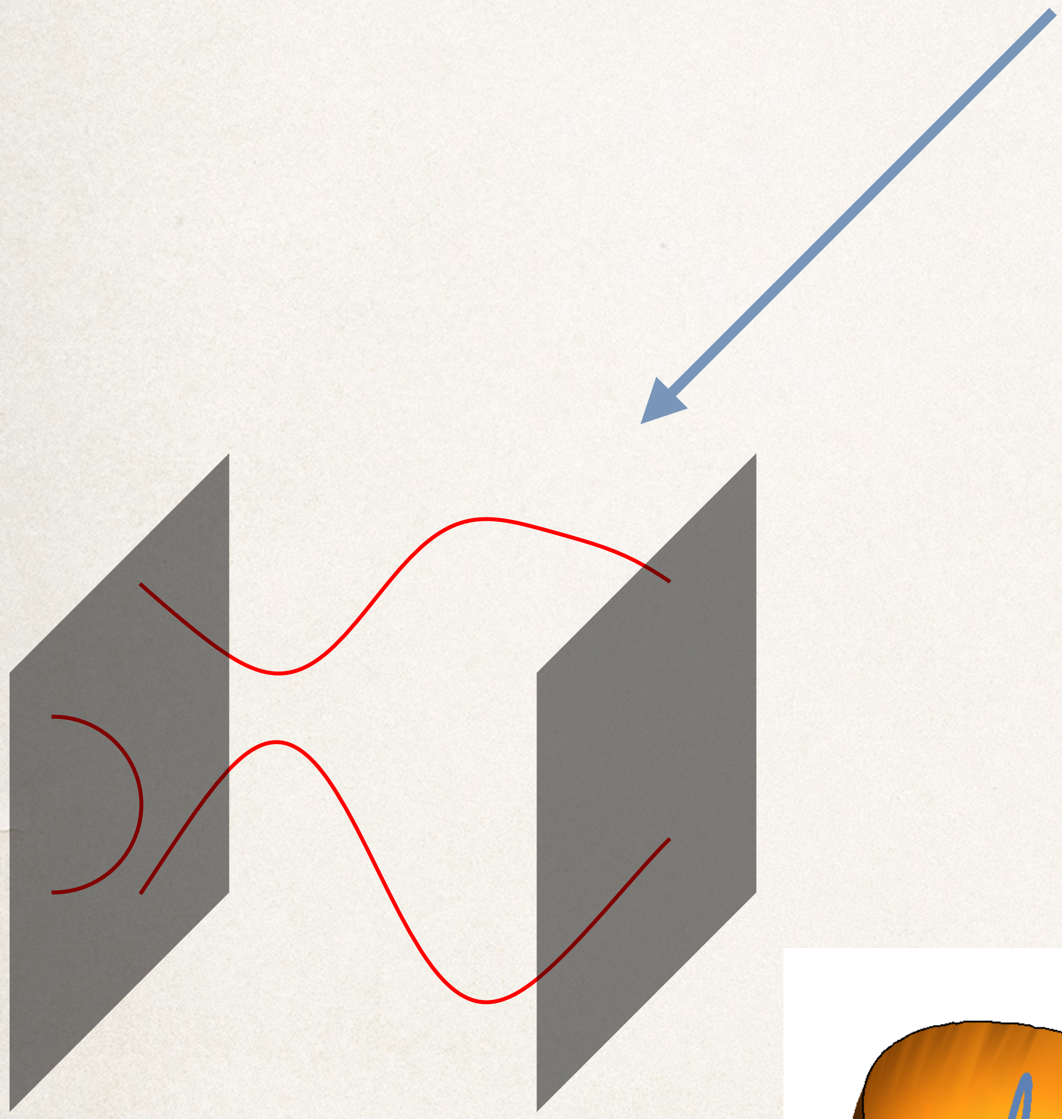
The importance of boundaries.

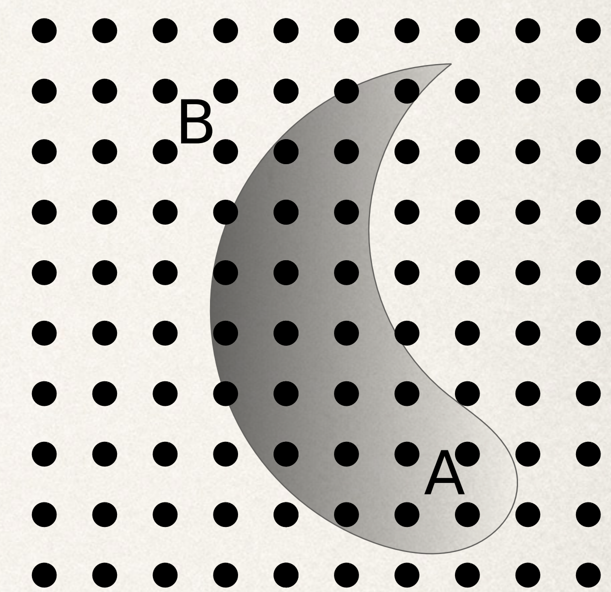


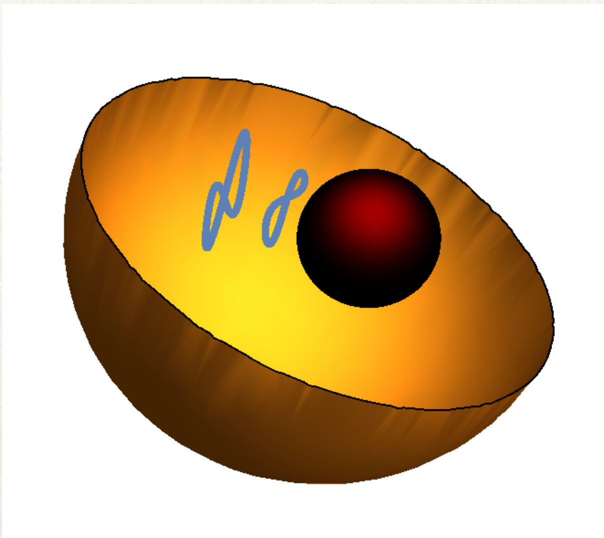


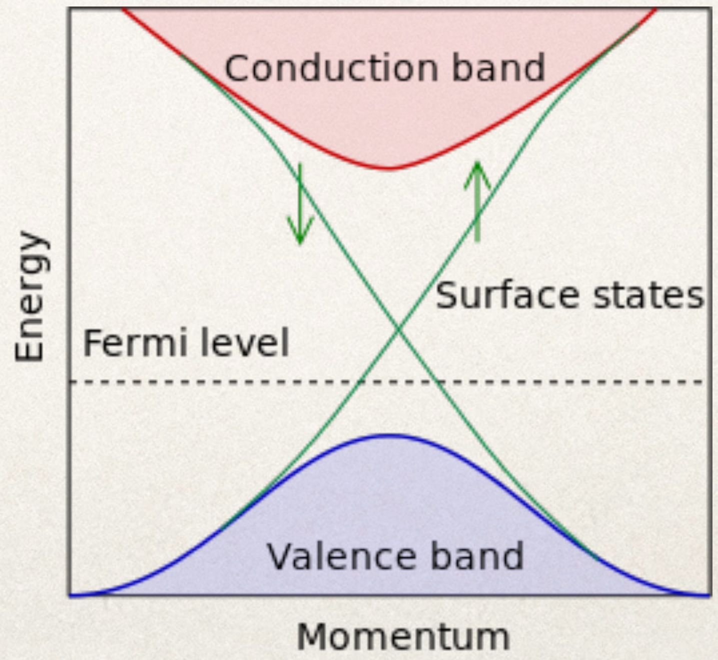


D-branes as boundary conditions for open strings

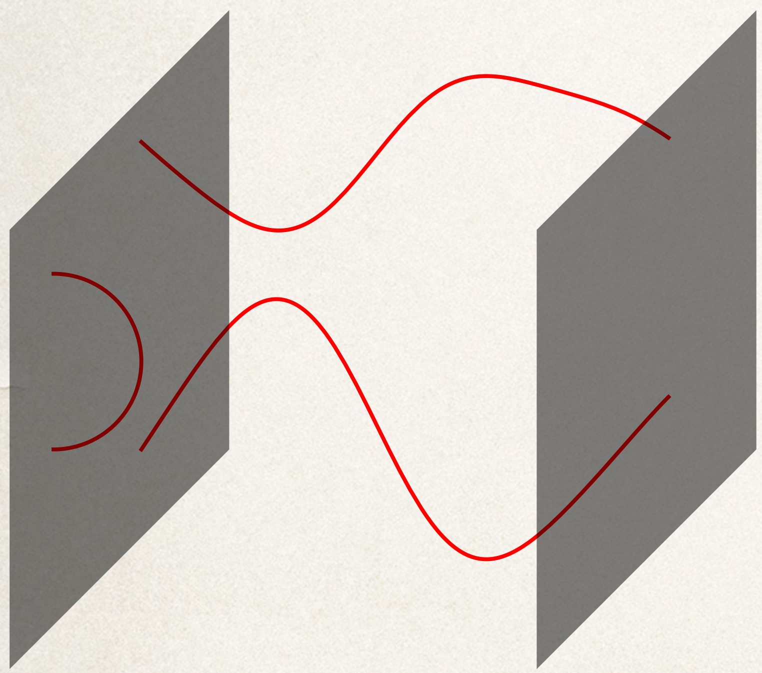


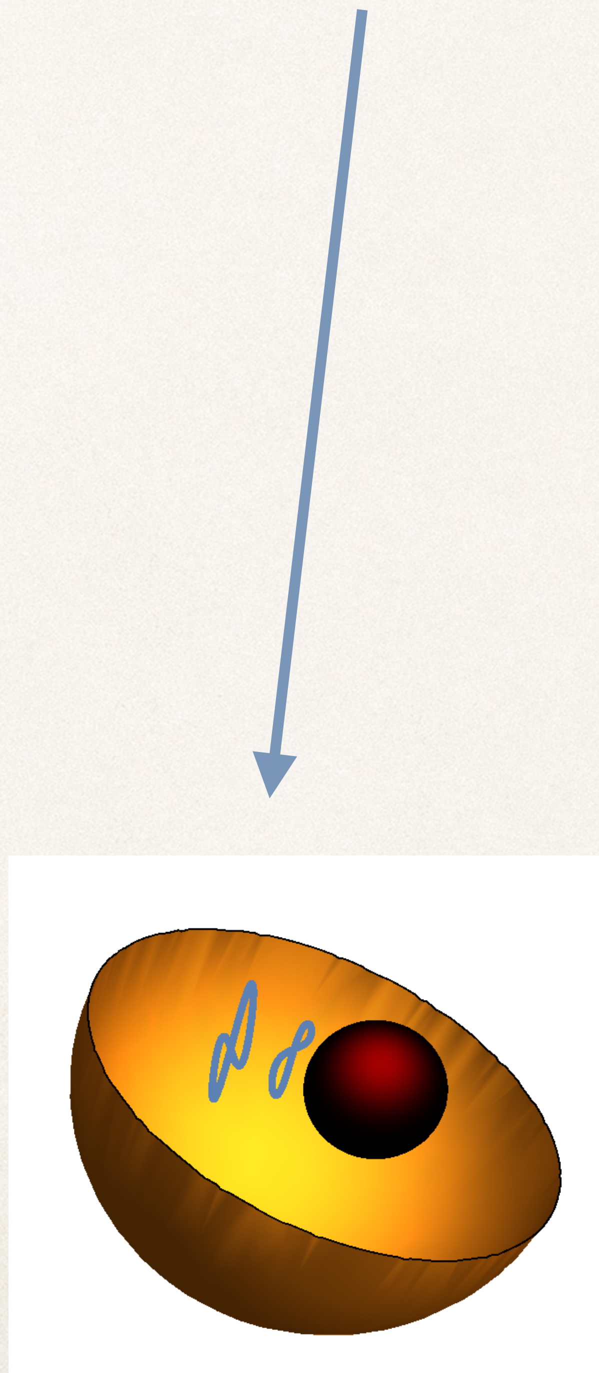


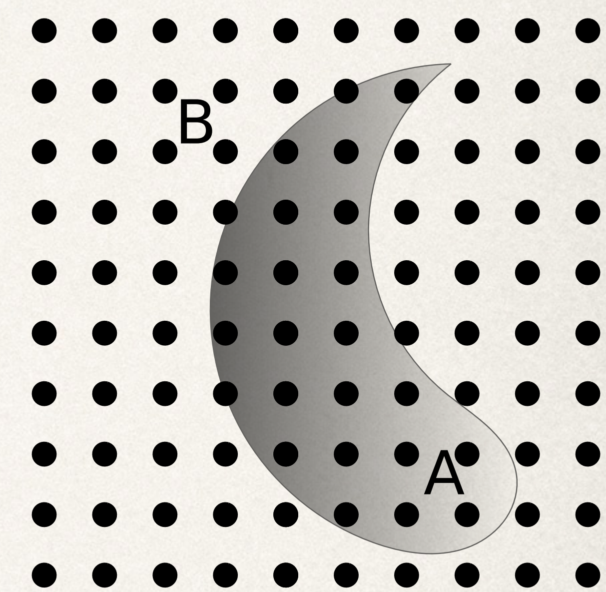


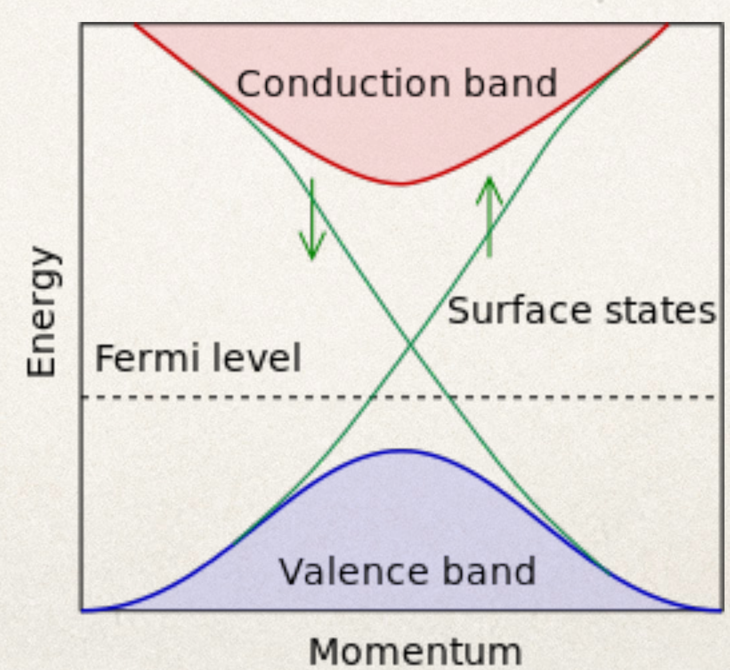


In AdS/CFT, conformal boundary of anti-de Sitter space is where the conformal field theory “lives”.

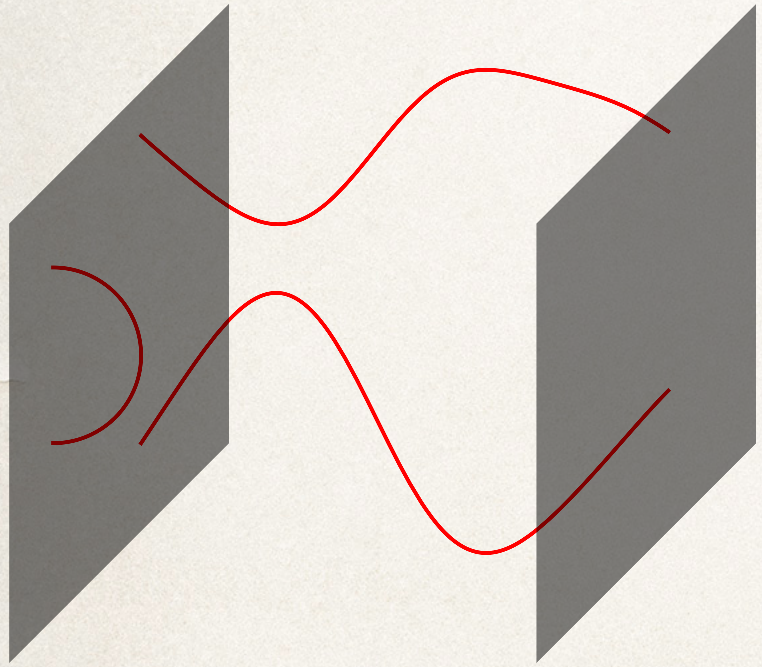


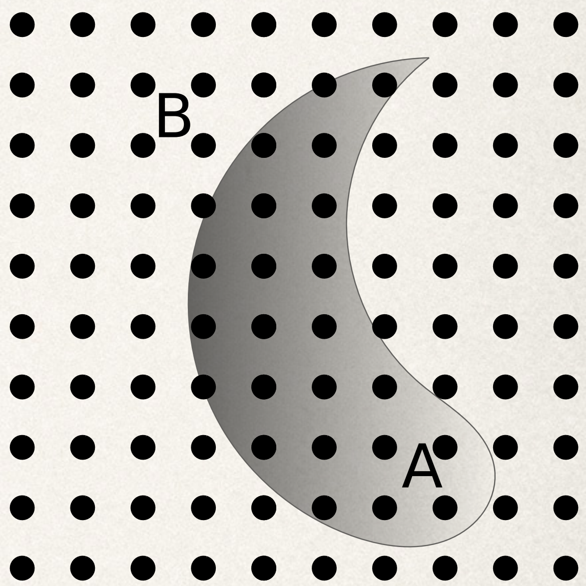


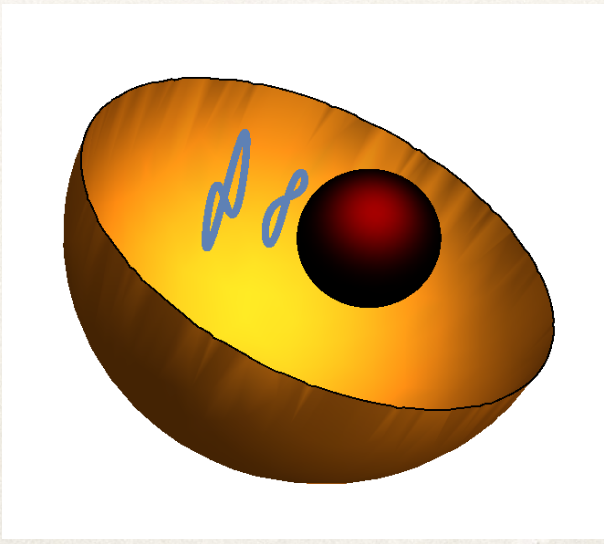


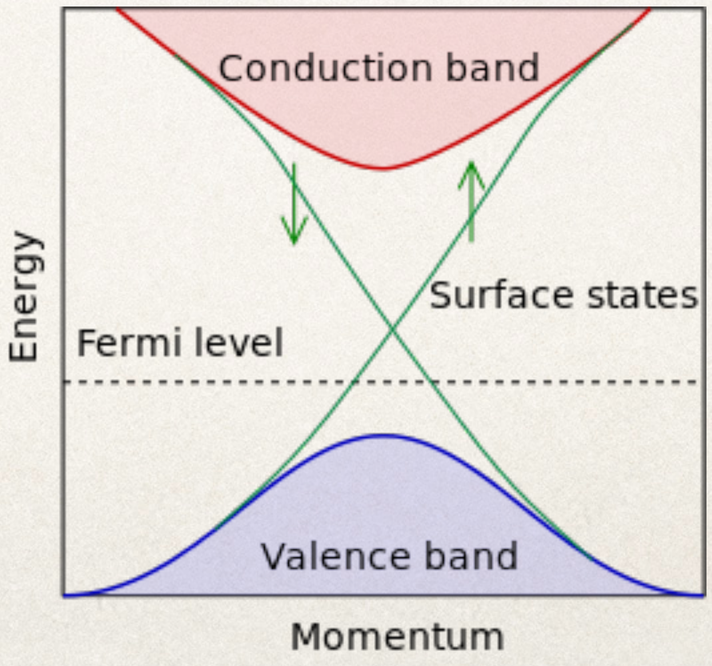


For topological insulators, the insulating bulk material has conducting (massless) surface states that are protected by symmetry.

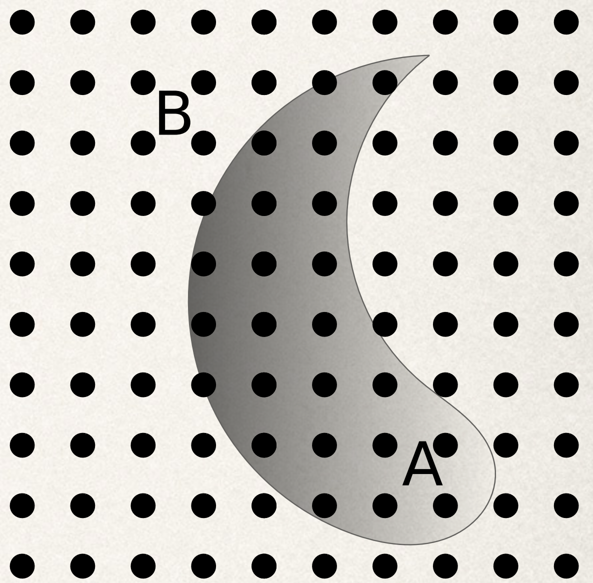


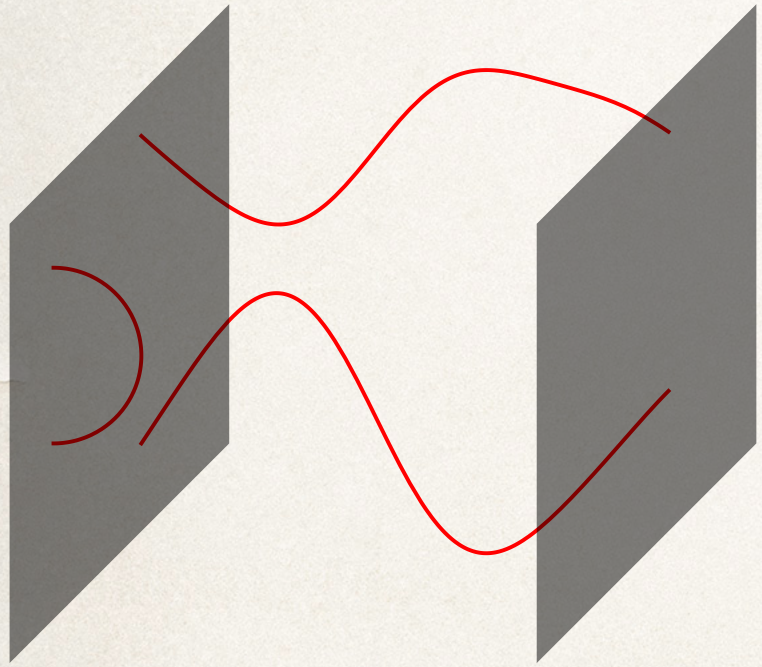


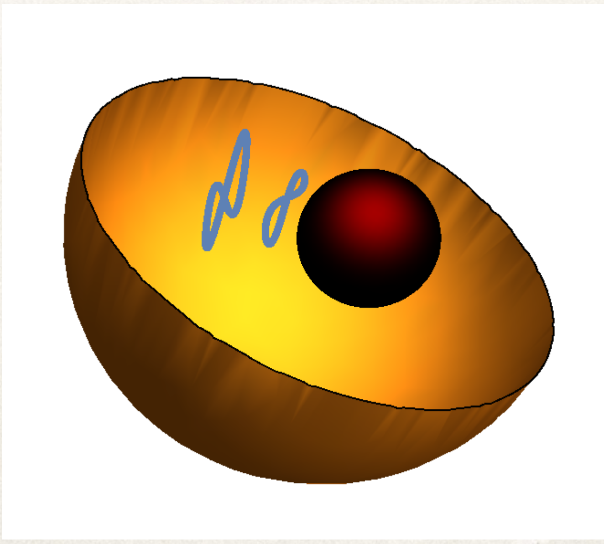


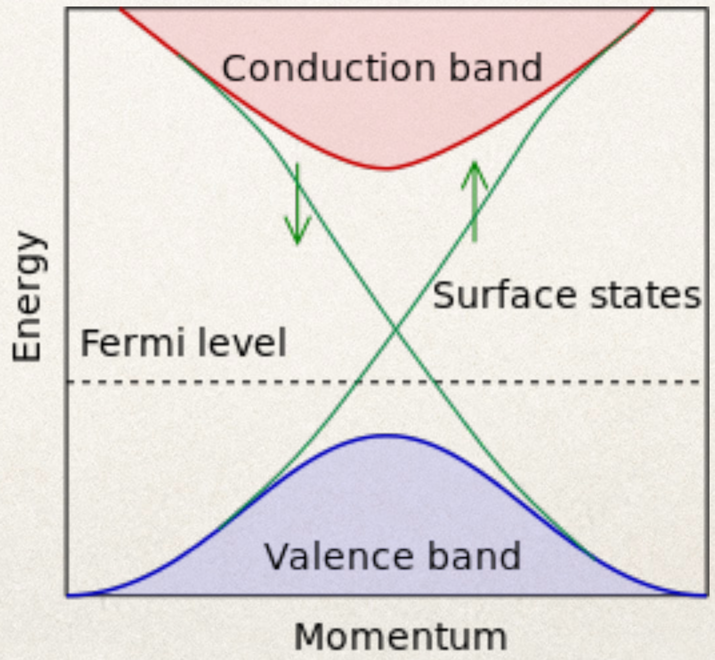


In field theory, entanglement is often measured with respect to spatial regions, leading to the importance of the “entangling surface”.









Would all of these developments have been "obvious" if we just understood quantum field theory in the presence of a boundary a little better to begin with?

- As CFT are fixed points in RG flow, it is natural to start a study of boundary / defect QFT with a study of boundary / defect CFT.
- Implicit importance of CFT in at least three of the four developments just mentioned — world sheet theory for strings, the boundary theory in AdS / CFT, the "a" and "c" theorems from entanglement entropy

Boundary Conformal Field Theory

- Surprisingly unexplored. McAvity and Osborn '93 and '95 papers on two point functions. The more recent just recently received more than 100 citations.
- Flat space: A planar boundary breaks the SO(d,2) symmetry to SO(d-1,2).
- Curved space: Require that the boundary and boundary terms in the action preserve Weyl invariance.
- In today's talk, a focus on the trace anomaly…

Stress Tensor Trace

Today's talk: To understand new terms in the trace anomaly associated with the presence of a boundary.

A quick review:

classically, Weyl invariance implies $S[\Omega(x) g_{\mu\nu}] = S[g_{\mu\nu}]$

infinitesimally $\Omega(x) = 1 + \delta\sigma(x)$ and hence

$$0 = \frac{\delta S}{\delta \sigma} = \frac{\delta S}{\delta g_{\mu\nu}} \frac{\partial g_{\mu\nu}}{\partial \sigma} = \frac{\delta S}{\delta g_{\mu\nu}} g_{\mu\nu} \sim T^{\mu}{}_{\mu}$$

quantum mechanically, there are anomalies…

Trace Anomaly in 2d (no boundary)

$$\langle T^{\mu}{}_{\mu} \rangle = \frac{c}{24\pi} R$$

Ricci scalar curvature

- c is also the coefficient of the $T_{\mu\nu}$ two-point function.
- Zamolodchikov c-theorem ('86), $c_{\mathrm{UV}} > c_{\mathrm{IR}}$
- Huerta-Casini ('04) entanglement entropy proof of c-theorem

—towards a map of 2d QFTs

Trace Anomaly in 4d (no boundary)

$$\langle T^{\mu}{}_{\mu} \rangle = \frac{1}{16\pi^2}(cW^2 - aE.D. + d\Box R)$$

Weyl curvature Euler density scheme dependent, ignore

- c is also the coefficient of the $T_{\mu\nu}$ two-point function.
- Cardy ('88) *a*-conjecture and later Komargodski-Schwimmer proof ('11) $a_{\rm UV} > a_{\rm IR}$
- Casini-Teste-Torroba ('17) entanglement entropy proof of *a*-theorem

—towards a map of 4d QFTs

Trace Anomaly in 6d (no boundary)

$$\langle T^\mu{}_\mu \rangle \sim a\, E.D. + c_1 W^3 + c_2 W^3 + c_3 W \Box W$$

hints from supersymmetry and AdS/CFT of a 6d *a*-theorem

Trace Anomaly with a Codimension One Boundary

K_{AB} extrinsic curvature
hat on K removes trace

2D $\quad \langle T^\mu{}_\mu \rangle = \dfrac{c}{24\pi}(R + 2K\delta(x^n))$

Jensen-O'Bannon ('15) b-theorem

$a_{\rm UV} > a_{\rm IR}$

3D $\quad \langle T^\mu{}_\mu \rangle = \dfrac{1}{4\pi}(-aR + b \, {\rm tr}\, \hat{K}^2)\delta(x^n)$

4D $\quad \langle T^\mu{}_\mu \rangle = \dfrac{1}{16\pi^2}(cW^2 - aE.D. + (-b_1 \, {\rm tr}\, \hat{K}^3 + b_2 K^{AB} W_{nAnB})\delta(x^n))$

Solodukhin-Fursaev ('16) conjecture

$b_2 = 8c$

5D $\quad \langle T^\mu{}_\mu \rangle \sim \delta(x^n)(b_1 W^2 + b_2 K^4 + \ldots)$

6D $\quad \langle T^\mu{}_\mu \rangle \sim a\, E.D. + c_1 W^3 + c_2 W^3 + c_3 W \Box W + \delta(x^n)(b_1 K W^2 + \ldots)$

Results for Boundary Charges

Can we say anything more about

3d: aR conjecture

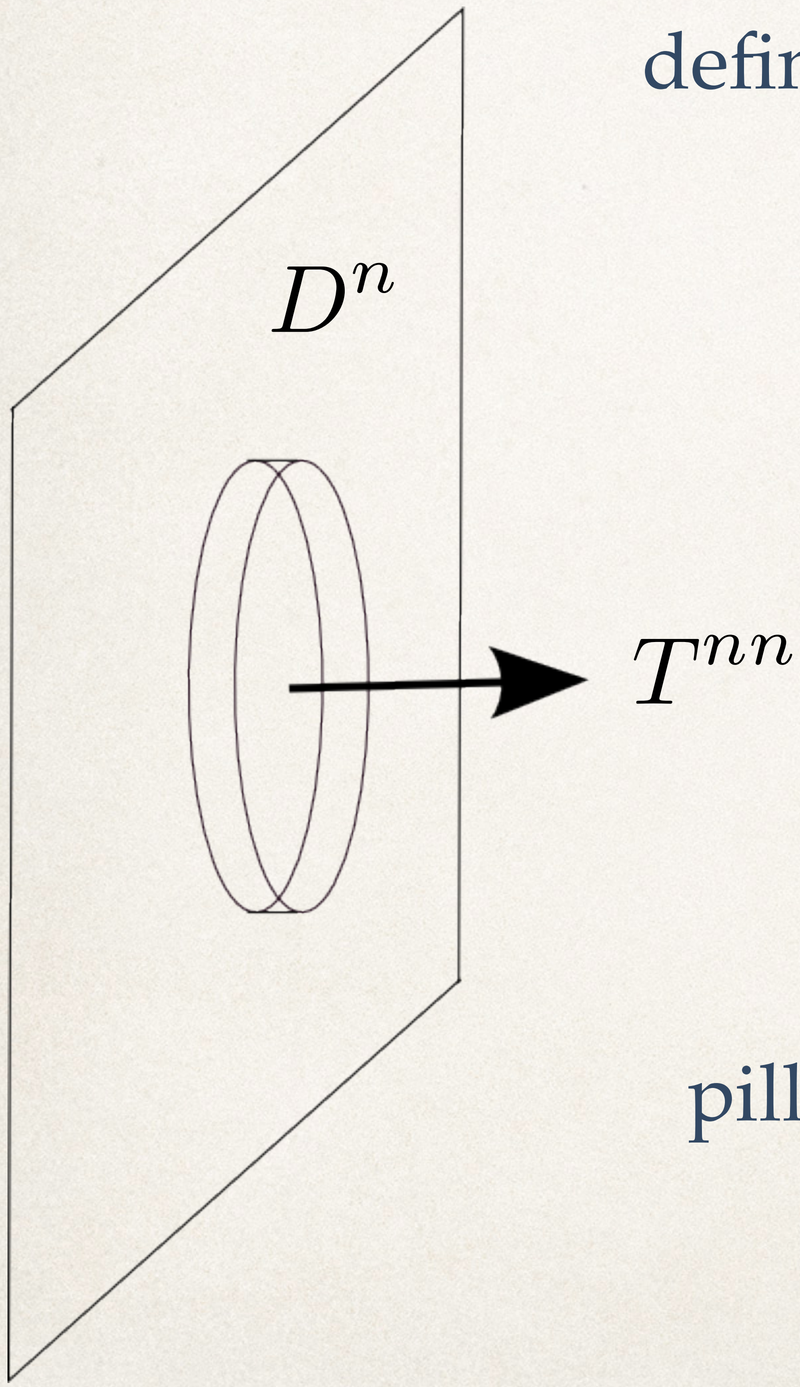
3d: $b\,\mathrm{tr}\,\hat{K}^2$ yes

4d: $b_1\,\mathrm{tr}\,\hat{K}^3$ yes

4d: $b_2 K^{AB} W_{AnBn}$ yes

Related to displacement operator two and three point functions

Displacement Operator



definition: operator sourced by small changes in the embedding

$$\frac{\delta I}{\delta x^n} \equiv D^n$$

diffeomorphism Ward identity:

$$\partial_\mu T^{\mu n} = D^n \delta(x^n)$$

$$\partial_\mu T^{\mu A} = 0$$

tangential components still conserved

pill box argument implies

$$T^{nn}(\vec{x}, x^n)|_{x^n=0} = D^n(\vec{x})$$

Results

$$\langle D^n(\vec{x}) D^n(0) \rangle = \frac{c_{nn}}{|\vec{x}|^{2d}}$$

$$\langle D^n(\vec{x}) D^n(\vec{x}') D^n(0) \rangle = \frac{c_{nnn}}{|\vec{x}|^d |\vec{x}'|^d |\vec{x} - \vec{x}'|^d}$$

3d

aR (?)

$b \, \mathrm{tr} \, \hat{K}^2$ $\quad b = \frac{\pi^2}{8} c_{nn}$

4d

$b_1 \, \mathrm{tr} \, \hat{K}^3$ $\quad b_1 = \frac{2\pi^3}{35} c_{nnn}$

$b_2 K^{AB} W_{AnBn}$ $\quad b_2 = \frac{2\pi^4}{15} c_{nn}$

Argument analogous to Osborn and Petkou's result for *c* in 4d. (Fails for *a* in 3d because R is topological.)

D in other contexts

Deutsch and Candelas '79 $$\langle T_{\mu\nu}\rangle = A\frac{\hat{K}_{\mu\nu}}{x_n^{d-1}} + \ldots$$

R.-X. Miao '18 showed that A is proportional to b in 3d and b_2 in 4d

Correa, Henn, Maldacena, Sever '12 (for line defects)

$$\int \langle D(x)D(0)\rangle dx$$ gives the cusp anomalous dimension in the small angle limit of a Wilson line in a gauge theory at a conformal fixed point

— related to power radiated by a moving quark

Faulkner, Leigh, Parrikar '16 (see also Myers et al.)

$\langle D(x)D(0)\rangle$ related to shape dependence of entanglement entropy

$b \,\mathrm{tr}\, \hat{K}^2$ in 3d

The term in the trace anomaly can be produced from an effective anomaly action in limit ε goes to zero where μ is a UV regulator:

$$I^{(b)} = \frac{b}{4\pi}\frac{\mu^\epsilon}{\epsilon}\int_{\partial M} \mathrm{tr}\, \hat{K}^2$$

For small deviations from planarity $K_{AB} \approx \partial_A \partial_B x^n$

$\Longrightarrow$ scale dependence of displacement 2-pt function

$$\mu\partial_\mu \langle D^n(\vec{x}) D^n(0)\rangle = \frac{b}{4\pi}\Box^2 \delta(\vec{x})$$

The short distance behavior of $\langle D^n(\vec{x}) D^n(0)\rangle = \frac{c_{nn}}{|\vec{x}|^6}$

can be regulated by writing instead $\langle D^n(\vec{x}) D^n(0)\rangle = \frac{c_{nn}}{512}\Box^3 (\log \mu^2 \vec{x}^2)^2$

(Freedman, Johnson, Latorre '92)

Checks for Free Fields

Using heat kernel methods, a number of these charges were computed for free fields in the late 80s and early 90s (Melmed, Moss, Dowker, Schofield) and later revisited in the last couple of years (Solodukhin, Fursaev, Jensen, Huang, CPH).

$$b^{s=0} = \frac{1}{64}\,(\text{D or R})\,, \quad b^{s=\frac{1}{2}} = \frac{1}{32}\,,$$

$$b_1^{s=0} = \frac{2}{35}(\text{D})\,, \quad b_1^{s=0} = \frac{2}{45}(\text{R})\,, \quad b_1^{s=\frac{1}{2}} = \frac{2}{7}\,, \quad b_1^{s=1} = \frac{16}{35}\,,$$

$$b_2 = 8c$$

The displacement operator correlation functions yield the same results!

Why is $b_2 = 8c$ for free fields?

$v \to 1$: boundary limit

$v \to 0$: coincident limit

For free theories $\alpha(v) \sim 1 + v^{2d}$

theory without boundary

effect of image points on other side of the boundary

$$\implies 2\alpha(0) = \alpha(1)$$

$\alpha(0) \sim c$ by the old Osborn-Petkou ('93) argument

What about interactions?

Wilson-Fisher fixed point for ϕ^4 scalar field theory, starting in 4d

McAvity and Osborn ('93, '95) showed, both in the ε expansion and in a large N expansion that

$$2\alpha(0) \neq \alpha(1)$$

Downside: We need to be in exactly 4d to connect to b_2, and in exactly 4d ϕ^4 scalar field theory is free

We need some more examples…

Mixed dimensional QED has something for everyone

$$S = -\frac{1}{4}\int_{\mathcal{M}} \mathrm{d}^4x\, F^{\mu\nu}F_{\mu\nu} + \int_{\partial\mathcal{M}} \mathrm{d}^3x (i\bar{\psi}\not{D}\psi)$$

where $D_\mu = \nabla_\mu - igA_\mu$ boundary conditions: $F_{nA} = g\bar{\psi}\gamma_A\psi$

- relation to graphene
- relation to large N_f QED$_3$ (Kotikov-Teber '13)
- behavior under electric-magnetic duality (Son '17)
- example of a bCFT with an exactly marginal coupling
- supersymmetric versions
- playground for computing trace anomalies

} our work

Relation to Graphene

Son's model of graphene (cond-mat/0701501):

$$-\sum_{a=1}^{N}\int dt\, d^2x(\bar{\psi}_a\gamma^0\partial_0\psi_a + v\bar{\psi}_a\gamma^i\partial_i\psi_a + iA_0\bar{\psi}_a\gamma^0\psi_a) + \frac{1}{2g^2}\int dt\, d^3x(\partial_i A_0)^2$$

things to note

- only electric interactions
- electrons travel at speed $v \approx c/300$

beta function for the electron velocity

$$p\frac{\partial v(p)}{\partial p} = -\frac{4}{\pi^2 N}v(p)$$

once v gets sufficiently large, can restore magnetic interactions and flow to a relativistic fixed point

Relation to large N_f QED$_3$

(Kotikov-Teber '13)

propagator for mixed dimensional QED (don't FT the normal direction y)

$$-i\frac{e^{-py}}{p}\eta^{AB}$$ (Feynman gauge)

propagator for large N_f QED$_3$, re-summed

$$-i\frac{\eta^{AB}}{p^2(1+\Pi(p))}$$ where $$\Pi(p) = \frac{N_f e^2}{8|p|} + O(N_f^0)$$

Compensated by vertices, 3d e drops out of the amplitudes.
For scattering processes on the boundary (y=0),
the Feynman rules are the same in the IR with the identification $\frac{1}{N_f} \sim g^2$

Behavior under EM Duality

(Hsiao-Son ’17)

Using recent progress in 2+1 dimensional non-SUSY dualities

$$\int d^3x \left[i\bar{\Psi}\gamma^A(\partial_A - ia_A)\Psi - \frac{1}{4\pi}\epsilon^{ABC}A_A\partial_B a_C \right] - \frac{1}{4g^2}\int d^4x F_{\mu\nu}^2$$

Integrating out a_B or A_μ yields same mixed QED theory but with a new

$$\tilde{g} = 8\pi/g$$

Can use the duality to calculate the current-current and stress tensor correlation function at the self-dual point and at infinite coupling — calculate transport coefficients.

(similar in spirit to H, Kovtun, Sachdev, Son ’07)

Mixed QED is a bCFT

$$S = -\frac{1}{4}\int_{\mathcal{M}} \mathrm{d}^4x\, F^{\mu\nu}F_{\mu\nu} + \int_{\partial\mathcal{M}} \mathrm{d}^3x(i\bar{\psi}\not{D}\psi) \qquad g_0 Z_{A_\mu}^{1/2} Z_\psi = g Z_g$$

The usual Ward identity for QED relates $Z_\psi = Z_g$

The superficial degree of divergence of the photon self energy is one (compared with two in four dimensional QED).

The gauge invariant prefactor $p_\mu p_\nu - \delta_{\mu\nu}p^2$ of $\Pi^{\mu\nu}(p)$ cuts down the degree of divergence to -1.

In other words, Z_γ is finite.

$\Longrightarrow$ coupling is not perturbatively renormalized.

Perturbative Results

$$b_1^{(\mathcal{N}=0)} = \frac{16}{35} - \frac{3g^2}{35} N_f + \mathcal{O}(g^4)$$
$$b_2^{(\mathcal{N}=0)} = \frac{4}{5} - \frac{g^2}{10} N_f + \mathcal{O}(g^4)$$

b_1 and b_2 depend on the coupling

a and c are unaffected by the coupling

$\implies$ We have an example where $b_2 \neq 8c$

Marginal Directions

b_1 and b_2 depend on the exactly marginal coupling!

Unlike the situation for the bulk charges a and c in 4d.

Wess-Zumino consistency forces a to be constant along marginal directions.

No such argument for c. However, SUSY fixes c to be a constant, and it's unknown how to construct 4d CFTs with marginal directions but without SUSY (and without boundaries).

Are b_1 and b_2 protected by supersymmetry?

$\mathcal{N} = 1$ Super Graphene

$$S_{\text{bulk}} = \int_{\mathcal{M}} \mathrm{d}^4 x \left(-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{g^2\theta}{16\pi^2} F^{\mu\nu} \widetilde{F}_{\mu\nu} + \frac{i}{2} \bar{\lambda} \gamma^\mu \partial_\mu \lambda + \frac{1}{2} D^2 \right)$$

$$S_{\text{bry}} = \int_{\partial\mathcal{M}} \mathrm{d}^3 x \Big(i\widetilde{\psi} \Gamma^A D_A \psi - |\mathrm{D}_A \phi|^2 + |F|^2 + ig\big(\widetilde{\lambda}_+ \psi \phi^* - \widetilde{\psi} \lambda_+ \phi\big) - \frac{1}{4} \bar{\lambda} \gamma^5 e^{\eta\gamma^5} \lambda - \frac{g^2\theta}{8\pi^2} \widetilde{\lambda}_+ \lambda_+ \Big)$$

$\mathcal{N} = 2$ Super Graphene

- The photino is symplectic Majorana instead of just Majorana
- Two extra bulk scalars, X and Y with corresponding extra Yukawa terms.
- The boundary multiplet can be kept the same, but there is now a preserved U(1) R-symmetry

$\mathcal{N} = 2$ Super Graphene

$$S_{\text{bulk}} = \int_{\mathcal{M}} \mathrm{d}^4 x \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{i}{2} \bar{\lambda}_i \gamma^\mu \partial_\mu \lambda^i - \frac{1}{2} (\partial_\mu X)^2 - \frac{1}{2} (\partial_\mu Y)^2 + \frac{1}{2} \vec{D}^2 \right)$$

$$S_{\text{bry}} = \int_{\partial\mathcal{M}} \mathrm{d}^3 x \left(-\frac{1}{4} \bar{\lambda}_i \, \vec{v} \cdot \vec{\tau}^{\,i}{}_j \gamma^5 e^{\eta\gamma^5} \lambda^j - X(\vec{v} \cdot \vec{D} + \partial_n X) \right.$$
$$+ i\widetilde{\psi} \Gamma^A D_A \psi - |D_A \phi|^2 + |F|^2 + \sqrt{2} i g \big(\phi^* \widetilde{\lambda}_+ \psi - \phi \, \widetilde{\psi} \lambda_+ \big)$$
$$\left. + g \widetilde{\psi} Y \psi - g^2 |\phi|^2 Y^2 - g(\vec{v} \cdot \vec{D} + \partial_n X) |\phi|^2 \right)$$

…we could do something similar with $\mathcal{N} = 4$ super graphene

Claim: Mixed dimensional QED along with $\mathcal{N} = 1, 2,$ and 4 super graphene are all bCFTs where the gauge coupling is exactly marginal.

Summary of Perturbative Results

$$b_1^{(\mathcal{N}=0)} = \frac{16}{35} - \frac{3g^2}{35} N_f + \mathcal{O}(g^4) \qquad b_1^{(\mathcal{N}=1)} = \frac{3}{5} - \frac{9g^2 N_f}{40} + \mathcal{O}(g^4)$$

$$b_2^{(\mathcal{N}=0)} = \frac{4}{5} - \frac{g^2}{10} N_f + \mathcal{O}(g^4) \qquad b_2^{(\mathcal{N}=1)} = 1 - \frac{g^2 N_f}{4} + \mathcal{O}(g^4)$$

$$b_1^{(\mathcal{N}=2)} = \frac{38}{45} - \frac{19g^2 N_f}{60} + \mathcal{O}(g^4) \qquad b_1^{(\mathcal{N}=4)} = \frac{4}{3} - \frac{g^2 N_f}{2} + \mathcal{O}(g^4)$$

$$b_2^{(\mathcal{N}=2)} = \frac{4}{3} - \frac{g^2 N_f}{3} + \mathcal{O}(g^4) \qquad b_2^{(\mathcal{N}=4)} = 2 - \frac{g^2 N_f}{2} + \mathcal{O}(g^4)$$

in all theories, b_1 and b_2 depend on the coupling (not protected by SUSY!)

a and c are unaffected by the coupling

(note however $b_1^{(\mathcal{N}=4)} - b_2^{(\mathcal{N}=4)}$ is coupling independent at one loop)

Summary of results

$$b \operatorname{tr} \hat{K}^2$$

$$b_1 \operatorname{tr} \hat{K}^3$$

$$b_2 K^{AB} W_{AnBn}$$

- Presented graphene like theories both with and without SUSY that are bCFTs with an exactly marginal coupling.
- Related boundary central charges in three and four dimensional bCFTs to two and three point functions of the displacement operator.
- Discussed graphene like theories as examples where $b_2 \neq 8c$ and where both b_1 and b_2 depend on a marginal coupling.

Future Projects

- Hemisphere localization for the $\mathcal{N}$=2 super graphene. (Gava, Narain, et al. '16)
- Higher codimension defects. (Billo, Goncalves et al. '16)
- Find bounds on these boundary central charges. (Hofman-Maldacena '08)
- Computation of these central charges in AdS/CFT in Janus solutions. (Takayanagi '11, Miao et al., Astaneh et al. '17)
- Models with only boundary interactions, like mixed QED.
- Boundary bootstrap. (Liendo et al. '12)

Larger Vision: Structure of QFT

- Constrain QFT by constraining CFTs
- Provide a more local view of QFT by figuring out how to deal with boundaries.

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